Chapter 4
Research on Individual Authority and Group Authority Relations in Collaborative Problem Solving in Middle School Mathematics

Jue Wu

4.1 Introduction

Classroom teaching practices have attracted the increasing attention of researchers (Li et al., 2022; Zhao et al., 2022). In teaching practice, front-line teachers are committed to promoting class management based on a fair and equitable system or the theory of teacher-centred authority in class management. Researchers have focused on examining teacher authority while paying insufficient attention to student authority. Teachers should not only pass knowledge to their students but also encourage them to think and learn by themselves, which is good for teaching and learning (Díez-Palomar et al., 2021). Good teacher-student relationships come from the integration of teacher authority and student authority. However, very little attention is paid to student authority in mathematics education. Collaborative problem solving (CPS) can be a good vehicle for exploring student authority. It provides a mathematical learning environment that involves non-teacher-led activities. In CPS, students use a range of mathematical and non-mathematical forms of language to gain authority, which influences the process and outcome of learning. For these reasons, the study of individual authority and group authority relations in CPS in mathematics is bound to become particularly important.

J. Wu (✉)
Changjun Banmahu School, Changsha, China
e-mail: 1181088483@qq.com

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4.1.1 Conceptual Framework

4.1.1.1 Authority

The term "authority" is widely used in social life and has different meanings in different cultures and contexts. In the Chinese dictionary (*Ci Hai*), it refers to two concepts: (1) power and prestige, and (2) a force of prestige and dominance developed in the course of human society. Also, there are different interpretations of authority in different disciplines. From a sociological perspective, authority is a force that convinces people without pure violence based on two elements: voluntary obedience and belief. In psychology, there are two manifestations of authority: formal and informal, which highlight the influence individuals and groups have on other people and groups.

There are also different interpretations of authority. Despite different disciplinary perspectives, all definitions reflect at least two characteristics. First, authority emphasises the relationship of obedience between authority objects and other authority subjects, based on value recognition. Second, the operation of authority produces a relationship between authority objects and other authority subjects in terms of influencing and being influenced. It is the recognition of authority objects to authority subjects in terms of ideology and obedience in terms of behaviour.

This study examines changes in the relationship between individual authority and group authority relations in CPS in mathematics. The definition of student authority used herein is based on the theory that authority emphasises the obedience of authority objects to authority subjects. This authority relationship between three and more people is the theoretical basis for the definition of group authority relations. Students' group authority relations change through interactions, and changes in authority are dynamic.

4.1.1.2 Student Authority in Mathematics Activities

This study examines the relationship between students' individual and group authority in CPS. In teacher-empowered student collaboration and management of student-led CPS activities, authority shifts from being unilaterally held by the teacher to being distributed between the teacher and the students. Mathematics activities research has begun to examine the authority relations between students.

In terms of research on authority in classrooms, several studies have pointed to definitions of authority in classrooms. Cohen (1994) defined authority as "an agreed-on rank order where it is generally felt to be better to be high than low rank." Status can be thought of as a relationship of power among peers. That power can be academic, as in status, derived from perceived smartness, or social, as in status derived from popularity. This somewhat vague positioning of authority highlights its intellectual and social categories. Ernest (2008) proposed that a teacher "has two overlapping roles—namely as director of the social organisation and interactions in

the classroom (i.e., social controller) and as director of the mathematical tasks (i.e., task controller). This distinction corresponds to the traditional separation between being 'in authority' (social regulator) and being 'an authority' (knowledge expert)." Researchers defined authority as a resource of control associated with the right to lead and the obligation to follow (Amit & Fried, 2005; Ernest, 2008; Pace & Hemmings, 2007). Teachers can be both an intellectual authority based on their knowledge and a social authority based on their power to issue instructions to students and control their behaviour. Boaler and Greeno (2000) believed that social authority always operates in classrooms, occurring wherever humans interact. Intellectual authority relations are at play when individuals are engaged in intellectual work, defined in schools as engaging in academic tasks.

From the above scholars' explanations of authority in mathematics education, student authority relations in CPS in mathematics can be divided into social authority and intellectual authority. The ACT21S project "CPS" consists of two main components, "Collaboration" and "Problem solving." The problem solving component consists mainly of skills required to solve problems. Intellectual skills reflect the management of tasks and include task management, learning, and knowledge building. This paper combines this perspective to define individual authority and group authority relations in CPS.

4.1.1.3 Student's Individual Authority in CPS in Mathematics

The definition of individual authority is derived from Langer-Osuna (2016). Students' individual authority refers to the students' personal intellectual and social authority in CPS activities. Students' individual intellectual authority refers to the fact that students' individual behaviour is a useful source of information (or lacks such credibility). A student's individual social (directive) authority refers to the student being deemed to have (or not have) the right to issue directives to group members.

4.1.1.4 Group Authority Relations in CPS in Mathematics

The definition of group authority is derived from Langer-Osuna et al. (2020b). Group authority relations are formed as a result of the authority of three or more people operating in the group. Group authority relations refer to an intellectual or social relation of submission between authority subjects and authority objects, which is also a relationship of influencing and being influenced. In the current research, seven types of group authority relations can be formed in cooperative groups as intellectual and social authority compete and disperse. Group authority relations are reflected in students' utterances and behaviours in classroom communication and are dynamic in their changes.

4.1.1.5 Conceptual Framework for Individual and Group Collective Authority Relations

This section defines students' individual authority and group authority relations in CPS in mathematics. This diagram explains how individual authority forms group authority relations and the types of group authority relations.

