Perspectives on Rethinking and Reforming Education

Yiming Cao *Editor*

Students' Collaborative Problem Solving in Mathematics Classrooms

An Empirical Study

Springer

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Editor

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To Professor David John Clarke

(1952–2020)

My best international collaborator, my friend

Preface

Human beings have never been isolated individuals; the development and continuity of humans are related to their interdependence with the external and internal environment. Human social activity promotes interpersonal interactions, and collaboration serves as a fundamental feature of human society. In *Cihai*, the word '合作' ('*collaboration*' in Chinese) is defined as creating or operating something together. The word '*collaborate*' in English is from *Latin* '*collaborare*' (with '*com-*' means '*with*', and '*laborare*' means '*to labor*'). It is obvious that the definitions of collaboration are consistent in East and West in general. From individuals to nations, collaboration is needed to address many common issues in human societies. Collaborative activities cover almost all aspects of modern human society: politics, economics, science and technology, education, medicine, etc., especially in the field of scientific research. With the development of science and technology, the time when researchers worked alone has gradually passed, and today's researchers need to join their wisdom and work together, which also raises new requirements for talent training.

Based on the development trend of human society, talents with collaboration ability are necessary in contemporary society. In 2002, the 21st Century Skills was launched in the United States, which introduced the 4C skills: Critical thinking and problem solving, Communication, Collaboration, and Creativity and innovation. In 2003, The United Nations Educational, Scientific and Cultural Organization (UNESCO) proposed and refined the Five Pillars of lifelong learning: Learning to know, Learning to do, Learning to live together, Learning to be, and Learning to change. In 2006, the European Commission proposed the Key Competences for Lifelong Learning, which was updated in 2018. In the latter version, collaboration was accentuated in Digital Competence as well as Personal, Social, and Learning to Learn Competence. The statements of teaching and learning approaches and environments also highlighted the irreplaceable place of collaborative learning. All of the above frameworks put emphasis on the necessity of mastering collaborative skills and abilities in the face of long-term (lifelong) development in the twenty-first century.

Collaborative learning was introduced and applied in Western classrooms earlier than that in China. In 2001, China's State Council issued *the Decision of the State*

Council on the Reform and Development of Basic Education, clearly encouraging collaborative learning. In 2017, the General Office of the CPC Central Committee and the General Office of the State Council issued *the Opinions on Deepening the Reform of the Education System*, which calls for the cultivation of four key competencies: cognitive ability, collaborative ability, innovative ability, and vocational ability. In 2019, the CPC Central Committee and State Council issued *the China Education Modernisation 2035*, regarding the development of world advanced quality education with Chinese characteristics as one of the important strategic tasks of modern education, stressing the cultivation of practical ability, collaborative ability, and innovation ability. The collaborative learning approach is the teaching model advocated by the Chinese education policies, and collaborative problem solving is a further requirement for the development of collaborative learning. However, the reform has also faced considerable challenges: due to the influence of traditional teaching principles, teaching has long been seen as the teacher-centred activity in Chinese classrooms, where the teachers play the dominant role, the systematic teaching knowledge by the teacher serving as the dominant and efficient way of teaching. PISA 2015 assessed collaborative problem-solving competency for the first time. Chinese students' performance was at an intermediate level, showing a clear deficit compared to their high performance in mathematics and science tests. The mathematics curriculum is an important platform for collaborative learning. Since the twenty-first century, the importance of collaborative learning has been underlined in all of China's mathematics curriculum standards. The newly promulgated Mathematics Curriculum Standards for Compulsory Education (2022 Edition) pays special attention to the heuristic, inquiry-based, participatory and interactive teaching and learning, cross-curricular thematic learning, and project-based learning. Collaboration can promote the implementation of these teaching and learning activities effectively.

One of the key issues in my 15-year collaborative research with Professor David Clarke of the University of Melbourne is to systematically explore collaborative problem solving. In 2005, I met Professor David Clarke of the University of Melbourne through the contact of Professor Rongjin Huang (now at Middle Tennessee State University), who was then teaching at East China Normal University. My main research interest at that time was mathematics classrooms. Professor David Clarke has been leading the Learner's Perspective Study for over five years. Our research interests are highly matched. I soon became a new member of the research team and undertook a series of collaborative studies over fifteen years, including the Alignment Project, The Lexicon Project: Analysing Pedagogical Naming Systems from Different Cultures to Reconceptualise Classroom Practice and Advance Educational Theory, Learning from Lessons ...... Professor David Clarke has presented our research in major international academic presentations and called my team as his best international partners.

