# BASIN-RING SPACING ON THE MOON, MERCURY, AND MARS 

RICHARD. J. PIKE<br>U.S. Geological Survey, Menlo Park, California, U.S.A. and<br>PAULD.SPUDIS<br>U.S. Geological Survey, Flagstaff, Arizona, U.S.A.

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#### Abstract

Radial spacing between concentric rings of impact basins that lack central peaks is statistically similar and nonrandom on the Moon, Mercury, and Mars, both inside and outside the main ring. One spacing interval, $(2.0 \pm 0.3)^{0.5} \mathrm{D}$, or an integer multiple of it, dominates most basin rings. Three analytical approaches yield similar results from 296 remapped or newly mapped rings of 67 multi-ringed basins: least-squares of rank-grouped rings, least-squares of rank and ring diameter for each basin, and averaged ratios of adjacent rings. Analysis of 106 rings of 53 two-ring basins by the first and third methods yields an integer multiple ( $2 \times$ ) of $2.0^{0.5} \mathrm{D}$. There are two exceptions: (1) Rings adjacent to the main ring of multi-ring basins are consistently spaced at a slightly, but significantly, larger interval, ( $2.1 \pm 0.3)^{0.5} \mathrm{D}$; (2) The 88 rings of 44 protobasins (large peak-plus-inner-ring craters) are spaced at an entirely different interval $(3.3 \pm 0.6)^{0.5} \mathrm{D}$.

The statistically constant and target-invariant spacing of so many rings suggests that this characteristic may constrain formational models of impact basins on the terrestrial planets. The key elements of such a constraint include: (1) ring positions may not have been located by the same process(es) that formed ring topography; (2) ring location and emplacement of ring topography need not be coeval; (3) ring location, but not necessarily the mode of ring emplacement, reflects one process that operated at the time of impact; and (4) the process yields similarly-disposed topographic features that are spatially discrete at $2^{0.5} \mathrm{D}$ intervals, or some multiple, rather than continuous. These four elements suggest that some type of wave mechanism dominates the location, but not necessarily the formation, of basin rings. The waves may be standing, rather than travelling. The ring topography itself may be emplaced at impact by this and/or other mechanisms and may reflect additional, including post-impact, influences.


## 1. Introduction

The origin of large concentric-ringed impact structures, or 'basins', remains an outstanding unsolved problem in planetology. Radial spacing of the ring diameters, $D$, may provide quantitative photogeologic constraints on geophysical models proposed to explain basins. In this paper, we try to ascertain whether or not the oftcited $2^{0.5} \mathrm{D}$ spacing interval between rings (Fielder, 1963) is valid for terrestrial planets, and hence worthy of consideration as a key ingredient in basin hypotheses.

Impact basins are ubiquitous on solid planets and large satellites throughout the solar system (Hartmann, 1981; Moore et al., 1984). Large multiple-ringed basins (Figures 1-3) are so common on the Moon, Mars, and Mercury that they constitute the broad-scale structural framework of the upper crust on these bodies (Wilhelms and McCauley, 1971; Schultz, 1984; Spudis and Strobell, 1984). It has been suggested, ever since basin ring diameters were first measured on the Moon (Hartmann and Kuiper, 1962; Hartmann and Wood, 1971; Howard et al., 1974), that
radial spacing of basin rings on the three planets is not random (Wood and Head, 1976).

### 1.1. THE PROBLEM

Opinion on the regularity of ring spacing, its relevance to basin origin, and even existence of many of the rings remains polarized. For example, contrast '... reality of the $2^{0.5}$ relation has been and remains obvious ... no one has ever doubted that.' (C. A. Wood, personal communication, dated $1 / 20 / 1986$ ) with 'I am unconvinced ... that more than 2 or 3 rings surround basins on the Moon, Mars, or Mercury'. and '... the putative $2^{0.5}$ ring spacing ...' (H. J. Melosh, personal communication, 11/18/85). The 1980 conference on multi-ring basins (Schultz and Merrill, 1981) did not even address, much less resolve these key issues.

The spacing of basin rings has been cited as evidence for strongly contrasting explanations to the problem of basin formation. A high degree of spatial orderliness, by which successively larger rings increase in D by a nearly constant multiplier, or spacing increment, invariably $2^{0.5} \mathrm{D}$ or 2 D , is claimed by some to support certain physical models for ring origin (Baldwin, 1963, 1974; Fielder, 1963; Van Dorn, 1968, 1969; Chadderton et al., 1969; Floran and Dence, 1976; Murray, 1980; Pike, 1981; Pike and Spudis, 1984a, b). However, the spacing of rings is not everywhere as systematic and clear-cut as at, say, Orientale on the Moon (Howard et al., 1974; Wilhelms, 1980a; Pike, 1985). Numerical values for the spacing increment and the degree of certainty with which they describe basin rings are both controversial. Accordingly, a low or moderate degree of spatial orderliness among rings also has been invoked in devising other, markedly different, hypotheses to explain ring and basin formation (Head, 1977a; Wilhelms et al., 1977; Hodges and Wilhelms, 1978; Melosh and McKinnon, 1978; Solomon and Head, 1980; McKinnon and Melosh, 1980; Croft, 1981a). We return to differences among these contending models later.

Past numerical treatment of ring spacing has been casual: typically by graphs, histograms, and lists of ratios. This had resulted partly from the small data set, largely from the Moon (e.g., Wilhelms et al., 1977). Past work also failed to recognize major size-dependent differences in ring geometry between some two- and multi-ring basins (Pike, 1983). Finally, earlier work emphasized the central tendency of ring ratios at certain values but ignored their dispersion. Thus, the nonrandomness of ring spacing has never been tested formally, and it has been difficult to judge the reality and strength of any spacing 'law'.

### 1.2. APPROACH AND SCOPE

This paper reexamines the problem of basin ring spacing using abundant new observations and a more rigorous analysis. Recently, the data set has been greatly augmented by the discovery of many old and/or degraded multi-ring basins on both Mars (Schultz and Glicken, 1979) and Mercury (Spudis, 1984). To address the uncertainties of basin-ring spacing we have (1) mapped or remapped almost 500 rings on the Moon, Mercury, and Mars; and (2) analyzed their diameters by three statistical
approaches: rank-grouped correlation, size/rank correlation for individual basins, and ring ratios. Formal statistical tests are carried out in all three analyses.

We have limited this investigation to the three planets that provide the most statistically adequate sample. Moreover, the Moon, Mercury, and Mars may differ sufficiently in crustal properties, thermal history, and to a lesser extent gravity (Carr et al., 1984), that planet-specific differences in ring spacing for terrestrial bodies should become evident. Brief summaries of some results have been published (Pike and Spudis, 1984a, b). What appear to be related types of basins also occur on Earth, Ganymede, Rhea, Tethys, and perhaps Callisto, but they are much less common, and the ring-spacing statistics are commensurately less robust. Preliminary formal study of their ring geometry has been reported elsewhere; first results on spacing intervals are consistent with those found here (Pike, 1985).

Current basin nomenclature is unsatisfactory: (1) The term 'peak-ring basin' (PRB; e.g. Wood and Head, 1976) for two-ring structures (Figure 4; Hartmann and Kuiper, 1962; Stuart-Alexander and Howard, 1970) implies the inner ring is analogous genetically to the central peak found in craters - a contention not yet demonstratcd. (2) 'Central-pcak basins’ (CPB) morphologically resemble large craters more than basins (Figure 5; Stuart-Alexander and Howard, 1970; Hartmann and Wood, 1971; Baldwin, 1974). Moreover, as we show here (see also Pike, 1983), the inner ring is systematically smaller than it is in two-ring basins, so much so that the two cannot be included in the same analysis. In this paper, we refer to a 'two-ring basin' as such and adopt the expressly transitional term 'protobasin' (Pike, 1983) for the large peak-plus-ring structures that are neither strictly craters nor basins. We retain the excellent term 'multi-ring basin' for structures with three or more rings (Figures 1-3; Hartmann and Wood, 1971).

This paper examines all three classes of basins. Two-ring basins and protobasins are particularly important because (1) they are related genetically to the larger, multiring, basins and their attributes surely bear on the problem of spacing between multiple rings (Pike, 1982, 1983, 1985), and (2) the existence and dimensions of their rings are more certain and widely accepted than are those of some of the multi-ring basins listed here.

## 2. Recognizing and Mapping Basin Rings

Concentric geologic structures and the impact process were first linked causally on Earth (Boon and Albritton, 1936). Only later were multi-ring and then two-ring basins recognized on the Moon (Figure 1) as large, concentric, and circular patterns that appeared to be loci of the dark, low-lying (and presumably volcanic) maria (Dietz, 1946; Baldwin, 1949, 1963; Hartmann and Kuiper, 1962). The process by which these broad-scale, fundamental structures were interpreted from the confusing detail of planetary surfaces has been described by Hartmann (1981). Progressively more degraded basins were recognized on the Moon as its geologic mapping continued. Interpretation has matured to the point where ringed lunar structures that


Fig. 1a. Six-ring interpretation of the Orientale basin, Moon, after Hartmann and Kuiper (1962). Outermost ring, about 1900 km in diameter (Table I), depicted only where photogcologic clements can be recognized. Stereographic-projection base map by U.S. Geological Survey.
are all but completely obliterated (e.g. the two- or three-ringed Al-Khwarizmi-King basin, not included here) have been recognized and mapped (El-Baz, 1972; Wilhelms and El-Baz, 1977). The total of 'known' lunar basins now stands at 30 to 40 (Wilhelms, 1980b, 1984), and a dozen more are 'possible' (Wilhelms, 1984).

It was helpful that the first extraterrestrial body studied geologically, the Moon, has many basins in various degrees of degradation, for it was the experience of lunar mapping that enabled similar structures to be quickly identified on more distant planets (Figures 2, 3; e.g. Wilhelms, 1973; Murray et al., 1974). Recently, more than


Fig. 1b. Eastern part of the Orientale basin region, showing evidence for its two most cryptic rings (see also Hartmann and Kuiper, (1962, Plate 12.42)). Main ring (Cordillera, rank IV) is 930 km in diameter (Table I). Ring just exterior to Cordillera indicated as rank V. Outermost Orientale ring (rank VI) delineated by massifs, arcuate topographic highs, and scarp-like features. North at top; portion of Lunar Orbiter IV image 167M.

20 old multi-ring basins have been discovered on both Mars (Schultz and Glicken, 1979; Schultz et al., 1982; Stam et al., 1984) and Mercury (Spudis, 1984; Spudis and Prosser, 1984). Many of these features are as highly degraded as the most obscure lunar basin. Recognition and delimitation of old multi-ring basins, specifically deciding on ring positions and dimensions, require detailed study and terrain mapping of the sort carried out by Schultz and Glicken (1979) and Schultz et al. (1982). The techniques are illustrated here by three large basins, on the Moon, Mercury, and Mars, whose preservation states range from obvious to cryptic. The following description of the mapping methods will also illustrate some of the uncertainties inherent in the data for multi-ring basins (Table I). Two-ring structures, a much simpler case (Table II), are discussed separately in Section 2.5.

### 2.1. ORIENTALE BASIN, MOON

Orientale is the youngest and freshest multi-ring basin on the Moon (Figure 1). A concentric, multi-ring form for this feature was first described in detail by Hartmann and Kuiper (1962) from rectified Earth-based telescopic photographs. Following acquisition of the Zond and Lunar Orbiter images, the geology of the basin was described repeatedly (e.g. McCauley, 1968, 1977; Hartmann and Wood, 1971; Moore et al., 1974; Head, 1974; Schultz, 1976). Figure 1a shows our 6-ring interpretation of the ringed structure of Orientale. It is based largely on that of Hartmann and Kuiper (1962).

Locations and dimensions of basin rings within the 930 km -diameter Cordillera scarp, the main ring (initially the 'Eichstadt' ring), are well accepted, but rings exterior to it are less so (Table I). The three interior rings are defined clearly by concentric alignments of massifs and scarps. Some irregularities are evident in the circular patterns, but most students of impact basins agree on diameters of the innermost four rings (cf. Head, 1974; Wilhelms et al., 1977; Croft, 1979; Wilhelms, 1984). The smallest and best exposed ring outside the Cordillera scarp (the 'Rocca' ring) was recognized by Hartmann and Kuiper (1962) and is listed by Croft (1979). Some later work (Head, 1974; Moore et al., 1974; Hodges and Wilhelms, 1978; Croft, 1981b; and Wilhelms, 1984) has not addressed the presence of this ring, but the evidence for it (Figure 1b; Hartmann and Kuiper, 1962, Plate 12.42) is compelling.

We have identified components of a sixth, discontinuous, Orientale ring beyond the 'Rocca' ring. Although detected by Gerard Kuiper in Hartmann and Kuiper (1962, Plate 12.42) and mentioned in the text, it was not listed explicitly in their summary table. Our evidence for a sixth ring consists of discontinuous massif elements (Figure 1b), undulating and concentric topographic highs that are buried by Orientale ejecta at about $-47^{\circ}, 114^{\circ} \mathrm{W}$ (near crater Mendel), and scarp-like terrain. The massif elements shown in Figure 1b do not appear to be associated with other, pre-Orientale features: they are not central peaks or parts of other basin rims.

The contrast in prominence between the crisp inner and vague outer rings of Orientale, the least modified lunar basin, has prompted the generalization (e.g.,
TABLE I
Ring diameters, ranks, and weights for 67 multi-ring basins ${ }^{\text {a }}$

| Basin ${ }^{\text {b }}$ | Center <br> Coordinates <br> (deg) | Mean observed ring diameters (km) ${ }^{\text {c, d }}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | "B" | "A" | I | II | III | IV | V | VI | VII |
| Moon |  |  |  |  |  |  |  |  |  |  |
| Orientale | -19, 95 W | - | - | 320 | 480 | 620 | 930 | 1300 | (1900) | - |
| Imbrium | +35, 17 W | - | - | - | 550 | 790 | 1160 | (1700) | (2250) | [3200] |
| Nectaris | -16, 34 E | - | 240 | - | 400 | 620 | 860 | (1320) | - | - |
| Moscoviense | +26,148 E | - | - | [140] | 220 | [300] | 420 | (630) | - | - |
| Mendel-Rydjerg | -50, 94 W | - | - | - | [200] | (300) | 420 | 630 | - | - |
| Hertzsprung | +02, 128 W | - | [150] | - | 255 | (380) | 570 | - | - | - |
| Humorum | -24, 39 W | - | - | - | [210] | [340] | 425 | (570) | 800 | (1195) |
| Smythii | $-02,87 \mathrm{E}$ | - | - | (260) | 370 | 540 | 740 | [1130] | - | - |
| Crisium | +18, 59 E | - | - | - | - | 360 | 540 | 740 | 1080 | (1600) |
| Grimaldi | -66, 68 W | - | - | - | 230 | 300 | 440 | - | - | - |
| Humboldtianum | +59, 82 E | - | - | (250) | 340 | 460 | 650 | 1050 | (1350) |  |
| Coulomb-Sarton | +52,123 W | - | - | (160) | (250) | - | (440) | (670) | - | - |
| Serenitatis South | +26, 18 E | - | - | - | 410 | 620 | 920 | 1300 | (1880) | - |
| Korolev | -04, 158 W | - | - | -- | 220 | - | 440 | 590 | [810] | - |
| Ingenii | -43,165 E | - | - | - | [165] | - | 315 | 450 | [660] | - |
| Apollo | -36, 151 W | - | - | - | 240 | - | 480 | (720) | - | - |
| Balmer | $-15,70 \mathrm{E}$ | - | - | - | - | 260 | 400 | (580) | - | - |
| Keeler-Heaviside | -10, 162 E | - | - | - | - | (325) | 500 | 750 | [1000] | - |

Table I (continued)

| Mercury |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basin ${ }^{\text {b }}$ | Center <br> Coordinates <br> (deg) | Mean observed ring diameters (km) ${ }^{\text {c }, ~ d ~}$ |  |  |  |  |  |  |  |  |
|  |  | "B" | "A" | I | I1 | III | IV | V | VI | VII |
| Borealis | + 73, 53 W | - | - | - | (860) | - | 1530 | [2230] | - | - |
| Gluck-Holbein | +35, 19 W | - | - | - | (240) | - | 500 | - | (950) | - |
| Derzhavin-Sor Juana | +51, 27 W | - | - | - | - | - | 560 | (810) | - | - |
| Sobkou | + 34, 132 W | - | - | - | (490) | - | 850 | [1385] | - | - |
| Brahms-Zola | +59,172 W | - | - | - | 325 | - | 620 | (840) | [1080] | - |
| Donne-Molière | +04, 10 W | - | - | 375 | - | (785) | 1060 | (1500) | - | - |
| Hiroshige-Mahler | $-16,23 \mathrm{~W}$ | - | - | - | 150 | - | 355 | - | (700) | - |
| Mena-Theophanes | -01, 129 W | - | - | (260) | - | [475] | 770 | 1200 | - | -- |
| Tir | +06, 168 W | - | -- | 380 | 660 | 950 | 1250 | - | - | - |
| Budh | +17, 151 W | - | - | - | - | 580 | 850 | (1140) | - | - |
| Ibsen-Petrarch | $-31,30 \mathrm{~W}$ | - | - | - | - | 425 | 640 | (930) | [1175] | - |
| Andal-Coleridge | -43, 49 W | - | - | [420] | (700) | 1030 | 1320 | (1750) | - | - |
| Matisse-Repin | -24, 75 W | - | - | - | (410) | - | 850 | 1250 | [1785] | - |
| Bartok-Ives | -33, 115 W | - | - | (480) | - | 790 | 1175 | [1500] | - | - |
| Hawthorne-Riemenschneider | -56, 105 W | - | - | - | 270 | - | 530 | 780 | 1050 | - |
| Vincente-Yakolev | -52, 162 W | - | - | - | [360] | - | 725 | 950 | [1550] | - |
| Eitoku-Milton | -23, 171 W | - | (290) | - | 590 | (850) | 1180 | - | - | - |
| Sadi-Scopas | -83, 44 W | - | - | 360 | - | (600) | 930 | [1310] | - | - |
| Tolstoy | -16, 164 W | -- | - | - | [260] | 330 | 510 | (720) | - | - |
| Caloris | +30, 195 W | - | - | - | [630] | (900) | 1340 | (2050) | [2700] | [3700] |
| Chong-Gauguin | + 57, 106 W | - | 220 | 350 | - | [580] | (940) | - | - | - |
| Shakespeare | +49, 151 W | - | - | - | [200] | - | 420 | 680 | - | -- |
| Van Eyck | +44, 159 W | - | - | - | 150 | - | 285 | [450] | [520] | - |


| Mars |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basin ${ }^{\text {b }}$ | Center <br> Coordinates <br> (deg) | Mean observed ring diameters (km) ${ }^{\text {c, d }}$ |  |  |  |  |  |  |  |  |
|  |  | "B" | "A" | I | II | III | IV | V | VI | VII |
| near Newcomb crater (1) | -22, 04 W | - | - | (290) | 380 | [580] | 870 | - | - | - |
| Ladon (3) | -18, 29 W | - | - | - | 325 | - | [550] | (760) | 1000 | (1600) |
| near Holden crater (4) | -25, 32 W | - | - | - | 300 | - | 580 | - | - | - |
| Chryse (5) | +24, 45 W | - | - | - | - | 840 | 1320 | 1920 | 2840 | [3940] |
| Mangala (6) | $+00,147 \mathrm{~W}$ |  |  | - | - | (500) | 660 | (1040) |  |  |
| Sirenum (7) | -44, 167 W | - | - | - | - | 320 | 500 | 810 | (1040) | - |
| so. of Hephaestus Fossae 1 (9) | -30, 180 W | - | - | [180] | - | 290 | 440 | - | - | (1540) |
| Al Qahira (10) | -20, 190 W | (125) | (210) | - | 340 | (490) | 720 | 1140 | - | - |
| so. of Hephaestus Fossae II (11) | -10, 233 W | - | - | (370) | (570) | - | 1040 | - | - | - |
| southeast of Hellas (12) | -58, 275 W | - | - | (200) | - | (360) | 550 | - | - | - |
| Nilosyrtis Mensae (13) | +32,281 W | - | - | - | - | - | 370 | - | (720) | - |
| south of Renaudot crater (14) | + 38, 297 W | - | 150 | - | (310) | (440) | 660 | - | - | - |
| south of Lyot crater (15) | +42,322 W | - | - | 145 | 200 | 260 | 400 | (565) | - | - |
| Cassini (16) | +24,331 W | - | - | 150 | 200 | (320) | 430 | - | 950 | - |
| Deuteronilus B (17) | +43,338 W | - | 44 | - | - | 130 | 200 | - | (405) | - |
| Deuteronilus A (18) | +44,342 W | 44 | - | 80 | - | 140 | 220 | [290] | - | - |
| near South crater (19) | -73, 214 W | - | - | - | (385) | - | (740) | 1205 | 1315 | [1890] |
| near Schiaparelli crater (20) | -05, 349 W | - | - | 140 | - | (300) | 420 | 700 | - | (1120) |
| near I.e Verrier crater (21) | -37, 356 W | - | - | [190] | - | - | 480 | [640] | (850) | - |
| Noctis Labyrinthus (Schultz and Glicken, 1979) | - 13, 105 W | - | - | - | (900) | - | (1800) | (2700) | (3900) | [5800] |
| South Polar (Croft, 1979) | -83, 267 W | - | - | - | (425) | (670) | 850 | - | - | - |
| Argyre | -50, 43 W | - | - | - | [410] | 630 | 800 | (1100) | (1360) | (1900) |
| Isidis | +13, 272 W | - | - | - | 700 | - | 1500 | - | 3000 | [3800] |
| Hellas | -41, 295 W | - | - | (840) | (1200) | (1700) | 2200 | 3100 | 4400 | [5500] |
| Huygens | -14,304 W | - | - | - | 240 | 340 | 465 | - | - | - |
| near Cassini (Stam et al., 1984) | +14,324 W | - | - | - | (170) | (250) | 355 | [520] | [680] | (1100) |