Figure 4.1 explains the relationship between individual authority and group collective authority. (1) Individual authority is composed of intellectual and social authority. In a group of three or more, individual authority forms group authority relations; specifically, individual intellectual authority forms group intellectual authority relations and individual social authority group collective social authority relations. (2) Group intellectual authority relations include shared and concentrated intellectual authority relations; contested intellectual authority relations are formed through students' sharing of, concentration of, or competition for intellectual authority. (3) Group social authority relations include shared, concentrated, and contested social authority relations, as well as disbanded social authority relations due to the sharing, concentration, competition, or dissolution of students' social authority. (4) These seven different group authority relations have conceptual crossovers. Shared intellectual and shared social authority relations belong to shared authority relations. Concentrated intellectual and concentrated social authority relations belong to concentrated authority relations. Contested intellectual and contested social authority relations belong to contested authority relations.

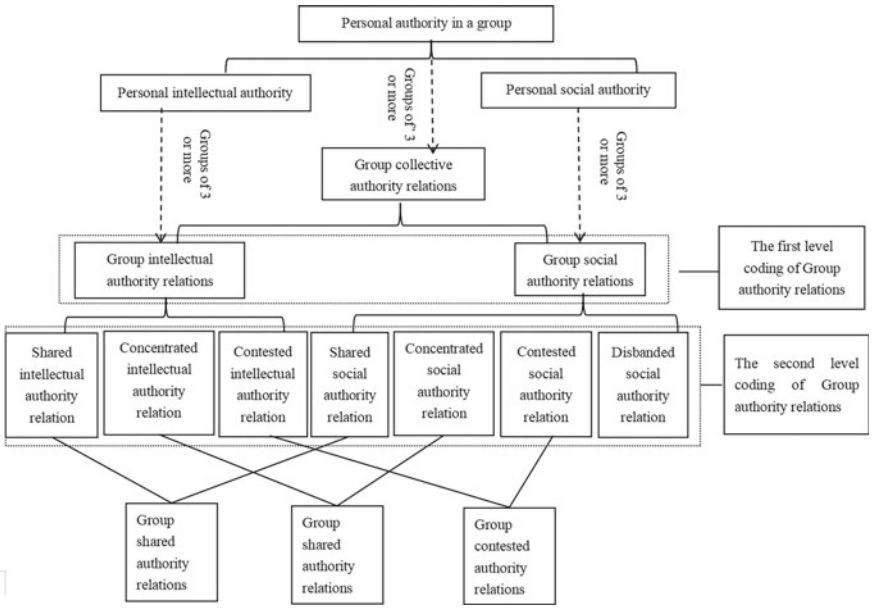

Fig. 4.1 Conceptual framework for individual authority and group authority relations

4.1.1.6 Research Questions

Through exploring the characteristics of authority within different structured groups, it is possible to explore the potential relationship between authority and performance. Further, the transformation of authority possibly influences students' activity during CPS (Langer-Osuna et al., 2020b). Based on a video analysis of CPS activities in a middle school, this study examined student authority relations in the CPS stage to answer two research sub-questions:

Question 1: What is students' individual authority in high- and low-scoring groups in CPS in middle school mathematics?

As this study explores individual and group authority relations, it first explores individual authority (individual intellectual authority and social authority) in high- and low-scoring groups in CPS in mathematics.

Question 2: How do students distribute and shift group authority relations in high- and low-scoring groups in CPS in middle school mathematics?

Individual authority interacts in groups of three or more to form group collective group authority relations. This question explores the specific distribution and variation of the seven different group authority relations.

4.2 Research Design

This study focused on the students' authority relations in CPS in middle school mathematics. Two sub-problems were used to explain the characteristics of individual and group authority in CPS. The overall idea of this research is shown in Fig. 4.2.

This research mentality diagram depicts the concepts: of (1) node (related group authority relations proposal negotiation unit); (2) coverage rate (the ratio of node dialogues generating group authority relations to all dialogues); and (3) individual authority rise and fall (the frequency with which an individual's proposal is accepted or rejected).

4.2.1 Data Sources

Purposeful sampling is applied for the current research. Six four-student groups were selected for the study. The six groups were drawn from Teacher A's classes (C01a and C01b) and Teacher B's classes (C02a and C02b) (see Table 4.1) within the same school. This study further divided the six groups into three high-scoring and three low-scoring groups for more pertinent analysis, based on the group score table for mathematical collaboration problem solving (Appendix 4.1). Students scoring above 6 were in the high-scoring group; students scoring below 6 were in the low-scoring

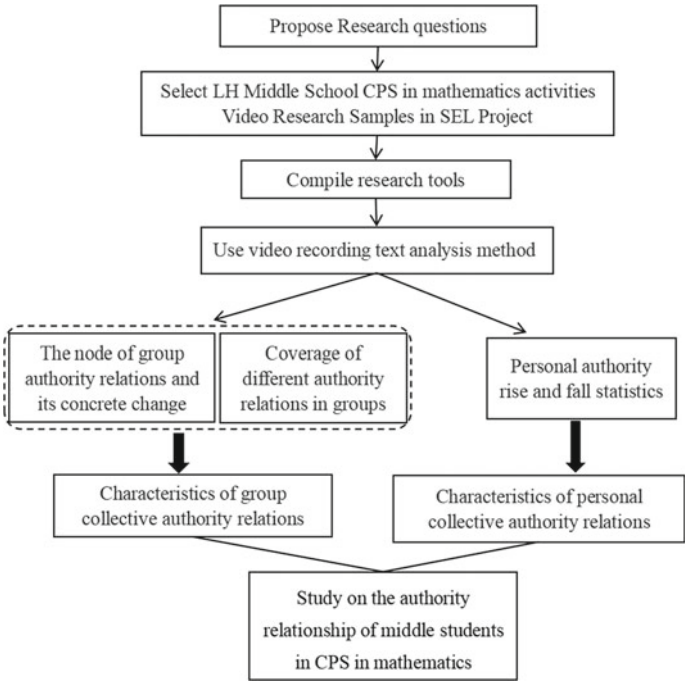

Fig. 4.2 The overall research roadmap of this study

Table 4.1 The attributes of the object group in this study

School	Group classification	Group name	Teacher	Score
LH middle school	High-scores groups	01b-02	Teacher A	10
		01a-02	Teacher A	7
		02a-04	Teacher B	9
	Low-scoring groups	01a-01	Teacher A	5
		01b-01	Teacher A	5
		02b-05	Teacher B	3

group. The three high-scoring groups received scores of 10, 9, and 7, respectively, while the three low-scoring groups received scores of 5, 5, and 3. The groups selected were those who spoke up most actively in classes.