This book is supported by two joint projects of me and Professor David Clarke: the Australian National Innovation Fund project Social Essentials of Learning: Collaborative Learning in Australia & China (Project number: DP170102541) and the Chinese National Social Science Foundation's Thirteenth Five-Year Plan 2018

General Project in Education: Cognitive and Social Interactions and Their Relationships in Collaborative Problem Solving for Secondary School Students. (Project number: BHA180157). Our two teams worked together on the proposal, discussion, design, data collection, and preliminary analysis of the data. Unfortunately, Professor Clarke passed away on 25 January 2020. All of the authors of this book and I are honoured to complete this work, which is one of the 'academic legacies' of Professor Clarke. We deeply miss Professor Clarke and the time we spent working with him.

The constructs and design of the whole book were conceived by Yiming Cao. Each chapter was developed based on discussions, investigation, collaboration, and revision, and finally edited by Yiming Cao. Rangmei Li and Yixuan Liu assisted with editing. Especially thank Dr. Yi Wang, Dr. Shu Zhang, Dr. Meng Guo, Mr. Tommy Tanu Wijaya, and Dr. Xiaoying Chen for proofreading several chapters.

Although we invested great effort into the book, there are inevitably some limitations in our research. We are sincerely open to hearing any criticism.

Beijing, China Prof. Yiming Cao

Contents

Chapter 1
Research on Collaborative Problem Solving Teaching in a Secondary School Mathematics Classroom

Yiming Cao

1.1 Introduction

At this time, the socio-economic background of globalization and information technology is presently increasing. Collaboration among team members is crucial to the success of groups, families, corporations, public institutions, organizations, and government agencies (OECD, 2017). Collaboration and communication skills are also considered as indispensable basic skills for future citizens' overall quality. At all societal levels, breakthroughs on many important issues are often due to teamwork and concerted action and are unable to be solely achieved by individual battles. To help the new generation adapt better to the profound changes in the future society, Chinese governments and international organizations have reportedly ordered the strengthening of basic education, emphasized the importance of inculcating students' collaborative problem solving abilities, and realized classroom transformation from subject-teaching interdisciplinary integration. In recent years, research has been conducted on teaching collaborative problem solving, with the support of the National Social Science Foundation of China and the Australian Research Council Innovation Fund.

1.1.1 Research Background

UNESCO (1996) proposed that school education in the twenty-first century should focus on four main goals, i.e., studying to learn, perform, unite, and live. This shows that collaboration and communication with others is the basic condition for learning

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to unite, becoming a new requirement for educating people. To deeply understand the primary skills students should possess in this era, the United States officially launched the 21st-Century Skills Research Project in 2002 and proposed the 4C skills, namely critical thinking and problem solving, communication, collaboration, and creativity and innovation. This proves that collaboration is considered the most important student ability development direction within American education in the twenty-first century (ZHANG, 2012). Furthermore, the Education 2030 project proposed by the Organization for Economic Co-operation and Development (OECD) has attracted much attention in the learning sector, with collaboration and problem solving being included in the predicted 28 competencies. This states that students should participate in teamwork and explore strategies and methods to solve mathematical problems (Cao et al., 2020). A complex and changeable social environment is also responsible for students' conduct, understanding of basic knowledge, and acquiring disciplinary thinking methods. This implies that the ability to communicate effectively and solve comprehensive problems is an important twenty-first-century school education goal.

The cultivation of collaborative problem solving ability has become more important in curriculum reform and practices in China since the beginning of the century, with the achievement of specific results. However, the collaborative solution ability of 15-year-old Chinese students in Beijing, Shanghai, Jiangsu, and Guangdong was lower than the OECD average (496/500) according to the PISA 2015 Collaboration Problem Solving Report (2017). China was ranked 20th out of 51 countries, with no correlation with reading, science, and math literacy performance (Wang, 2018). Despite this, this sample did not fully represent the country's overall situation, as the national level was far below these performance stages in many regions. It also reflected the poor ability of mainland students, leading to severe issues for collaborative problem solving learning in China. This was due to the emphasis of the existing traditional teaching methods on teacher explanations while neglecting students' exploration, specifically the main orientation of subsequent education goals. These observations led to an overemphasis on independent paper and pencil test scores and exam competitions, neglecting peer collaboration and practical innovation. Therefore, teachers only focus on the performance of high-interest exams (high school and college entrance exams), with other factors being closely related.

