NUMERICAL INVESTIGATION OF COLLISION ORBITS OF LUNAR SATELLITES

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Abstract. In the present study an investigation of the collision orbits of natural satellites of the Moon (considered to be of finite dimensions) is developed, and the tendency of natural satellites of the Moon to collide on the visible or the far side of the Moon is studied. The collision course of the satellite is studied up to its impact on the lunar surface for perturbations of its initial orbit arbitrarily induced, for example, by the explosion of a meteorite. Several initial conditions regarding the position of the satellite to collide with the Moon on its near (visible) or far (invisible) side is examined in connection to the initial conditions and the direction of the motion of the satellite. The distribution of the lunar craters–originating impact of lunar satellites or celestial bodies which followed a course around the Moon and lost their stability – is examined. First, we consider the planar motion of the natural satellite and its collision on the Moon's surface without the presence of the Earth and Sun. The initial velocities of the satellite are determined in such a way so its impact on the lunar surface takes place on the visible side of the Moon. Then, we continue imparting these velocities to the satellite, but now in the presence of the Earth and Sun; and study the forementioned impacts of the satellites but now in the Earth-Moon-Satellite system influenced also by the Sun. The initial distances of the satellite are taken as the distances which have been used to compute periodic orbits in the planar restricted three-body problem (cf. Gousidou-Koutita, 1980) and its direction takes different angles with the x-axis (Earth-Moon axis). Finally, we summarise the tendency of the satellite's impact on the visible or invisible side of the Moon.

1. Introduction

The external theory of the lunar's craters is connected with the effects which are produced by the impacts of other celestial bodies, for example, meteorites, asteroids or comets on the Moon's surface. So, the lunar surface can be considered as an 'impact counter' of external bodies collided with the Moon cf. Kopal (1966), as well as, a boundary condition of all internal processes which may have been taken place in lunar interior.

Galileo Galilei was the first telescopic observer of the Moon and he has recorded in his 'Sidereal Messenger' (1610), that, the surface of the Moon is 'full of inequalities'.

Robert Hooke (1667), dropped bullets into a pipe clay and water mixture and saw formations arise which one could call 'impact craters'. But Hooke also boiled a mixture of powdered alabaster with water and observed that this too produced transient craterlike structures on the surface of the liquid. Hooke himself rejected the impact analogy because it would be difficult to imagine whence those bodies should come.

Gilbert (1893), who reviewed the characteristics of Moon's craters to those of the various types of terrestrial volcanoes, concluded that the differences in form were so great that a volcanic origin for the Moon's craters seemed improbable. So, Gilbert developed an

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impact hypothesis for the origin of the lunar craters, which was based on some acute telescopic observations of the Moon as well as upon laboratory experiments. During the 20th century (until now) the astronomers began to incline towards the impact hypothesis. It is known that there are bodies in space small compared with the planets which move around the Sun as the planets do. The interplanetary space between the motions of the Earth, Moon around the Sun contains a wide number of particles; from the elementary micrometeors to major meteorites, asteroids or comets whose orbits may intersect the path of the Moon and occasionally collide with it; such bodies created most of the craters, as Kopal mentions in his book 'Introduction to the Study of the Moon' (1966).

In the present work, the collision courses of such meteorites are numerically computed when these bodies lose their stability. Arbitrary initial conditions are applied for the case when the presence of the Earth and Sun does not affect the meteorite's path. The evolution of collision courses of the Moon's natural satellites are subsequently computed in the presence of the Earth and Sun giving, now, the previous initial condition for the two-body problem.

First of all, we apply such initial conditions to the satellite that permit it to fall on the visible side of the Moon, experimenting with a large number of starting directions. These conditions, which lead to collision on the visible side of the Moon in the two-body problem, are reapplied to the satellite for the case for the existence of the Earth and Sun. Now, the perturbed satellite's orbit follows a new collision path and it is investigated if this collision continues to take place or not on the visible side of the Moon.

2. Initial Conditions for Collision Orbits in the Two-Body Problem (Moon and its Satellite)

We consider the Moon-Satellite system assuming that the satellite's orbit around the Moon is elliptic. The equations of motion in polar coordinates are given by the relations

$$\ddot{r} = r\dot{\theta}^2 - \frac{G(m_1 + m_2)}{r^2}, \tag{1}$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}. \tag{2}$$

The satellite starts its motion at $t = 0$ from a point on the x-axis with distances from the centre of the Moon equal to the distances that gave periodic orbits in the three-body problem for the system Earth-Moon-Satellite (Gousidou-Koutita, 1980). The initial velocity V_0 has been taken in such a way that the meteorite's ellipse is tangent to the Moon's surface in the visible side, that is, the pericentre is equal to the Moon's radius R (Figure 1). This condition is expressed by the equation

$$a(1 - e) = R. \tag{3}$$

The velocity V_0 satisfies the equations of angular momentum and the kinetic energy

$$L = |r_0 \times \mathbf{V}| = r_0 V \sin\varphi, \tag{4}$$

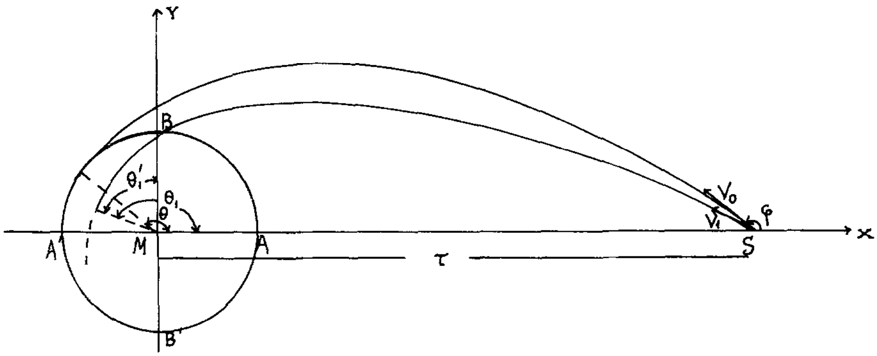

Fig. 1.

$$E = \tfrac{1}{2}V^2 - \frac{Gm_M}{\tau_0}, \tag{5}$$

where φ is the angle of ejection of the satellite, τ_0 the initial distance equal to the distance which gave periodic orbit in the three-body problem and m_M is the Moon's mass. According to the relations

$$a = -K/2E, (K = Gm_M), \tag{6}$$

$$e = (1 + 2EL^2/K^2)^{1/2}; \tag{7}$$

and the relations (3), (4), and (5), we compute the initial velocity $V_0(\tau_0, \varphi)$ from

$$|V_0(\tau_0, \varphi)| = [[-K(2R^2 - \tau_0^2 \sin^2 \varphi - \tau_0 R) - K[(2R^2 - \tau_0^2 \sin^2 \varphi - \tau_0 R)^2 - 4R(\tau_0 - R)(\tau_0^2 \sin^2 \varphi - R^2)]^{1/2}]/[\tau_0(\tau_0^2 \sin^2 \varphi - R^2)]]^{1/2}. \tag{8}$$

By means of the relation

$$\tau_0 = a(1 - e^2)/(1 + e \cos \theta_0), \tag{9}$$

we compute the angle θ_0 between the x-axis and the radius R (Figure 1) for different vaues of φ and consequently for different values of $V_0(\varphi)$. The angles of ejection φ of the satellite have been taken equal to 30°, 45°, 60°, 90°, 120°, 150°, 210°, 240°, 270°, 300°, 330° for $\tau_0 = 0.043865$. The corresponding values of $V_0(\tau_0, \varphi)$ and $\theta_0(V_0)$ are given in Table I.

For the above values of φ and τ_0, the corresponding values of $V_1(\tau_0, \varphi)$ – where $V_1(\tau_0, \varphi)$ represents the values of initial velocities of the meteorite leading it on orbits meeting the Moon on its pole B (Figure 1) – have been calculated.

This velocity $V_1(\tau_0, \varphi)$ can be calculated from Equations (4) and (5). Then, the semi-major axis a and the eccentricity e are computed from Equations (6) and (7). From the hypothesis, that V_1 is the initial velocity that constrains the meteorite to move in an ellipse meeting the Moon on its pole B and the Figure 1 we can, easily, obtain the relation

TABLE I

Values of φ and the corresponding values of velocities $V_0(\varphi)$ and $V_1(\varphi)$ for distance $\tau_0 = 0.043\,865\,014$.

φ	V_0	θ_0	V_1	θ_1
30°	0.46256	− 202°.5	0.3113	− 189°.425
45°	0.32351	− 193°.	0.22752	− 185°.885
60°	0.26319	− 188°.3	0.18955	− 183°.5
90°	0.22752	180°	0.16897	− 180°
120°	0.26319	172°.4	0.20119	− 175°.9
150°	0.46256	− 157°.5	0.37286	− 166°.05
210°	0.46256	+ 157°.5	0.37286	+ 166°.05
240°	0.26319	− 188°.3	0.20119	+ 175°.9
270°	0.22752	180°	0.16897	+ 180°
300°	0.26319	− 172°.4	0.18955	+ 183°.5
330°	0.46256	+ 157°.5	0.3113	+ 189°.425

$$\frac{\tau_0}{R} = \frac{1 + e\cos\theta_1'}{1 + e\cos\theta_1}, \tag{10}$$

with $\theta_1 = \theta_1' + \pi/2$. Consequently, we can take the term $\cos\theta_1$ as a function of V_1 as

$$\cos\theta_1 = [-\tau_0(\tau_0 - R) \pm R[e^2(R^2 + \tau_0^2) - (\tau_0 - R)^2]^{1/2}]/[e(R^2 + \tau_0^2)], \tag{11}$$

and the initial velocity $V_1(\tau_0, \varphi)$ can be calculated from the relation

$$\tau_0 = a(1 - e^2)/(1 + e\cos\theta_1), \tag{12}$$

in accordance with the relations (4), (5), (6) and (7): the result is

$$|V_1(\tau_0, \varphi)| = [[[K\tau_0 \sin^2\varphi(R^4 + \tau_0^2 R^2 + \tau_0(\tau_0 - R)(R^2 + \tau_0^2) - (R^2 + \tau_0^2)^2] \pm [(K\tau_0 \sin^2\varphi(R^4 + \tau_0^2 R^2 + \tau_0(\tau_0 - R)(R^2 + \tau_0^2)^2 - (R^2 + \tau_0^2)^2)^2 + \tau_0^2(R^2 + \tau_0^2)\sin^2\varphi\,(R^2 - \sin^2\varphi(R^2 + \tau_0^2))K^2R^2(\tau_0^2 + R^2)]^{1/2}]/[\tau_0^2(R^2 + \tau_0^2) \times \sin^2\varphi(R^2 - \sin^2\varphi(R^2 - \sin^2\varphi(R^2 + \tau_0^2))]^{1/2}. \tag{13}$$