4.2.2 Data Analysis

This study chose a qualitative research method based on video recordings. After transcribing the video dialogues, video text analysis methods were used to generate statistics on the rise and fall of individual students' authority in the high- and low-scoring groups. Student authority relations in CPS in mathematical activities were explored by counting group authority relation nodes and different group authority relations as a percentage of coverage (as a ratio of total discourse). This study was conducted using NVivo version 12.

4.2.3 Coding Scheme

This sub-section describes two research tools used to explore the individual authority and group authority relations in CPS in mathematics. There were several reasons for selecting and adapting these two research instruments. The first reason was the similarity of the two research samples. The original scales were initially used to study 10–11-year-old students' CPS in mathematics activities; this study involved Grade 7 students' CPS in mathematics activities. The second reason was the operability of the research codes. The research tools provide operational definitions that can be analysed in classroom videos, making both individual and group authority relations more visible. Third, both research tools have a strong theoretical basis, making them persuasive.

(1) An analytical tool for coding individual authority in CPS in middle school mathematics

This study drew on Langer-Osuna's (2016) findings on individual authority, adapting them accordingly to form an individual authority coding analysis tool in conjunction with this study (see Table 4.2). This study omitted the coding of individual authority statistics related to teachers assessing the quality of students' arguments and the merits of students' behaviour. The groups consisted of four people, two boys and two girls, coded as B1, B2, G1, and G2.

This study was conducted to represent the dynamics of students' individual authority and authoritative characteristics through their individual intellectual and social authority statistics. Individual authority rise and fall statistics refer to the frequency of each group member's successful and devalued bids for intellectual authority and social authority. Engle et al. (2014), in their work, *Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument*, referred to a proposal negotiation unit (PNU). A PNU is a set of interactions that begin with a discourse that makes a suggestion around a problem (for example, presenting an idea to be evaluated or giving an instruction). The data for this study were analysed at the event level of the PNU, where a PNU is a group authority relation node. Table 4.3 explains the methodology

Table 4.2 Coding of individual authority statistics for CPS in mathematics in this study

Coding	Definition	Sample
Impact on problem solving	The student's idea is positioned to be part of the completion of the problem solving pathway or final answer (or is rejected)	G2's contribution was written on a shared task list as part of the final answer "I think it should include the kitchen, the toilet, the two bedrooms, the balcony and the living room "Right, right, right" B1 and G2 started drawing the five rooms In the task list it is possible to see the results of their correspondence with the dialogue
Individual intellectual authority	Student proposals are used as a source of applicable information (or lack credibility) and have an impact on problem solving outcomes	"First you have to draw a good scale" "Right" (or someone else indicates the default) or "There should be another aisle drawn" "Just draw the room directly"
Individual social authority	Students are seen as having (or not having) the right to issue instructions to group members that have an impact on problem solving	Respond to an instruction: "First, make the picture bigger" G2 responds to related instructions and enlarges the picture

for the rise and fall statistics of individual authority in a given PNU. For example, a suggestion might include, "I know what to do, let's add numbers." This would be coded as a bid for intellectual authority; a group response such as "yes" after adding the numbers would be coded as a successful acceptance of the bid, positioning the first speaker's intellectual authority. Conversely, a response such as "no" would be coded as a rejection of the bid, thereby devaluing the first speaker's perceived authority. In this study, B1 represents the first male, G1 the first female, and so on.

Figure 4.3 shows the results of CPS in mathematics for the groups corresponding to the authority relation nodes in Tables 4.2 and 4.3. Students' individual authority statistics were based on students' conversations and the results of CPS in mathematics.

Students' interactions were qualitatively analysed after coding. The interplay of absorption frequency (represented by positive signs) and rejection frequency (represented by negative signs) were analysed through a specific CPS in mathematical videos.

Table 4.3 Schematic table of the way individual authority statistics for CPS in mathematics are presented

Contested authority PNU: Area of Apartment toilets, kitchens, living rooms	
Authoritative relation events	Changes in individual authority
B1: For example, the toilet and the kitchen are 20 m^2 in total G2: 20 m^2 in total? G1: 10 m^2, or else it's gone B1: So the kitchen is only 5 m^2 in total	The kitchen and toilet areas in the task list total 20 m^2 [B1 authority increased, recorded as B1 (+1)]
G1: The toilet is smaller G1: The bedroom is 10 m^2 and the toilet is 5, that's 15 in total, how many bedrooms? B1: That's only 1	The bedroom in the task list is 15 m^2 and the toilet is 10 m^2 [The authority of the G1 is reduced and is recorded as G1 (−1)]
B1: Living room assumed to be 15 G1: It's a bit small B1: That's all that's left, how much more, 60 m^2 in total, your living room takes up 50 m^2	The living room in the task list is not 15 [The authority of the B1 decreases, The authority of the G1 increases, note as B1(−1), G1(+1)]