## Table I (continued)

${ }^{\text {a }}$ Source: measured or remeasured by authors on spacecraft images, photomosaics, and maps, particularly after lists in Wilhelms (1984, Table 6.4), Schultz et al. (1982, Table 2), and Spudis (1984).
${ }^{b}$ Many names are provisional and do not constitute official nomenclature. Parenthesized identification numbers after Martian basins are those of Schultz et al. (1982, Table 2); their Arcadia (8) and Aram Chaos (2) basins exist, but the rings are too ambiguously defined for satisfactory measurements.
${ }^{c}$ Rings assigned to seven relative radial positions, or ranks, within each basin and two provisional ranks (see text). Weight values: []$=1,()=2$, all others $=3$ (see text), reflect quality of observations.
${ }^{d}$ Italic printed diameters are weighted geometric means (in km ) of two (in one case threc) closcly spaced (split) rings that occupy one rank (see text), as follows:

|  | Mean diameter | Constituent Diameters |
| :---: | :---: | :---: |
| Moon | (1195) | 1120, (1340) |
|  | [810] | [720], (850) |
|  | [660] | (620), [740] |
| Mercury | (810) | (740), 890 |
|  | [1385] | [1280], 1420 |
|  | (785) | 700, (825) |
|  | [1785] | (1550), [1900] |
|  | [1550] | (1250), (1700) |
| Mars | [550] | 490, 570, 600 |
|  | (3940) | (3600), (4300) |
|  | (565) | 510, 620 |
|  | (290) | 280, (312) |
|  | (385) | 360, 415 |
|  | (740) | 685, 800 |

Wilhelms et al., 1977) that rings outside the main rim may never have been as well expressed as those within. (We exclude rings perhaps accentuated by later endogenic development; Schultz, 1979.) Thus, rings of older, more degraded, basins may not always be visible, especially outside the main ring. The implication is that apparently 'missing' rings may well be present at older basins, but are not now recognizable.

### 2.2. Tir basin, mercury

Ancient, multi-ring basins on Mercury have been tabulated by Wood and Head (1976), Schaber et al. (1977), De Hon (1978), Frey and Lowry (1979), Croft (1979), Spudis (1984), and Spudis and Strobell (1984). They are recognized by four criteria: (1) isolated massifs and massif chains in circular patterns; (2) arcuate ridges aligned with massifs; (3) scarps aligned with (1) and (2) above; and (4) anomalously high terrain within the Mercurian inter-crater areas that align with all of the above. Most basins on Mercury are exceedingly degraded, and careful geologic mapping (Spudis and Prosser, 1984), assisted by topographic profiles from Earth-based radar (Spudis and Strobell, 1984), is required to establish their presence. The Mercurian measurements in Table I are largely from recent work by Spudis (1984).

The Tir basin lies within a structural trough associated with the Caloris basin, and is filled with smooth plains materials (Figure $2 \mathrm{a}, \mathrm{b}$ ). Tir is very old, predating the Mercurian intercrater plains, one of the oldest geologic units on the planet (Trask and Guest, 1975). Numerous massifs (Figure 2a, b) align with circular mare-type ridge patterns to outline the ancient structure of Tir. The mapping of basin-associated topographic and structural elements leads to the basin terrain map (Figure 2b). From these observations, rings are fitted that best explain the observed topographic pattern (Figure 2c).

Mapping the Tir basin illustrates several problems associated with determining precise ring diameters. Missing parts of rings (e.g. the northeast sector of the main ring), massif elements that lie outside an otherwise mapped ring (at $-02.5^{\circ}$, $162.5^{\circ} \mathrm{W}$.), and diverging ridge clements (at $+02^{\circ}, 174^{\circ} \mathrm{W}$.) all contributed to errors in ring placement. Thus, our estimates of ring diameters (Table I) may be considered to carry a precision of $\pm \mathrm{ca} .50 \mathrm{~km}$. for diameters on the order of 1000 km . We emphasize that these problems also attend rings even in the freshest basins (e.g. Cordillera ring of Orientale), so that the precision of ring data can be fairly similar from basin to basin.


Fig. 2a. Site of the subdued Tir basin, Mercury. Rim of Caloris basin in upper left. Mariner 10 photomosaic of the Tolstoy (H-8) quadrangle of Mercury. Scene is 2000 km across; north at top.


Fig. 2b. Terrain map of basin-associated elements for the Tir basin of Spudis (1984). Location of multiple rings indicated by massifs, mare-ridge alignments, and topographic highs. Base is part of shaded relied map of Tolstoy (H-8) quadrangle by U.S. Geological Survey.

### 2.3. AL QAHIRA BASIN, MARS

Schultz et al. (1982) first recognized the multi-ringed Al Qahira structure, along with 20 other large but fragmented basins on Mars (Figure 3). It lies within the southern uplands and has been degraded by large superposed impact craters and embayed by plains materials. Numerous large massifs and ridges within overlying smooth plains outline the structure (Figure 3a, b). Al-Qahira basin appears to have partly controlled the direction of subsequent large-scale channels, such as Ma'adim Vallis (Schultz et al., 1982). Our ring interpretation (Figure 3c) is based on independent terrain mapping (Figure 3b), and thus the 112 Martian ring diameters in Table I differ somewhat from the 55 measured initially by Schultz et al. (1982). We also include two very small inner rings not listed by Schultz et al. (1982, Table II). Their omission would not affect the conclusions reached in this paper.


Fig. 2c. Four-ring interpretation of the Tir basin derived from terrain map in (b). Largest ring is 1250 km across (see Table l). Base map same as in (b).

Problems in mapping the Al Qahira basin resemble those encountered on both the Moon and Mercury. Because reliable topographic data are absent, the choice of the most prominent ring must rest on a qualitative, photogeologic assessment. In this case, the largest massifs appear to represent the original basin rim ( 720 km in diameter). Complicating the delineation of basin structure are groups of massifs, located between latitude $-26^{\circ}$ and $-28^{\circ}$ and W . longitude $185^{\circ}$ to $187^{\circ}$, that may be related to another basin just to the south, centered at $-29.5^{\circ}, 180^{\circ} \mathrm{W}$. (Schultz et al., 1982). Unusually prominent massifs at $-21^{\circ}, 187^{\circ} \mathrm{W}$. lie at the intersection of rings from both A1 Qahira and the basin south of it. This topographic enhancement of intersecting rings and intervening troughs on Mars (Wilhelms, 1973), which is well known from lunar work (Wilhelms and McCauley, 1971), also may be observed at overlapping basins on Mercury. We found such enhancement to be an important tool in discovering additional basins.


Fig. 3a. Site of southern two-thirds of the Al Qahira basin, Mars. Large channel at right is Ma'adim Vallis. Viking photomosaic of the MC-23 SE (Aeolis) quadrangle of Mars. Width of scene is about 1300 km ; north at top.

### 2.4. UnCERTAINTIES IN MEASUREMENT OF MULTIPLE RINGS

The three multi-ring basins (Figures 1-3) illustrate various difficulties faced in assembling the sample of ring diameters from photogeologic evidence (Table I). For each of 67 basins we examined the available photographs and photomosaics, performed terrain mapping from the criteria described above, and attempted to fit circles that include most of the concentric features mapped. Because this sequence of steps will not lead to unique results in every case, some of our basin rings and/or their diameters differ from previously published results (cf. Schultz et al., 1982; Croft, 1979; Wilhelms, 1984). Nonetheless, agreement between our estimates and those of others is substantial. For example, of the 55 Martian rings mapped by Schultz et al. (1982), $79 \%$ of our measured diameters lie within $10 \%$ of theirs. The different values for some basins partly reflect our access to more maps in the 1:2 000000 -scale Viking photomosaic series than were available for the earlier study by Schultz et al. (1982). Thus we consider our measurements in Table I to be as accurate as any others, considering the inherent uncertainties.

Three basins in Table I, Nilosyrtis Mensae and that near the crater Holden (Mars) and Derzhavin-Sor Juana (Mercury), have only two observed rings each. We include them because (1) the rings are so large that those basins almost certainly had at least


Fig. 3b. Terrain map of basin-derived features associated with the Al-Qahira basin of Schultz et al. (1982). Massifs and ridges outline the ancient basin rings. Base is part of shaded relief map (MC-23 quadrangle) by U.S. Geological Survey.
three rings initially (true two-ring basins are systematically smaller; see Table II), and (2) the second ring of two of these basins is larger than the main ring, not smaller as is observed for all true two-ring basins.

In some cases, two closely spaced rings or a group of ring arcs share one radial ring position in a basin. These 'split' rings have been described in basins on both the Moon (Wilhelms et al., 1977; Wilhelms, 1980a) and Mars (Schultz et al., 1982). One of the best examples of a split ring is in the Ladon Basin on Mars. Three closely spaced rings (Schultz et al., 1982) make up the main rim of Ladon, as evidenced by topographic data from Earth-based radar (Saunders et al., 1978). The chaotic arrangement of massifs that transect the split rings in many places suggests that the three rings are not separate and distinct. Similar geologic relations are evident within the Martian Argyre and lunar Crisium basins. We have computed the geometric mean for


Fig. 3c. Six-ring interpretation of the Al-Qahira basin derived from terrain map in (b). Largest ring is 1140 km in diameter (see Table I). Base map same as in (b). Evidence for two smallest rings, not mapped by Schultz et al. (1982), appears in (b) from interpretation of physiography in (a).
diameters of broken arcs to assign a single diameter to split rings. This averaging introduces observational bias into the statistical analysis of ring spacing, but it is minimal because split rings are such a small percentage (about $5 \%$ of the rings of just multi-ring basins) of the total sample.

### 2.5. Two-RING basins and protobasins

Concentric structures with fewer than three rings pose greatly reduced problems of recognition and measurement compared to those attending multi-ring basins. Consequently, most two-ring structures in Table II are well known from past work, and their diameters are uncontroversial. We examined on images and maps all tworing basins (Figure 4) and protobasins (Figure 5) listed previously for the Moon, Mercury, and Mars (e.g., Croft, 1979; Wood, 1980; Wilhelms, 1984), checking and
TABLE II
Ring diameters of 97 protobasins and two-ring basins ${ }^{\text {a }}$

| Protobasins |  |  |  |  |  | Two-ring basins |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Name | Center coordinates (deg) | Observed diameter ${ }^{\text {b }}$ |  |  |  | Name | Center coordinates (deg) | Diameter ${ }^{\text {b }}$ |  |  |
|  |  | Rim | Inner <br> ring | Base of central peak | $\mathrm{W}^{\text {c }}$ |  |  | Rim | Inner ring | W |
| Moon |  |  |  |  |  |  |  |  |  |  |
| Antoniadi | $-70,188 \mathrm{E}$ | 135 | 60 | 8 | (3) | Amundsen- |  |  |  |  |
| Bailly | -07, 291 E | 290 | 125 | $-{ }^{\text {d }}$ | (3) | -Ganswindt | $-81,120 \mathrm{E}$ | 355 | 175 | (1) |
| Campbell | +45, 153 E | 230 | 90 | 9 | (2) | Birkhoff | $+59,214 \mathrm{E}$ | 325 | 150 | (1) |
| Compton | +56,105 E | 175 | 75 | 20 | (3) | Lorentz | $+34,263 \mathrm{E}$ | 365 | 170 | (3) |
| D'Alembert | $+51,165 \mathrm{E}$ | 23 ? | 95 | - | (3) | Planck | $-58,135 \mathrm{E}$ | 325 | 160 | (1) |
| Fermi | -20, 123 E | 238 | 90 | 40 | (2) | Poincaré | -57, 164 E | 325 | 160 | (1) |
| Gagarin | $-20,149 \mathrm{E}$ | 272 | 110 | - | (2) | Schiller- |  |  |  |  |
| Hipparchus | $-06,05 \mathrm{E}$ | 151 | 58 | 9 | (1) | Zucchius | $-56,316 \mathrm{E}$ | 335 | 175 | (3) |
| Janssen | -45, 42 E | 180 | 70 | - | (1) | Schrödinger | $-76,134 \mathrm{E}$ | 320 | 150 | (3) |
| Landau | +43,241E | 221 | 90 | - | (1) |  |  |  |  |  |
| Mendeleev | +06, 141E | 365 | 140 | 21 | (2) |  |  |  |  |  |
| Milne | -31, 113E | 262 | 108 | - | (2) |  |  |  |  |  |
| Oppenheimer | -35, 194 E | 215 | 85 | - | (1) |  |  |  |  |  |
| Pasteur | -11, 105 E | 245 | 85 | - | (1) |  |  |  |  |  |
| Schickard | -44, 306 E | 22? | 87 | - | (2) |  |  |  |  |  |
| Schwarzschild | +71, 120 E | 220 | 80 | - | (3) |  |  |  |  |  |
| Zeeman | $+75,225 \mathrm{E}$ | 184 | 74 | - | (1) |  |  |  |  |  |

Table II (continued)

| Mercury |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $-28,20 \mathrm{~W}$ | 72 | 36 | 7 | (2) | Ahmad Baba | +59, 127 W | 132 | 60 | (3) |
| - | +01, 17 W | 76 | 25 | 15 | (3) | Al-Hamadhani | +39, 90 W | 155 | 75 | (3) |
| - | +13, 27 W | 95 | 32 | 15 | (2) | Bach | -69, 103 W | 210 | 103 | (3) |
| Asvaghosa | +11, 21 W | 86 | 29 | 13 | (3) | Botticelli | +64, 110 W | 148 | 70 | (2) |
| Bernini | $-79,135 \mathrm{~W}$ | 165 | 65 | 10 | (3) | Cervantes | $-75,122 \mathrm{~W}$ | 203 | 106 | (3) |
| Boethius | -01, 74 W | 122 | 50 | 12 | (3) | Chekhov | -36, 62 W | 205 | 100 | (3) |
| Brunelleschi ${ }^{\text {e }}$ | -09, 23 W | 128 | 34 | 17 | (3) | Dorsum Schiapa- |  |  |  |  |
| Chong Chol | +47, 116 W | 138 | 58 | 11 | (2) | relli Basin | + 17, 166 W | 225 | 120 | (2) |
| Equiano | -39, 31 W | 105 | 46 | 18 | (2) | Dürer | +22, 119 W | 190 | 95 | (3) |
| Hawthorne | -51, 116 W | 125 | 55 | 25 | (2) | Handel | +04, 34 W | 153 | 68 | (2) |
| Hitomaro | -16, 16 W | 115 | 40 | 14 | (3) | Hitomaro Basin | $-15,15 \mathrm{~W}$ | 255 | 125 | (3) |
| Jo' Kai | +73, 136 W | 86 | 38 | 12 | (3) | Homer | -01, 37 W | 310 | 172 | (2) |
| Lu Hsun | 00, 24 W | 101 | 41 | 22 | (2) | Ma Chih-Yuan | -59, 78 W | 190 | 90 | (3) |
| Mansur | +48, 163 W | 95 | 38 | 12 | (2) | Mark Twain | -11, 138 W | 150 | 80 | (2) |
| Scarlatti | +40, 100 W | 152 | 69 | 6 | (3) | Mendes-Pinto | -61, 20 W | 195 | 100 | (2) |
| Sinan | +16, 30 W | 136 | 60 | 7 | (3) | Michelangelo | -45, 110 W | 225 | 105 | (3) |
| Ts'ai Wen Chi | +23, 23 W | 123 | 54 | 14 | (3) | Mozart | +08, 191 W | 255 | 130 | (3) |
| Van Gogh | -76, 136 W | 98 | 37 | 18 | (2) | No. Pole Basin | +85, 171 W | 170 | 85 | (2) |
| Verdi | +65,169 W | 140 | 50 | 35 | (1) | Pushkin | $-65,24 \mathrm{~W}$ | 235 | 125 | (3) |
| Zeami | -03, 147 W | 130 | 44 | 12 | (1) | Renoir | $-18,52 \mathrm{~W}$ | 225 | 110 | (3) |
|  |  |  |  |  |  | Rodin | $+22,18 \mathrm{~W}$ | 235 | 117 | (3) |
|  |  |  |  |  |  | Shelley-Delacroix Basin | - 48, 136 W | 222 | 105 | (2) |
|  |  |  |  |  |  | Sotatsu | -48, 19 W | 160 | 74 | (2) |
|  |  |  |  |  |  | So. of Molière | +10, 16 W | 295 | 145 | (1) |
|  |  |  |  |  |  | Strindberg | + 54, 134 W | 190 | 90 | (2) |
|  |  |  |  |  |  | Surikov | -37, 125 W | 235 | 110 | (2) |
|  |  |  |  |  |  | Valmiki | -23, 142 W | 210 | 105 | (2) |
|  |  |  |  |  |  | Vivaldi | +14, 86 W | 205 | 100 | (3) |
|  |  |  |  |  |  | Vyasa | +49, 81 W | 280 | 145 | (1) |
|  |  |  |  |  |  | Wang Meng | $+09,104 \mathrm{~W}$ | 165 | 80 | (3) |
|  |  |  |  |  |  | Wren | +25, 36 W | 208 | 120 | (2) |
|  |  |  |  |  |  |  | $+48,169 \mathrm{~W}$ | 175 | 80 | (2) |