Based on practice, collaborative problem solving is an important method for cultivating innovative talents. This is due to the implementation of the "Lide and cultivates people" requirements and the development of students' core literacy. This method has reportedly been considered in Chinese educational research and teaching practices in recent years. On February 23, 2019, the Central Committee of the Chinese Communist Party and the State Council issued *Education Modernization of China 2035,* based on developing world-class, high-quality learning. This program was launched with Chinese characteristics as an important strategic learning task. It also exhibited the necessity to innovate talent training, implement teaching techniques (heuristic, inquiry, participatory, and collaborative), and focus on cultivating students' innovative spirit and practical and collaborative abilities (Party and Council, 2019a). The classroom is also the main front of future talent training, indicating the urgency to improve the quality of direct teaching. On June 23, 2019, the Chinese government subsequently issued the *Opinions on Deepening Education and Teaching Reform, as well as Comprehensively Improving the Quality of Compulsory Education*, explaining the need to optimize teaching methods. It also evaluated the focus on mutual, heuristic, interactive, and inquiry-based teachings, explored the comprehensive learning of subject-based curriculum, and implemented research-based, project-based, and collaborative education (Party and Council, 2019b).

As an important subject in the basic education stage, mathematics plays an important role in cultivating students' collaborative consciousness and problem solving ability. Since 2000, the evolution of key competencies in the PISA mathematics assessment framework showed that communication has always been an important skill. This states that students must be able to appropriately use symbols, language, etc., to express their thoughts and interact with others. According to the New PISA (2021), communication was one of the eight 21st-Century Skills in the Mathematical Literacy Assessment Framework (Sun et al., 2019). With the reform of the Chinese basic education curriculum, the cultivation of collaborative problem solving ability has gradually penetrated different levels of mathematics education. This requires students to learn the methods of cooperating with others and experience the process of collaborative communication and problem solving. Teachers' educational concepts are also constantly improved, with students' learning methods in the classroom being gradually diversified. Besides receptive learning, hands-on practice, independent exploration, and collaborative exchanges have subsequently become important mathematical learning methods, nevertheless, many collaborative teachings are only superficial and mere formalities in the classroom (Wang and Wang, 2022). This is partly because teachers' organization and implementation strategies are still inadequate due to the lack of collaborative knowledge, even when students' discussions are found to be very lively (Kirschner et al., 2006). Therefore, more students should participate in the collaborative process and improve learning effectiveness, which has always troubled most front-line mathematics teachers. Although many teachers conceptually recognize this teaching method, it is still rarely used in daily learning due to the pressure of periodical task completion. This explains that teachers only attempt to develop this method for a few minutes when classes open, leading to unsatisfactory and maximal teaching effects (Wang and Wang, 2022).

Based on the international educational background, learning policies, and practical needs, the characteristics of collaborative research and the problem solving process of Chinese middle school students should be deeply investigated. The collaborative problems in real mathematics classrooms should also be analysed, with the methods by which teachers adequately play the roles of educators, guides, and collaborators being explored. To optimize the interaction process in the mathematics classroom environment and cultivate students' problem solving ability, the National Innovation Fund Project (co-chaired by Professors Cao Yiming and David Clark of Beijing Normal University, China and the University of Melbourne, Australia) launched a program known as "Social Elements in Learning: An Experimental Investigation of Collaborative Problem solving and Knowledge Construction in Chinese-Australian Mathematics Classrooms," which was approved by the Australian Research Council (ARC) in January 2017. This was based on carrying out the "Empirical Research on Cognitive and Social Interactions, and Their Relationships in Collaborative Problem solving Abilities of Middle School Students." For secondary school mathematics classrooms, the characteristics of cognitive and social interactions should be explored, with the teaching of collaborative problem solving being analysed. This is based on promoting the cultivation of students' collaborative problem solving ability and improving the quality of mathematics classroom teaching.