As we have seen from Figure 1 for values of velocity between the values V_1 and V_0 the meteorite's orbit intersects the lunar surface on its visible side. On the other hand, for values of V outside of that region the meteorite does not collide with the Moon on its visible side. The figure 2a and 2b represents the relation between the angle φ and the velocities $V_0(\varphi)$ and $V_1(\varphi)$ for a constant value of τ_0. The point $P(\varphi_p, V_p)$ is the point with velocity equal to velocity giving periodic orbit in the Earth-Moon-Satellite system, with initial distance $\tau_0 = 0.043\,865\,014$. Table II gives the values of φ and the corresponding values of velocities $V_0(\varphi)$ and $V_1(\varphi)$ for distance $\tau_0 = 0.024\,963\,723\,073$ equal

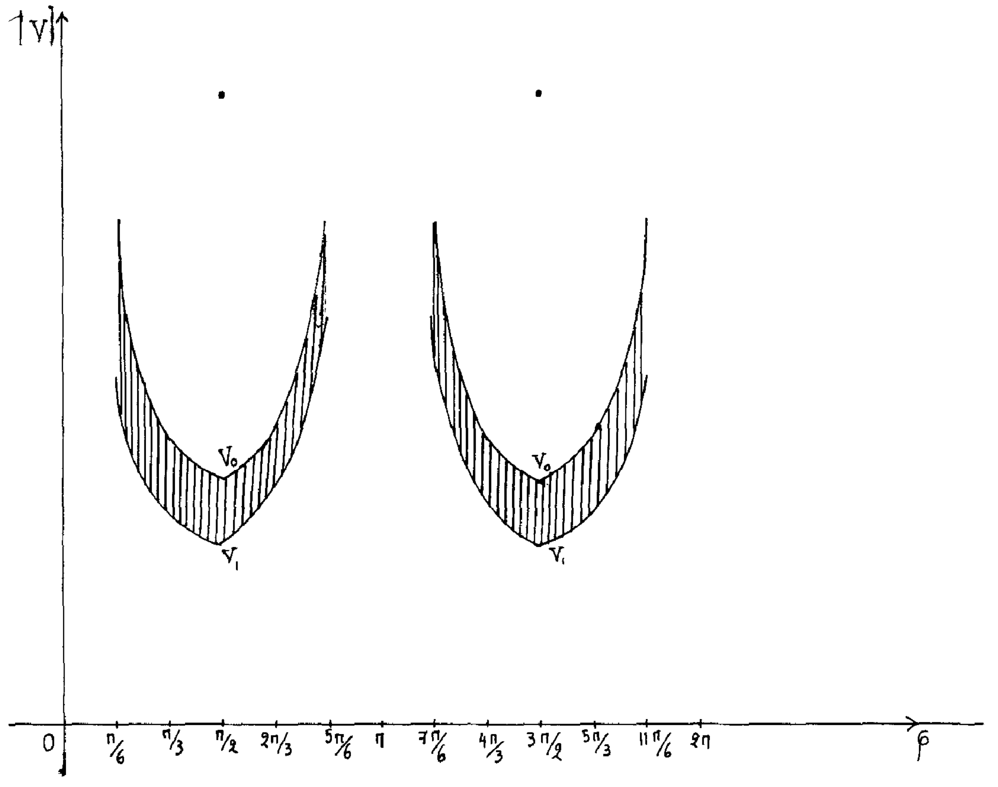
|V|
V0
V1
V0
V1
0
π/6
π/3
π/2
2π/3
5π/6
π
7π/6
4π/3
3π/2
5π/3
11π/6
2π
φ

Fig. 2a.

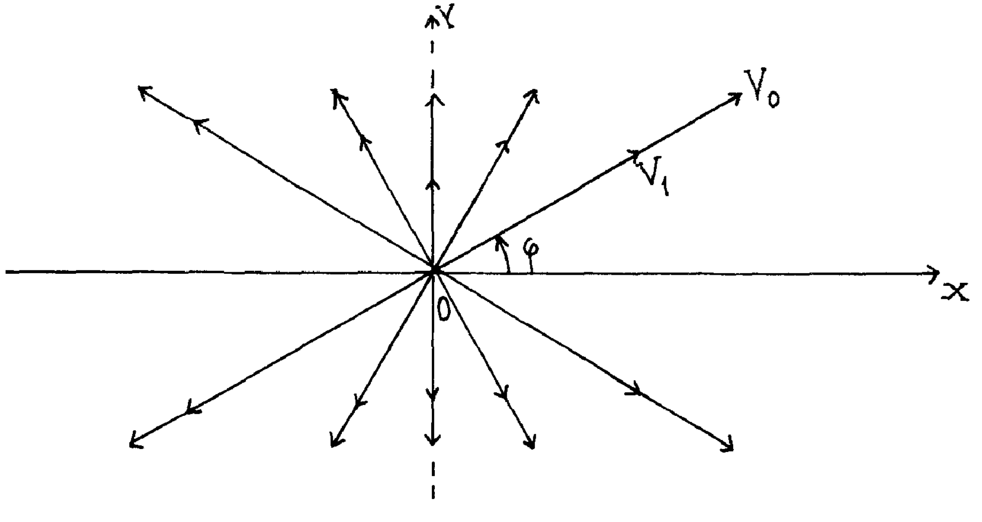
Y
V0
V1
φ
x
0

Fig. 2b.

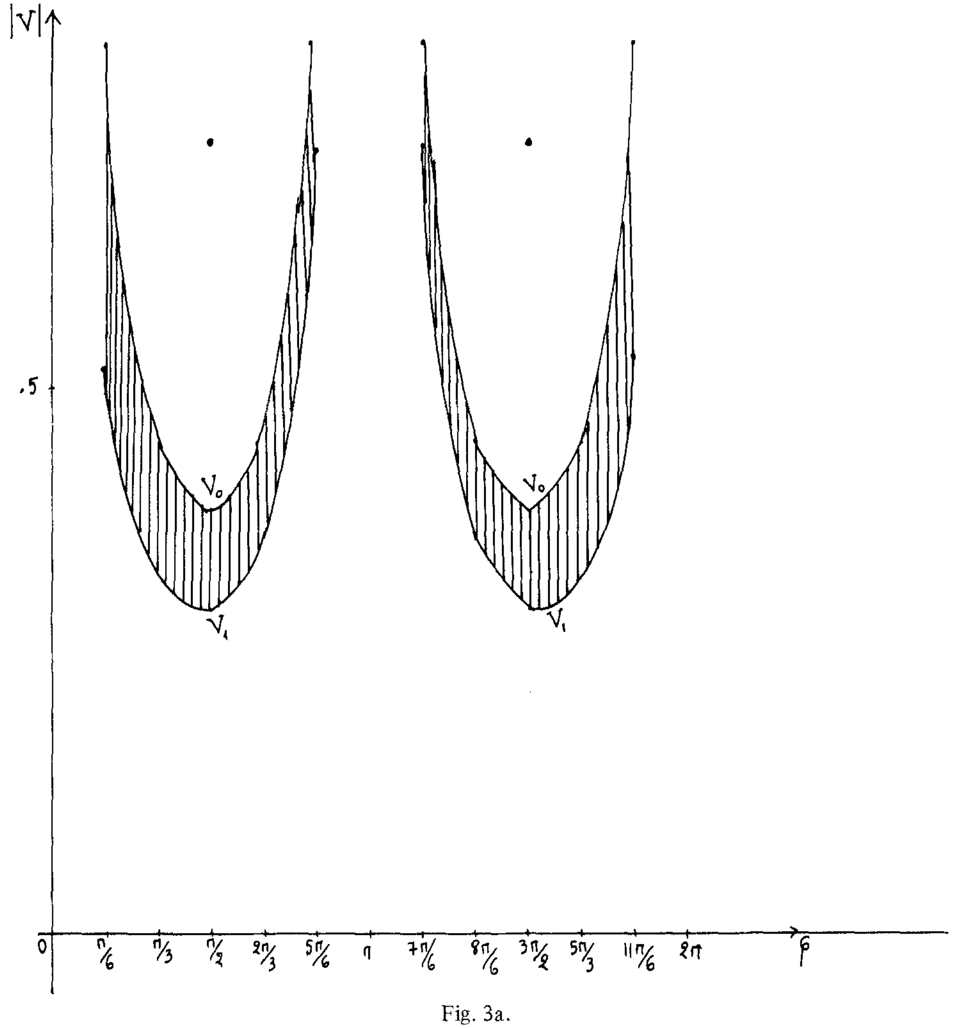

Fig. 3a.

TABLE II

φ	V_0	V_1
30°	0.81531	0.51809
45°	0.55590	0.38636
60°	0.44867	0.32621
90°	0.38636	0.29691
120°	0.44867	0.36231
135°	0.55590	0.46401
150°	0.81531	0.71680
210°	0.81531	0.71680
225°	0.55590	0.46401
240°	0.44867	0.36231
270°	0.38636	0.29691
300°	0.44867	0.32621
315°	0.55590	0.38636
330°	0.81531	0.51809

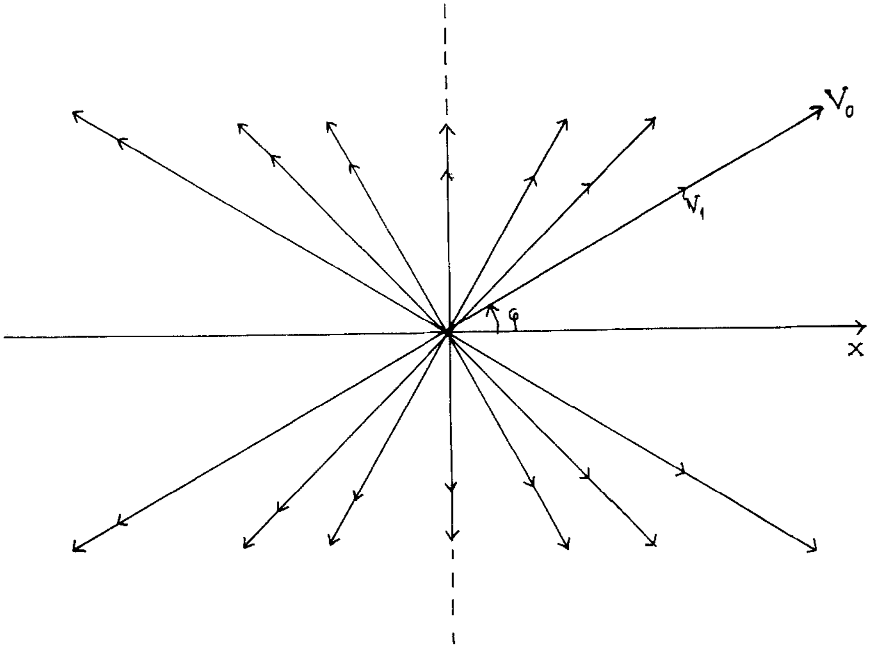

Fig. 3b.

TABLE III

φ	V_0	V_1
60°	1.37868	0.96304
90°	1.08316	0.96545
120°	1.37868	1.37225
240°	1.37868	1.37225
270°	1.08316	0.96545
300°	1.37868	0.96304

to distance for which we had periodic orbit in the Earth-Moon-Satellite system. Figures 3a and 3b give the relation between φ and velocities $V_0(\varphi)$ and $V_1(\varphi)$ for the above distance. Table III gives the values of φ and the corresponding values of initial velocities $V_0(\varphi)$ and $V_1(\varphi)$ for initial distance $r_0 = 0.007\,677\,215\,777$ at which we also have taken periodic orbit in the E-M-S system. Figures 4a and 4b represent the curves of velocities $V_0(\varphi)$ and $V_1(\varphi)$.