Table II (continued)

| Mars |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22-Gt | -09, 237 W | 64 | 26 | 11 | (3) | $5-\mathrm{Kd}$ | +35,325 W | 96 | 46 | (3) |
| Bakhuysen | - 22, 344 W | 153 | 60 | 32 | (2) | $12-\mathrm{Xq}$ | +16,355 W | 52 | 24 | (2) |
| Barnard | -61,298 W | 123 | 39 | 24 | (3) | $16-\mathrm{Re}$ | -25, 164 W | 151 | 68 | (3) |
| Bjerknes | -43, 189 W | 90 | 35 | 17 | (2) | $16-\mathrm{Qg}$ | -23, 163 W | 57 | 27 | (1) |
| Holden | $-26,34 \mathrm{~W}$ | 147 | 61 | 20 | (1) | 20-Yp | - 13, 357 W | 70 | 38 | (3) |
| Liu-Hsin | -55, 172 W | 135 | 51 | 23 | (3) | 22-Dt | -09, 231 W | 90 | 41 | (2) |
| Trouvelot | +16, 13 W | 148 | 58 | 18 | (1) | Arrhenius | -40, 237 W | 122 | 70 | (2) |
|  |  |  |  |  |  | Galle | -51, 31 W | 214 | 104 | (2) |
|  |  |  |  |  |  | Kaiser | -46, 341 W | 204 | 100 | (1) |
| Herschel ${ }^{\text {l }}$ | -15,230 W | 301 | 145 | 40 | (2) | Kepler | -47, 219 W | 224 | 122 | (3) |
| Lyot ${ }^{\text {f }}$ | +51,331 W | 224 | 113 | 27 | (2) | Lowell | -52, 81 W | 198 | 94 | (3) |
|  |  |  |  |  |  | Molesworth | $-28,211 \mathrm{~W}$ | 179 | 91 | (3) |
|  |  |  |  |  |  | Phillips | -67, 45 W | 175 | 90 | (2) |
|  |  |  |  |  |  | Ptolemaeus | -47, 158 W | 167 | 78 | (3) |
|  |  |  |  |  |  | Schiaparelli | -03, 343 W | 442 | 225 | (3) |

[^0]

Fig. 4. The two-ring basin (alternatively 'peak-ring basin') Bach, 210 km across at $-69^{\circ}, 103^{\circ} \mathrm{W}$, on Mariner-10 photomosaic 15-C (Davies et al., 1976), of the Bach area of Mercury (see also Spudis and Prosser, 1984). Similar two-ring basins are observed elsewhere on Mercury and on the Moon and Mars (Table II). North at top.
remeasuring the diameters of both rim and inner ring (Table II). Ring diameters were easier to define accurately and precisely (generally less than about $\pm 5 \% \mathrm{D}$ ) than were those of multi-ring basins, because most two-ring structures are so much smaller, more continuous, and (usually) younger and less degraded. Inner rings of protobasins such as Van Gogh on Mercury (Figure 5), may be incomplete and thus difficult to measure accurately. Their central peaks are unusually small (Pike, 1982, 1983), and can be absent entirely (Table II; Pike, 1983). The inner ring of the two-ring basin Bach, on Mercury (Figure 4), is complete and exceptionally conspicuous, which is not always the case (cf. the Moon; Wilhelms, 1984). As more and better source materials have become available, especially for Mars, we discarded some structures as lacking definable and measurable inner rings (e.g., Mie, Moreaux, and Gale) and added others for which good data could be obtained unambiguously (e.g., Bjerknes, Trouvelot, and 22-Dt). Similarly, at least three rings now are mapped for the formerly two-ring lunar basins Korolev, Apollo, and Balmer (Table I).

## 3. Ring Observations and Their Weighting

The raw data are average diameters of 490 rings mapped by the foregoing procedures for 164 basins and protobasins on the Moon, Mars, and Mercury. Protobasins


Fig. 5. Two protobasins (alternatively 'central-peak basins'), large impact craters that have a small central peak and a small inner ring. Van Gogh, 98 km across at $-76^{\circ}, 136^{\circ} \mathrm{W}$, and Bernini, 165 km across at $-79^{\circ}, 135^{\circ} \mathrm{W}$, both on Mariner-10 photomosaic 15-B (Davies et al., 1976), of the Bach area of Mercury. Similar protobasins are observed elsewhere on Mercury (e.g., De Hon et al., 1981) and on the Moon and Mars (Table II). North at top.
( $n=44$ ) and two-ring basins ( $n=53$ ) account for 194 rings (Table II). The 67 multiring basins share the remaining 296 rings (Table I). Ring ranks, radial positions of rings within each multi-ring basin, are explained below. Each ring diameter (inner and outer rings together for two-ring basins) also is assigned a relative weight of 1 , 2 , or 3 for subsequent analysis. The weighting value reflects both (1) likelihood that the ring exists, and (2) accuracy with which the ring could be measured, and thus it roughly indicates the ring's physiographic prominence. Some low weights simply reflect poor photographs and images. The diameters of most of the doubtful, or 'weakest', rings in Table I (weight $=1$ ) are bracketed, and those of only 'moderately strong' rings (weight 2) are parenthesized; the remainder are diameters of the
'strongest' rings, in which we have the most confidence (weight 3). Weights are listed separately in Table II. The relative weights are roughly comparable from planet to planet.

Fourteen ring positions in Table I contain two and in one case (Mars) three split segments, whose individual diameters are given in a footnote. The presence of split rings reduces 296 mapped rings to 281 ranked rings (We recognized split rings before the ranking step). Geometric mean diameters of split rings were calculated from the constituent diameters using the 1-2-3 weights. In subsequent calculations, split basin rings are assigned the weight of the weaker constituent. The weighting is purposely conservative to reflect the uncertainties inherent in measuring split rings and partial arcs.

## 4. Multiple-Ring Position and Ring Rank

The rings of two-ring basins and protobasins clearly are either 'outer' (main) or 'inner'. This lack of ambiguity does not extend to multi-ring basins. Therefore, rings of each multi-ring basin are assigned a numerical rank that corresponds to their relative radial position (Pike, 1981; Pike and Spudis, 1984a, b). Ranking enables the rings of many basins, including those with only two rings, to be combined in testing for statistical and spatial regularities (Wilhelms et al., 1977; Croft, 1981b; Schultz et al., 1982). Its effects are twofold: (1) Ranking facilitates comparisons among basins, by establishing a simple geometric correspondence of rings referenced to the most important ring in each basin. (2) Ranking surmounts the problem of possibly 'missing' rings, which severely complicates any type of statistical analysis of ring spacing (see discussion in Sections 5.1.2 and 5.1.3.), by establishing for each basin a similar suite of ring positions, which may or may not be 'filled' by observed features.

### 4.1. RATIONALE AND PROCEDURE

The basis for ranking the rings of multi-ring basins (Pike, 1981) derives from four observations, first made on lunar data: (1) adjacent rings of the fresh Orientale basin and the older basins Nectaris, Humorum, and Imbrium are spaced almost exactly at $2.0^{0.5} \mathrm{D}$ (Fielder, 1963; Van Dorn, 1968); (2) many rings of other, less well preserved, basins on the Moon have approximately this same spacing (Hartmann and Kuiper, 1962); (3) non- $2^{0.5} \mathrm{D}$ spacing intervals, commonly 2.0 D , tend to be integer multiples of $2^{0.5}$ D (Hartmann and Kuiper, 1962; Hartmann and Wood, 1971), rather than some unrelated value; and (4) one ring in a basin tends to be more prominent than the others (Hartmann and Wood, 1971).

The 281 ranked rings of the 67 multi-ring basins belong to at least seven welldefined groups of ring diameters (Figures 6-8, Table I). Spacing between adjacent ring groups increases incrementally outward by about $2^{0.5} \mathrm{D}$ or an integer multiple of it. This value was first proposed for ring spacing of the four lunar basins measured in Hartmann and Kuiper (1962) by Fielder (1963), in a seldom-cited paper in which
he interpreted basin rings as endogenic. Hartmann and Kuiper (1962) themselves had given the interval virtually the same value, '.. about $1.5 \mathrm{D} \ldots$..

We recognized groups of ring diameters and their constant spacing on separate planets from graphic analysis of combined data for many multi-ring basins (Figures 6-8). Each ring (diameter $=D_{n}$ ) in a basin was assigned a rank number on a common assumption and observation: One ring (diameter $=D_{m}$ ), rather more prominent and/or continuous than the others (see Hartmann and Wood, 1971, Table II and Figure 27 and discussion in Hodges and Wilhelms, 1978, and Schultz et al., 1982), is equivalent to the rim crest of nonringed craters, up to the size of protobasins on that planet. The topographically 'strongest' ring is the 'main ring' or 'topographic rim' (e.g. Cordillera of Orientale). Its selection, made only after all rings of that basin had been mapped and measured, usually was straightforward but can be equivocal in the case of degraded basins.

Plotting ring diameter, $D_{n}$, against that of the main ring, $D_{m}$, for each multi-ring basin, in the $\log D$ domain (after Figure 11 of Wilhelms et al., 1977) separates most of the observed rings into linear concentrations of points (Figures 6a-8a). (Comparison of the diameter of any basin ring with that of the main ring originated with Hartmann and Wood, 1971, Figure 27; the $\log \mathrm{D}$ transformation is required by skewness of both variates toward high values; Baldwin, 1949.) The resulting point clusters are subparallel, slope at about unity, and are spaced at similar intervals, about $2^{0.5} \mathrm{D}$. That the $2^{0.5} \mathrm{D}$ interval is optimum for these data is immediateiy evident by overlaying Figures $6 \mathrm{a}-8 \mathrm{a}$ with different templates of equispaced parallel lines. More points lie close to lines with the $2^{0.5}$ spacing than they do for any other. The linear groups of basin rings are most discrete on the Moon and least discrete on Mars. Our assignments of outliers, ambiguous points between two distinct clusters, to one group or the other were aided by (1) drawing a polygon around each cluster and/or (2) fitting it visually with a least-squares line. The number of points in each cluster ranges from one to 18 .

### 4.2. Resulting ranks of multiple rings

The $D_{n} / D_{m}$ plots in Figures $6 \mathrm{a}-8 \mathrm{a}$ reveal six principal groups of rings for multi-ring basins on each planet. Probably one (all three bodies) and perhaps two (Mars only) more small ( $n=2$ or 3 ) groups lie at the lower end of the diameter range (columns ' $A$ ' and ' $B$ ' in Table I). A seventh principal cluster, which falls in the middle of the other six and thus occupies the fourth position, is simply the strongest ring plotted against itself. The resulting three sets of plots constitute empirical statistical models for average basin-ring spacing on the Moon, Mercury, and Mars. The three patterns are similar, including the observation that the uppermost ring group is the weakest.

The seven $D_{n} / D_{m}$ distributions of basin rings in Figures 6a-8a are numbered $\mathrm{I}-$ VII, in order of increasing ring size on the vertical axis, in Figures $6 \mathrm{~b}-8 \mathrm{~b}$. The main ring, or topographic rim ( $D_{m}$ ), thus is IV. The corresponding taxonomy followed here for the six other ring groups is: (I) inner, (II) 'peak' (sic), (III) intermediate, (V) outer-1, (VI) outer-2, and (VII) outer-3. The equivalent ring


Fig. 6a. Ring-diameter correlations for 18 lunar multi-ring basins: unranked observations. Dots: diameter of each ring in a basin, $D_{n}$, plotted against that of the main ring, $D_{m}$ (Table I). Circles: diameters of partial arcs and less-certain rings. Short vertical lines join two closely spaced rings or arcs, presumably split segments of one ring (see text). Discreteness of linear clusters most evident here, least so for Mars. Compare with Figures 7a and 8a.
diameters are $D_{\mathrm{I} . . . \mathrm{vII}}$. This system is provisional in that the possible ring cluster below rank I on all three planets ('A') may eventually warrant the I designation, in which case the main ring would become ring $V$.

Seven $D_{\text {l-vII }} / D_{\text {IV }}$ ring positions are not evident on all planetary bodies. Only abbreviated versions of the same patterns have been identified elsewhere, presumably because the available measurements are far fewer. The five multi-ring basins thus far discovered on the outer-planet satellites Rhea, Tethys, and Ganymede have a total of only 24 rings, which appear to occupy six clusters (Moore et al., 1984). The 26 structural and topographic rings of the six terrestrial impact structures that contain three or more rings belong to just five groups (Pike, 1985).


Fig. 6b. Resultant ring ranks for the Moon. Dots: basin ring diameter, $D_{\mathrm{I} . . . \mathrm{V}_{I I}}(n=85)$, as a function of main-ring diameter, $D_{\text {IV }}$ (Table I), plotted in (a). Shaded pairs of curves enclose $95 \%$ confidence intervals for linear fits to five of the six major clusters of rings (C.I. not plotted for rank VII, where $n<5$; upper dashed line is equation). Analysis of covariance shows that adjacent equations here and in Figures 7 b and 8 b differ significantly at $1 \%$. Least-squares fits, (Table III) slope at about unity and are spaced at about $2^{0.5} \mathrm{D}$ intervals. Intervals III-IV and IV-V are slightly wider here, also on Mercury (Figure 7b) and Mars (Figure 8b). Two points belong to a rank below I (lower dashed line). Partial arcs and less-certain rings (circles) weighted less in correlations. Split segments (joined by short vertical lines, as in a), which occupy the same rank in a basin (see text), have been averaged for least-squares fits. Compare with Figures 7 b and 8 b .

At least five of the ring groups derived in Figures 6-8 may be familiar from previous work, especially on the Moon, although true and detailed 'structural equivalence' (e.g., Croft, 1979, p. 135; Schultz et al., 1982, Figure 10) is less easily established (see discussion in Pike, 1985). Our ranks II-VI are essentially the five clusters of lunar rings evident in Figure 27 of Hartmann and Wood (1971) and in Figure 6 of Howard et al. (1974). The same five ranks also seem to be the classes of


Fig. 7a. Ring-diameter correlations for 23 multi-ring basins on Mcrcury: unranked observations. Dots: diameter of each ring, $D_{n}$, as a function of main-ring diameter, $D_{m}$ (Table I). Circles are partial arcs and less certain rings. Short vertical lines are split rings. Linear ctusters less distinct here than on the Moon, but more so than on Mars. Compare with Figures 6 a and 8 a.
lunar rings identified by Wilhelms et al. (1977, Figure 11), whose broad 'inner ring' group is equivalent to both ranks II and III in our scheme. Ring ranks II, III, and IV correspond, respectively, to the lunar 'central-peak' ring, intermediate ring, and 'outer' ring of Head (1977a). Our ranks I-V also correspond to the six lunar classes of Croft (1981a), his two types of 'intermediate' rings being equivalent to our rank III. The sixth, and especially the seventh, ranks commonly are tenuous and fragmentary outer arcs whose lunar counterparts may be represented by the 'outer ring' of Imbrium (Wilhelms and McCauley, 1971) and our ring VI of Orientale. These two ranks also are less common than most (Croft, 1981a), especially VII on Mercury and the Moon.


Fig. 7b. Resulting ring ranks for Mercury. Dots: basin ring diameter, $D_{\text {1...VII }}(n=92)$, as a function of main-ring diameter, $D_{\mathrm{IV}}$ (Table I), plotted in (a). Shaded pairs of curves enclose $95 \%$ confidence intervals for linear fits to the five major clusters of rings. Least-squares fits, spaced at roughly $2^{0.5} \mathrm{D}$ intervals, are given in Table III. The greater width of intervals III-IV and IV-V is most evident on Mercury. Ring VII is represented by one point only (upper dashed line: not a mathematical fit); two low points belong to a rank $<1$ (lower dashed line). Partial arcs weighted less in fits (see text). Split rings and arcs (see text) averaged for least-squares fits. Compare with Figures 6 b and 8 b .

Three general relations are evident among the 281 ranked rings for the multi-ring basins in Table I: (1) those outside the main ring $(n=93)$ are only $1 / 4$ less frequent than those within ( $n=121$ ); (2) $70 \%$ to $100 \%$ of the rings in ranks V to VII are weighted ' 1 ' or ' 2 ', whereas only $45 \%$ to $60 \%$ of those in ranks I to III carry such low weights; and (3) split exterior rings are much more common ( $11 \%$ ) than split interior rings ( $2 \%$ ). The first observation is particularly important: It suggests that exterior rings are nearly as prevalent as interior ones, despite their weaker


Fig. 8a. Ring-diameter correlations for 26 multi-ring basins on Mars: unranked observations. Dots: ring diameter, $D_{n}$, plotted against main-ring diameter, $D_{m}$ (Table I). Circles are partial arcs and less certain rings. Short vertical lines are split rings; short horizontal lines bracket mean $D_{\mathrm{IV}}$ values derived from two or three closely spaced rings. Linear clusters less discrete here than on the Moon and Mercury. Compare with Figures 6a and 7a.
topographic expression, and thus that any explanation for ring spacing must be equally plausible for rings in both locations. The second relation introduces some measure to the well-known qualitative observation that physiographic prominence of rings decreases with increasing rank. Finally, our data indicate that prevalence of split rings increases with rank, even within interior and exterior groups: from $0 \%$ at ranks $<$ I and I to $2 \%$ at II and III, and from $8 \%$ at rank V to $13 \%$ and $14 \%$ at VI and VII ( $3 \%$ of rank IV rings are split). Thus exterior rings are much more prone to splitting than interior rings, as well as being more vaguely defined. We examine the implications of this relation for process at the close of the paper.


Fig. 8b. Resulting ring ranks for Mars. Dots: basin ring diameter, $D_{\mathrm{I} . . . \mathrm{VII}}(n=119)$, as a function of main-ring diameter, $D_{\text {IV }}$ (Table I), plotted in (a). Shaded pairs of curves enclose $95 \%$ confidence intervals for linear fits to the six major clusters of rings (five low 'orphan' points belong to two ranks $<\mathrm{I}-$ dashed lines). Least-squares fits, spaced at roughly $2^{0.5}$ intervals, are given in Table III (intervals for ranks III-IV and IV-V are wider). Partial arcs weighted less, and split rings were averaged for fits. Compare with Figures 6 b and 7 b . Regressions differ significantly at the $1 \%$ level.

## 5. Statistics of Ring Spacing

This section presents a thorough treatment of ring spacing by simple statistics of central tendency and dispersion, using three different approaches. The parallel analyses are not redundant, but rather extract the maximum amount of information from the large sample of basins and protobasins. We computed mean spacing and its standard deviation and/or $95 \%$ confidence interval for ranked rings arranged in three ways: (1) by groups of similarly ranked diameters correlated with those of basin
main rings, (2) as diameters correlated with rank number for each basin (multi-ring basins only), and (3) as ratios of diameters of adjacent and alternately ranked rings. The resulting values of ring spacing were converted to a common variable, $x$, which enables spacings to be compared directly, regardless of their actual values. Results of the three complementary approaches for both multi-ring and two-ring structures yield a mean increment of ring spacing, $\bar{x}$, of $(2.0 \pm 0.3)^{0.5} \mathrm{D}$, at $95 \%$ C.I. on each planet. Protobasins, however, differ significantly; their $\bar{x}=(3.3 \pm 0.6)^{0.5} \mathrm{D}$ at $95 \%$ C.I.