1.1.2 Research Problem

How Did the Learning Activities Occur in the Problem Solving Process?

This problem is the basis of the whole research, with the effective implementation of teaching only being observed after a specific degree of in-depth analysis and knowledge of students' learning process. This explains that the psychological mechanism of students' learning process is complex, with problem solving being a high-order thinking and innovative activity affected by many aspects. Much existing research has reportedly been analysed through two aspects, namely cognitive and non-cognitive factors and internal and external variables (Hesse et al., 2015; OECD, 2017). Meanwhile, collaboration is mainly a learning method in teaching scenarios, where students and teachers interact with each other to achieve a common purpose. This process requires the guidance and support of teachers and the participation of families and society. Although the quality of collaborative problem solving depended on the quality of group members' interactions, much research still proved that not all communications positively impacted student development (Johnson and Johnson, 2009). Teachers' intervention guidance also has an important impact on student communication (van Leeuwen and Janssen, 2019a, 2019b), with the main question of the research based on the methods by which the social and cognitive interactions were optimized in the collaborative problem solving among middle school students. This cultivates problem solving ability and promotes classroom learning through cognitive and social interactions and teacher guidance. Therefore, the main problem was divided into the following three sub-problems:

(1) What are the characteristics of cognitive interaction and knowledge construction in the collaborative problem solving activities of middle school students?
(2) What are the characteristics of social interaction and its changing process in the collaborative problem solving activities of middle school students?
(3) How do teachers guide students to optimize efficiency and facilitate classroom learning in the collaborative problem solving process?

Based on these research questions, there were two concerned modules, namely theoretical construction and empirical research, which had a total of nine topics as follows:

(1) The analytical framework and behavioural process of knowledge construction in collaborative mathematics problem solving for middle school students,

(2) The theoretical framework and qualitative research on middle school student's participation in collaborative mathematics problem solving,
(3) The authority relationship of middle school students in collaborative problem solving in mathematics classrooms,
(4) The characteristics of mathematical communication in the collaborative problem solving of middle school students,
(5) The conflict discourse among secondary school students in collaborative problem solving in mathematics classrooms,
(6) Students' interaction in the middle school's peer collaborative mathematics problem solving,
(7) Teacher Noticing in Collaborative Problem solving in secondary mathematics classrooms,
(8) Teacher Intervention in Collaborative Problem Solving in secondary mathematics classrooms,
(9) The ability evaluation in Collaborative Problem Solving in secondary mathematics classrooms.

1.2 Literature Review

As the core of human learning and thinking, problem solving symbolizes the cognition of mankind and has reportedly been extended to many disciplines such as science, technology, and engineering. Since the 1980s, team-based problem solving has highly been considered, with national education strongly recommending the utilization of group classroom works for learning activities. Meanwhile, different people have increasingly emphasized sharing and win–win results through teamwork in the twenty-first century, with interpersonal and problem solving skills subsequently obtaining unprecedented attention.

Collaborative problem solving ability is used to effectively participate in team activities, establish common understanding, determine solutions, and obtain collaborative knowledge to solve problems. This ability includes two elements, namely "collaboration" and "problem solving," as it is also one of the key capabilities necessary for the lifelong development of talents in the twenty-first century. Based on the collaboration level, the social interaction between group members is emphasized, with mutual understanding, team organization, and consensus established, maintained, and achieved through effective communication. However, the problem solving level highlights the cognitive interaction of a task, reflecting the psychological attributes of human beings, including a series of processes such as information extraction, exploration and understanding, and plan execution. In PISA 2015, collaborative problem solving was defined as the ability of an individual to effectively participate in a team of two or more members by sharing understanding, achieving consensus, determining solutions, and uniting collaborative knowledge, skills, and actions to solve problems (OECD, 2017). Based on ATC21S, the team's activities were wholly highlighted, with collaborative problem solving being defined as "a common activity where group members perform a series of steps to complete the transition from a realistic state to an ideal goal" (Zhang et al., 2017). Irrespective of the definition, this ability is still a kind of socialization process, where individuals are found to obtain a high level of cognition through interaction with the social environment. This explains that collaboration and problem solving are not separated, as both are found to often integrate and promote each other. Through the effective collaboration of group members, the complex problem solving transformation process from a realistic state to an ideal goal was also found to be achieved. However, the process is not simply the application of knowledge in a specific discipline or field.