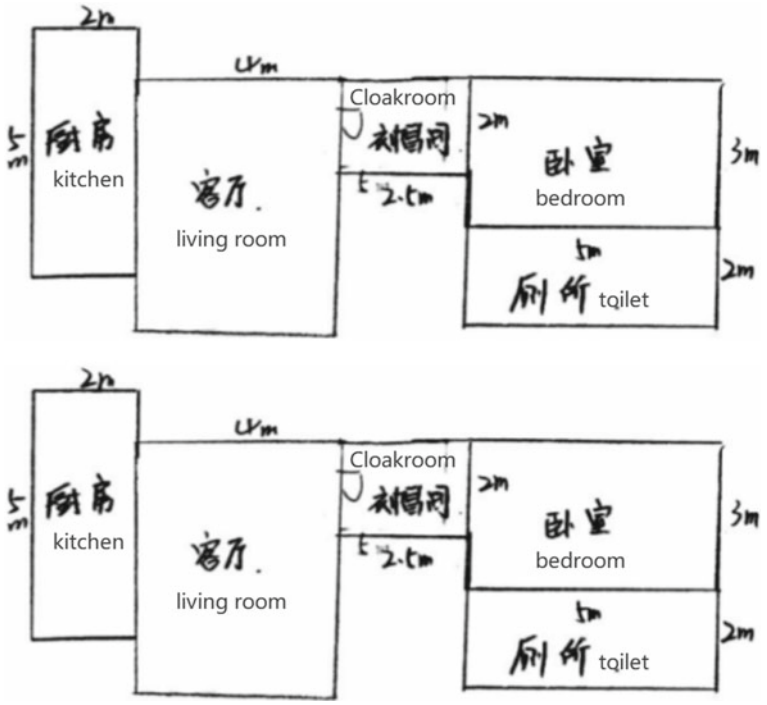

Fig. 4.3 Apartment layout task sheet in Table 4.3

(1) Tools for analysing group collective authority coding in CPS in middle school mathematics

Based on Langer-Osuna's (2020a) coding of group collaborative authority relations in CPS in mathematics, this study developed the group authority relations coding analysis tool. The individual authority mentioned above forms group authority relations in groups of three or more. Students' individual intellectual authority forms intellectual authority relations in groups of three or more. The group's intellectual and social authority can be transformed between shared authority, concentrated authority, contested authority, and disbanded authority. There are seven different group authority relations. This study examined the different social and intellectual authority relation nodes and coverage and their translation between collaborative group collectives. Table 4.4 shows an adaptation of Langer-Osuna's (2020a) coding of group authority relations to form an analysis tool for this study. The study was interpreted in the context of a CPS task in middle school mathematics called "Xiao Ming's Apartment".

This study first set intellectual and social authority relations as the primary codes. Shared, contested, and concentrated intellectual authority relations; shared, contested, and concentrated social authority relations; and disbanded social authority relations were set as secondary codes. Conceptual crossover occurred between the seven different group authority relations. Shared intellectual and social authority relations belong to shared authority relations, concentrated intellectual and social authority relations belong to concentrated authority relations, and contested intellectual and social authority relations belong to contested authority relations.

The data in this research were analysed at the PNU level. A PNU is a group authority relation node. For details, see the example in Table 4.3. This study counted the variation in social and intellectual authority relations across different groups and the nodes and continuity accounted for different types in the 15 min CPS video.

4.3 Results

4.3.1 Individual Authority Study Results of the High- and Low-Scoring Groups

This section explores statistics on the rise and fall of students' individual authority in CPS, referring to the frequency of each group member's successful and devalued bids for intellectual and social authority. B1 and G1 represent the first boy and first girl in the group, and so on. Specific statistical methods are explained in Table 4.3. For example, B1's suggestion, "I know what to do, let's add numbers," would be coded as a bid for intellectual authority; positive group responses (e.g., "Yes") after adding the numbers would be coded as a successful acceptance of the bid and recorded as B1 (+1), locating the first speaker's knowledge authority. Conversely, negative responses (e.g., "No") would be coded as a rejection of the bid and recorded as B1

Table 4.4 Coding of collective group authority relations in CPS in mathematics in this study

Distribution	Social	Intellectual
Shared	Multiple students' bids to manage their own and others' participation are taken up. This includes voiced negotiation of roles and distribution or management of tasks Example: the boys put the girls in charge of drawing pictures at the beginning of the task and the girls put the boys in charge of coming up with ideas	Multiple students' bids to contribute to the intellectual work are taken up. Disagreement about a mathematical idea or solution is in the service of reaching a consensus Example: a student is marking the area of each part of the kitchen and living room and discussing the agreement with the rest of the class to complete it
Concentrated	Bids to manage participation are taken up only in relation to one student. This includes instances where only one student successfully issues directives in the group Example: a student assigns roles to others in the group. One student instructs another student to write the name of the group and the student is instructed to write the name of the group This will increase the instructional authority of the instructing student as described in the table above	Bids to lead the intellectual work are only taken up in relation to one student. This includes instances where only one student's mathematical contributions are considered in the group Example: one student declared that the group would use blocks to solve. Other suggestions are rejected and the group continues with the discussion
Contested	Multiple bids to manage participation are rejected such that there is no settled authority Example: one student told another student to draw a toilet and a bedroom, as well as a bedroom with a toilet inside. The peer refuses and some other students suggest other options, some of which are accepted and some of which are ignored	Multiple bids to author ideas, offer help or lead the work are rejected such that there is no settled authority Example: one student told another student that he should draw the scale first and his companion refused. Other students suggested options such as drawing the outer frame of the plane first, some of which were accepted by their peers and some were ignored by them
Disbanded	N/a[1]	When the collaboration disbands into independent or off-task activity

(−1). The frequency of absorption (a positive sign) and rejection (a negative sign) are shown in Table 4.5.