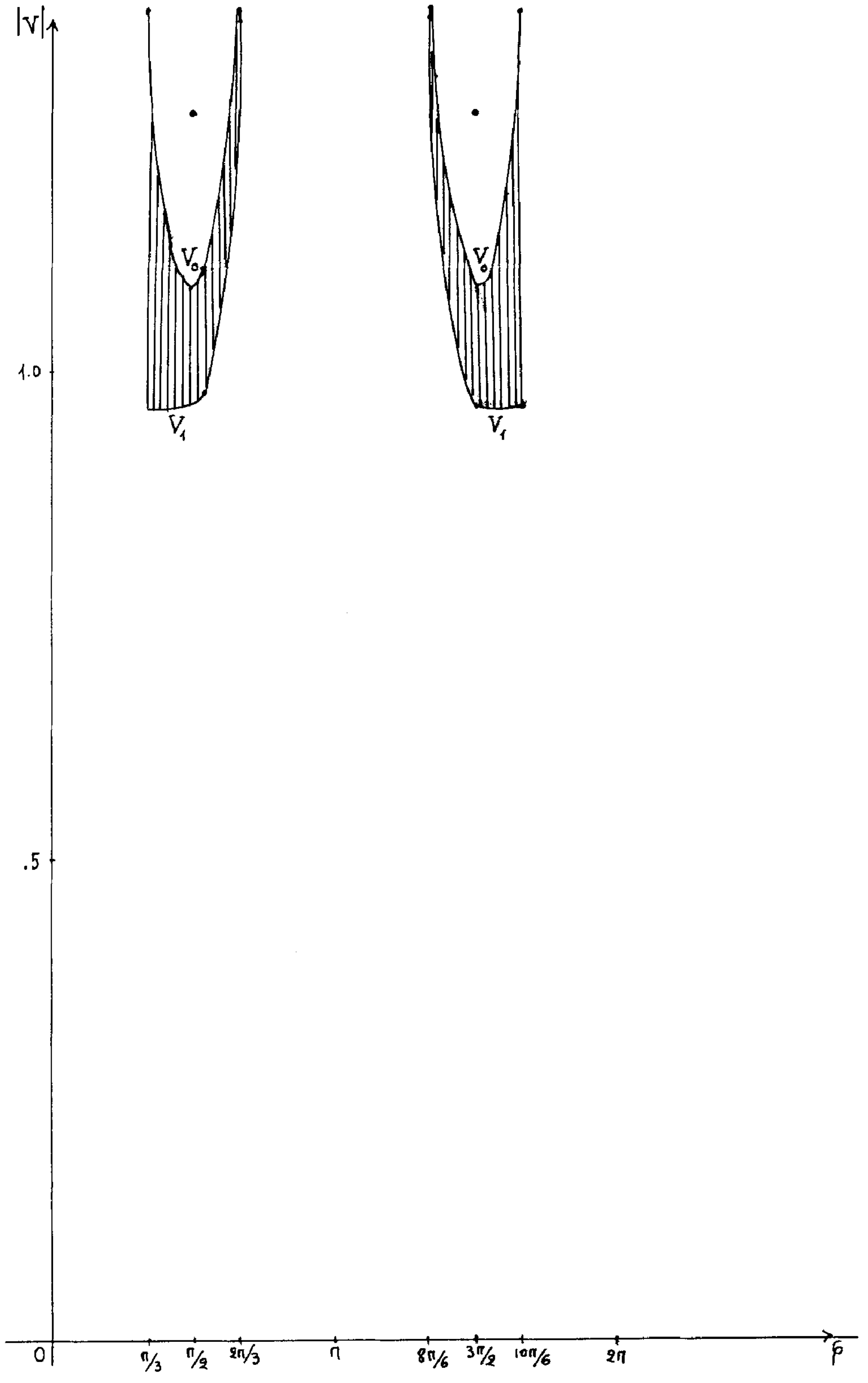

Fig. 4a.

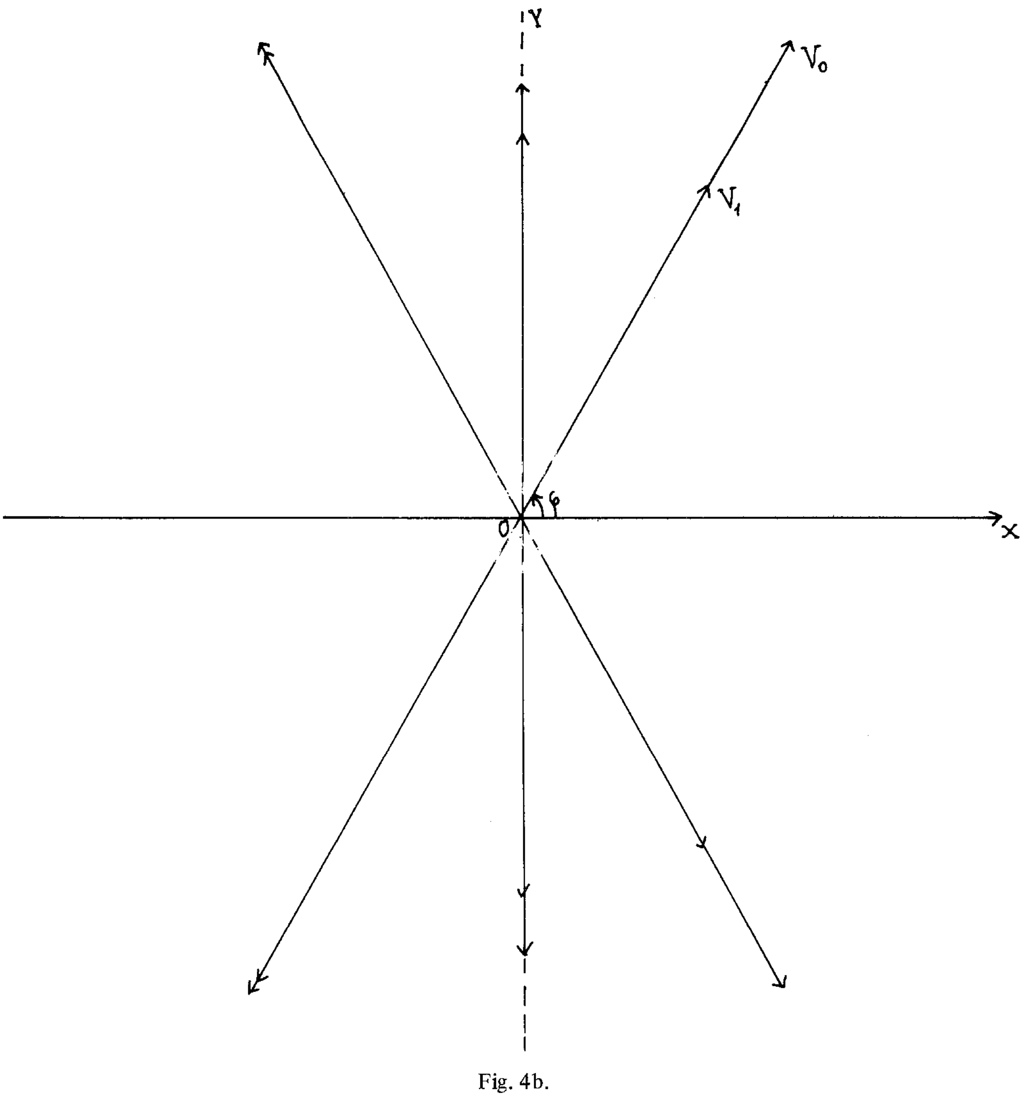

Fig. 4b.

3. The Behavior of the Collision Orbits of the Moon's Satellite in the Presence of the Earth and Sun

Let us consider first, the E-M-S system as an elliptic restricted three-body problem with the satellite's mass very small in comparison to the masses of Earth and Moon. The equations of motion in dimensionless rotating coordinate system *Oxy* with the two masses lying always on the rotating x-axis and oscillating on it, have been given by Hadjidemetriou (1975) as

$$\ddot{x} - 2\dot{y}\dot{\theta} - \ddot{\theta}y - x\dot{\theta}^2 = -(1-\mu)\frac{x-\mu\tau}{\tau_1^3} - \frac{x-(1-\mu)\tau}{\tau_2^3}, \tag{14a}$$

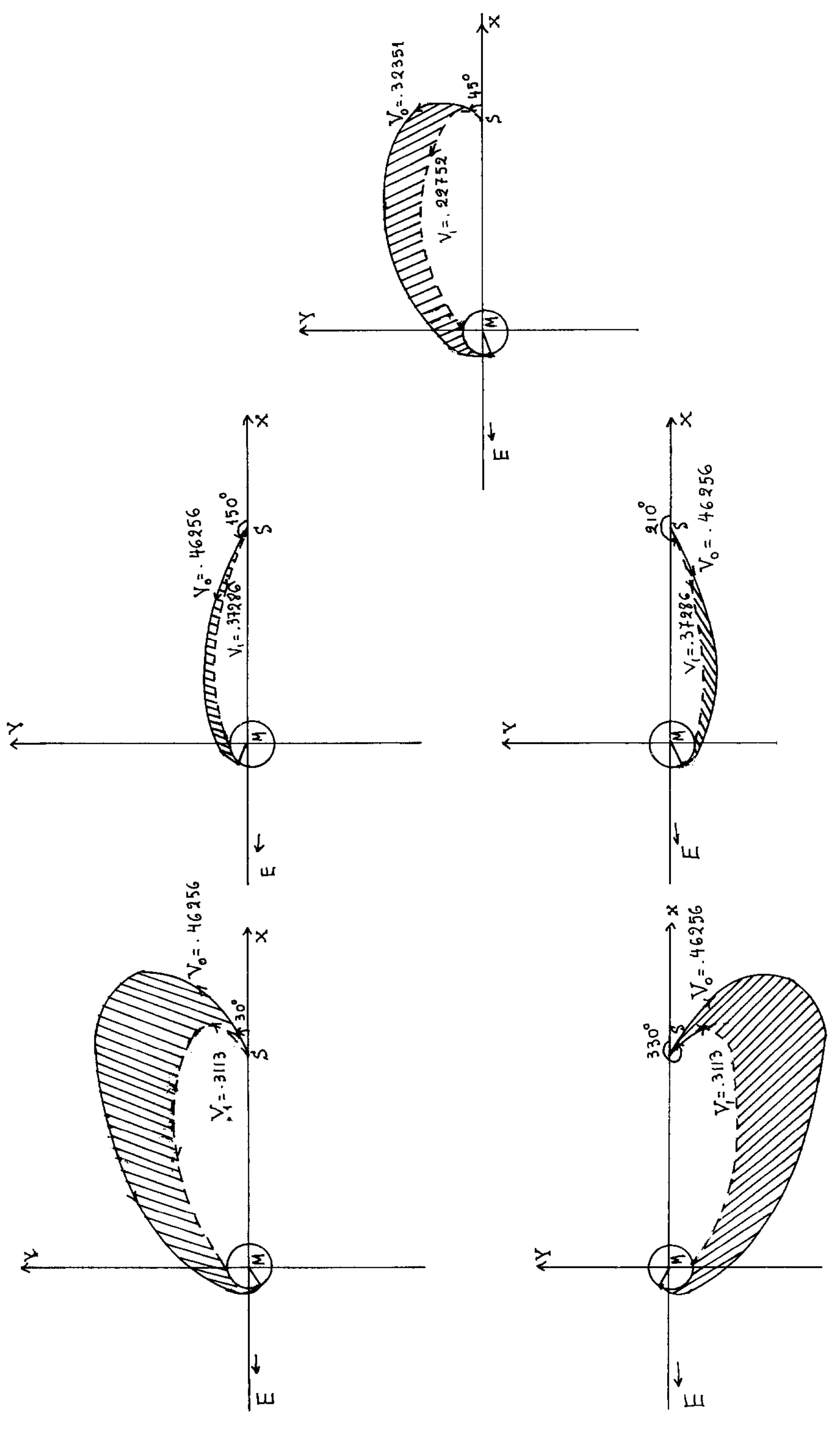

Fig. 5. Velocity regions in two-body problem (Moon-Satellite) for different values of the angle of ejection φ that resulting to collisions on the visible side of the Moon. (Initial distance of Moon-Satellite is $r_o = 0.043\,865\,014$).