### 5.1. LEAST-SQUARES GROUPS OF RANKED RINGS

The first analysis of ring spacing follows immediately from the method by which ring ranks were recognized for multi-ring basins (Figures 6-8). It determines the spacing between each ring group and the main basin ring (Table III), and then repeats the calculations for paired adjacent groups of multiple rings (Table IV). A similar treatment of least-squares groups of ring diameters for two-ring basins and protobasins (Figure 10, Table V) is deferred to Sections 5.1.3 and 5.1.4.

### 5.1.1. Adjacent Ring Positions of Multi-Ring Basins

We summarized each $D_{n} / D_{m}$ distribution in Figures 6a-8a statistically and evaluated the dispersion of the data by linear least-squares analysis (minimum $n=3$ ), in the $\log D$ domain, at the $95 \%$ confidence interval in Table III. The C.I. envelopes

TABLE III
Spacing of basin rings referenced to main ring, from linear least-squares fits to diameters of rings grouped by rank: multi-ring basins

| Results of regression analysis ${ }^{\text {a }}$ |  |  | Statistics of ring spacing |  |
| :--- | :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Table III (continued)

| Results of regression analysis ${ }^{\text {a }}$ |  |  |  |  | Statistics of ring spacing |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ring groups $^{\text {b }}$ | $a$ | $r^{\text {b }}$ | $b$, Slope | $\begin{aligned} & a, Y_{c} \text { at } \\ & X=1 \mathrm{~km}, \\ & \mathrm{~km} \end{aligned}$ | $\begin{aligned} & Y_{c}, 95 \% \\ & \text { C.I. at } \\ & \bar{X}_{g}, \mathrm{~km}^{\mathrm{c}} \end{aligned}$ | $Y_{c} / \bar{X}_{g}, 95 \% \text { C.I. }$ |  | Form ${ }^{\text {d }}$ | Observed$x, 95 \% \text { C.I. }{ }^{\mathrm{e}}$ |
|  |  |  |  |  | Mercury |  |  |  |  |
| ' $\mathrm{A}^{\prime} / \mathrm{IV} \mathrm{V}^{\text {f }}$ | 2 | - | - | - | - | 0.250 | 0.24 - | $x / 8$ | 1.8 - |
| I/IV | 7 | 0.81 | $0.798 \pm 0.601$ | 1.437 | 359 ${ }_{-34}+72$ | 0.354 | $0.356_{-0.034}^{+0.037}$ | $x^{0.5 / 4}$ | $2.028_{-0.366}^{+0.448}$ |
| IL/IV | 15 | 0.99 | $1.065 \pm 0.082$ | 0.332 | $334{ }_{-14}^{+15}$ | 0.500 | $0.506 \pm 0.022$ | $x / 4$ | $2.047_{-0.170}^{+0.185}$ |
| III/IV | 12 | 0.99 | $1.142 \pm 0.122$ | 0.261 | $559{ }_{-23}^{+24}$ | 0.707 | $0.676_{-0.028}^{+0.029}$ | $x^{0.5} / 2$ | $1.828_{-0.149}^{+0.161}$ |
| IV/IV | 23 | - | (1.000) | (1.000) | 762 | 1.000 | (1.000) | $x / 2$ | (2.000) |
| V/IV | 18 | 0.98 | $0.934 \pm 0.089$ | 2.235 | $963{ }_{-34}^{+35}$ | 1.414 | $1.458_{-0.051}^{+0.053}$ | $x^{0.5}$ | $2.124_{-0.147}^{+0.157}$ |
| VI/IV | 9 | 0.99 | $1.065 \pm 0.100$ | 1.286 | $1208{ }_{-55}^{+58}$ | 2.000 | $1.955_{-0.090}^{+0.094}$ | $x$ | $1.912_{-0.172}^{+0.189}$ |
| VII/IV ${ }^{\text {f }}$ | 1 | - | - | - | - | 2.828 | 2.8 - | $2 x^{0.5}$ | $1.9-$ |
|  |  |  |  |  | Mars |  |  |  |  |
| ${ }^{\prime} B^{\prime} / / V^{f}$ | 2 | - | - | - | - | 0.177 | 0.19 - | $x^{0.5 / 8}$ | $2.2-$ |
| 'A'/IV | 3 | 0.99 | $1.12 \pm 1.98$ | 0.12 | $89^{+209}$ | 0.250 | $0.24{ }_{-0.17}^{+0.55}$ | $x / 8$ | 1.8 |
| I/IV | 10 | 0.99 | $1.010 \pm 0.072$ | 0.336 | $250 \pm 12$ | 0.354 | $0.360 \pm 0.017$ | $x^{0.5} / 4$ | $2.068_{-0.190}^{+0.209}$ |
| II/IV | 16 | 0.99 | $1.013 \pm 0.076$ | 0.456 | $438{ }_{-18}^{+19}$ | 0.500 | $0.496_{-0.020}^{+0.021}$ | $x / 4$ | $1.967_{-0.156}^{+0.169}$ |
| III/IV | 18 | 0.99 | $1.055 \pm 0.064$ | 0.490 | $464_{-18}^{+19}$ | 0.707 | $0.699_{-0.027}^{+0.028}$ | $x^{0.5} / 2$ | $1.953_{-0.144}^{+0.161}$ |
| IV/IV | 26 | - | (1.000) | (1.000) | 620 | 1.000 | (1.000) | $x / 2$ | (2.000) |
| V/IV | 14 | 0.99 | $0.978 \pm 0.075$ | 1.736 | $1047_{-43}^{+49}$ | 1.414 | $1.505_{-0.061}^{+0.070}$ | $x^{0.5}$ | $2.265_{-0.180}^{+0.216}$ |
| VI/IV | 13 | 0.99 | $1.016 \pm 0.075$ | 1.785 | $1313_{-69}^{173}$ | 2.000 | $1.980_{-0.104}^{10.110}$ | $x$ | $1.961_{-0.200}^{+0.224}$ |
| VII/IV | 10 | 0.98 | $0.924 \pm 0.162$ | 4.655 | $2462{ }_{-244}^{+271}$ | 2.828 | $2.786_{-0.276}^{+0.307}$ | $2 x^{0.5}$ | $1.941_{-0.365}^{+0.451}$ |

${ }^{\text {a }}$ Equations of form $Y=a X^{b}$, where $Y=$ observed $D_{\mathrm{I} . . \mathrm{VII}}, X=$ observed $D_{\mathrm{IV}}, Y_{c}=$ calculated $D_{\mathrm{I} . . . \mathrm{VII}}$, and $\pm$ values of $b$ are at the $95 \%$ confidence interval; values of $Y_{c}$ and $\bar{X}_{g}$ are given here in the alog domain.
${ }^{6}$ Source: calculated by authors from data in Table I only; no two-ring basins or protobasins; provisional ranks ' A ' and ' B ' lie inside rank I. All regressions significant at $1 \%$ level. ${ }^{\text {c }} \bar{X}_{g}=$ geometric mid-point of observed $D_{\mathrm{IV}}$ for each ring group.
${ }^{d}$ Expresses spacing between each equation and that for the main ring (IV/IV) in terms of adjacent $2^{0.5}$ groups, through a variable $x$ raised to power of the constant 0.5 .
${ }^{\mathrm{e}} x=\left(\operatorname{alog}\left[\log \left(Y_{c} / \bar{X}_{g}\right)+(5-R) \log 2^{0.5}\right]\right)^{2}$, where $R=$ rank of rings for which $Y_{c}$ is calculated; $\pm x$ values are at the $95 \%$ confidence interval; model $x$ for all ring groups $=2.000$ (Fielder, 1963).
${ }^{\mathrm{f}}$ No least-squares fit; $Y_{c} / \bar{X}_{g}$ and $x$ are geometric means or single values only.
(for $n \geq 5$ ) are shown in Figures $6 b-8 b$, where the inverse correlation between sample size and dispersion of the $D_{\mathrm{I}-\mathrm{VII}} / D_{\mathrm{IV}}$ data in each group of rings is clearly evident. Rings are not evenly distributed across all seven ranks, but range from 5 to 18 (excluding all ranks IV and rank VII on Mercury and the Moon). All ranks on the three planets are not equally robust. A few rings clearly fall between the betterdefined trends, and in 14 instances two (in one case three) closely spaced ('split') rings from the same basin occupy a position normally filled by only one ring. Slopes of the 18 non- $D_{\text {IV }} / D_{\text {IV }}$ distributions in Table III range from 0.80 to 1.14 , and average $1.00 \pm 0.15\left(\mathrm{n}=207\right.$ rings, statistics weighted by number of rings per $D_{\mathrm{I}-\mathrm{VII}} / D_{\mathrm{IV}}$ distribution). The mean slope of unity was expected from past study of the dimensions of impact craters and other geometrically-similar landforms (Baldwin, 1963; Wilhelms et al., 1977; Pike, 1982). All regressions in Table III are significant at the $1 \%$ confidence level ( $n \geq 5$ ).

The spacing of multiple rings on the three planets, as derived specifically for grouped data, correspond to those predicted by the $2.0^{0.5} \mathrm{D}$ lunar model (Fielder, 1963). However, the agreement is not perfect, and the scatter evident within the ranked groups in Figures $6-8$ precludes treating the $2^{0.5} \mathrm{D}$ increment as a 'law'. The statistics of the agreement are detailed in Tables III-V and summarized for basins and protobasins on several planets and satellites in Table VI.

Observed departures from the $2^{0.5} \mathrm{D}$ spacing that result from dispersion in the data for the ring groups are evaluated at the center of each least-squares fit. We do this by calculating the ratio $Y_{c} / \bar{X}_{g}$ and then measuring the width of the fitted $95 \%$ confidence interval at $\bar{X}_{g} . Y_{c}$ is the point estimate of $D_{\text {I...VII }}$, here at $\bar{X}_{g} ; \bar{X}_{g}$ is the geometric mid-point of $D_{\text {IV }}$, calculated separately for each $D_{\mathrm{I} . . \mathrm{VIII}} / D_{\mathrm{IV}}$ ring group. Dispersion in the observations from the three planets is sufficiently low that both point and interval (error-bound) estimates of ring diameter from the regression analysis (Tables III, V) are statistically robust for most ranks (Figures $6 \mathrm{~b}-8 \mathrm{~b}$ ). Analysis of covariance reveals that the parallel $D_{\text {I-vII }} / D_{\mathrm{IV}}$ distributions differ significantly from their neighbors at the $1 \%$ level. However, only the point estimates of $Y_{c}$ are satisfactory for some ranks constructed from a small sample (I and VII on the Moon and Mercury).

Departures of the observed $Y_{c} / \tilde{X}_{g}$ (Table III, column 8) from the model ratio, defined as the closest fraction or integer multiple of $2.0^{0.5} \mathrm{D}$ (Table III, column 7), are minor. $Y_{c} / \bar{X}_{g}$ differs with ring rank, but deviations for all ranked ring diameters can be compared directly by expressing $Y_{c} / \bar{X}_{g}$ ratios in terms of a variable, $x$, that is raised to the constant power 0.5 , such that model $x=2.0$ (our convention; alternatively, $x$ could be held constant and the exponent varied). To do this we equated each observed $Y_{c} / \bar{X}_{g}$ with an algebraic expression containing $x$ (under 'Form', column 9 in Table III) so that the model spacing, regardless of ring rank, always is 2.0 . Thus, similarity/difference of any ring rank to/from the hypothesized model is evident immediately by proximity of $x$ (Tables III -V , last columns) to 2.0 . For example, because $D_{\mathrm{IV}} / D_{\mathrm{IV}}=1.0$, the expression required to make $x=2.0$ is $x / 2$. For $D_{\mathrm{V}} / D_{\mathrm{IV}}$, which the model predicts to be 1.414 (i.e. $2.0^{0.5}$ ), the
commensurate expression is $x^{0.5}$; the observed 1.505 value for the $Y_{c} / \bar{X}_{g}$ ratio of rings V/IV on Mars, for which $x=2.3$, in Table III exceeds the hypothesized spacing.

We similarly analyzed pairs of adjacent-ring groups (Table IV) to eliminate bias introduced by correlation with rank IV. This alternative has the advantage of yielding $x$ without the different algebraic conversions required in Table III (column 9), but it does not preserve the accompanying confidence interval. Instead, the standard deviation has been calculated for averaged $x$ values in Table VI. The spacings in Table IV were derived from the results of least-squares regression in Table III: Values of $x$ in Table IV were extracted from observed ratios of $\left(Y_{c} / \bar{X}_{g}\right)$ values for adjacent ring ranks in Table III. Comparison of results from the two analytic modes in Table VI shows them to be similar.

Valucs of $x$ corrcsponding to the 22 non- $D_{\mathrm{IV}} / D_{\mathrm{IV}}$ measured $Y_{c} / \bar{X}_{g}$ ratios of ring groups on the three planets vary from 1.8 to 2.4 in Table III and from 1.6 to 2.3 in Table IV. (Comparable terrestrial values are 1.8 to 2.1 ; Pike, 1985). Mean $x$ for all three planets (Table III results) is $2.011+0.195,-0.176,95 \%$ C.I. The average departures of the observed $x$ from 2.0 in Table III are roughly comparable for the Moon (0.012), Mercury ( 0.002 ), and Mars ( 0.019 ); values for Earth and the outer planet satellites, derived from fewer data, are expectedly high: 0.070 and 0.090 , respectively.

Differences between observed $Y_{c} / \bar{X}_{g}$ and model ratios of basin rings are both systematic and nonsystematic. The latter are evident in the nonunity slopes of the least-squares fits in Table III and scatter of $D_{\mathrm{I}-\mathrm{VII}} / D_{\mathrm{IV}}$ values above and below the lines, both for each planet and across all three bodies. We ascribe the nonsystematic variations to random causes of little genetic significance: small sample sizes, the


Fig. 9. Agreement of observed with model intervals of spacing for multi-ring basins, from correlation analysis of six ranked ring groups, on the Moon, Mercury, and Mars (Figures 6-8, data from Table III). Averaged spacings of observed ring groups cluster about model value ( $\bar{x}=2.0$ ), and avoid intervals between groups. Rings III and V lie farther from model value than other rings. Best fit to $2.0^{0.5} \mathrm{D}$ model is $x=2.0$; worst fits are extreme values of $x, 1.414$ and 2.828 , which are equivalent to distances midway between 'model' $D_{n} / D_{1 \mathrm{~V}}$ equations spaced exactly at $2.0^{0.5} \mathrm{D}$ intervals. Horizontal axis: Rank-grouped ring diameters paired in least-squares fits (except IV/IV). Vertical axis (left): Intervals of mean radial separation between $D_{\mathrm{I} . . \mathrm{VII}}$ and $D_{\mathrm{IV}}$ observed for seven groups of basin rings on the Moon (dots), Mercury (triangles), and Mars (circles). Values of $\bar{x}$ and $95 \%$ C.I. (error bars) calculated in log domain (see Table III, footnote). Vertical axis (right): Raw-ratio equivalents of $x$ for adjacent ring positions (cf.

TABLE IV
Spacing of adjacent basin rings, from linear least-squares fits to rings grouped by rank: multi-ring basins

| Adjacent | Combined | Ratio of observed | Observed |
| :--- | :--- | :--- | :--- |
| ring | $n^{\mathrm{b}}$ | $Y_{\mathrm{c}} / \bar{X}_{\mathrm{g}}$ for | $x^{\mathrm{d}}$ |
| groups $^{\mathrm{a}}$ |  | adjacent ranks ${ }^{\varepsilon}$ |  |


|  | Moon |  |  |
| :--- | :--- | :---: | :--- |
| 'A', I | 7 | 1.3 | 1.7 |
| I, II | 20 | 1.392 | 1.938 |
| II, III | 29 | 1.387 | 1.923 |
| III, IV | 32 | 1.453 | 2.111 |
| IV, V | 34 | 1.457 | 2.124 |
| V, VI | 25 | 1.362 | 1.854 |
| VI, VII | 12 | 1.4 | 2.1 |

Mercury

| 'A', I | 9 | 1.5 | 2.2 |
| :--- | :--- | :--- | :--- |
| I, II | 22 | 1.421 | 2.019 |
| II, III | 27 | 1.336 | 1.786 |
| III, IV | 35 | 1.479 | 2.188 |
| IV, V | 41 | 1.458 | 2.124 |
| V, VI | 27 | 1.342 | 1.800 |
| VI, VII | 10 | 1.4 | 2.0 |


|  | Mars |  |  |
| :--- | :--- | :--- | :--- |
| 'B', 'A' | 5 | 1.3 | 1.6 |
| 'A', I | 13 | 1.53 | 2.33 |
| I, II | 26 | 1.379 | 1.903 |
| II, III | 34 | 1.409 | 1.985 |
| III, IV | 44 | 1.431 | 2.048 |
| IV, V | 40 | 1.505 | 2.265 |
| V, VI | 27 | 1.316 | 1.731 |
| VI, VII | 23 | 1.407 | 1.980 |

[^1]degraded or otherwise nonoriginal state of many of the basins (including possible topographic reversal on Mars), and various random influences on the ring-forming process. We have identified no planet-specific differences in ring spacing from the data in Tables III and IV, thus confirming the findings of Wood and Head (1976) from a much smaller data set.

There is, however, a curious symmetry about the main ring on all three planets (first identified in Pike and Spudis, 1984b). Ring groups III and V both lie farther from ring IV, and commensurately closer to rings II and VI respectively, than the $2.0^{0.5} \mathrm{D}$ interval. Values of $x$ in Table III range from 1.83 to 1.95 for rank III and from 2.12 to 2.27 for rank V. The displacement of rings III and V shows up even more clearly in the results for adjacent groups of paired rings in Table IV, which gives an explicit value for the narrow spacing between rings II and III and between rings V and VI (averaged $x=1.853$, Table VI). This value is masked by the referenced-to-ring-IV technique (Table III).

Figures 6-8 show that the displacement of rings III and V can not have resulted from the few split rings in ranks III ( $2 \%$ ) and V ( $8 \%$ ). The effect, which was evident - albeit unnoticed at the time - for ring V in Figure 27 of Hartmann and Wood (1971), is weaker on the Moon than elsewhere, and it is particularly strong on Mercury. (Also evident on Mercury - but only slightly on Mars and absent on the Moon - is a tendency for ranks III and V to converge around IV with increasing basin size). This important second-order variation in ring spacing does not show up in the meager data thus far available for Earth and the outer-planet satellites.

Overall similarity in the geometry of grouped rings of multi-ring basins on Mars, Mercury, and the Moon enables all 296 rings ( 67 basins) to be combined into a general model for ring spacing on the terrestrial planets (Table VI; Figure 9). We have done this by computing means, standard deviations, and $95 \%$ confidence intervals of $x$ (non- $D_{\mathrm{IV}} / D_{\mathrm{IV}}$ values from Tables III and IV) for the three planets. Statistics for the combined model were weighted by the number of rings for each planet. The calculations are in two sets: Data for rings III and V, which have a slightly different spacing, are separated from those for the other four ranks. (The three values of $x$ for rings III/IV are converted, by [4.0-x], to express their greater distance from ring IV in the same direction as values for V/IV, and thus not cancel out their larger absolute values).