The "input-process-output" model is the basis of group collaborative learning, with the interaction process being closely related to the collaborative effectiveness, as an intermediate link between input and output (Tempelaar, 2004). When the group has external support conditions, its internal interaction process plays an important role in improving teamwork performance. In line with the composition of group members, the collaboration process shows more complicated, multi-dimensional, and dynamic interweaving. Furthermore, the interaction process is more conducive to understanding the essence of group collaboration, for effective guidance towards appropriate promotion (Johnson and Johnson, 2009). Collaborative learning has been widely accepted and drawn attention by academics since 1980s (Bruffee, 1984; Johnson and Johnson, 2009). Decades ago, the shared or situated cognition approach emphasizes social structures where interaction takes place. In this approach, the environment is considered as an integral part of cognitive activities, and knowledge is seen as transferred from one to another, rather, knowledge is constructed through interactions among collaborators (Lai, 2011).

The cognitive interaction research also had a macro description and micro-perspective investigation (Peng and Liu, 2009), with public constructivism stating that knowledge was socially constructed through the collaborative communication between the learners and the social environment. This was due to cognitive support and conflict being considered as two important aspects of psychological interaction. Team members scaffolded each other, argued and negotiated, questioned explanations, understood one another, and constructed interpretations to achieve high levels of cognitive processing and high-quality decisions and practices (Palincsar, 1998). In addition, cognition is the basis of learning, through the exploration of the psychological interaction and advanced thinking process among team members. This had great significance in collaborative problem solving, based on understanding the learning process. This was due to grasping the nature of learning interaction and promoting efficient education.

In the 1990s, the important role of context-based negotiation and renegotiation in constructing classroom knowledge was emphasized, where the research of Vygotsky stated that students' communication highly promoted the development of individual psychological functions when participating in a task with peers having greater abilities (Chan and Clarke, 2017). This reflected that the process of students' learning was to realize the construction of knowledge, through interaction with the classroom environment and participation in social activities. Social interaction also plays a key role in knowledge construction and public classroom teaching, as interdependent social interaction between individuals or groups through information dissemination meets several specific needs. The interaction mainly includes five basic elements—subject, carrier, goal, norm, and environment (Zhang, 2012). However, no research has been observed on social interaction in classroom teaching.

Based on the relationship between cognitive and social interaction, previous reports often attached student communication's social elements to its psychological research elements which subsequently emphasized the complementarity of social and cognitive elements (Cobb and Bowers, 1999), with students opting for the public actions having immediate interaction inducements among group members. In these activities, every student within each group constantly focused on the impact of their behaviours on others and themselves, due to being an important scaffold for reducing cognitive load and promoting classroom knowledge construction. From collaborative problem solving, each involved member expressed personal ideas in their discourse systems, communicated and interacted with other students, and involved the cognitive and social attributes of groups and individuals in the negotiation process. The learning method also focused on the dynamic interplay between problem solving and collaboration, emphasizing the appropriate integration of collaborative social literacy at the individual level. This showed that collaborative problem solving activities supported effective teaching in the classroom, due to being a reliable research hotspot. Presently, few researchers have deeply investigated the classroom field, explored the interactive nature of collaborative problem solving students, and understood the occurrence mode of teamwork learning. During students' collaborative learning, the essential characteristics and relationship between cognitive and social interaction provided theoretical support for teaching practice and measurement evaluation.

Moreover, teachers' organization and implementation of group collaboration directly affected students' participation and interaction effects. According to van Leeuwen (2019), 66 quantitative and qualitative researches on collaborative learning were synthesized to examine the relationship between teachers' instructional strategies and students' collaborative processes and effects (Leeuwen and Janssen, 2019a, 2019b). This proved that teachers focused on students' problem solving strategical feedbacks, helped them plan task progress, and coordinated group collaboration and member participation, positively impacting collaborative processes and results. Therefore, teachers played an important guiding role in group collaborative learning, with proper guidance subsequently promoting the smooth development of collaborative activities. Collaborative teaching also presented different characteristics from traditional classrooms due to their being more complex and teachers' roles highly multiple. This subsequently led to higher requirements for teachers' professional ability. Most research on Chinese collaborative learning presently focuses on students, leading to less consideration of teachers' roles, where a lack of effective empirical analysis has been observed. Based on inefficient collaboration status, there were still many challenges to how teachers can play better roles as educators, organizers, and facilitators.