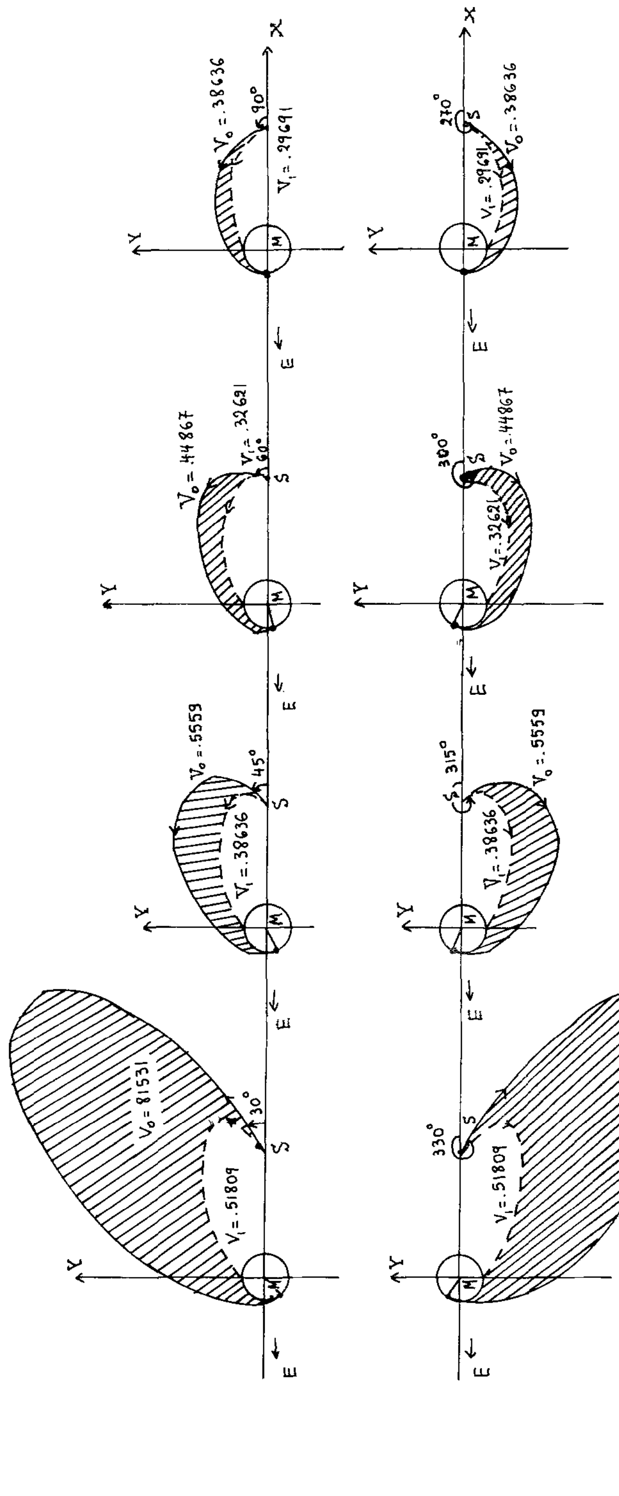

Fig. 6. Velocity regions in Moon-Satellite system for $r_o = 0.024\,963\,723\,073$, which produce collisions on the visible side of the Moon.

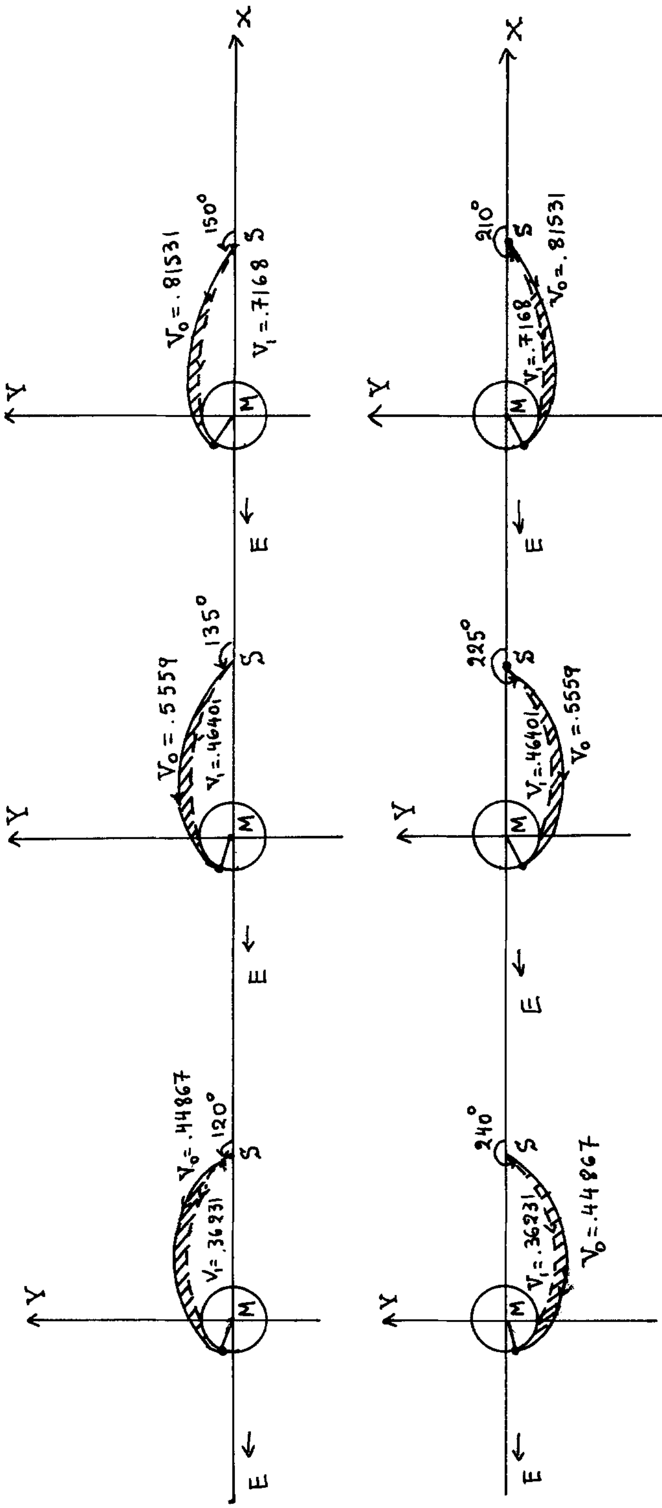

Fig. 6. (continued).

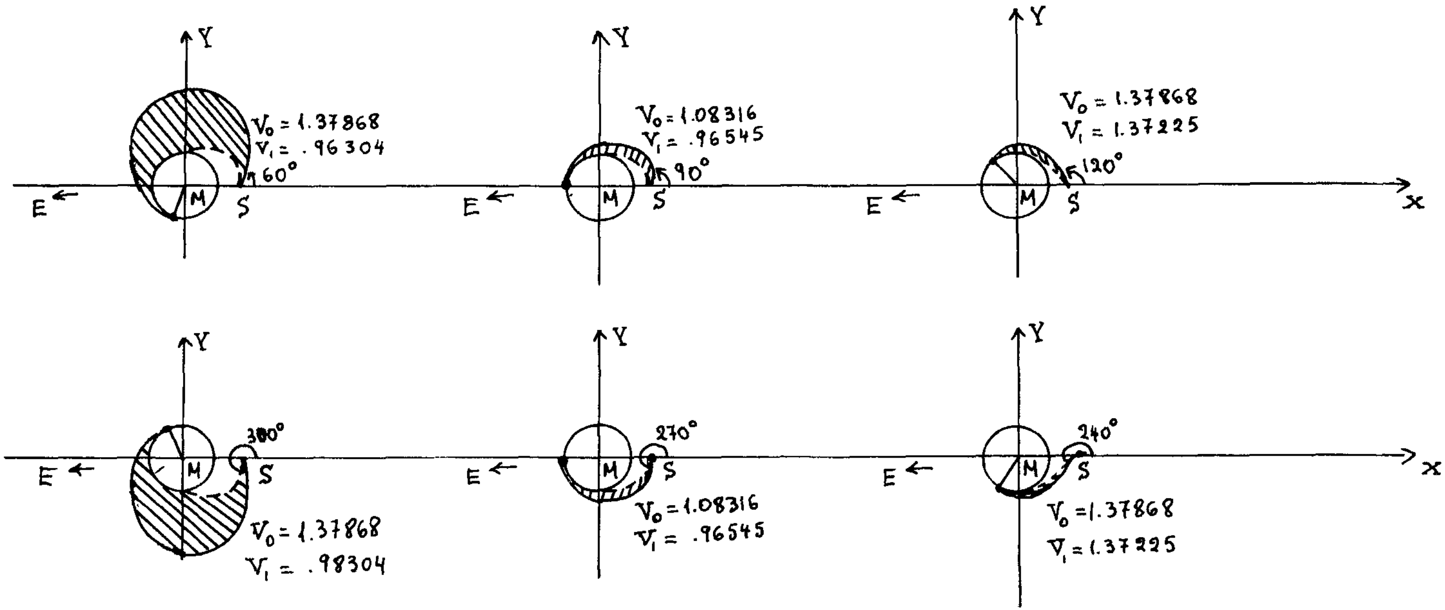

Fig. 7. Velocity regions for collisions on the visible side of the Moon for r_0 = 0.007 677 215 7 (Moon-Satellite distance).

$$\ddot{y} + 2\dot{x}\dot{\theta} + \ddot{\theta}x - y\dot{\theta}^2 = -(1-\mu)\frac{y}{\tau_1^3} - \mu\frac{y}{\tau_2^3}, \tag{14b}$$

where $\mu = m_2/(m_1 + m_2)$ and τ_1, τ_2 are the distances of the satellite from the Earth and Moon, respectively, given by the relations

$$\tau_1^2 = (x + \mu\tau)^2 + y^2, \tag{15a}$$

$$\tau_2^2 = [x - (1-\mu)\tau]^2 + y^2. \tag{15b}$$

By use of the above equations, symmetric periodic orbits have been found in a distance from the Moon equal to a small fraction of the Earth-Moon's distance (cf. Gousidou-Koutita, 1980).