In high-scoring group 01a-02, B1 had intellectual authority bids accepted six times, while G1 had intellectual authority bids accepted eight times. While the difference in intellectual authority between the two was not significant, their authority bids were accepted significantly more often than those of the other two students in their group. In groups 01b-02, B2's intellectual authority bid was accepted seven times and G2's social authority bid was accepted six times. Group 02a-04's B2 and G2's intellectual and social authority bids were also accepted more often than the

Table 4.5 Statistics on the rise and fall of individual authority in the high- and low-scoring groups

	Group	Member	Intellectual authority	Social authority		Group	Member	Intellectual authority	Social authority
High-scoring group	01a-02	B1	+6 (−2)	+3 (−2)	Low-scoring group	02b-05	B1	+1 (−3)	+1
		B2	+1	+2 (−3)			B2	+3 (−4)	+2
		G1	+8 (−4)	+3			B3	+2	−1
		G2	+4 (−2)	+1			B4	+4 (−3)	+2
	01b-02	B1	−1	0		02b-01	G1	+3 (−2)	+3 (−1)
		B2	+7 (−2)	−2			B1	+4 (−3)	−1
		G1	+5 (−1)	+2 (−2)			B2	+3 (−1)	+3 (−1)
		G2	+2 (−4)	+6 (−4)			B3	+1 (−2)	−2
	02a-04	B1	+1	−1		01a-01	G1	+4 (−3)	+2 (−2)
		B2	+6	+4 (−3)			G2	+4 (−2)	+1 (−1)
		G1	+2 (−1)	+1 (−1)			B1	+3 (−3)	−3
		G2	+4 (−3)	+3 (−2)			B2	+4 (−2)	+4 (−4)

other students in this group. Therefore, for the high-scoring groups sampled, group members' authority bids were accepted more often, with one or two students having significantly higher individual intellectual or social authority than the other students in their group.

In the low-scoring group, the frequency of authority bids being accepted and rejected authority bids was more balanced across group members. For example, in group 02b-05, B4 made four intellectual authority bids, but they were rejected three times in favour of other group members' authority bids. Based on Tables 4.5 and 4.6 offers a statistical summation for the individual social and intellectual authority for the high- and low-scoring groups.

Table 4.6 shows that there were 111 authority bids in the high-scoring group and 98 in the low-scoring group, indicating that individuals in the high-scoring group were more inclined to contribute to CPS in mathematics. There were more intellectual authority bids than social authority bids in both the high-scoring (64 vs 45) and low-scoring (64 vs 34) groups. The high-scoring group had a higher intellectual authority summation (26) than the low-scoring group (10). It also had a significantly higher total authority summation (31) than the low-scoring group (12). Thus, it can be concluded that students in the high-scoring group were able to have their proposals endorsed more often than the low-scoring group.

4.3.2 Group Collective Authority Study Results for the High- and Low-Scoring Groups

The first part analyses the intellectual and social authority relations in the high- and low-scoring groups under primary coding. The second part offers a comparative analysis of the different authority relations in the high- and low-scoring groups under secondary coding.

4.3.2.1 Nodal Analysis of Intellectual Authority Relations and Social Authority Relations in High- and Low-Scoring Groups

In this study, relevant authority relation nodes were counted and coded. Teacher intervention discourses were not included into coding. The coding process removed non-responsive self-monologues and conversations after members had stopped using their pencils for the task.

In this article, the layout of the apartment, dimensions of the rooms, and names of the rooms (kitchen, living room, bedroom) are in the category of intellectual authority relations. Specific functions (toilet shower, bedroom sleeping) belong to the category of social authority relations. The PNU mentioned above is a set of interactions that begins with a discourse making a proposal around a coded component (e.g., presenting an idea to be evaluated or giving an instruction). The table below

Table 4.6 Statistical table of individual authority results for high- and low-scoring groups

High-scoring group					Low-scoring group				
Group	Member	Intellectual authority	Social authority	Authority	group	Member	Intellectual authority	Social authority	Authority
01a-02	B1	+4	+1	+5	02b-05	B1	−2	+1	−1
	B2	+1	−1	0		B2	+1	+2	+3
	G1	+4	+3	+7		B3	+2	−1	+1
	G2	+2	+1	+3		B4	+1	+2	+3
	Summation	+11	+4	+15		Summation	+2	+4	+6
01b-02	B1	−1	0	−1	02b-01	G1	+1	+2	+3
	B2	+5	−2	−3		B1	+1	−1	0
	G1	+4	0	+4		B2	+2	+2	+4
	G2	−2	+2	0		B3	−1	−2	−3
	Sum	+6	0	+6		Sum	+3	+1	+4
02a-04	B1	+1	−1	0	02a-01	G1	+1	0	+1
	B2	+6	+1	+7		G2	+2	0	+2
	G1	+1	0	+1		B1	0	−3	−3
	G2	+1	+1	+2		B2	+2	0	+2
	Sum	+9	+1	+10		Sum	+5	−3	+2
High-scoring group authority bid summation (times)		66	45	111	Low-scoring group authority bid summation (times)		64	34	98
Total number of authorities in high-scoring group		+26	+5	+31	Total number of authorities in the low-scoring group		+10	+2	+12

shows an example of a contested authority negotiation unit, where a PNU represents an authority relation node.

Contested authority PNU: Area of apartment toilets, kitchens, living rooms
B1: Let's say the toilet and the kitchen are 20 m^2 in total
G2: 20 m^2 in total?
G1: 10 m^2, otherwise there would be no more space to use
B1: So the kitchen is only 5 m^2 in total
G1: The toilet is a bit smaller
G1: The bedroom is 10 m^2 and the toilet is 5 m^2, that's a total of 15 m^2, how many bedrooms?
B1: That's only 1, right?
B1: Let's say the living room is 15
G1: It's a bit small
B1: That's all that's left, how much more does the living room take up, 60 m^2 in total, your living room takes up 50 m^2

In the last authority relation event, it can be concluded that B1 and G1 created a relevant dispute over the size of the apartment and proposed separate solutions to the problem. Some were accepted and some were rejected. Events like these were defined as contested intellectual authority. Similarly, authority relation PNUs identified throughout the CPS in the mathematics process were noted as authority relation nodes (Table 4.7).

The high-scoring group had 38 authority relation nodes, compared to 57 for the low-scoring group. The high-scoring group had more intellectual authority relation nodes than social authority relation nodes, while the low-scoring group had the opposite. The difference in the number of intellectual and social authority relation nodes between the high- and low-scoring groups was not significant. Also, based on the specific video recording text analysis, this study found that in terms of intellectual authority relations, shared authority was the most frequent, and concentrated authority was the least.