The $2^{0.5} \mathrm{D}$ spacing constant proposed by Fielder (1963) for four lunar basins is at once both confirmed in a much wider application and modified. The results are summarized in Table VI: (1) ring groups ranked as I, II, VI, and VII are spaced from the main ring, rank IV, at average $x$ increments of $(1.990 \pm 0.154)^{0.5} \mathrm{D}$, C.I. $=+0.229-0.203$, or their integer multiples; (2) grouped rings occupying positions III and V are spaced from ring IV at a larger average increment $(2.134 \pm 0.071)^{0.5}$ D, C.I. $=+0.157-0.145$. Interval estimates of $x$ at two sigma (from the $95 \%$ C.I.), 1.781 to 2.219 and 1.989 to 2.291 , respectively, overlap so much that the mean values of $x$ seem to differ little. However, a $t$-test reveals that the means differ at any level of significance. Thus the slightly different geometry of rings III and $V$ differs so systematically from that of other basin rings on all three bodies, that the spacing increment for grouped basin rings on the inner planets could be regarded as a compound, rather than a single, value.

### 5.1.2. Alternate Ring Positions of Multi-Ring Basins

The spacing interval between observed neighboring rings within individual basins commonly is about 0.5 D (or 2.0 D , by reversing order of the rings) rather than $2.0^{0.5} \mathrm{D}$ (Hartmann and Kuiper, 1962). This systematic difference is explained by the recognition, anticipated by Hartmann and Kuiper (1962, Table 2), that adjacent rings mapped for a basin do not always occupy adjacent ring positions, or ranks, in Figures 6-8. For example, the four rings identified at the Eitoku-Milton basin on Mercury (Spudis, 1984) fall into ranks, II, IV, V, and VI, but not III. However, the $2^{0.5} \mathrm{D}$ spacing still describes this basin, and the many analogous cases, because all its ring groups belong to integer multiples of $2^{0.5} \mathrm{D}$. The $\mathrm{D}_{\text {III }}$ position simply is unoccupied, and the adjacent observed rings II and IV are spaced at a multiple or fraction of $2^{0.5}$ D, such as 0.5 D (Hartmann and Wood, 1971). The 'missing' ring III of EitokuMilton evidently did not form or has been buried or obliterated. Analogous 'missing' structural rings in terrestrial meteorite craters may not exist, or they may have formed but have not yet been recognized (Pike, 1985).

### 5.1.3. Two-Ring Basins

The rings of all double-ring basins that lack central peaks also are spaced at an interval of approximately 0.5 D , not $2^{0.5} \mathrm{D}$. According to this observation of Hartmann and Wood (1971) (see also Baldwin, 1963; Howard et al., 1974; and Wood, 1980), which is elaborated in Pike (1985), the inner rings ( $D_{i}$ ) are essentially similar to ring II of multi-ring structures. The resemblence in Figure 10 is supported by the least-squares results (Tables V, VI) from the observations in Table II. $D$ and $D_{i}$ on the Moon, Mercury, and Mars $(n=53)$ are spaced at an average $x$ increment of $(1.948 \pm 0.022)^{0.5}$ D, C.I. $=+0.112-0.104$ (Table VI). Comparable figures for $D_{\mathrm{II}} / D_{\mathrm{IV}}$ (averaged from Table III) are $(1.995 \pm 0.045)^{0.5} \mathrm{D}$, C.I. $=+0.164-0.151$ ( $n=46$ ). A $t$-test reveals that mean $D_{i} / D$ and $D_{\mathrm{II}} / D_{\mathrm{IV}}$, while close, are not statistically similar at the $5 \%$ level of significance. Two-ring basins have slightly smaller inner rings.

From the discussion of alternate ring positions for multi-ring basins, in Section 5.1.2., it follows that the 0.5D spacing of two-ring basins in Figure 10 probably is a variant of the $2^{0.5} \mathrm{D}$-spacing - characteristic of small basins only (Hartmann and Kuiper, 1962; Hartmann and Wood, 1971) - wherein a potential ring position always remains unfilled or a ring's structure is so subdued in amplitude as to be unrecognizable. Thus, the 0.5 D -interval of two-ring basins almost certainly arose similarly to that of larger, multi-ring basins, and is not a separate spacing 'rule' that requires a different formational mechanism. More importantly, the similarity suggests no genetic difference between large and small basins save that of scale (Wood, 1980), a generalization that extends to protobasins (Pike, 1982; 1985), despite their quantitative difference in ring spacing, and large complex craters as well. We emphasize a final, and most important, implication of the close correlation between $D_{i} / D$ and $D_{\mathrm{II}} / D_{\mathrm{IV}}$ in Figure 10: The near similarity argues strongly against the 'intermediate' ring (rank III) of multi-ring basins being the main ring, an


Fig. 10. Ring-diameter correlations for protobasins (circles) and two-ring basins (dots) on the Moon ( $n-17,7$ ), Mercury ( $n-20,31$ ), and Mars ( $n-7,15$ ). Data (Table II) are values of inner ring diameter, $D_{i}$, as a function of main ring (rim) diameter, $D$. The nine paired curves enclose $95 \%$ confidence intervals for linear log-log fits to the three plots of protobasins (lower, shaded), the three plots of two-ring basins (unshaded), and to the three plots of rings II/IV for multi-ring basins (upper, shaded; see Figures 6b-8b). Six dashed lines are linear fits to the data (Table V). Ring geometry of two-ring basins is quite close to that of rings II/IV of multi-ring basins, but differs significantly (at the $1 \%$ confidence level) from that of protobasins.

TABLE V
Spacing between inner $\left(D_{i}\right)$ and outer $(D)$ rings, from linear least-squares fits to ring diameters ${ }^{\text {a }}$

| Results of regression analysis ${ }^{\text {b }}$ |  |  |  |  | Statistics of ring spacing |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & D_{i} / D \\ & \text { ring } \\ & \text { groups } \end{aligned}$ | $n$ | $r$ | $b$, Slope | $\begin{aligned} & a, Y_{c} \text { at } \\ & X=1 \mathrm{~km}, \\ & \mathrm{~km} \end{aligned}$ | $\bar{X}_{g}{ }^{d}$ | $\begin{aligned} & Y_{c}, 95 \% \\ & \text { C.I. at } \\ & \bar{X}_{g}, \mathrm{~km}^{\mathrm{d}} \end{aligned}$ | Observed $\begin{aligned} & Y_{c} / \bar{X}_{g}, \\ & 95 \% \text { C.I. } \end{aligned}$ | Observed $x$ $95 \%$ C.I. ${ }^{\text {f }}$ |
| Moon |  |  |  |  |  |  |  |  |
| 2-Ring basins | 7 | 0.70 | $0.88 \pm 1.04$ | 1.005 | 342 | $165 \pm 9$ | $0.484 \pm 0.026$ | $1.874_{-0.196}^{+0.215}$ |
| Protobasins | 17 | 0.96 | $0.913 \pm 0.141$ | 0.640 | 222 | $88 \pm 3$ | $0.400{ }_{-0.013}^{+0.014}$ | $1.280-0.091$ |

Mercury

| 2-Ring basins | 31 | 0.98 | $1.130 \pm 0.096$ | 0.247 | 202 | $100 \pm 2$ | $0.494 \pm 0.009$ |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| Protobasins | 20 | 0.85 | $1.044 \pm 0.378$ | 0.316 | 109 | $42 \pm 3$ | $0.987_{-0.072}^{+0.030}$ |


| Mars |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2-Ring basins | 15 | 0.99 | $1.018 \pm 0.078$ | 0.454 | 152 | $75 \pm 3$ | $0.496_{-0.020}^{+0.021}$ | $1.968_{-0.155}^{0.170}$ |
| Protobasins | 7 | 0.95 | $0.920 \pm 0.366$ | 0.545 | 99 | $37 \pm 5$ | $0.377_{-0.043}^{+0.049}$ | $1.137_{-0.245}^{+0.315}$ |

a Source: calculated by authors from ring diameters in Table II only; no multi-ring basins; identical calculations (not shown here) carried out on same observations, but in $D / D_{i}$ format, summarized in Table VI.
${ }^{\text {b }}$ Equations of form $Y=a X^{h}$, where $Y=$ obscrved $D_{i}, X=$ obscrved $D, Y_{c}=$ calculated $D_{i}$, and $\perp$ valucs of $b$ are at the $95 \%$ confidence interval; values of $Y_{c}$ and $\bar{X}_{g}$ given here in the alog domain. All regressions except that for lunar 2 -ring basins ( $5 \%$ ) are significant at $1 \%$ level of confidence.
${ }^{\text {c }}$ Protobasins are large craters with an inner ring $\left(D_{i}\right)$ and a central peak; 2-ring basins have no central peak.
${ }^{\text {d }} \bar{X}_{g}=$ geometric mid-point of observed $D$ for each subset of paired rings.
e $Y_{c}$ calculated at $\bar{X}_{g} ;$ model $Y_{c} / \bar{X}_{g}$ for all subsets shown here $=0.500$.
${ }^{f} x=\left(a \log \left[\log \left(Y_{c} / \bar{X}_{g}\right)+3 \log 2^{0.5}\right]\right)^{2}$, where $\pm x$ values are at the $95 \%$ confidence interval; model $\bar{x}=2.000$ (Fielder, 1963).
equivalence that is a key element in some models of ring genesis (e.g., Head, 1974, 1977a).

### 5.1.4. Protobasins

The ring spacing in protobasins, two-ring structures that also have central peaks, clearly differs from that of peakless two-ring basins (Pike, 1982; 1985, Figure 3). The inner rings of protobasins are proportionally smaller. Protobasins have been regarded as small and morphologically immature peak-plus-inner-ring basins, but really are mature 'super-craters'. (It is no accident that Hartmann and Wood, 1971, initially termed them 'central-peak craters', not 'basins'). The difference in ring
spacing between protobasins and mature basins on all three planets is evident in our least-squares results (Figure 10, Tables V, VI) from the measurements in Table II. $D$ and $D_{i}$ of protobasins $(n=44)$ are spaced at an average $x$ increment of only $(1.220 \pm 0.053)^{0.5} \mathrm{D}$, C.I. at $95 \%=+0.188-0.157$, whereas comparable values for two-ring basins are virtually 2.0 (see discussion above and Tables III, V, and VI). Formal statistical tests comparing both $D / D_{i}$ ratios and the regression equations show that protobasins differ significantly from two- and multi-ring basins ( $D_{\mathrm{IV}} / D_{\mathrm{II}}$ ) at any level.

The morphometric contrast between protobasins and mature two-ring basins (Figure 10), which span the size-morphologic transition from the largest complex craters to the smallest multi-ring basins (Stuart-Alexander and Howard, 1970), can be explaincd by presence of the undersized central peak in protobasins: If a peak and an inner ring form in a crater, each is smaller than it would be if the other were absent (Hale and Grieve, 1982; Pike, 1983, 1985). Thus the mechanism(s) of ring formation within two-ring and multi-ring basins may apply equally to protobasins.

### 5.1.5. Summary of Least-Squares Ring Groups

The weighted mean value of $x$ for the 32 non- $D_{\text {IV }} / D_{\text {IV }}$ groups of basin rings on all bodies examined thus far, including Earth and three outer-planet satellites, is essentially 2.0: $1.998 \pm 0.22$ (Table VI). Adding the three groups of two-ring basins (Table V) does not alter these figures: $1.989 \pm 0.20$ C.I. (Table VI). The deviation of rings III and $V$ about ring IV are so symmetric, when expressed in this form, that they sum to zero and hence do not affect $\bar{x}$. Thus the mean spacing increment observed for grouped basin rings, specifically when referenced to the main ring, varies within a fairly restricted range, from $1.8^{0.5} \mathrm{D}$ to $2.2^{0.5} \mathrm{D}$, never far from $2.0^{0.5} \mathrm{D}$. The $95 \%$ confidence interval lies well within one $2.0^{0.5} \mathrm{D}$ interval - the distance separating adjacent ring groups. However, comparable $x$ values for the three groups of protobasins (Table V), $1.220 \pm 0.16$, are so unlike those for basins that the two sets of ring data differ significantly at any level. The disparity indicates that calculations including protobasins with two- and multi-ring basins can yield erroneous conclusions (cf. Wood and Head, 1976, Figure 14; Head, 1977, Figure 5).

### 5.2. AdJacent rings of individual basins

The spacing of multiple rings can be examined basin-by-basin, if enough rings are present. In this section we present statistics of ring spacing for single multi-ring basins on Mars, Mercury, and the Moon (Pike, 1981; Clow and Pike, 1982). Two-ring basins and protobasins are necessarily excluded. Measurements of average ring diameters and their rank are from Table I. Analysis of ring rank:ring diameter for adjacent-ranked rings of individual basins yields an average spacing of $(2.01 \pm 0.26)^{0.5} \mathrm{D}$, in substantial agreement with the results for rings in least-squares groups (Section 5.1).

We estimated a value of the spacing increment, $x$, for each basin from a functional dependence of log ring size ( $Y$ ) upon radial ring position, or rank $(X)$. This relation
was established by a linear least-squares fit for each basin, initially with three or more rings (total $n=64$ basins, 278 rings): on the Moon ( $n=18,85$ ), Mercury ( $n=22,85$ ), and Mars ( $n=24,108$ ). The equations, after Clow and Pike (1982) from Fielder (1963), are of the form $\log D_{n}=\log D_{\mathrm{IV}}+(n-4) \log b$, where $D_{n}$ is diameter of a ring of any rank $n$ and $D_{\mathrm{IV}}$ is diameter of the main ring, both in km. Ring ranks are similarly spaced integers in arbitrary units, and $\log b$ is slope. Diameters for the correlations were weighted 1,2 , or 3 , according to their quality (Table I). Figure 11 shows three examples of the observations and the resulting least-squares lines.

Fits of the equation to each basin yielded statistical estimates of the slope $b$ and diameter of the main basin ring $D_{1 v}{ }^{*}$ (Table VII). The average spacing increment for adjacent (ranked) rings within a basin, $x$, is the square of alog slope, $(\operatorname{alog} b)^{2}$. Rings III and V, which lie farther from ring IV than the nominal 2.0 spacing increment in grouped-data results, above, were not separated from those in other ranks in the analysis of individual basins. (Otherwise, the number of basins remaining with enough rings for the fits would have been too few.) We had anticipated that this forced inclusion of rings III and V (e.g., rings III and V of Al Qahira, Figure 11 and Table I) would yield fits with slopes uniformly greater than $2.0^{0.5}$. However, the average observed excess of slope over $2.000,0.012$, is so much smaller than the accompanying confidence interval that it is not statistically significant. Values of $x$ for individual basins (Table VII) range from 1.913 (Grimaldi) to 2.241 (Balmer) on the Moon, from 1.800 (Bartok-Ives) to 2.282 (Shakespeare) on Mercury, and from 1.768 (Argyre) to 2.270 (Sirenum) on Mars.

Mean values of $x$ for each planet lie close to the model spacing-increment of 2.000 . However, the average dispersion of alog slope, $b$, given here by the mean $95 \%$ confidence interval (C.I.) around $\bar{x}$, is high: Moon, $2.034+0.323-0.299$; Mercury, $2.006+0.667-0.579$; and Mars, $2.004+0.523-0.449$. We examined C.I. values as a function of the number of rings in each basin, and found an inverse, strongly nonlinear, dependency (Figure 12). The relation between sample size and dispersion of slope is approximately linear for basins with 4 to 7 rings, but three-ring

TABLE VI
Mean spacing increment, $\bar{x}$, for grouped basin rings analyzed by least squares ${ }^{\text {a }}$

| Planet or | Referenced to main ring (rank IV) |  |  | Adjacent ring ranks |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (and multi-ring groups) | Number <br> of rings (no. IV) ${ }^{\text {b }}$ | Number of ring groups ${ }^{\text {© }}$ | $\bar{x}$ <br> and $95 \% \text { C.1. }{ }^{\mathrm{d}}$ | $\begin{aligned} & \text { Com- } \\ & \text { bined } \\ & n^{\mathrm{e}} \end{aligned}$ | $\bar{x}$ and $s^{\text {f }}$ |


|  | Two rings $\left(D_{i} / D\right):$ Moon + Mercury + Mars (Tablc V) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2-ring basins | $53 / 53$ | 3 | $1.946_{-0.112}^{+0.120 \mathrm{~g}}$ | 106 | $1.946 \pm 0.029$ |
| Protobasins | $44 / 44$ | 3 | $1.220_{-0.147}^{+0.179} \mathrm{~g}$ | 88 | $1.220 \pm 0.053$ |


| Planet or satellite <br> (and multi-ring groups) | Referenced to main ring (rank IV) |  |  | Adjacent ring ranks |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number <br> of rings <br> (no. IV) ${ }^{\text {b }}$ | Number of ring groups ${ }^{\text {c }}$ | $\bar{x}$ <br> and $95 \%$ C.I. ${ }^{\text {d }}$ | Com- <br> bined $n^{\mathrm{e}}$ | $\bar{x}$ and $s^{f}$ |
| Three or more rings (Tables III, IV) |  |  |  |  |  |
| Moon (all ring groups) | 64/59 | 7 | $2.012_{-0.138}^{+0.151}$ | 159 | $1.996 \pm 0.120$ |
| Mercury (all groups) | 64/61 | 7 | $1.998_{-0.182}^{+0.203}$ | 171 | $2.015 \pm 0.162$ |
| Mars (all groups) | 86/81 | 8 | $2.019^{+0.224}{ }_{-0.197}$ | 212 | $2.020 \pm 0.185$ |
| Farth (all groups) ${ }^{\text {h }}$ | 18/18 | 4 | $1.93{ }_{-0.30}^{+0.37}$ | 42 | $1.98 \pm 0.12$ |
| Outer Satellites (all groups) ${ }^{\text {i }}$ | 19/14 | 6 | $1.91{ }_{-0.50}^{+0.75}$ | 44 | $1.96 \pm 0.14$ |
| $\begin{aligned} & \text { Moon }+ \text { Mercury }+ \text { Mars: } \\ & \text { (all ring groups) } \end{aligned}$ | 214/201 | 22 | $2.011_{-0.176}^{+0.195}$ | 542 | $2.011 \pm 0.159$ |
| $\begin{aligned} & \text { ‘Nominal’ spacing: } \\ & \text { (no III/IV, V/IV) } \\ & \text { ('A'/I, I/II, VI/VII) } \end{aligned}$ | 122/109 | $10$ | $\begin{gathered} 1.990_{-0.203}^{+0.229} \\ C_{-} \end{gathered}$ | $147$ | $1.991 \pm 0.154$ |
| 'Wide' spacing: (III/IV, V/IV) | 92/92 | 6 | $2.134_{-0.145}^{+0.157}$ | 226 | $2.142 \pm 0.071$ |
| ‘Close’ spacing: (II/III, V/VI) | - | - | - | 169 | $1.853 \pm 0.089$ |

Summary: Two- and muiti-ring basins (above)