Normalised distances giving symmetric periodic orbits in the Earth-Moon-Satellite system have been found equal to 0.043 865 014, 0.024 963 723 073, 0.007 677 215 777. These periodic orbits have been studied, next, in the presence of the Sun. Thus, the three-body problem (Earth-Moon-Satellite) is transformed to a four-body problem (Sun-Earth-Moon-Satellite) incorporating the perturbations induced by the Sun on the E-M-S system; since the Earth-Sun distance is very large in comparison to Earth-Moon distance.

Let R_D be the disturbing function, arising from the attraction of the Sun on the Moon. The expression for R_D is of the form

$$\begin{aligned} R_D = {} & n'^2a^2[\tfrac{1}{4} + \tfrac{3}{4}\cos(2\lambda - 2\lambda') - \tfrac{1}{2}e\cos(\lambda - \tilde{\omega}) - \tfrac{9}{4}e\cos(\lambda - 2\lambda' + \\ & + \tilde{\omega}) + \tfrac{3}{4}e\cos(3\lambda - 2\lambda' - \tilde{\omega}) + \tfrac{3}{4}e'\cos(\lambda' - \tilde{\omega}') + \tfrac{3}{8}e^2 + \\ & + \tfrac{15}{8}e^2\cos(2\lambda' - 2\tilde{\omega}) + \tfrac{3}{8}e'^2 - \tfrac{3}{8}\gamma^2 + \tfrac{3}{8}\gamma^2\cos(2\lambda' - 2\Omega) + \\ & + \tfrac{3}{8}\frac{a}{a'}\cos(\lambda - \lambda') + \tfrac{5}{8}\frac{a}{a'}\cos(3\lambda - 3\lambda') - \tfrac{15}{16}\frac{a}{a'}e\cos(\lambda' - \tilde{\omega}) - \\ & - \tfrac{15}{16}\frac{a}{a'}ee'\cos(\tilde{\omega} - \tilde{\omega}')]. \end{aligned} \tag{16}$$

Only the most significant terms of the Moon's motion are retained (cf. Brouwer and Clemence, 1961).

The term with argument $2\lambda - 2\lambda'$ in the disturbing function is known as 'variation' and its period is $2\pi/2(n - n') = T/2(1 - m) = 14.765\,294$ days $(m = n'/n)$. This term is equal to $\frac{3}{4}n'^2a^2\cos(2\lambda - 2\lambda')$ and a first approximation to the variation is given by Brouwer and Clemence (op. cit.) as

$$\delta\psi = +\tfrac{11}{8}m^2\sin(2\lambda - 2\lambda'), \tag{17a}$$

$$\delta\tau = -am^2\cos(2\lambda - 2\lambda'). \tag{17b}$$

The term with argument $2\lambda' - 2\tilde{\omega}$ in the disturbing function is known as 'evection' and it has the form $+\frac{15}{8}n'^2a^2e^2\cos(2\lambda' - 2\tilde{\omega})$. A first approximation of the evection is

$$\delta\psi = +\tfrac{15}{4}me\sin(\lambda - 2\lambda' + \tilde{\omega}), \tag{18a}$$

$$\frac{\delta\tau}{a} = -\tfrac{15}{8}me\cos(\lambda - 2\lambda' + \tilde{\omega}); \tag{18b}$$

and the period of evection is $2\pi/(2n - 2n' - cn) = T/(1 - 2m + 1 - c) = 31.807\,472$ days. The effect of the action of the Sun is producing evection, is to cause periodic variations of the eccentricity and of the longitude of perigee of the Moon. The evection is the largest periodic perturbation in the Moon's longitude.

We have taken into account the above two perturbations in the motion of the Moon around the Earth as the most important perturbations in this motion.

The initial conditions for the position of the Moon's satellite and its velocity have been taken identical to those giving periodic orbits in the unperturbed E-M-S system (cf. Gousidou-Koutita, 1980).

Keeping the initial distance of the satellite from the centre of Moon's mass the same as above, we gave such velocities to the satellite that led to collisions on the visible side of the Moon in the two-body problem M-S system, that is, in the area $[V_1, V_0]$ as we have mentioned in Section 2. Thus, here we investigated whether or not these collisions occur preferentially on the visible side of the Moon. The procedure has been repeated for values of the velocity outside this area for different values of the angle φ, supposing that the meteorite undergoes an instantaneous explosion at $t = 0$, so that the meteorite's pieces begin their motions at that time with different values of φ and V. Their collision orbits and their collapses on the lunar surface have been studied.

The collision points of the meteorite with the lunar surface – with the presence of the Earth and Sun exhibit a transition on the lunar surface relatively to the collisions in absence of the Earth and Sun, and with the collision orbits which the meteorites execute for several revolutions around the Moon before falling on the lunar surface for some values of V.

These collisions with different values of φ and V and the number of revolutions around the Moon before the meteorite fall on the lunar surface are given in next section.

4. Collisions of the Satellites on the Visible Side of the Moon. Numerical Results.

Initial velocity values have been given, for each value of φ, to lunar satellites with values inside the region $[V_1, V_0]$ and outside of it. Numerical results for all cases are given in the following Tables IV, V, VI.

As we have mentioned in the previous paragraph, in the case of the perturbed three-body problem the satellite having the same initial conditions as in the case of two-body problem, falls on the lunar surface, for most of the initial conditions, after some revolutions around the Moon. The collision points are in different positions on the Moon's surface relatively to the two-body problem. Now, in order to locate the collision points of the meteorite on the lunar surface we have considered the Moon as a circle in the orbital plane with radius $R = 0.004\,521\,332$ in nondimensional units. The number of revolutions of the satellite around the Moon before collision is given in the next tables.

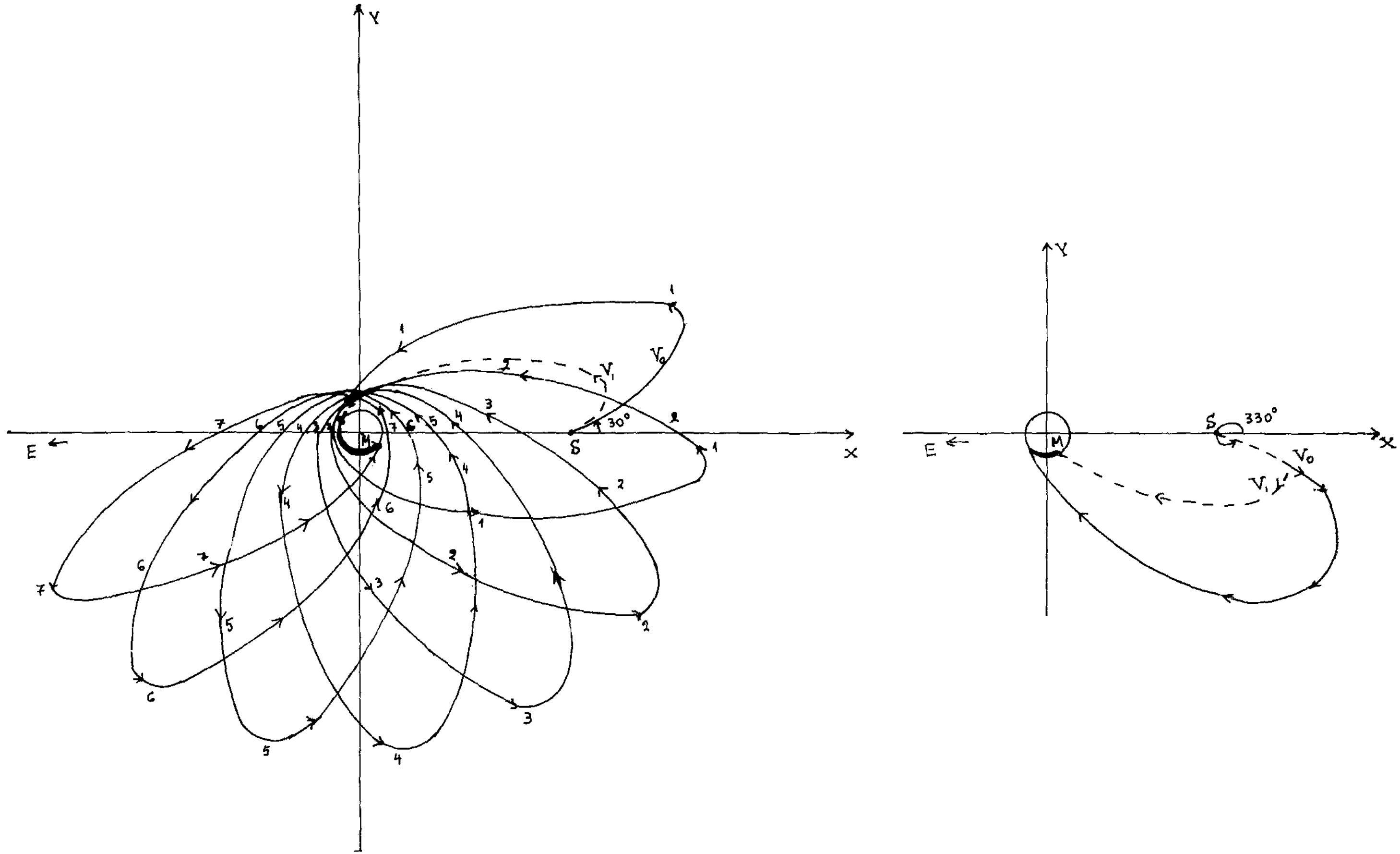

Fig. 8. The evolution of the collision orbits for two angles of ejection in the perturbed Earth-Moon-Satellite system with the same initial velocities as in the corresponding two-body problem ($r_0 = 0.043\,865\,014$).

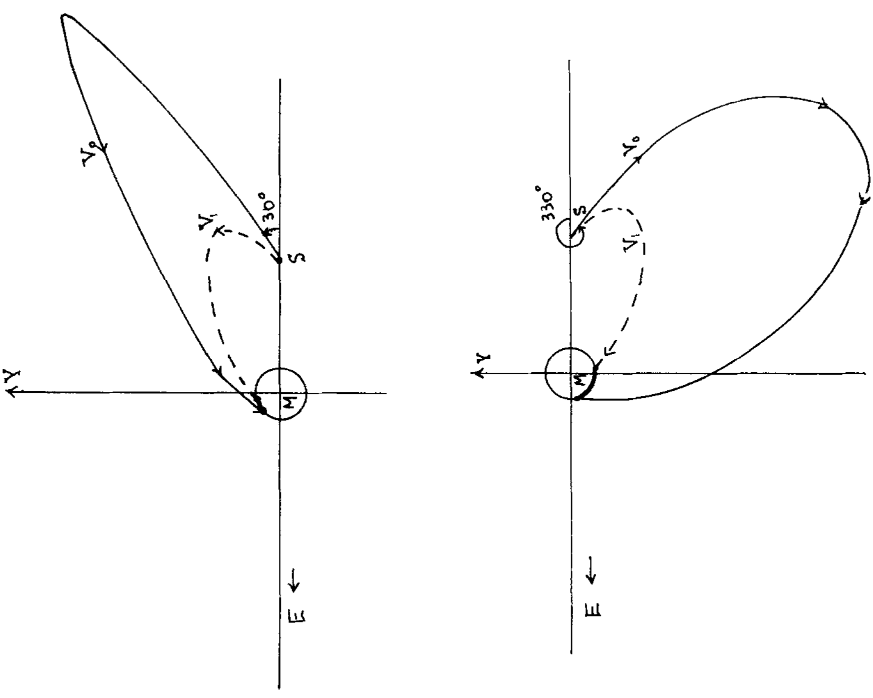

Fig. 9. Some of the orbits of Figure 6 in the perturbed three-body problem (Earth-Moon-Satellite).