Table 4.7 Nodal table of intellectual authority relation and social authority relation for high- and low-scoring groups

	Group	Total authority relations node	Intellectual authority relation nodes	Social authority relation nodes
High-scoring group	01b-02	12	8	4
	01a-02	16	8	8
	02a-04	10	6	4
	Sum	38	22	16
Low scores Group	01a-01	25	11	14
	01b-01	9	4	5
	02b-05	23	11	12
	Sum	57	26	31

Table 4.8 Table of coverage of intellectual authority relation and social authority relations for high- and low-scoring groups

	Group	Total authority relations Coverage (%)	Intellectual authority relations coverage (%)	Social authority relations coverage (%)
High-scoring group	01b-02	69.61	47.71	21.90
	01a-02	73.82	50.48	23.34
	02a-04	71.66	58.33	13.33
	Average	71.70	52.17	19.53
Low-scoring Group	01a-01	61.40	30.92	30.48
	01b-01	70.76	47.93	22.83
	02b-05	81.10	32.2	50.88
	Average	71.08	37.02	34.73

The volume of discourse was counted using NVivo application to determine related coverage, referring to the ratio of conversations corresponding to nodes of authority relations to total conversations. Table 4.8 counts the ratio of group social authority relations and social authority relations under the group authority relations level code.

The average authority relation coverage for the high-scoring group was 71.7%, roughly the same as for the low-scoring group (71.08%). Both the high- and low-scoring groups had more intellectual than social authority relations. In terms of average coverage, both groups had more intellectual than social authority relations. The high-scoring group's higher average intellectual authority coverage was more pronounced than its higher coverage of social authority relations.

4.3.2.2 Analysis of Specific Changes in Different Authority Relation Nodes in High- and Low-Scoring Groups

Relevant authority relation nodes were counted in this study. The high-scoring group had a total of 38 authority relation nodes, of which 22 were intellectual authority relation nodes and 16 were social authority relation nodes. The low-scoring group had 57 authority relation nodes, of which 26 were intellectual authority relation nodes and 31 were social authority relation nodes. This section lists all authority relation nodes by names in chronological order. A line graph is used to illustrate the changes in their specific authority relations. The horizontal axis has 25 authority relation nodes. The vertical axis represents the seven authority relations: shared, contested, and concentrated intellectual authority relations; shared, contested, and concentrated social authority relations; and disbanded social authority relations. This diagram facilitates observation of the specific changes and continuity of different group authority relations, based on Table 4.7 (Figs. 4.4 and 4.5).

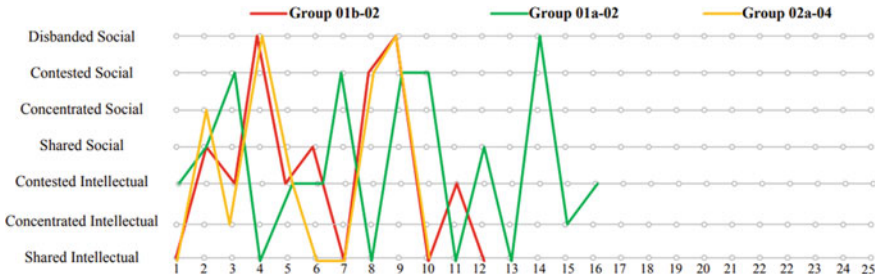

Fig. 4.4 Change in authority node for high-scoring group

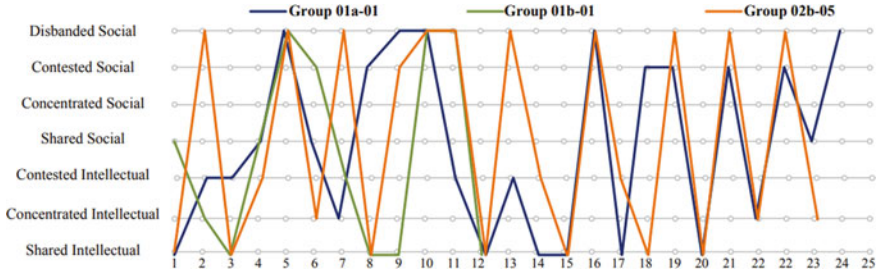

Fig. 4.5 Change in authority node for low-scoring group

From the previous section, it can be concluded that there were more intellectual than social authority relations in the high-scoring group. In terms of stability, the high-scoring group had less variation in authority relation nodes. In both the high- and low-scoring groups, shared intellectual authority nodes were the most common and concentrated authority was the least common throughout the 15-min mathematical CPS task.

In summation, there were 12 shared intellectual authority nodes in the high-scoring group and 17 in the low-scoring group. In terms of specific changes in group authority relations, the high-scoring group's 12 shared intellectual authority nodes produced contested and concentrated social authority relations without obvious tendency. However, the low-scoring group's shared intellectual authority relations were very likely to develop into disbanded social authority relations. In the 02b-05 group, there were five "shared intellectual authority relation → disbanded social authority relation" changes, compared to only two in the 01b-01 group with only seven authority relation changes. In terms of specific changes in their successive intellectual authority, contested authority and shared authority always alternated over time, with shared intellectual authority relations developing into contested intellectual authority relations and contested intellectual authority relations developing into shared intellectual and shared social authority relations. In terms of social authority relations, contested and disbanded social authority alternated. Contested social authority was prone to becoming disbanded social authority in the group.

4.3.2.3 Comparison of Shared, Contested, Concentrated, and Disbanded Authority Relations in the High- and Low-Scoring Groups

This study proposes a crossover between primary and secondary codes in the conceptual framework for group authority relations. There was conceptual crossover in the seven different group authority relations (shared intellectual and social authority relations belong to shared authority relations; concentrated intellectual and social authority relations belong to intellectual authority relations; and contested intellectual and social authority relations belong to contested authority relations), thereby forming four different group authority relations: shared authority relations, contested authority relations, concentrated authority relations, and disbanded authority relations (Fig. 4.6).