Cultivating students' collaborative problem solving ability is an inevitable requirement for implementing the strategy of rejuvenating and strengthening the country through science, technology, and talent. As the core of teaching, the classroom is an important place for cultivating middle school students' collaborative problem solving ability. This research focuses on secondary school mathematics classroom teaching and performs collaborative learning based on mathematical problem solving. It also deeply explores the nature of teacher–student and student–student communication, optimizes middle school students' learning in the classroom, evaluates cognitive and social interactions, and creates an autonomous, collaborative, inquiry-based educational environment. In addition, the promotion, innovation, and cultivation of transformation, teaching methods, and students' collaborative problem solving ability are the core goals of project research and practices.

A combined qualitative and quantitative method was adopted in this research, where collaboratively solvable mathematical tasks were developed to explore the cognitive and social interaction characteristics of collaborative problem solving. This was carried out through several case research and quantitative analyses.

1.3 Methodology

1.3.1 Methods

1.3.1.1 Literature Review and Expert Discussion

By analysing relevant reports in the literature, frontier trends were accurately obtained to determine the plan and questions. Thirty experts with in-depth knowledge of problem solving, collaborative learning, and STEM education were selected, accompanied by the determination of 50 teachers interested in being project participants. Subsequent in-depth evaluations of the problems and analytical frameworks were conducted with the urgency to be solved in collaborative problem solving. This was based on three different backgrounds, namely science, society, and occupation. Suitable mathematical tasks were also designed for middle school students, to carry out collaborative problem solving activities.

1.3.1.2 Classroom Recording

Multi-camera tracking shooting mode was used in the teaching classes, including close-up images of teachers and groups and a panoramic view of students. Two wireless microphones were placed in each group, with teachers utilizing one to ensure clear and complete speech information.

1.3.1.3 Teacher–Student Interviews

Semi-structured interviews were conducted to address the research questions and teachers' understanding of collaborative problem solving before class activities. After class, students were found to have an in-depth understanding of teachers' intuitive feelings on teaching, such as their thoughts and reflections on classroom form, collaborative tasks, and students' performance. Subsequently, students' first-hand information regarding the collaborative problem solving process (students' feelings, task-answering situations, etc.) was obtained, which was a valuable supplement to the analysis of video and data.

1.3.1.4 Research Comparison

An in-depth analysis and comparison of the teacher–student, student–student interaction processes were conducted and captured by the video. This showed that the problem solving strategies and methods summarized and extracted the characteristics of cognitive and social interactions, and explored a society for the efficient collaborative solution process.

1.3.2 Research Design

1.3.2.1 The Tasks

With the support of corresponding experts and teachers, we developed a dozen of tasks suitable for collaborative problem solving. Nevertheless, the tasks involved in the current research are mostly open-ended tasks (except Task 3 in the Appendix), which have multiple solutions and methods for the solutions. Open-ended tasks with a low floor to get started and a high ceiling to achievement are suitable for collaboration (Li et al., 2022). The tasks in the current study have symbols and graphic elements consistent with the mathematics curriculum as well as connections with social context.

For example, the "Xiao Ming's apartment" task led to the following specific problem:

> *Xiao Ming's apartment has an area of 60 square metres. There are five rooms in Xiao Ming's apartment. Draw a possible plan of Xiao Ming's apartment. Label all rooms and show the dimensions (length and width) of each room.*

This task was closely related to the students' actual lives, requiring them to use existing experiences to solve problems. The problematic task was open-ended and situational and had many possible answers with diverse solution methods. This problem was designed from the geometry content in Chap. 5 in the initial volume of the seventh grade PG (Preliminary Geometry) textbook, People's Education Edition.

Completing the problem required group collaboration of four to six students, with the task being to apply the basic principles of collaborative problem solving design—situational, challenging, ideological, and diverse—to analyse the power of teamwork and strongly support cultivating students' collaborative problem solving ability (Li and Cao, 2019).