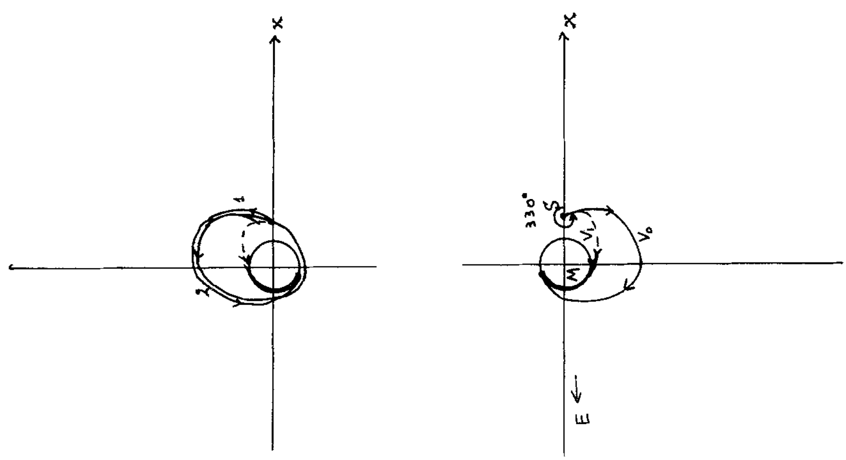

Fig. 10. Some of the orbits of Figure 7 in the perturbed three-body problem (Earth-Moon-Satellite).

TABLE IV

The Cartesian coordinates of collision points in the perturbed three-body problem (Earth-Moon-Satellite) for each value of velocity V and ejections angle φ. Initial Moon-Satellite distance equal to $r_0 = 0.043\,865\,014$.

				$r_0 = 0.043\,865\,014$					
φ	V	x	y	n_0	φ	V	x	y	n_θ number of revolutions
30°	0.24	0.000 192 260	0.004 499 107		330°	0.24	0.002 400 906	−0.002 740 816	
	0.311 3	−0.002 522 784	0.003 431 046			0.311 3	0.000 917 835	−0.003 389 186	
	0.36	−0.004 180 140	0.001 074 479			0.36	0.000 397 961	−0.004 393 519	
	0.462 56	0.003 756 295	−0.001 983 965	6		0.462 56	−0.002 638 840	−0.002 819 472	
	0.50	0.002 131 851	−0.003 873 347	5		0.50	−0.003 273 838	−0.002 544 991	
150°	0.32	−0.002 195 453	0.003 523 933		210°	0.32	0.002 706 447	−0.003 453 053	
	0.36	−0.004 163 966	0.001 674 216			0.36	0.002 076 183	−0.003 818 771	
	0.372 86	0.004 359 816	−0.000 223 228	6		0.372 86	0.001 672 555	−0.003 825 421	
	0.42	0.004 485 113	0.000 507 608	5		0.42	0.000 887 155	−0.004 171 441	
	0.462 56	0.004 249 027	0.001 510 786	4		0.462 56	−0.000 001 390	−0.004 337 047	
	0.50	0.001 255 639	−0.003 586 402	4		0.50	−0.001 064 558	−0.004 211 893	
45°	0.18	−0.000 505 313	0.004 203 589						
	0.227 52	−0.003 024 693	0.002 986 549						
	0.26	0.003 415 762	0.002 786 405	10					
	0.30	0.004 437 509	−0.000 078 947	13					
	0.323 51	−0.008 356 19	−0.003 362 850	65					

TABLE V

The same as in Table IV but with initial Moon-Satellite distance $r_0 = 0.024\,963\,723\,073$

φ	V	x	y	n_0 (number of revolutions)
30°	0.48	− 0.000 119 850	0.004 123 793	
	0.518 09	− 0.000 726 065	0.004 197 543	
	0.66	− 0.003 346 504	0.002 775 042	
	0.70	− 0.003 851 402	0.001 971 156	
	0.815 31	− 0.000 136 853	0.004 943 889	
45°	0.34	− 0.000 070 677	0.003 905 378	
	0.386 36	− 0.000 823 319	0.004 336 969	
	0.50	− 0.004 339 422	0.000 662 493	
	0.555 90	0.003 824 411	0.002 233 502	19
	0.58	0.004 499 563	0.000 277 289	21
60°	0.30	− 0.000 260 351	0.004 173 674	
	0.326 21	− 0.001 476 278	0.003 646 521	
	0.40	− 0.004 103 613	0.001 448 382	
90°	0.26	0.000 333 692	0.004 340 265	
	0.296 91	− 0.001 323 009	0.004 003 547	
	0.34	− 0.003 314 243	0.002 948 716	
	0.36	− 0.000 347 197	0.004 496 744	12
120°	0.32	− 0.000 085 410	0.004 153 204	
	0.362 31	− 0.001 310 073	0.004 144 189	
	0.40	− 0.002 924 594	0.003 418 972	
	0.448 67	0.003 534 590	0.002 793 715	17
135°	0.42	− 0.000 604 618	0.003 999 45	
	0.464 01	− 0.001 575 35	0.003 991 731	
	0.50	− 0.002 965 836	0.003 285 749	
	0.5559	0.003 443 402	0.002 857 735	11
150°	0.62	− 0.000 099 608	0.004 149 755	
	0.7168	− 0.001 673 754	0.004 096 696	
	0.74	− 0.003 022 613	0.003 194 845	
	0.815 31	0.002 230 676	− 0.003 078 318	2
330°	0.48	0.000 984 864	− 0.004 163 046	
	0.518 09	0.000 379 920	− 0.004 364 596	
	0.66	− 0.002 602 270	− 0.002 315 311	
	0.70	− 0.003 303 626	− 0.002 004 187	
	0.78	− 0.004 142 034	− 0.001 074 753	
315°	0.34	0.001 687 353	− 0.004 190 242	
	0.386 36	0.000 159 978	− 0.004 018 839	
	0.50	− 0.002 862 277	− 0.003 049 675	
	0.555 90	− 0.004 187 437	− 0.001 458 213	
	0.58	− 0.004 234 119	0.000 787 450	

TABLE V (continued)

φ	V	x	y	n_0
300°	0.30	0.001 328 604	− 0.004 244 581	
	0.326 21	0.000 110 118	− 0.003 953 539	
	0.40	− 0.002 277 498	− 0.003 315 660	
	0.448 67	− 0.003 770 270	− 0.002 405 736	
270°	0.26	0.001 549 244	− 0.004 010 897	
	0.296 91	0.000 056 219	− 0.003 911 975	
	0.34	− 0.001 507 286	− 0.003 626 983	
	0.386 36	− 0.003 406 540	− 0.002 400 599	
240°	0.32	0.000 887 845	− 0.003 674 999	
	0.362 31	0.000 202 589	− 0.004 049 327	
	0.40	− 0.000 280 943	− 0.004 511 865	
	0.448 67	− 0.002 256 112	− 0.003 855 906	
225°	0.42	0.000 750 999	− 0.003 767 408	
	0.464 01	0.000 488 158	− 0.004 245 450	
	0.50	− 0.000 411 217	− 0.004 203 186	
	0.5559	− 0.001 596 107	− 0.004 223 422	
	0.58	− 0.002 777 063	− 0.003 499 833	
210°	0.62	0.000 947 946	− 0.003 692 582	
	0.716 8	0.000 228 134	− 0.004 117 519	
	0.74	− 0.000 383 273	− 0.003 972 167	
	0.815 31	− 0.001 201 479	− 0.004 147 291	

Numerical results have been also obtained for collision orbits of the meteorite in the case that the satelite begins its motion when it lies between Earth and Moon – i.e. we consider the system E-S-M at $t = 0$.

In that case we give the results for initial distance equal to $r_0 = -0.024\,963\,723\,073$ (Table VII).

TABLE VI

As in Table IV but with initial distance $r_0 = 0.007\,677\,215\,777$

				$r_0 = 0.007\,677\,215\,777$					
φ	V	x	y	n_0	φ	V	x	y	n_0
60°	0.90	0.000 326 897	0.004 077 107		300°	0.90	.0000 824 411	−0.004 292 476	
	0.963 04	−0.005 005 062	0.005 818 834			0.963 04	−0.000 136 322	−0.004 355 557	
	1.0	−0.000 980 778	0.004 212 378			1.0	−0.000 690 991	−0.004 362 565	
	1.2	−0.004 236 439	0.000 919 856			1.2	−0.004 116 475	−0.000 983 495	
						1.378 68	−0.001 525 814	0.004 226 817	
120°	1.0	0.001 807 044	0.002 967 576		240°	1.0	0.001 746 261	−0.002 882 546	
	1.372 25	0.001 191 407	0.004 321 906			1.372 25	0.001 133 775	−0.004 216 044	
	1.378 68	0.001 180 034	0.004 348 245			1.378 68	0.001 122 350	−0.004 242 357	
90°	0.94	0.000 179 670	0.004 427 675		270°	0.94	0.000 011 162	−0.004 238 444	
	0.965 45	−0.000 506 006	0.004 359 851			0.965 45	−0.000 687 577	−0.004 118 971	
	1.083 16	−0.001 828 565	0.003 861 711			1.083 16	−0.004 186 881	−0.001 592 65	

TABLE VII

The same as in Table IV but in the system Earth-Satellite-Moon and initial Moon-Satellite distance equal to $r_0 = -0.024\,963\,723\,073$

φ	V	x	y	n_0	φ	V	x	y	n_0
60°	0.32	−0.000 968 270	0.003 722 04		300°	0.32	0.000 005 57	−0.004 207 206	
	0.362 31	−0.000 275 290	0.004 098 391			0.362 31	0.001 239 487	−0.004 204 58	
	0.40	0.001 046 889	0.003 833 07			0.40	0.002 867 221	−0.003 487 98	
	0.448 67	0.002 200 865	0.003 914 523						
	0.48	0.004 363 913	0.000 918 68						
120°	0.26	−0.001 705 035	0.003 519 592		240°	0.26	−0.000 976 025	−0.004 083 264	
	0.326 21	−0.000 327 724	0.004 143 558			0.326 21	0.001 268 361	−0.003 894 124	
	0.40	0.002 095 152	0.003 630 75			0.40	0.004 006 409	−0.001 825 44	
	0.448 67	0.003 840 289	0.001 717 075						
	0.48	0.004 285 931	−0.001 086 37						
150°	0.46	−0.001 420 659	0.004 122 530		210°	0.46	−0.000 186 175	−0.004 023 254	
	0.518 09	−0.000 516 258	0.004 456 602			0.518 09	0.000 452 193	−0.004 457 03	
	0.70	0.003 000 152	0.003 170 262			0.70	0.003 494 447	−0.002 276 992	
	0.815 31	0.003 505 239	0.002 146 83			0.815 31	0.000 456 202	−0.003 574 232	