Shared authority was the highest of all authority relations for both the high- and low-scoring groups. Each group worked together to make intellectual or social contributions to solve the problem. Concentrated authority relation nodes were the least represented, accounting for only 4.22% of all nodes in the low-scoring group and reflecting that students shared authority during CPS in mathematics rather than clustering authority on the same person. The low-scoring group had a 23.27% rate of disbanded authority, second only to shared authority. The differences in the percentage of coverage of this component between the high- and low-scores groups were significant. Engaging in off-task activities significantly negatively influenced the low-scoring group's CPS outcomes.

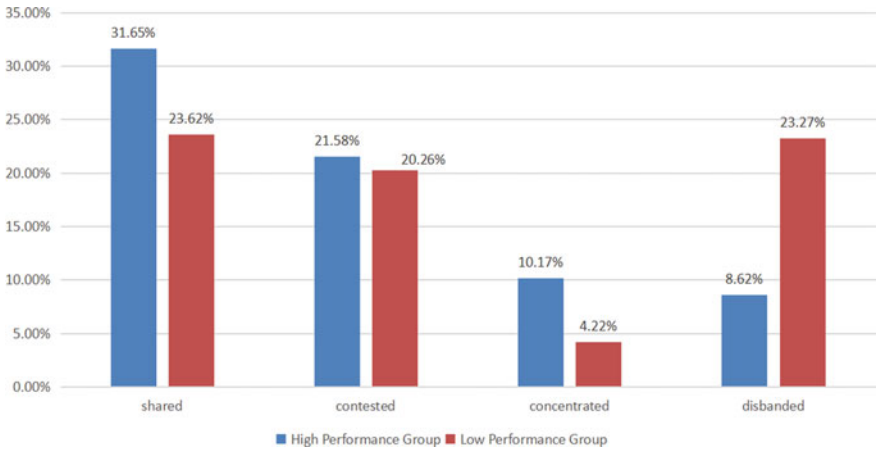

Fig. 4.6 Comparison of shared, contested, concentrated, and disbanded authority relations coverage for high- and low-scoring groups

4.3.2.4 Comparison of Shared, Contested, Concentrated, and Disbanded Authority Relations Under Intellectual and Social Authority in the High- and Low-Scoring Groups

Figure 4.7 shows a comparison of authority relation coverage for the seven different group authority relations under Level 2 coding.

After secondary coding, intellectual authority relations represented a very high proportion of shared authority relations, 28.08% in the high-scoring group and 17.79% in the low-scoring group. Shared, focused, and contested intellectual authority relations were higher in the high-scoring group than in the low-scoring group. There was little difference between the high- and low-scoring groups in contested intellectual authority. The low-scoring group had the highest disbanded social authority coverage. Disbanded authority relations are not conducive to high scores in CPS.

4.4 Conclusion

The study found that students in high-scoring groups had more individual authority bids and acceptances. The imbalances in individual authority predisposed to high scores in CPS outcomes.

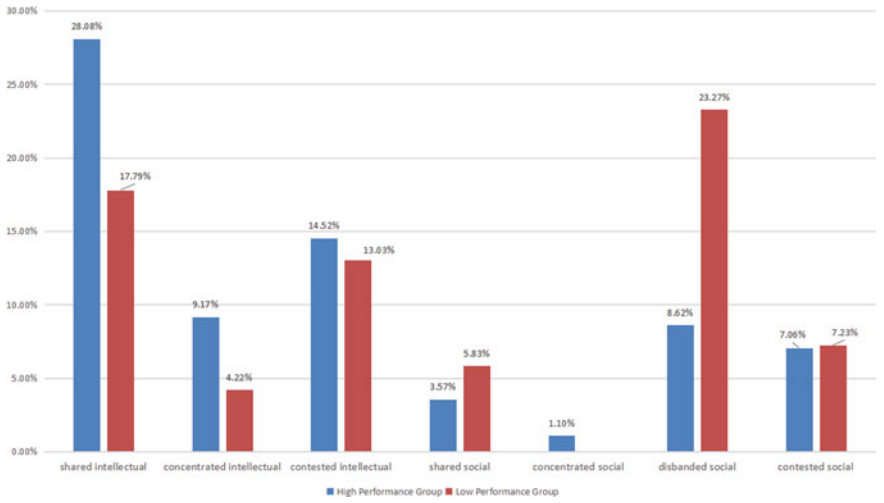

Fig. 4.7 Comparison of coverage of different authority relations under Level 2 coding for high- and low-scoring groups

4.4.1 Analysis of Individual Authority in CPS in Middle School Mathematics

4.4.1.1 More Individual Student Authority Bids and Acceptances in High-Scoring Groups Than in Low-Scoring Groups

The total number of bids was greater for the high-scoring group members, indicating they were more likely to contribute to CPS in mathematics than their low-scoring peers. Members made more intellectual than social authority bids in both groups. The authority summation was significantly higher in the high-scoring group, indicating its members' proposals were approved more often than in the low-scoring group.

Students in high-scoring groups were more likely to express and address task-related ideas, accept criticism, and respect other group members' opinions. The results indicated that timely feedback is an important feature of deep discussion. Group members' ability to give reasoned explanations for others' questions or timely feedback on others' suggestions facilitated deeper group discussion.

The unidirectional and bidirectional connections arising between the four factors are presented in Engle et al. (2014) influence model. The more verbal or eye contact is made, the more students enter the conversational layer and interaction space, making it easier to produce high-quality arguments and thereby increase authority. This is a cyclical process that ultimately affects CPS outcomes. In conjunction with this study, it can be concluded that a bold approach to the articulation of mathematical reasoning ideas and an openness to criticism are factors that promote good CPS outcomes in middle school mathematics.