1.3.2.2 Participants

Using purposive sampling, two to four schools representing local average levels were selected in Beijing, Jiangsu, Guangdong, Sichuan, Shanxi, and other provinces/cities. Two to three classes were collected in each school that included a total of 30 classrooms and 1200 students. The sample of teachers has various profiles in gender, age, working experience, and title. All teachers and students have experience in collaborative mathematics learning. Each teacher taught two classes at the same time, and they are similar in size and mathematical achievement on average. All teachers implemented two types of instruction—minimal and structured—in two classes. The structured model provided scaffolding for students' metacognition skills, although the direct provision of mathematical facts or problem solving procedures differed. The minimal type required little performance from teachers. Under the structured intervention, three forms of teacher–student interaction were permitted: (1) Mathematical, (2) Social-Mathematical, and (3) Social. When teachers evaluated students' performance, their main goal was to promote students' working effectively with their peers and groups without directly providing them with the steps to complete the mathematic tasks. This highlighted that students were encouraged to explain and illustrate their ideas and points through diagrams or tables. Besides introducing the task before commencement, teachers should avoid teaching the whole class or assessing the correctness of students' answers/methods.

1.3.2.3 Role of Researchers

All researchers get permission for collecting and analysing data from headmasters, teachers, and student's parents. The researchers in this research had no conflict of interest. Although researchers designed tasks and processes of lessons, all teachers could make their own adaptions before and during lessons and express their own ideas and feelings during the interviews.

1.3.3 Data Collection

The process of data collection is shown in Fig. 1.1, which contains five stages. Before formal data collection, the process has been tested and validated in pilot studies. The experts and teachers gave suggestions for optimizing processes after pilot studies.

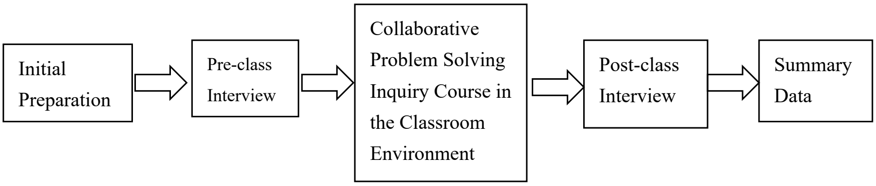

Fig. 1.1 Flow chart of research on collaborative problem solving among middle school students in the classroom environment

Each teacher in this study carried out the designs to two parallel classes, with minimal or structured teacher interventions. The students' task list was distributed to the classroom teacher during the preparation stage. It asked teachers to consider and evaluate the procedural steps for categorizing students into groups. Before the lessons, pre-class interviews were conducted to understand teachers' and students' familiarity with collaborative problem solving activities and participation. This was accompanied by placing cameras and microphones in appropriate positions, based on the group situation and seat layout (see Fig. 1.2). The camera's shooting angle is key to capturing students' interaction process in the classroom. Based on students' seats, each group's cameras were placed so that all members could be captured without hindering teachers' normal movement and actions. The classrooms in current study are prepared ahead for capturing teachers' and students' interaction by video recording.

After completion of the preparatory stage, the students' mathematical collaborative problem solving activities inquiry class was officially implemented. After a few prior plot studies and taking into account teachers' feedback, researchers worked together to decide on a collaborative problem solving classroom organization process, as shown in Table 1.1. The process contains four sections: Independent thinking, Peer collaboration (two-students groups), Group collaboration (four-students groups), and Summarization. Durations in Table 1 are estimated by previous experience in prior studies, and they are provided for reference to teachers. Teachers were required to carry out designed mathematical collaborative problem solving inquiry courses in their classes.

Audio and video recordings captured students' interactions and teachers' behaviours to ensure that the most realistic student–student and teacher–student interaction models became the main focus of data generation. After the course, post-class interviews were conducted with teachers and individual students to understand their feelings, experiences, and strategies regarding this inquiry class. The final data set obtained included all video and audio data of teachers, the group as a unit, the whole class, the recorded teacher and student interviews, and the task lists, including independent thinking and peer and group collaborations.