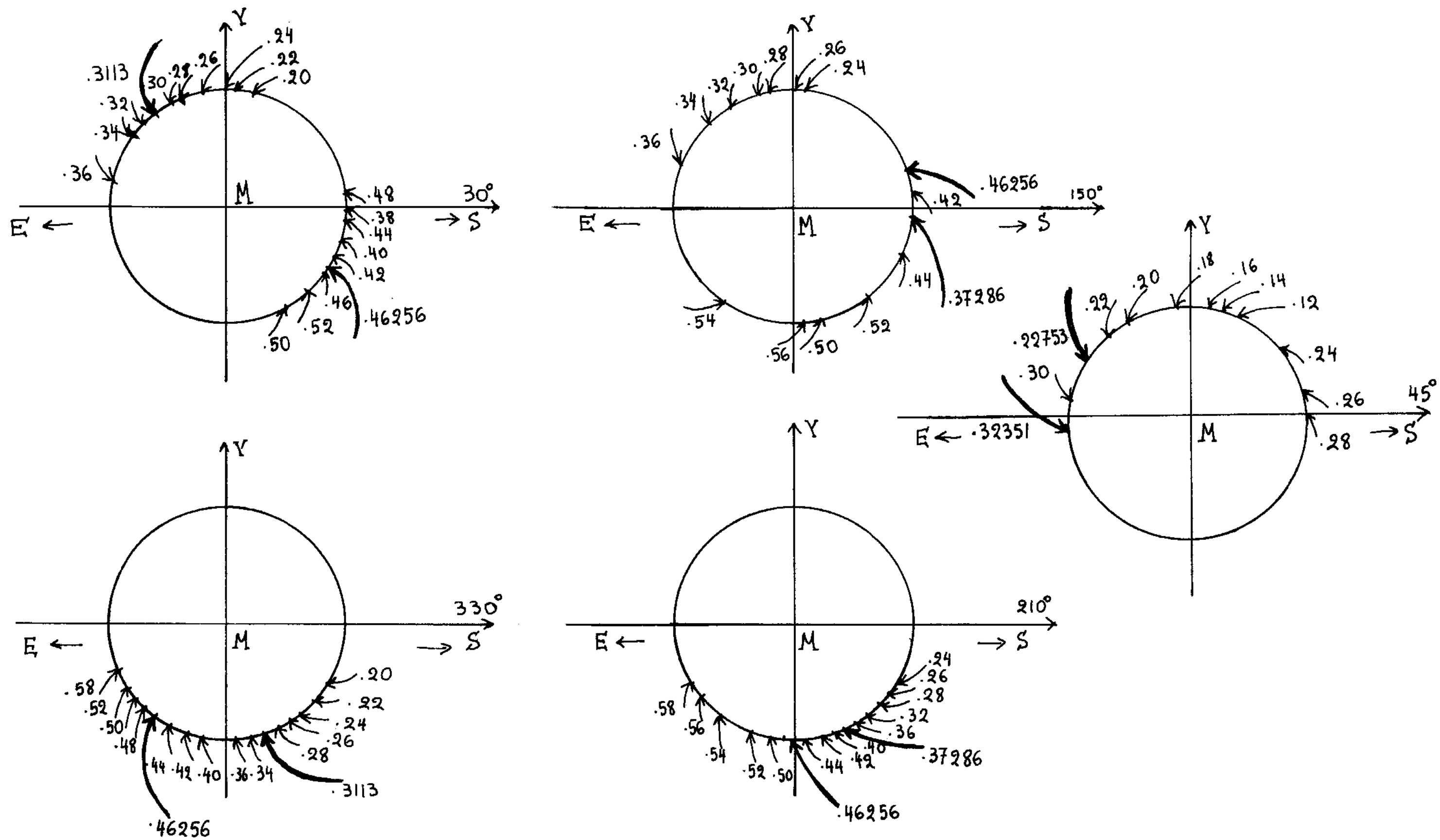

Fig. 11. Collision points on the Moon's surface in the perturbed three-body problem for different value of ejection φ and initial Moon-Satellite distance equal to $r_0 = 0.043\,860\,14$. The numbers around the Moon indicate the velocities of Satellite's ejection and the numbers denoted by darker arrows correspond to the boundary values of velocities which gave collisions only on the visible side of the Moon in the two-body problem as in Figure 5.

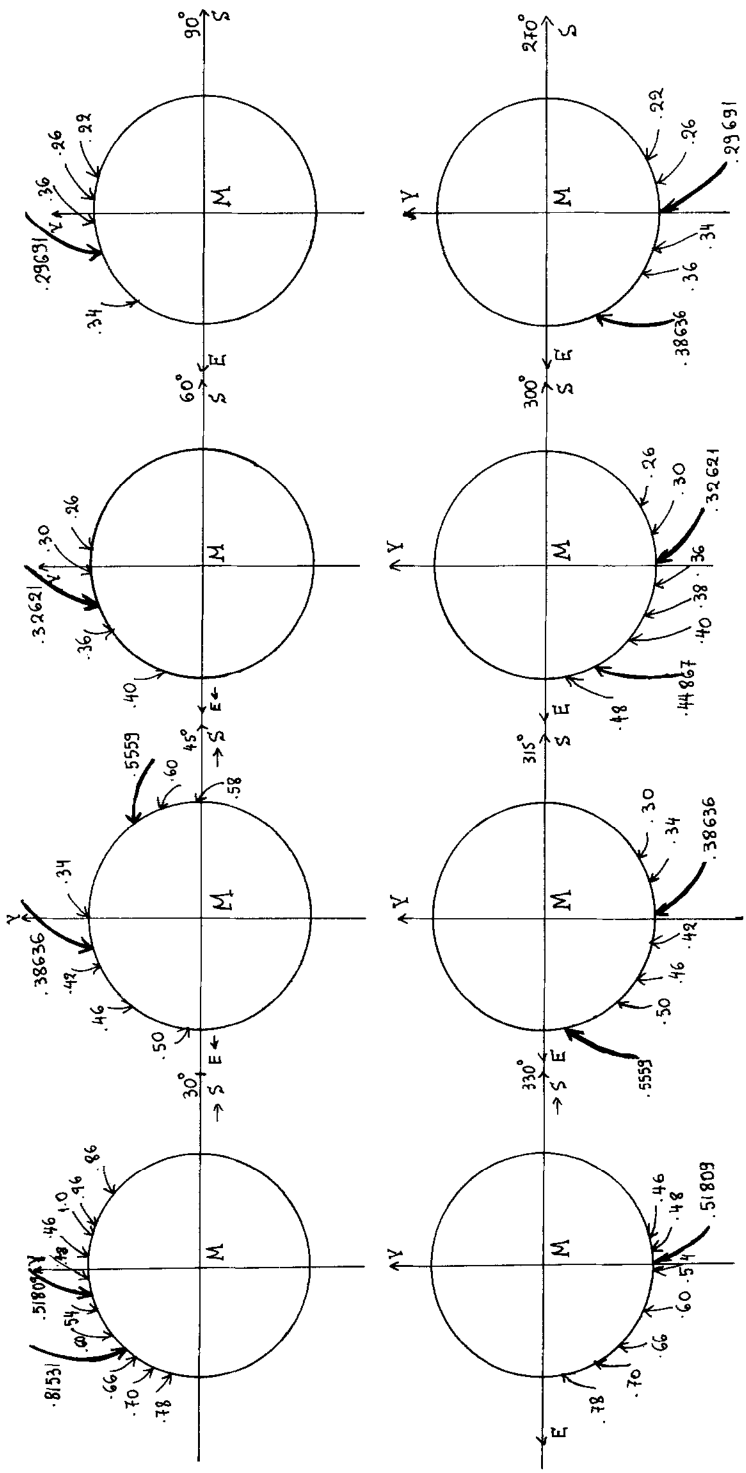

Fig. 12. As in Figure 11 but with initial Moon-Satellite distance equal to $r_0 = 0.024\,963\,723\,073$.

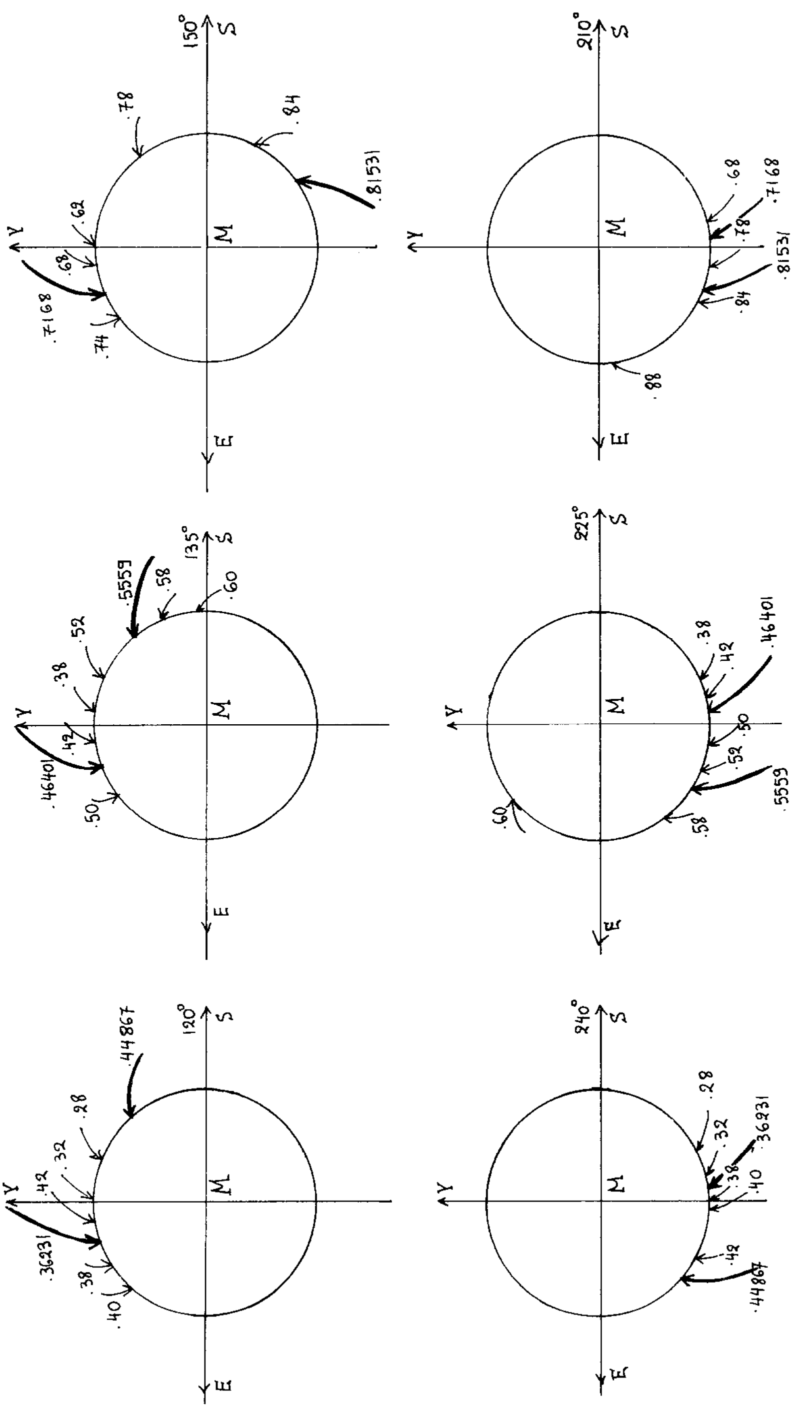

Figure 12. (continued).