[^2]TABLE VII
Least-squares fits to basin-ring rank and diameter ${ }^{\text {a }}$

| Basin ${ }^{\text {b }}$ | Number | $r$ | $\begin{gathered} D_{\mathrm{lV} *} \\ (\mathrm{~km}) \end{gathered}$ | Slope, $b$ |  |  | $5 \%$ C.I. ${ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moon |  |  |  |  |  |  |  |
| Orientale | 6 | 0.999 | 924 | 1.420 | 2.016 | +0.10 | -0.098 |
| Imbrium | 6 | 0.999 | 1137 | 1.427 | 2.036 | +0.09 | -0.091 |
| Nectaris | 5 | 0.994 | 871 | 1.403 | 1.967 | +0.29 | $-0.225$ |
| Moscoviense | 5 | 0.998 | 432 | 1.428 | 2.040 | +0.196 | -0.179 |
| Mendel-Rydberg | 4 | 0.999 | 428 | 1.458 | 2.126 | +0.220 | -0.199 |
| Hertzsprung | 4 | 0.992 | 548 | 1.419 | 2.013 | +0.639 | -0.485 |
| Humorum | 6 | 0.996 | 424 | 1.386 | 1.920 | +0.16 | -0.152 |
| Smythii | 5 | 0.999 | 760 | 1.431 | 2.047 | +0.12 | -0.116 |
| Crisium | 5 | 0.999 | 524 | 1.443 | 2.081 | +0.11 | -0.105 |
| Grimaldi | 3 | 0.995 | 432 | 1.383 | 1.913 | $+2.610$ | -1.104 |
| Humboldtianum | 6 | 0.996 | 684 | 1.420 | 2.016 | +0.179 | -0.164 |
| Coulomb-Sarton | 4 | 0.996 | 461 | 1.406 | 1.977 | +0.420 | -0.347 |
| Serenitatis South | 5 | 0.999 | 895 | 1.462 | 2.137 | +0.11 | -0.105 |
| Korolev | 4 | 0.999 | 429 | 1.388 | 1.927 | +0.15 | -0.143 |
| Ingenii | 4 | 0.999 | 320 | 1.412 | 1.994 | +0.21 | -0.191 |
| Apollo | 3 | 0.999 | 491 | 1.435 | 2.060 | +0.93 | -0.643 |
| Balmer | 3 | 0.999 | 393 | 1.497 | 2.241 | $+1.31$ | -0.831 |
| Keeler-Heaviside | 4 | 0.996 | 493 | 1.476 | 2.179 | $+0.52$ | $-0.420$ |
| Mercury |  |  |  |  |  |  |  |
| Borealis | 3 | 0.997 | 1567 | 1.358 | 1.843 | +1.519 | $-0.833$ |
| Gluck-Holbein | 3 | 0.999 | 487 | 1.411 | 1.990 | +0.929 | -0.633 |
| Sobkou | 3 | 0.984 | 900 | 1.374 | 1.889 | +6.100 | $-1.442$ |
| Brahms-Zola | 4 | 0.999 | 609 | 1.362 | 1.854 | $+0.169$ | -0.155 |
| Donne-Molière | 4 | 0.999 | 1066 | 1.414 | 1.999 | +0.113 | $-0.107$ |
| Hiroshige-Mahler | 3 | 0.997 | 336 | 1.475 | 2.176 | $+2.26$ | -1.110 |
| Mena-Theophanes | 4 | 0.996 | 788 | 1.466 | 2.150 | +0.50 | -0.409 |
| Tir | 4 | 0.988 | 1333 | 1.482 | 2.198 | + 1.02 | -0.699 |
| Budh | 3 | 0.997 | 827 | 1.407 | 1.979 | +1.980 | -0.990 |
| Ibsen-Petrarch | 4 | 0.994 | 624 | 1.430 | 2.044 | +0.56 | -0.442 |
| Andal Coleridge | 5 | 0.986 | 1329 | 1.396 | 1.948 | +0.440 | -0.359 |
| Matisse-Repin | 4 | 0.999 | 857 | 1.448 | 2.096 | $+0.06$ | -0.067 |
| Bartok-Ives | 4 | 0.993 | 1125 | 1.341 | 1.800 | +0.42 | $-0.345$ |
| Hawthorne-Riemenschneider | 4 | 0.999 | 537 | 1.410 | 1.987 | +0.15 | -0.144 |
| Vincente-Yakolev | 4 | 0.992 | 710 | 1.412 | 1.993 | +0.598 | $-0.460$ |
| Eitoku-Milton | 4 | 0.999 | 1189 | 1.420 | 2.017 | +0.08 | -0.085 |
| Sadi-Scopas | 4 | 0.993 | 909 | 1.374 | 1.888 | +0.469 | -0.376 |
| Tolstoy | 4 | 0.993 | 499 | 1.439 | 2.072 | $+0.615$ | $-0.474$ |
| Caloris | 6 | 0.997 | 1330 | 1.438 | 2.067 | +0.17 | -0.158 |
| Chong-Gauguin | 4 | 0.992 | 900 | 1.407 | 1.979 | +0.61 | -0.469 |
| Shakespeare | 3 | 0.996 | 438 | 1.511 | 2.282 | +3.512 | -1.383 |
| Van Eyck | 4 | 0.995 | 286 | 1.372 | 1.884 | $+0.382$ | -0.317 |

Table VII (continued)

| Basin ${ }^{\text {b }}$ | Number | $r$ | $\begin{gathered} D_{\mathrm{IV} *} \\ (\mathrm{~km}) \end{gathered}$ | Slope, b |  |  | $5 \%$ C.I. ${ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mars |  |  |  |  |  |  |  |
| Al Qahira | 6 | 0.994 | 742 | 1.420 | 2.017 | $+0.22$ | -0.199 |
| near Newcomb crater | 4 | 0.996 | 853 | 1.459 | 2.129 | +0.51 | -0.416 |
| Ladon | 5 | 0.994 | 579 | 1.356 | 1.838 | +0.23 | -0.208 |
| Chryse | 5 | 0.999 | 1282 | 1.482 | 2.196 | +0.17 | $-0.162$ |
| Mangala | 3 | 0.988 | 694 | 1.442 | 2.080 | +6.99 | -1.603 |
| Sirenum | 4 | 0.992 | 497 | 1.507 | 2.270 | $+0.85$ | -0.621 |
| so. of Hephaestus Fossae I | 4 | 0.991 | 458 | 1.465 | 2.145 | +0.766 | -0.565 |
| no. of Hephaestus Fossae II | 3 | 0.996 | 1053 | 1.399 | 1.958 | +2.174 | -1.030 |
| southeast of Hellas | 3 | 0.996 | 535 | 1.398 | 1.954 | $+2.37$ | -1.073 |
| south of Renaudot crater | 4 | 0.999 | 651 | 1.446 | 2.090 | +0.110 | -0.104 |
| south of Lyot crater | 5 | 0.996 | 389 | 1.400 | 1.961 | +0.228 | -0.204 |
| Cassini | 5 | 0.998 | 444 | 1.452 | 2.108 | +0.168 | -0.155 |
| Deuteronilus-B | 4 | 0.999 | 194 | 1.451 | 2.105 | $+0.140$ | $-0.131$ |
| Deuteronilus-A | 5 | 0.997 | 207 | 1.370 | 1.876 | +0.173 | -0.158 |
| near South crater | 5 | 0.974 | 774 | 1.359 | 1.846 | +0.55 | -0.426 |
| near Schiaparelli crater | 5 | 0.993 | 428 | 1.434 | 2.056 | +0.338 | -0.291 |
| near La Verrier crater | 4 | 0.999 | 474 | 1.348 | 1.818 | +0.093 | -0.088 |
| Noctis Labyrinthus | 5 | 0.999 | 1851 | 1.451 | 2.105 | +0.103 | -0.098 |
| South Polar | 3 | 0.986 | 872 | 1.405 | 1.975 | $+6.770$ | -1.529 |
| Argyre | 6 | 0.996 | 803 | 1.330 | 1.768 | +0.13 | -0.124 |
| Isidis | 4 | 0.998 | 1451 | 1.422 | 2.022 | +0.28 | -0.252 |
| Hellas | 7 | 0.999 | 2252 | 1.378 | 1.898 | +0.07 | -0.069 |
| Huygens | 3 | 0.999 | 468 | 1.392 | 1.937 | +0.57 | -0.441 |
| near Cassini | 6 | 0.999 | 356 | 1.443 | 2.083 | $+0.10$ | -0.098 |

Summary ${ }^{\text {d }}$

| Moon | $(15$ basins $)$ | 73 | - | - | 1.424 | $2.029+0.220-0.192$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mercury ( | 16 basins $)$ | 67 | - | - | 1.414 | $2.000+0.394-0.312$ |  |
| Mars | $(19$ basins) | 93 | - | - | 1.417 | $2.008+0.260$ | -0.217 |

${ }^{\text {a }}$ From observations in Table I; values of $D_{\mathrm{IV}}$ and $b$ shown here in the alog domain; fits are of form $\log$ $D_{\mathrm{n}}=\operatorname{lng} D_{\mathrm{IV}}+(n-4) \operatorname{lng} b$, where $D_{n}=$ ring diameter at any rank $n, D_{\mathrm{IV}}=$ diameter of main ring (both in km ), and $\log b=$ slope. All regressions significant at $1 \%$ level.
${ }^{b}$ Many names are provisional, and do not constitute official nomenclature.
c $x=(a \log b)^{2}$.
${ }^{d}$ Means weighted by number of rings ( $\geq 4$ ).
basins have a dispersion about four times that expected from an extrapolation from basins with four or more rings. Thus diameter:rank fits to three-ring basins are too poorly constrained to yield robust estimates of the dispersion for slope of the equations (and hence basin-ring spacing). Accordingly, we deleted the 14 three-ring basins and recalculated mean slope and its dispersion for the remaining 50 basins ( 233 rings). The resulting values for $\bar{x}$ and mean C.I., weighted by number of rings per


Fig. 11. Log ring diameter (dots) as a function of ring rank (arbitrary units) for the three basins shown in Figures 1-3 (Table VII): Orientale (Moon), Tir (Mercury), and Al Qahira (Mars). All data from Table I. Solid lines, linear least-squares fits of the form $\log D_{n}=\log D_{\mathrm{IV}}+(n-4) \log b$ (Equation 4 of Fielder, 1963, recast by Clow and Pike, 1982), where $n=$ any ring rank, $D_{\mathrm{IV}}=$ diameter of the main basin rim, and $\log b=$ slope. Ring diameters weighted by quality of observations (see text). Error bars define $95 \%$ confidence interval. Calculated slope, $b$, yields average spacing increment between adjacent rings of each basin. Dashed lines, spacing model, where $b=1.4142(\bar{x}=2.000)$. Observed values of $b$ for the three basins are respectively, $1.42000 .036(x=2.016 \pm 0.101), 1.482 \pm 0.286(x=2.198 \pm 0.862)$ and 1.420 $\pm 0.074(x=2.017 \pm 0.210)$. Table VII gives results for the other 61 basins. Average $x$ for each planet (Table X) lies close to the model value of 2.0 .


Fig. 12. Nonlinear inverse dependency of the $95 \%$ confidence interval, which shows dispersion of alog slope, $b$, for least-squares fits to multi-ring basin ring diameter and ring rank, upon number of rings per basin (Table VII). Diameter/ring fits to three-ring basins are poorly constrained. Dots: Moon, triangles:

Mercury, circles: Mars. Dashes connect geometric means at 0.95 C.I.
basin, are (Table VII): Moon, $2.029+0.220-0.192$ ( $n=15$ basins, 73 rings); Mercury, $2.000+0.394-0.312(n=16,67)$; and Mars, $2.008+0.260-0.217$ ( $n=19,93$ ).

The revised average-spacing increments for single basins are barely changed from the initial ones. However, the values of dispersion are reduced substantially, and approach the intrinsically lower ones derived from analyses of grouped basins in Section 5.1 (Table VI). The weighted mean $\bar{x}$ and C.I. values for all three bodies from Table VII are $2.012+0.286-0.236$. This observed $95 \% \%$ C.I., 1.776 to 2.298 , lies well within one $2.0^{0.5} \mathrm{D}$ interval, 1.414 to 2.829 (Defined by distances midway between $\log x$ model ring-spacing values $1.0,1.414,2.0,2.828$, etc.). Whether the observed mean $95 \%$ C.I. - although narrow - is small enough to differ significantly from one that might arise from random processes (cf. Clow and Pike, 1982) remains to be tested (as in the next section). However, this is unnecessary. A $t$-test for the correspondence of $\log b$ and $\log 2.0^{0.5}$ reveals no difference between the two, for all 50 basins, at the $1 \%$ level of significance. Thus, basin rings are spaced at $2.0^{0.5} \mathrm{D}$ intervals.

### 5.3. Ring-Diameter Ratios

Finally, we present an analysis of ratios of ring diameters, the first and mostcommonly applied parameter of ring spacing (Hartmann and Kuiper, 1962; Fielder, 1963; Hartmann and Wood, 1971; Howard et al., 1974; Wood and Head, 1976). Table VIII summarizes the scope of our observations for multi-ring basins from the average ring diameters of 67 basins in Table I. The basic data are 214 ratios of diameters of adjacent observed basin rings, $D_{n} / D_{n-1}$ (where n is ring rank). The rings include both adjacent ranked and alternate ranked rings. The number of ratios for two-ring basins and protobasins, 97 , is the same as the number of structures (Tables II, V).

### 5.3.1. Multi-Ring Basins

First, we examine multi-ring basins (Figure 13). The 214 ratios fall into the two clusters of values found by Hartmann and Kuiper (1962) for multi-ring and doublering basins on the Moon: The first cluster, centered at about 1.4 to 1.5 , usually describes adjacent. ranks ( $n=169$ ); the second, centered at about 2.0 , usually is for alternate ranks $(n=45)$. Data on the second cluster are sparse (three of the 45 ratios span more than one basin-ring rank and hence are omitted from further calculations). To supplement the few observed alternate-rank ratios we generated a second set of 103 'alternate ring-rank' ratios by deliberately skipping one observed ring in calculating each ratio (Table VIII, Figure 13). Additionally, the three subsets of ratios were grouped according to inclusion or exclusion of rings III or V , established in Section 5.1 to be systematically displaced. The total number of possible subsets for multi-ring basins (Table IX) is 30 , six of which are not subdivided according to rings III and V.


Fig. 13. Ratios of diameters ( $D$ ) of observed adjacent rings for multi-ring basins on the three planets (larger $D /$ smaller $D$, from Table I). Most of the 314 observed ratios fall into two groups ( Cable IX) : those for adjacent ranked rings (histograms on left, $n=169$; cf. Figure 9) and those for alternate ranked rings (historgrams on right, $n=145$ ). (Three ratios skip $>1$ ring and are omitted here). Most ratios in each group lie wholly within one of two $2^{0.5}$ D intervals (ef. Figure 9): either 1.189 to 1.682 or 1.682 to 2.378 (upper scale), and cluster around the respective spacing-model values of 1.414 for adjacent ranked rings and 2.000 for alternative ranked rings ( $x=2.0$ in both cases). Two frequency distributions of ratios were drawn at random (RAND Corp., 1955) for the same two $2^{0.5} \mathrm{D}$ intervals, 1.189 to 1.681 and 1.682 to 2.378, to statistically test the groups of observed ratios for nonrandomness (see text and Table IX). (Dispersion of the $n=42$ sample is identical to that of the other, despite the 'peak' at $D_{n} / D_{n-1}=2.05$ ).

Mean and standard deviation were calculated, in the $\log \left(D_{n} / D_{n-1}\right)$ domain (basinring ratios also are skewed strongly toward high values), for each of the 21 groups of observed ratios for multi-ring basins (Table IX). We judged that nine other groups had too few $(<10)$ ratios for stable statistics. For subsequent tests, the same two statistics were calculated for two sets of ratios drawn from a standard table of random numbers (RAND Corp., 1955), one set ( $n=58$ ) corresponding to ring ratios for adjacent ranks ( 1.189 t0 1.682), the other $(n=42$ ) to ratios for alternate ranks ( 1.682 to 2.378 ). (The intervals are defined by distances midway between two log

TABLE VIII
Numbers of multi-ring basins, ring diameters, and their ratios ${ }^{\text {a }}$

| Planet | Multi- <br> ring <br> basins | Rings <br> as <br> mapped | Ring <br> positions <br> ranked | Ratios: <br> rings in <br> adjacent <br> ranks |  |
| :--- | :--- | :--- | :--- | :--- | :--- | | Ratios: rings in alternate <br> ranks |  |
| :--- | :---: |

${ }^{a}$ Raw data from Table I.
${ }^{b}$ See Table X.
c Three ratios omitted (see text).
$D_{n} / D_{n-1}$ ) model ring-spacing values, respectively $1.0,1.414$, and 2.0 and $1.414,2.0$, and 2.828). We transformed all statistics of $D_{n} / D_{n-1}$ ) to the domain of the spacing increment, $x$, so that (1) statistics for both adjacent and alternate ranks are directly comparable (Table IX), and (2) the results are comparable with those from sections 5.1 and 5.2. Calculations were weighted 1 to 5 by combining weights assigned to individual rings in Table I.

The 267 observed ratios ( $x$ equivalents) in the 21 multi-ring basin categories where $n>10$ (Table IX) cluster around the model spacing increment of 2.000 . The combined weighted mean for all three planets is $2.039+0.382-0.284$, close to the values obtained above both for least-squares groups and for ranked rings of individual basins. The net excess of observed $x$ over model $\bar{x},+0.040$, lies well within the accompanying error bounds. It is statistically insignificant. Weighted mean values for individual planets, excluding subsets 'All' (multi-ring plus two-ring basins) and those with $<10$ ratios in Table IX, are $2.091+0.248-0.221$ (Moon, $n=82$ ratios); $1.997+0.319-0,274$ (Mercury, $n=67$ ); and $2.027+0.393-0.333$ (Mars, $n=118$ ).