4.4.1.2 Imbalance in Individual Authority Makes It Easy for CPS to Result in High Scores

Authority bids were accepted more often in the high-scoring group, with one or two students within each high-scoring group having significantly more individual intellectual or social authority than the other students. Authority bids were more balanced across group members in the low-scoring groups. For example, in the 02b-05 group, B4 made five intellectual authority bids but was rejected four times, while other group members made fewer authority bids. The low-scoring group was more balanced in terms of the frequency of authority bids being accepted and rejected. Other research on the role of social skills in middle school leadership in problem solving activities (Sun et al., 2017) indicates that groups that experience discussion achieve better problem solutions. In conjunction with the findings of this study, this shows that one to two leaders must be present in each group to lead other members through CPS and make it more effective.

4.4.2 Analysis of Group Authority Relations in CPS in Middle School Mathematics

The characteristics of group authority relations in CPS in mathematics in middle school were derived by counting the group authority relation nodes (authority nodes) and the discourse coverage of different authority relations in students' groups throughout the CPS, and then classified and interpreted in this study.

4.4.2.1 Shared Intellectual Authority Relations Are Most Conducive to Producing CPS in Mathematics Results

The specific variation in nodes showed that the number of shared intellectual authority nodes was highest across authority relations for both the high- and low-scoring groups. The frequency of intellectual authority relations was generally higher than for social authority relations in both the high- and low-scoring groups, so the group contributed intellectually or socially to problem solving. Video text analysis revealed that the shared intellectual authority relations period produced the most CPS results.

4.4.2.2 Least Occurrence of Concentrated Authority Relations in CPS in Middle School Mathematics

The lowest percentage of concentrated authority was found in the high-scoring group, with 1.1% of concentrated social authority. No concentrated social authority relations nodes appeared in the low-scoring group, reflecting that students shared authority during CPS in mathematics and teachers dispersed student authority during CPS.

4.4.2.3 Contested Authority as a Catalyst for Authority Change in CPS in Mathematics

Contested authority was second only to shared authority in both the high- and low-scoring groups, while contested intellectual authority was more predominant in terms of its secondary coding. This study has shown that contested and shared authority always alternated over time. Specifically, shared intellectual authority relations later developed into contested intellectual authority, while contested intellectual authority relations later developed into shared intellectual authority relations, facilitating CPS outcomes.

Cobb (1995) noted that argument for authority can be productive in the classroom, facilitating the more equitable distribution of authority and supporting different students' opportunities to learn. However, from an intellectual perspective, that can hinder the formation of CPS outcomes. Previous studies (Langer-Osuna, 2011; Langer-Osuna et al., 2020a) have shown that contested social authority may disrupt the process in ways that reshape the dynamics of cooperation. Taken together, the existing research and the present study's findings suggest that contested authority catalyses excellent CPS outcomes with contested intellectual authority and low CPS outcome scores with contested social authority under secondary coding.

4.4.2.4 More Stable Changes of Authority Relations Promote High Scores CPS in Mathematics Outcomes

The high-scoring group had more intellectual authority relation nodes than social authority relation nodes, while the opposite was true in the low-scoring group. The high-scoring group also had more stable authority relation performance, with more coverage of social authority relations than the low-scoring group. Shared intellectual authority relations were more unstable in the low-scoring group. The "shared intellectual authority relation → disbanded social authority relation" process changed several times during the task solving process. Non-engagement with on-task activities occurred after shared cooperation. Stable shared intellectual authority relations were most conducive to producing CPS in mathematics outcomes. As Langer-Osuna et al. (2020a) pointed out, shifts in social authority relations are more dynamic than shifts in perceived authority. Combined with the characteristics of group authority relations in this study, the high-scoring group had a higher proportion of intellectual authority relations.

4.4.2.5 Summary

The current research has revealed the characteristics of high/low-scoring groups. In the current research, more individual student authority bids and acceptances and more imbalance of authority within groups are found in the high-scoring groups, which insights teachers to consider the authority distributions within groups. Teachers can encourage students to more generate shared intellectual authority and contested authority in process of collaborative activities in classroom.

Appendix 4.1: Scores Rating Scale for CPS in Mathematics Outcomes in Middle School

Scoring dimensions	Scoring rules	Score (marks)
Overall requirements (2 marks)	The group agrees and submits a final copy of the problem solving results	2
	The group was not in agreement and there were several task list issue resolution results	0
If there are multiple task order resolution results, the first one (below the task order question) will be scored		
Apartment length and width (2 marks)	Complete labelling of the length and width of each room (with scale or side length units)	2
	Only the length and width of individual rooms are indicated or the length and width of each room are indicated in full but without a scale or side units	1
	No room lengths and widths are indicated or only the area of each room is indicated	0
Apartment size (2 marks)	Complete with the area or length and width of each room, the sum of the individual rooms is 60 m^2	2
	The area or length and width of each room are fully indicated and the sum of the individual rooms is not equal to 60 m^2	1
	Only individual room dimensions are indicated or no room dimensions are indicated to give a total area of 60 m^2	0
Number of rooms (1 mark)	It can be clearly seen that there are five rooms	1
	There are no 5 rooms or the picture is confusing so you can't tell there are 5 rooms	0
Apartment features (2 marks)	Complete labelling of room functions, e.g. kitchen, bathroom, etc	2
	Not fully labelled room features	1
	No room features marked	0
Apartment layout (1 mark)	Apartment layouts are sensible shapes: quadrilateral, triangular, circular, etc	1
	The layout of the Apartment is completely unreasonable and unrealistic and cannot be designed properly	0

Scoring criteria: The high- and low-scoring groups in this study were assessed in absolute terms. The current absolute curriculum assessment in our schools is a percentage system, with a passing mark of 60 (60% of the total score). This system has been used in China's schools for roughly a century, for several reasons. First, it was developed in connection with an educational reform movement that took place in China in the late nineteenth and early twentieth centuries. Through this reform movement, Chinese education began to move away from the shackles of feudal education and towards modern education, as a result of learning from European

and American education. Second, it was a result of convention. Therefore, as the maximum total score in this study was 10, a score greater than 6 identified the high-scoring group and less than 6 the low-scoring group.

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