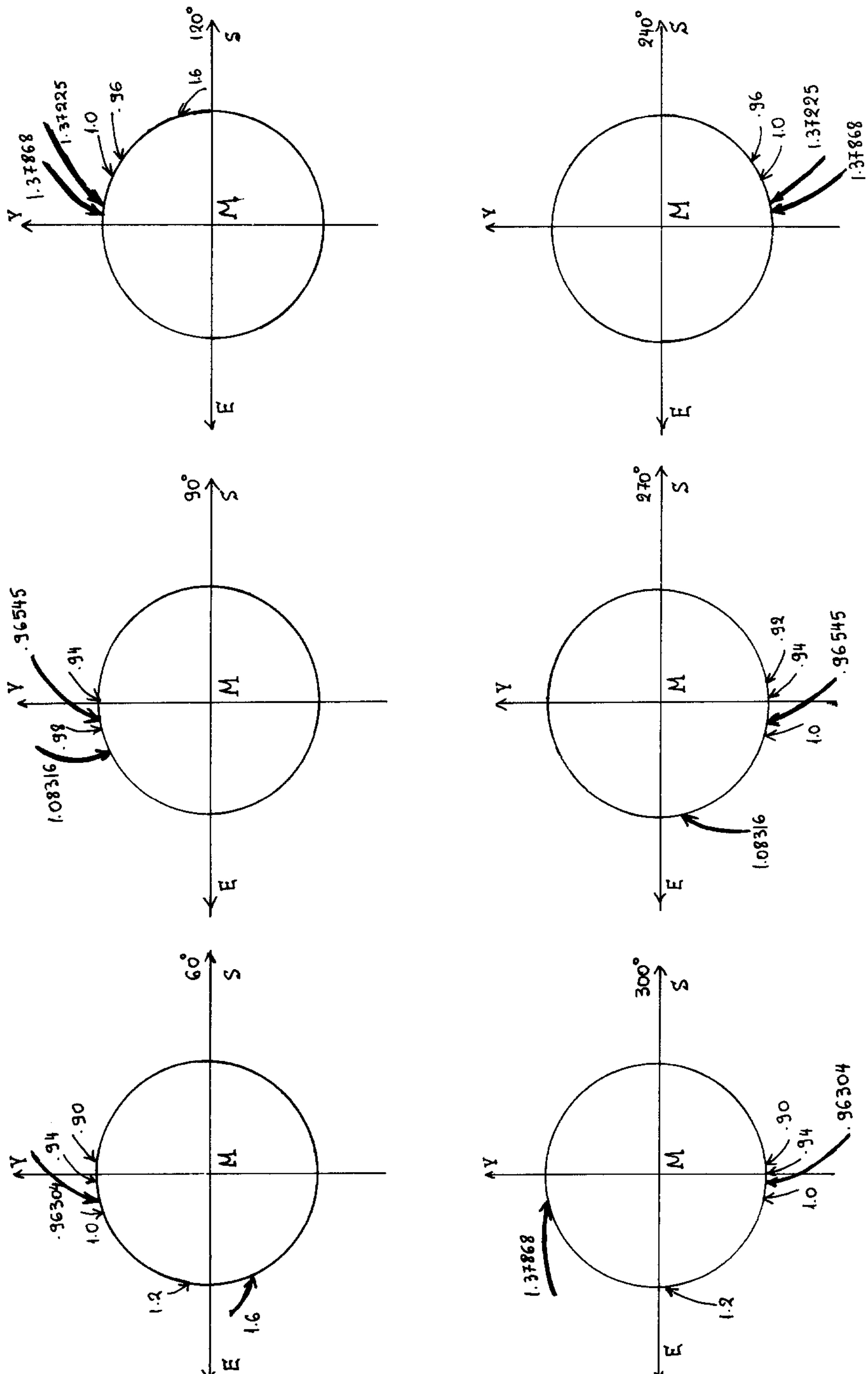

Fig. 13. As in Figure 11 but now with initial Moon-Satellite distance equal to $r_0 = 0.007\,677\,215\,7$.

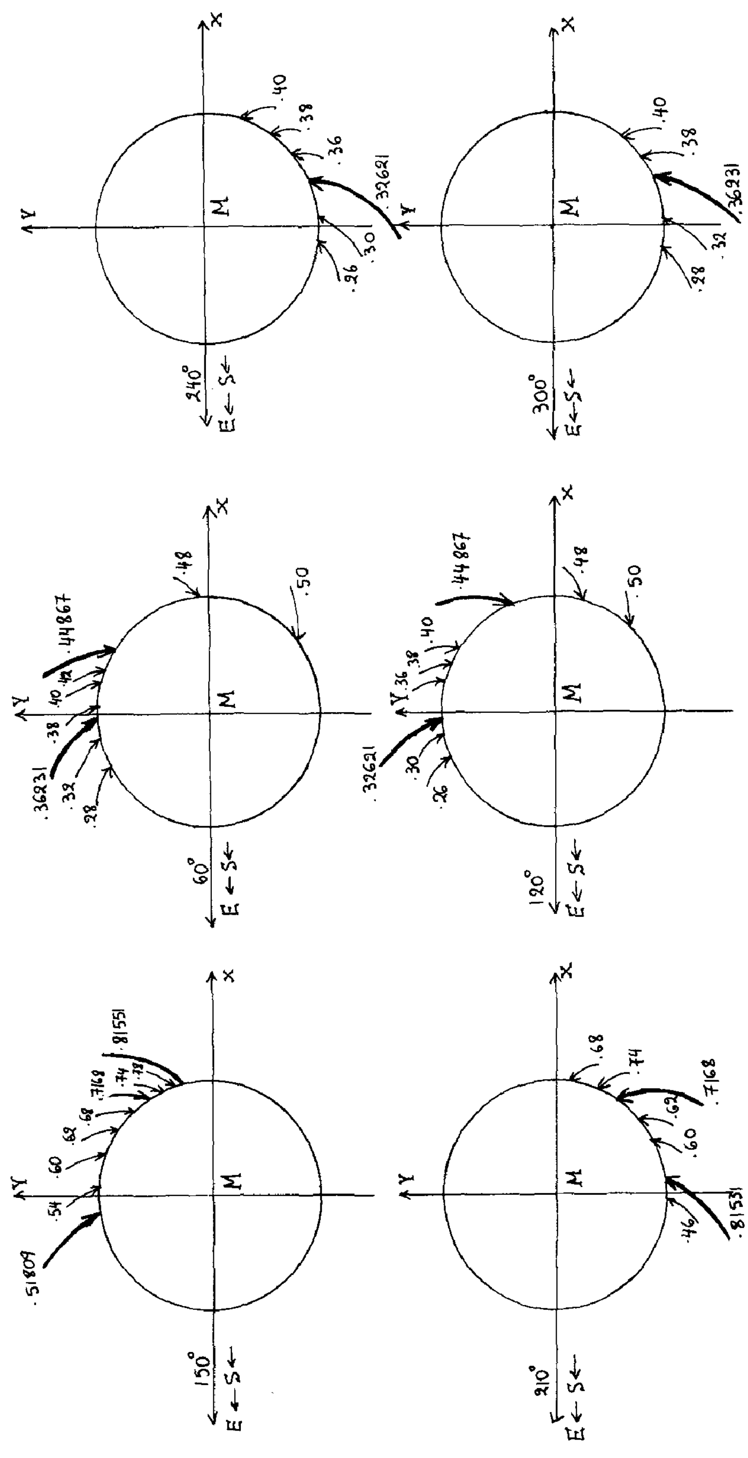

Fig. 14. As in Figure 12 but the Satellite lies at $t = 0$ between Earth and Moon.

From the numerical results we observe that we have collisions on the visible side of the Moon for the following regions of velocities:

TABLE VIII

Regions of velocities and the corresponding angles of ejection for collisions on the visible side of the Moon.

$r_0 = 0.043\,865\,014$		$r_0 = 0.007\,677\,215\,777$	
φ	V	φ	V
30°	[0.24 , 0.36]	60°	[0.94, 1.6]
45°	[0.18 , 0.323 51]	90°	[0.92, 1.083 16]
150°	[0.26 , 0.36]	270°	[0.94, 1.083 16]
210°	[0.462 56, 0.56]	300°	[0.94, 1.378 68]
330°	[0.38 , 0.58]		

$r_0 = 0.024\,963\,723\,073$		$r_0 = -0.024\,963\,723\,073$	
φ	V	φ	V Velocity values less than
30°	[0.48 , 0.78]	60°	0.38
45°	[0.34 , 0.50]	120°	0.33
60°	[0.30 , 0.40]	150°	0.54
90°	[0.26 , 0.34]	210°	0.50
120°	[0.32 , 0.40]	240°	0.30
135°	[0.40 , 0.50]	300°	0.32
150°	[0.62 , 0.74]		
210°	[0.72 , 0.84]		
225°	[0.48 , 0.58]		
240°	[0.38 , 0.48]		
270°	[0.269 91, 0.386 36]		
300°	[0.34 , 0.48]		
315°	[0.40 , 0.60]		
330°	[0.52 , 0.78]		

5. Conclusions

From all the above it is obvious that the presence of the Earth and Sun has an important influence on the meteorite's collision orbits and, therefore, to the collision points on the lunar surface. For all initial distances of the lunar satellite in both the E-M-S of E-S-M systems and for all satellite launch angles a shift of the collision points is observed as it is seen from the numerical results. The shift occurs in a counterclockwise direction.

For values of the velocity beyond a certain bound the satellite makes a number of revolutions around the Moon before its eventual impact. Certainly, in that case the collision may not occur on the visible side of the Moon, as it happens in the case of launch angles inside the range $(\pi, 2\pi)$. For launch angles $\varphi \in (\pi, 2\pi)$ the collision points are displaced in a counterclockwise direction on the Moon's surface resulting in colisions on the far side (in presence of the Earth and Sun) – contrary to the case when Earth and

Sun is not taken in consideration. From the distribution of the collision points on the Moon we conclude that a tendency exists for collisions to occur along the ancient equatorial plane of the Moon.

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