Differences in mean spacing among 'close', 'wide', and 'neither close nor wide' ring ratios, although not tested for significance, are strong enough to show up clearly in averaged $x$ values of these groups in Table IX: respectively $1.928+0.353-0.305$ ( $n=62$ ratios), $2.117+0.304-0.265(n=92)$, and $1.976+0.314-0.270$ ( $n=91$ ). The mean spacing for ring ratios V/III is, expectedly, much larger than 2.0: $\bar{x}=2.287+0.441-0.364(n=22)$, because both rings are spaced abnormally far from ring IV.
TABLE IX
Ratios of ring diameters: Statistics of central tendency and dispersion for basins and protobasins ${ }^{\text {a }}$

| Groups of ring ratios ${ }^{\text {d }}$ (outer/inner) | Mean, $\pm$ standard deviation ${ }^{b}$, (number of ratios) [results for raw ratios below, and their $x$ equivalents above ${ }^{c}$ ] |  |  |
| :---: | :---: | :---: | :---: |
|  | Moon | Mercury | Mars |
| Adjacent ring ranks |  |  |  |
| All (ignoring rank and sample size, $n$ ) | $\begin{aligned} 2.082 & +0.243-0.218 \\ {[1.443} & +0.082-0.078] \end{aligned}$ | $\begin{gathered} 2.085+0.390-0.328 \\ {[1.444+0.129-0.119]} \end{gathered}$ | $\begin{gather*} 2.056+0.392-0.329(65)  \tag{19}\\ {[1.434+0.131-0.120]} \end{gather*}$ |
| II/I, VII/VI (spacing neither wide nor close) | $-(8)^{\mathrm{f}}$ | $-(4)^{f}$ | $\begin{gathered} 2.017+0.383-0.323(14) \\ {[1.420+0.129-0.119]} \end{gathered}$ |
| [1//II, VI/V close spacing) | $\begin{aligned} & 1.995+0.256-0.225(20)^{\mathrm{e}} \\ & {[1.412+0.088-0.082] } \end{aligned}$ | $\begin{gathered} 1.888+0.352-0.297 \\ {[1.374+0.123-0.133]} \end{gathered}$ | $\begin{gathered} 1.929+0.412-0.371 \\ {[1.389+0.141-0.128]} \end{gathered}$ |
| IV/III, V/IV (widẹ spacing) | $\begin{gathered} 2.116+0.224-0.204(30)^{\mathrm{e}} \\ {[1.455+0.075-0.072]} \end{gathered}$ | $\begin{gather*} 2.097+0.327-0.283(30)^{\mathrm{e}}  \tag{46}\\ {[1.448+0.109-0.101]} \tag{12} \end{gather*}$ | $\begin{aligned} & 2.138+0.356-0.306(32)^{\mathrm{e}} \\ & {[1.462+0.117-0.109]} \end{aligned}$ |
| Randomly-chosen values from 1.189 to 1.682 |  | $\begin{aligned} 2.069 & +0.475 \end{aligned}-0.387(105)$ | $\longrightarrow \longrightarrow \longrightarrow$ |

Table IX (continued)

Table IX (continued)

| Groups of ring ratios ${ }^{d}$ (outer/inner) | Mean, $\pm$ standard deviation ${ }^{\text {b }}$, (number of ratios) [results for raw ratios below, and their $x$ equivalents above ${ }^{c}$ ] |  |  |
| :---: | :---: | :---: | :---: |
|  | Moon | Mercury | Mars |
| Alternate ring ranks: One observed ring skipped |  |  |  |
| IV/II, VI/IV (spacing neither wide nor close) | $\begin{aligned} & 2.044+0.240-0.214(20)^{e} \\ & {[2.022+0.115-0.109]} \end{aligned}$ | $\begin{aligned} & 1.897+0.237-0.211(12)^{\mathrm{e}} \\ & {[1.948+0.118-0.111]} \end{aligned}$ | $\begin{gather*} 1.997+0.345-0.294  \tag{10}\\ {[1.999+0.166-0.153]} \end{gather*}$ |
| III/I, VII/V (close spacing) | $-(7)^{f}$ | $\begin{aligned} & -(3)^{\mathrm{f}} \\ & - \end{aligned}$ | $\begin{gather*} 1.849+0.421-0.342(11)  \tag{19}\\ {[1.923+0.208-0.188]} \end{gather*}$ |
| Rings V/III <br> (doubly wide spacing) | $\begin{aligned} & 2.263+0.306-0.269(12)^{\mathrm{e}} \\ & {[2.127+0.139-0.131]} \end{aligned}$ | $-(9)^{\mathrm{f}}$ | $\begin{gathered} 2.316+0.603-0.478 \\ {[2.152+0.264-0.235]} \end{gathered}$ |
| Summary (adjacent + atternate ring ranks) ${ }^{\mathrm{k}}$ |  |  |  |
| All (ignoring rank, $n$ ) but protobasins | $2.049+0.34-0.29$ (74) | $2.027+0.39-0.33 \quad$ (95) | $2.001+0.39-0.33$ (98) |

[^3]
### 5.3.2. Two-Ring Structures

The 97 ratios for two-ring basins and protobasins fall into two clusters (Figure 14). One, for two-ring basins, is the Hartmann-Kuiper (1962) group (same as that for alternate ranks of multi-ring basins) centered at a $D_{n} / D_{n-1}$ of about 2.0. The other, for protobasins, is recognized here for the first time. It is centered at about 2.5 $D_{n} / D_{n-1}$.

The 53 ratios ( $x$ equivalents) for the three two-ring basin categories (Table IX) cluster around the model spacing increment of 2.000 . The combined weighted mean for all three planets is $2.058+0.258-0.229$, close to the values found above for ranked ring groups, single basins, and basin ring ratios. The close geometric correspondence, which bears out results obtained above from least-squares analysis of ring diameters (Table V ), suggests that two-ring basins differ from their multi-ring counterparts only in size and number of rings.

The 44 ratios ( $x$ equivalents) for the three protobasin categories (Table IX) do not cluster around the model spacing increment of $D_{n} / D_{n-1}=2.000$. Rather, the combined weighted mean for all three planets is $3.253+0.787-0.625$, quite unlike our values obtained for least-squares groups, ranked rings of individual basins, and ring ratios of two- and multi-ring basins. The disparity indicates that protobasins differ from larger basins in geometry. In fact, the difference between ring ratios for protobasins and both two- and multi-ring basins is statistically significant (by a twosided normal test) at the $1 \%$ level. The result supports that obtained above from least-squares analysis of ring diameters (Table V) and the accompanying two implications for protobasins (Sections 5.1.4 and 5.1.5).

### 5.3.3. Statistical Tests

The observed average values of $x$ for two- and multi-ring basins, although close to 2.0, support the spacing model only when a second condition is met: Dispersion about the mean must occupy limits too narrow to have occurred by chance. This requirement is necessary because $\bar{x}$ for randomly chosen ratios within one $2.0^{0.5}$ interval also is 2.0 (Table IX). Accordingly, we compared, by an $F$-test at the $5 \%$ level of significance (Natrella, 1963), the standard deviation, $s$, of each subset in Table IX with a value of $s$ from one of the two sets of randomly generated ratios mentioned above. Calculations were carried out in the $\log \left(D_{n} / D_{n-1}\right)$ domain. Table IX has 23 testable subsets: Those 10 with $\leq 9$ ratios and the three for protobasins were excluded.

All-in-all, the spread of observed basin-ring ratios about the mean is significantly less than that for the randomly drawn ratios, i.e., dispersion that might arise from random processes. This difference, with qualifications noted below, indicates that the $2.0^{0.5} \mathrm{D}$ ring spacing is real.

Results of the $F$-test are systematic, by ring-sample composition, by ring-sample size, and by planet (Table IX). The $2.1^{0.5} \mathrm{D}$ spacing of ring ranks III and V , although differing little from the $2^{0.5} \mathrm{D}$ of other rings, greatly affects test results. For the six subsets that mix rings of all ranks, the observed dispersion is significantly less than


Fig. 14. Ratios of diameters $(D)$ of paired rings for two-ring basins and protobasins on the three planets ( $D / D_{i}$ from Table II). The 97 ratios fall into two groups (Table IX): those for two-ring basins (open histograms, on left, $n=53$ ) and those for protobasins (filled histograms, on right, $n=44$ ). Ratios for tworing basins lie wholly within the same $2^{0.5} \mathrm{D}$ interval, 1.682 to 2.378 , occupied by alternate ranks of multiring bains (cf. Figure 9,13 ), and cluster around the spacing-model value of 2.000 for alternate ranked rings $(\bar{x}=2.0)$. Ratios for protobasins do not occupy a $2^{0.5} \mathrm{D}$ interval that is related to the spacing of two- or multi-ring basins, but rather cluster around a value of ca. $2.6(\bar{x}=3.3)$ that is unique to protobasins and differs significantly from those for two- and multi-ring basins (Table IX). Random distribution is that in

Figure 13.
random dispersion (i.e., a 'pass') for only two. However, for the 17 subsets that are constituted according to presence/absence of multi-ring ranks III and V or contain only two-ring basins, 9 pass. The effect of mixing ring ranks is so marked that $F$-test results omitting these six subsets are included (in brackets) for comparison, below.
Test results are highly sensitive to sample size. Of the 11 [6] subsets having $\geq 20$ ratios, including two that describe two-ring basins, the observed dispersion is significantly less than random dispersion in eight [all 6] cases. Of the 12 [11] subsets containing $\leq 19$ ratios, only three [3] pass. Clearly, when the sample is sufficiently large, dispersion of observed ring ratios is significantly less than that ascribed to chance. Most subsets 'failing' the $F$-test simply are too small - so small that their
difference from random is 'not proven' rather than 'disproven' a critical distinction suggesting that given adequate basin-ring observations, any subset could pass.

Planet-specific effects on our $F$-test results may include overall geologic history, efficacy of surface processes, and quality of spacecraft images. Five [5] out of six [5] subsets of lunar ring ratios pass the test; four [3] of the seven [5] Mercurian subsets pass; only two [1] of the 10 [8] Martian subsets (one of them, at $n=32$, the largest subset tested) pass(es). We conclude that the $2.0^{0.5} \mathrm{D}$ spacing is strongest on the Moon, where basin rings are best exposed and most closely studied; weakest on Mars, where most rings are severely degraded and poorly exposed; and intermediate (but still much weaker than on the Moon) on Mercury, where degradation is slow but where the rings are not now well displayed, and may never have been strongly developed.

In sum, ratios excluding rings ranked III and V , but including two-ring basins, have a mean spacing increment ( $\bar{x}$ ) of about 2.0 (Table IX). Corresponding raw ratios are approximately either 1.4 or 2.0 . Ratios including rings ranked III and V , as for the least-squares groups described in Section 5.1, have a larger increment, $x$

TABLE X
Summary statistics for ring spacing: Moon, Mercury, Mars

| Analytic method and ring groups | Mean spacing increment $\bar{x}$, <br> $95 \%$ confidence interval |
| :--- | :--- |

Least-Squares Groups of Ranked Rings ${ }^{\text {a }}$ :

| All multi-ring groups + two-ring basins | $1.998_{-0.160}^{+0.177}$ |
| :--- | :--- |
| Protobasins $\left(D / D_{i}\right)$ | $3.433_{-0.409}^{+0.478}$ |

Diameter: Rank Correlated for Single Basins ${ }^{\text {b }}$ :
All multi-ring basins with $\geq 4$ rings
$2.012_{-0.236}^{+0.286}$
Ratios of Ring Diameters ${ }^{\text {c }}$ :
$\begin{array}{ll}\text { All two- and multi-ring basins }{ }^{\mathrm{d}} & 2.024_{-0.32}^{+0.38} \\ \text { Protobasins }(D / D) & 3.253^{+0.787}\end{array}$
Protobasins ( $D / D_{i}$ )
$3.253_{-0.625}^{+0.787}$
Gcncralized Ring Observations for Inner Planets:

| All two- and multi-ring basins ${ }^{\mathrm{e}}$ | $2.01_{-0.24}^{+0.28}$ |
| :--- | :--- |
| Protobasins $^{\mathrm{e}}$ | $3.34_{-0.52}^{+0.63}$ |
| basin ring-spacing model of Fielder (1963) | 2.00 |

[^4]TABLE XI
Contrasting models for basin rings

| Models | Group A | Group B | Group C |
| :--- | :--- | :--- | :--- |
| Summary | Deep-seated structural <br> changes during modification <br> stage: 'Megaterracing'. | Impact-driven wave <br> mechanism at penetration or <br> excavation stage; <br> Dominantly surficial | Differences in target <br> strength or thickness; |
| Proponents |  | Syn- or post impact. |  |

averaging 2.1. The $F$-test indicates, with $95 \%$ confidence, that these spacings did not arise by chance. Finally, ring spacing for the 44 protobasins, weighted $x$ averaging $3.3 \pm 0.6$, C.I. at $95 \%$, differs from both of the latter spacings significantly.

### 5.4. Summary of Ring Spacing

We have demonstrated that basin-ring spacing is nonrandom. Three different analyses of a large data set, much of it new, yield similar results on three terrestrial planets, regardless of their crustal structure and composition: One spacing increment dominates the ring geometry of two- and multi-ring basins. The spacing is the same both inside and outside the main basin ring. Exterior rings, once thought to be rare, are shown here to be nearly as common as interior rings. Protobasins have an entirely different ring spacing, commensurate with their morphologic immaturity and the presence of a small craterlike central peak.

Our summary descriptive model is as follows: Basin rings excluding ranks III and V , which immediately flank the main ring (IV), are spaced at an average increment, $\bar{x}$, of $(2.0 \pm 0.3)^{0.5} \mathrm{D}$. Rings of two-ring basins are spaced at exactly twice this interval. Ranks III and $V$ are separated from ring IV by an average increment of (2.1 $\pm 0.3)^{0.5} \mathrm{D}$. (Error bounds are at $95 \%$ confidence interval). Thus the observed spacing of most basin rings is found to average ( 1.7 to 2.3$)^{0.5} \mathrm{D}$, and in two restricted cases ( 1.8 to 2.4$)^{0.5} \mathrm{D}$. Ring spacing effectively straddles the $2.0^{0.5} \mathrm{D}$ value proposed initially from data for a few lunar bsins by Fielder (1963). Rings of protobasins are spaced at a much larger mean $x$ increment, $3.3 \pm 0.6$. Table X summarizes averaged values of mean spacing for grouped rings (Section 5.1), individual basins (Section 5.2), and ring ratios (Section 5.3) on the Moon, Mercury, and Mars. Within the conservative error limits applied here, the results are remarkably consistent.

## 6. Explaining Basin Rings

We think that the spacing of paired and multiple rings is sufficiently constant to impose some limits on formational hypotheses for impact basins and protobasins. The evidence assembled here, for many structures on three quite different geologic bodies, enables speculations on ring genesis to proceed from a firmer observational foundation. Ring-forming hypotheses fall into at least three groups (Table XI): structural, wave-form, and target-strength. Because the ring problem has been linked to that of basin excavation, these three models, in turn, are related to two classes of hypotheses for basin excavation (Table XII): proportional and non-proportional growth of the transient cavity.

### 6.1. MODELS INDEPENDENT OF THE $2^{0.5} \mathrm{D}$ SPACING

The possibility of a constant $2.0^{0.5} \mathrm{D}$ spacing has not strongly influenced most explanations of basin rings listed in Table XI. Rather, the problem of basin-ring origin has been linked closely with that of basin excavation (Table XII), specifically the nature of the transient-crater rim (Wilhelms, 1984). Although neither size nor shape of the transient cavity is known, proponents of basin models frequently have attempted to identify a specific basin ring (usually of what we now call rank II, III, or IV) as the topographic rim of the excavation cavity.

In general, those workers favoring a small initial cavity (Table XII: Group A Models) have tended to appeal to structural origins for ring systems (Table XI: Group A), whereas those preferring a large cavity (Table XII: Group B) have held that physical properties of the target were responsible for rings (Table XI: Group C). The two classes of ring mechanisms are not mutually exclusive, and some hypotheses incorporate features of both. Perhaps the most comprehensive scheme is that of Grieve et al. (1981), who also included aspects of a third group of ring-forming hypotheses, the 'fluidization' model (Table XI: Group B). We now examine some examples of each of the three categories of ring models in Table XI.

The first approach is structural. The concept of a 'megaterrace', a large concentric

TABLE XII
Contrasting models of basin excavation

| Model | Group A | Group B |
| :---: | :---: | :---: |
| Summary | Proportional growth of excavation cavity. Basins are scaled-up versions of smaller impact craters. | Non-proportional cavity growth. Basins form by at least some fundamentally different. mechanisms than do small craters |
| Proponents | Gault, 1974 <br> Croft, 1980, 1981, 1985 <br> Grieve et al., 1981 <br> Schultz et al., 1981 <br> Spudis et al., 1984 <br> Spudis, 1986 | Baldwin, 1963, 1974, 1981 <br> Hodges and Wilhelms, 1978 <br> Murray, 1980 <br> Wilhelms, 1984, 1986 <br> Schultz and Gault, 1986 |
| Arguments for | (1) Predicts excavated depths and volumes that agree with lunar sample data | (1) Predicts shallow excavation cavity, in agreement with lunar sample data |
|  | (2) Does not invoke hypothetical and untestable physical mechanisms | (2) Explains basin inner rings by either strength differences or oscillatory uplift and collapse |
| Arguments against | (1) Experiments indicate flow fields might change with increasing crater size | (1) Cratering process changes in some unknown way above certain energy level (deus ex machina) |
|  | (2) Does not explain basin rings, let alone the $2^{0.5} \mathrm{D}$ spacing | (2) Does not explain basin outer rings |

slump feature surrounding an initially smaller excavated crater (Table XI: Group A), was one of the earliest ideas to be advanced for the origin of basin rings on the Moon (Mackin, 1969; Gault, 1974; Head, 1974; Dence, 1976, 1977; McCauley, 1977; Melosh and McKinnon, 1978; and Croft, 1981b). The structural hypothesis is based in part on purportedly analogous rim failure evident in smaller lunar and terrestrial complex craters (Dence, 1964; Quaide et al., 1965), although it has yet to be shown that such modest slumping can be scaled directly to basin-sized events. The basin workers above all agreed on a megaterrace origin for the outer ring(s) of basins, although not necessarily for all the rings shown here. Gault (1974) and Dence (1976, 1977) envisioned this mechanism forming all outer rings around a small, proportional-growth transient cavity. Those authors who equated the intermediate ring (usually our rank III) with the transient basin cavity suggested a dual mechanism whereby the outer ring was produced by megaterracing, but the inner rings formed by rebound, analogous to the central peaks of lunar craters (Head, 1974; McCauley, 1977; Scott et al., 1978).

A completely different approach was taken by Baldwin $(1963,1974)$, after Boon and Albritton (1936), and then by Murray (1980). They argued that a basin-size impact would 'fluidize' the lunar crust in the target region, and that traveling waves would produce concentric rings like the ripples in a pond after a stone has been dropped into it (Table XI, Group B). The term 'fluidization' is used here in the sense that target materials behave like a fluid (Melosh, 1979); it does not imply any volatile content of the target as it does for the terrestrial process of fluidization. Baldwin (1963; 1974) suggested that lunar 'rock-tsunami' waveforms, propagated outward by the collision of a basin-forming projectile, would freeze in place as concentric raised rings. Murray (1980) elaborated Baldwin's (1974) idea of an oscillatory mechanism, whereby 'fluidized' lunar crust would produce ring systems by continuous back-andforth overthrust of ripples until the deformed crust increased in strength and solidified, preserving the multi-ring pattern (see also Wilhelms, 1984).

The third approach appeals to target rheology, thickness, and gross structure. Hodges and Wilhelms (1978) and Wilhelms et al. (1977) advocated a nested-crater mechanism for basins whereby interior rings formed from intersection of global compositional layers in the lunar crust by the shock wave (Table XI, Group C). Each successively smaller basin ring represents the boundary of a crater of excavation in the next-deeper crustal layer, and the basin topographic rim marks the boundary of excavation of the largest, outermost crater. The existence of outer rings, which was questioned in the nested-crater model, was ascribed to structural deformation outside the transient-cavity rim (Wilhelms et al., 1977). Melosh and McKinnon (1978) and McKinnon (1981) applied a theoretical approach to ring genesis in which they equated ring spacing with lithospheric thickness and rheology. The resulting model invokes a variable ring spacing, rather than the $2^{0.5} \mathrm{D}$ interval, on both inner planets and outer satellites. This 'ring-tectonic hypothesis' explains only rings inside the topographic rim.

We do not think that any of these models completely explains the several evenly spaced rings of large impact basins, particularly those outside the main ring, although some specific elements of each hypothesis may be valid. We especially question the dogma - generally, and hitherto uncritically, accepted - that one of the observed basin rings somehow must equate with the rim of the transient or excavation cavity. There simply is no evidence for this assumed, but very convenient, equivalence, nor is it required to effectively model the formation of multi-ring basins.

### 6.2. New CONSTRAINTS ON ORIGIN

Our observations impose limits on any model of basin ring location and emplacement (Table XIII). Analytical results for the Moon, Mercury, and Mars indicate that: (1) basins may have up to seven concentric rings or ring positions; (2) the spacing of rings is not random, but rather has a fairly constant interval; (3) this interval prevails both inside and outside the main ring; (4) the interval lies at or close to $2^{0.5}$ (Table X); and thus (5) the process responsible for locating basin rings is spatially discrete rather than continuous.

The suggestion that basin rings are spaced at $2^{0.5} \mathrm{D}$ is not new (Fielder, 1963; Hartmann and Wood, 1971). What we have confirmed here is that this spacing factor prevails in over 100 basins on three different planets (cf. Wood and Head, 1976), not on the Moon alone, or specifically at Orientale. Each of the three planets has a unique geologic history, thermal evolution, and possibly bombardment history. By contrast, the similar spacing of rings despite interplanetary differences in targets poses difficulties for basin-ring models that require specially defined target layering (Hodges and Wilhelms, 1978) or planetary thermal conditions (Van Dorn, 1968; Melosh and McKinnon, 1978).

We have found that the spacing of basin rings is constant both within (rings I-III) and outside (rings V-VII) the main topographic rim (ring IV). This important result indicates that basin rings form - or at least that their locations are determined penecontemporaneously with the basin-forming impact. It does not preclude subsequent modification of ring morphology by either passive or active processes. Similar spacing inside and outside the main ring merely suggests that (1) rings in both locales probably formed by the same processes and perhaps at the same time, and (2) the structural pattern of basin rings was already cstablished before ringmodification processes operated.

Our results indicate that planetary target conditions may have only second-order effects on basin-ring topography. The constant spacing factor of about $2^{0.5} \mathrm{D}$ applies to basins of such widely varying geologic age and target rheology that, again, the basic structural pattern of basin ring formation must have been established at the time of the impact and in association with it.

The fact that all ring positions predicted for a given basin are not filled does not preclude one model spacing, because the rings that are observed are spaced at multiples of $2^{0.5} \mathrm{D}$. The absence of a ring at any predicted position within a basin may be a consequence of several factors, including: (1) burial of low-relief ring topography by subsequent and thick geologic units, (2) erasure of created rings by either internal or external processes, and (3) unique target conditions that prevented the formation of a ring at the time of impact.

### 6.3. PREVIOUS MODELS DEPENDENT ON $2^{0.5}$ D SPACING

Two principal explanations have been proposed for the constant mean spacing of about $2^{0.5} \mathrm{D}$ for rings on the Moon (Hartmann and Wood, 1971, p. 59), and for those now described on Mercury and Mars (Table X; Pike and Spudis, 1984). The mechanisms differ in every way: One appeals to post-impact concentric fracturing of a basin-interior margin that has been loaded statically by volcanic fill; it belongs tentatively to Group C in Table XI. The other entails propagation of a 'rock tsunami' outward through a planetary crust at impact (Table XI, Group B).

Fielder (1963, p. 1259), Hartmann and Wood (1971), p. 57-59), and Schultz (1976, p. 8 and 34) all have suggested that patterns of shear stress observed by Lance and Onat (1962) in experimentally deformed steel plates might explain the $2^{0.5} \mathrm{D}$ value for lunar rings. Rigidly supported plates $5^{\prime \prime}$ and $10^{\prime \prime}$ in diameter were loaded
hydrostatically, and the resulting radial and spiral stress patterns compared with those predicted from solution to the theory of rigid/plastic bending of circular plates. No shear strain was observed. Lance and Onat verified a predicted circumferential zone of transition between two contrasting stress regimes situated at 0.73 R from the center of the plate. This distance lies well within our error bounds for the nominal spacing increment of basin rings $(0.71 \mathrm{R} \pm 0.11 \mathrm{R}$, from Table X$)$.

Five problems attend the proposed mechanism. First, and fatally, the oft-quoted '... concentration of fractures at 0.73 R ...' described by Hartmann and Wood (1971, Figure 36) simply does not exist. Close reading of Lance and Onat (1962) reveals instead that the '.. concentration of shear fractures resembling those found in a steel specimen under pure bending'. (Hartmann and Wood, 1971, p. 57) is located '... near the support of the (rigidly clamped) plate'. (Lance and Onat, 1962, p. 310); i.e., at radial distance $R$, not $0.73 R$.

The remaining problems are worth mentioning, for even without a fatal flaw, Lance-Onat deformation is unworkable for impact basins: The second problem is that the rigidly held plate, which is an essential condition for the 0.73 R interface of stress patlerns, is a particularly poor analog of the crustal configuration in and around a multi-ring basin. Third, the stress fields generated by Lance-Onat deformation require hydrostatic loading of the central basin region. Such loading undoubtedly occurs in deeply flooded or otherwise modified basins on the Moon (e.g. the mascon in Imbrium). However, the observation that fresh, little-flooded, and relatively unmodified lunar basins (e.g. Orientale) and also terrestrial multi-ring craters (Pike, 1985) - none of which are loaded in this way - display the same ring spacing argues against pure Lance-Onat deformation. Fourth, Lance-Onat fracturing specifically generates only one circumferential feature. A preposterously complex, multisequence history of repeated loading (e.g., Fielder, 1963) would be required for each basin to explain the multiple-ring pattern observed on all the planets and satellites. Finally, the lone circular pattern purportedly formed by the Lance-Onat mechanism would lie within the basin topographic rim; we have shown here that rings and arcs spaced at the $2^{0.5}$ increment are common outside ring IV.

The spacing of several rings at about $2^{0.5} \mathrm{D}$ has suggested to other authors, correctly we think, that the physical processes controlling ring locus and perhaps formation must be periodic (Van Dorn, 1968; 1969). The 'tsunami' mechanism of ring formation discussed above in its pure form (Baldwin, 1974) is analogous to water waves propagating from a point-source disturbance. As such, each successive ring is separated by the same distance from the ring immediately within it. This problem was recognized by Van Dorn (1968). He attemped to reconcile the tsunami model with the observed increasing-outward spacing on the Moon by invoking ad hoc a 50 km thick layer of loosely compacted debris overlying a rigid basement.

We find the most obvious drawback to such a wave model to be the requirement for a similar crustal structure at each basin-impact site on all three planets. The possibility seems vanishingly small. Another unrealistic attribute of the tsumani model is the dominantly surficial character of its resulting ring topography. Rather,
geologic studies have accumulated considerable evidence for deep structural deformation associated with basin rings (Hartmann and Wood, 1971; Schultz, 1976; McCauley, 1977). Finally, we find implausible the notion that elements of a traveling wave form would always 'freeze' at a similar and discrete ( $2^{0.5} \mathrm{D}$ ) interval (see also, Chadderton et al., 1969, p. 261). Instead, a variety of spacings would be expected as the outward-propagating wave 'froze' at different distances from the impact focus at different basins.

A possible solution to problems with existing wave models invokes standing (or 'stationary'), rather than traveling waves. First, a standing wave pattern can have an increasing-outward spacing of adjacent rings. Second, a standing wave is more consistent with the discrete interval observed between basin rings than is a traveling wave. Chadderton et al. (1969) attempted to explain the $2^{0.5}$ spacing of Orientale basin rings as the result of standing waves produced by free-body vibrations of the entire Moon.

Although we think that adaptation of the standing-wave mechanism was an important development, this specific model has several difficulties. Perhaps most damaging is the observation that multi-ring basins occupy the same, continuous, geometric sequence as smaller two-ring basins (Pike, 1983, 1985). The smaller impacts, which produced basins a third the size of Orientale, (to say nothing of the even smaller ones on Earth), almost certainly carry too little energy to induce the whole-planet vibrations required to form rings. Moreover, standing-wave vibration requires a wave guide, in this case a resonant cavity, to set up constructive interference. Such a cavity must be available on all planets and at each basin site. Its geometry must be everywhere the same to produce the similar ring spacing observed at each basin.

### 6.4. Speculations on a Ring-Forming mechanism

That some part of the basin itself might provide the resonant cavity is suggested by the radially symmetric pattern of damage to structures in the Mexico City earthquake of 19 September 1985 (Flores et al., 1987). Overall damage was disposed in alternating, similarly spaced annuli of heavy and light destruction, but only within a local region of water-saturated sediments. The concentric zonation indicates that seismic longitudinal ( $p$ ) waves reflected internally within the low-strength region (the resonant cavity), interfered with each other, and generated standing waves. Both maxima and minima of standing-wave solutions to the Poisson equation correspond spatially to the large surface accelerations, according to the geophysical model by Flores et al. (1987). Many areas of greatest damage from the 19 September 1985 earthquake are located in these concentric zones of maximal acceleration.

We find remarkable the several parallels between this terrestrial seismic event and ringed impact basins. They include: (1) an initiating disturbance of high energy and short duration; (2) a compact low-strength region enclosed by rigid materials; (3) concentric zoning at the surface; (4) a similar spacing of concentric elements. These similarities raise several issues: Might a large impact generate longitudinal waves that
interfere and create a ring-locating resonance within the low-strength ('fluidized'?) target bounded by high-strength planetary crust? Could ring positions thus determined also develop outside the low-strength center? What part of the basin might serve as the resonant cavity? The answers to such questions undoubtedly will be more complicated than a simple appeal to resonance of the reflected shock wave within a basin's 'strength cavity' (e.g., Pike and Spudis, 1984b). Although clearly intriguing, this new analog requires much further investigation before it can be developed into a plausible model for basin-ring spacing.

## 7. Discussion

A model for concentric rings (Table XIII) must be consistent with existing evidence and constraints on basin formation as well as those developed here. Accordingly, it should have at least the following attributes: (1) the basic impact mechanics are similar overall to those forming smaller craters, differing mostly in the total energy input; (2) ring spacing develops independently of gravity, location within a basin, and conditions of target or projectile; (3) the ring-locating processes invoked accommodate discrete ring positions, of which seven are now known; (4) the model accounts for the many associations of deep-seated geologic structure with topographic rings; (5) ring location is imprinted at the time of impact, and special circumstances at most only modify this early- established concentric pattern; (6) ring topography may not have formed by the same process(es) that determined ring position; and (7) ring location and emplacement of ring topography need not be coeval. Other constraints are summarized in Table XIII.

The average $2.0^{0.5} \mathrm{D}$ spacing of basin rings almost certainly was established at impact, although this characteristic precludes neither modulation of ring geometry by target inhomogeneities nor postimpact geologic changes in basin topography. However, we assert that these complicating factors modify ring morphology much more than ring position, and account for only some of the scatter evident in the statistics of dispersion for mean ring spacing. Among target effects that may modulate basin-ring morphology are pre-impact structure and topography (Head, 1977b), broad-scale crustal layering (Hodges and Wilhelms, 1978), and lithospheric thickness (Melosh and McKinnon, 1978). Post-impact changes are mainly volcanic and tectonic. Annular zones of structural weakness formed during basin excavation, which could persist long after the initial impact, may remain loci of later endogenic modification (Schultz, 1979; Schultz and Glicken, 1979; Schultz et al., 1982).

We attribute split basin rings to the increasing influence of local crustal structure on ring morphology, but not necessarily position, with distance outward. First, splitring morphology frequently coincides with weak topography outside the main ring: rings are increasingly scarce in ranks V to VII, and tend to be fragmented partial arcs where they occur (Table I). That such rings also occupy discrete $2^{0.5} \mathrm{D}$-spaced positions is consistent with a decay in energy at greater distances (Chadderton et al., 1969). We propose that these weaker ring-forming disruptions were more easily

TABLE XIII
Proposed synthesis of basin-ring origin

## Constraints on Excavation

(1) Most lunar basin ejecta from crust; little evidence for mantle material.
(2) Excavated volume at Orientale $=4$ to $8 \times 10^{6} \mathrm{~km}^{3}$ (gravity, photogeology).
(3) Pre-basin structures preserved within main rim (Orientale, Imbrium).
(4) Basin impact melts (LKFM) are more mafic than composition of surface rocks.

## Constraints on ring formation

(1) Main rim (rank IV) equivalent to rims of smaller (complex) craters.
(2) Many outer rings exist; age relation of their topography to basin unsure.
(3) Inner and outer basin rings on 3 planets are spaced at $2 .^{0.5} \mathrm{D}$ intervals.
(4) Deep-seated structures are associated with basin rings.

## Toward a model of basin and ring formation

(1) Proportional growth of cavity is valid: $D_{t c}=(0.5 \pm 0.1) \mathrm{D}, \mathrm{km}$.
(2) Maximum depth of excavation $=(0.1 \pm 0.02) D_{t c}, \mathrm{~km}$.
(3) Volume of ejecta for Orientale $=7.7 \times 10^{6} \mathrm{~km}^{3}$.
(4) Basin impact melts are generated at crustal depths of 30 to 60 km (LKFM).
(5) Main rim forms by collapse around gravity-scaled excavation cavity.
(6) Ring positions (not necessarily topography) determined during impact.
(7) Ring locations and ring topography may not reflect same process(es).
(8) Ring loci due to wave phenomenon, possibly oscillation or resonance effect.
(9) Outer rings of largest impacts may arise from lithospheric fracturing
modulated by layering and other crustal inhomogeneities (e.g., Hodges and Wilhelms, 1978), thus frequently creating conditions favorable for the development of split rings.

The systematic displacement of ranks III and V from the main ring (Figures 6-8) is less readily explained. We suspect that the symmetry about ring IV is related to its topographic prominence (which suggests that most of our choices for the main ring were correct). Undoubtedly, the symmetry is a clue to physical processes operating during basin formation, perhaps 'shedding' of rings III and V from IV during oscillatory movements of the sort envisioned by Baldwin (1974) and Murray (1980). We have no explanation for the diameter-dependent convergence of ranks III and V for basins on Mercury (Figure 7b).

A tentative model for concentric rings that emerges from the constraints summarized here (Table XIII) resembles none of the existing hypotheses (Tables XI, XII) in pure form. Rather, it embodies elements of several models. We think prevailing evidence favors a proportional-growth transient cavity about half the diameter of the final main ring, which forms by collapse around a gravity-scaled excavation cavity no deeper than about 0.1 its diameter. Up to seven major ring positions spaced at $2^{0.5} \mathrm{D}$ are determined early during the excavation stage. We think this imprinting occurs through a wave mechanism that operates in an oscillatory or
resonant mode and can establish ring loci in both low- (fluidized?) and high-strength (unfluidized) target material (Melosh, 1979). The topography of inner rings may form at this time by the same mechanism. The topography of some, or even many, outer ring and arcs, particularly those of the largest impacts, may be emplaced postimpact, likely as the result of faulting at the loci determined early in the impact.

The synthesis outlined here represents only a first attempt on the problem of basin genesis from an analysis of basin rings. It still leaves unanswered some important questions about concentric-ringed structures on the planets. Perhaps the most intriguing of all is why rings are spaced at $2^{0.5} \mathrm{D}$ rather than at some other number.

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[^0]:    ${ }^{\text {a }}$ Source: measured or remeasured by authors on spacecraft images, photomosaics, and maps, mostly after lists of Wood and Head (1976, Table 2), Croft (1979, Tables 5.1-5.3), Wood (1980, Table 1), and Pike (1983, Figure 1).
    c Walues for rim and inner ring are means, values for peak base are
    ${ }^{\text {d }}$ Central peaks not observed (probably buried) for 11 presumed protobasins, which were recognized on basis of high $D / D_{i}$, morphologic dominance of rim over inner ring, and $D$ range: $\geq$ Antoniadi but $\leq$ Mendeleev.
    ${ }^{\text {c }}$ Unusally small inner ring possibly an open or expanded cluster of central-peak elements (Pike, 1983).
    ' 3 rd ring between $D$ and $D_{i}, 220 \mathrm{~km}$ (Herschel) and 158 km (Lyot), thus making these anomalous features 'multi-ring protobasins'; omitted from statistics.

[^1]:    ${ }^{\text {a }}$ From Table I; multi-ring basins only.
    b See Table III.
    ${ }^{c}$ From results of least-squares fits and statistics of ring spacing in Table III; model value $=1.414$.
    ${ }^{\mathrm{d}} x$ for adjacent ranks $=\left(Y_{\mathrm{c}} / \bar{X}_{g}\right)^{2} ;$ model value $=2.000($ Fielder, 1963 $)$.

[^2]:    ${ }^{\text {a }}$ Source: Tables III-V; for derivation of $x$ see Table III.
    ${ }^{b}$ Number of rings correlated with ring IV; first value $=n$ for $\bar{x}$; second value $=n$ for $95 \%$ C.I.
    ${ }^{\text {c }}$ Including one-ring groups; IV/IV excluded.
    ${ }^{d}$ Weighted by number of rings ( $n \geq 5$ ) in each group.
    ${ }^{e}$ Summed numbers of rings in paired adjacent ranks ( $\geq 3$ rings; Table IV); all rings ( 2 rings; Table V).
    ${ }^{r}$ Weighted by number of rings in each paired group (Tables IV, V).
    $\because$ Respecive values from identical calculations in $D / D_{i}$ format are: $2.049_{-0.102}^{+0.108}$ and $3.433_{-0.409}^{+0.478}$.
    ${ }^{\text {h }}$ Pike (1985).
    ${ }^{i}$ R. J. Pike, unpublished preliminary ring diameters for basins on Rhea, Ganymede, and Tethys.

[^3]:    a Source: calculated by authors from data in Tables I, II.
    b Calculations in $\log$ domain; ratios weighted 1 to 5 according to quality of data (see text).
    ${ }^{c} x=\left(Y_{c} / \bar{X}_{g}\right)^{2}$ for adjacent ring ranks, and $\left(Y_{c} / \bar{X}_{g}\right)^{2 / 2}$ for alternate ring ranks and for two-ring basins and protobasins.
    ${ }^{d}$ Include ratios formed from a few rings ranked below I ('A', 'B'). in the log $x$ domain at the $5 \%$ level of significance.
    f Too few data for stable statistics ( $n \leq 9$ ), except protobasins.
    g $D$ equivalent to ring IV, $D_{i}$ to ring II; protobasins have a central peak in addition to the inner ring.
    h 'Doubly-wide' spacings (rings V/III) not observed (Ring IV intervenes).
    ${ }^{\text {i }}$ Mean $D / D_{i}$ values for protobasins differ significantly from those for two-ring basins, by a 2 -sided normal test carried out in the log domain at the $5 \%$ level of significance.
    j Values for the six confirmed lunar protobasins (see Table II): $3.110+0.451-0.223$ (6) ${ }^{\mathrm{i}}$
    ${ }^{\mathrm{k}} x$ equivalents only: standard deviations are estimates; no $F$-tests.

[^4]:    ${ }^{\text {a }}$ Weighted means in Table VI. ( $D / D_{i}$ calculations for protobasins not shown).
    ${ }^{b}$ Weighted mean of three summary results in Table VII.
    ${ }^{c}$ Weighted means of two sets of results (three values each) in Table IX.
    ${ }^{d}$ Standard deviations are unweighted means of estimates.
    ${ }^{\text {e }}$ Unweighted means of, respectively, three and two results listed above.