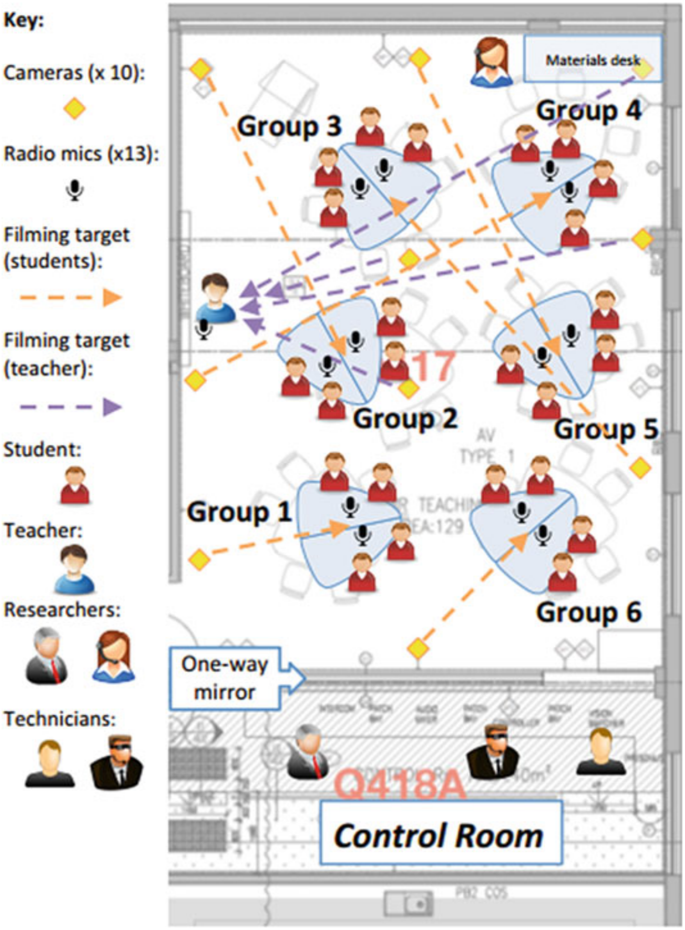

Fig. 1.2 Schematic diagram of collaborative problem solving teaching grouping (Chan et al., 2018)

1.3.4 *Data Analysis*

Considering that the research questions and focuses varied in different aspects, all authors in the book developed their own methods for data analysis, including analysing task sheets, interactive dialogues within student–student interaction or teacher–student interaction, and interviews in quantitative, qualitative, and mixed-method manners. Multiple software tools, such as NVivo 11, MAXQDA 2018, and SPSS 22.0 were used to restore students' collaborative problem solving process in the classrooms, comprehend what teachers and students assumed in the collaborative procedure, and ensure valid data analysis. One of the key features of the research is *Triangulation,* which is a method to increase the credibility and validity of research findings (Cohen & Crabtree, 2008). Through enriched data from different resources, researchers in this research could explore, analyse, and clarify teachers'/students' behaviour and its reasons.

1.4 The Main Research Results

There are three special focuses in this research: (1) "the path and performance of collaborative knowledge construction for middle school students," (2) "the social interaction mode in middle school students, based on the collaborative problem solving in mathematics," and (3) "teachers intervention strategies for middle school students' collaborative problem solving abilities." A combined qualitative and quantitative model was designed through video recording, teacher–student interviews, physical collection, and other methods to obtain materials and data from multiple perspectives. The analyses were based on Chinese middle school students' collaborative problem solving practices and included macro- and micro-level statistical inference analyses to deeply depict the nature of students' social and cognitive interactions.

1.4.1 Construction of Design-Based Mathematical Collaborative Problem Solving Teaching Model

After repeated evaluations by several peer experts and front-line teachers, 12 suitable mathematical tasks were eventually designed and developed based on three different backgrounds—science, society, and occupation. These mathematical tasks utilized were as follows: (1) open-ended, where multiple solutions enabled different correct answers, (2) symbols and graphic elements, and (3) association with situations outside the classroom.

A design-based teaching model for middle school students was also developed in this research, with a teacher arrangement time in the classroom of approximately 45 min, as shown in Table 1.1. In the class, students completed the prescribed math tasks based on different organizational forms, including independent thinking (one student), peer (two students), and group (four to six students) collaborations. From the collaborative tasks, limited mathematics tools were also allocated to each group, creating and establishing resource interdependence and a positive relationship among the members. For example, a group only provided a task list, without guaranteeing everyone the required tools, so members would have to combine resources to achieve the group goal.

Collaborative learning groups often include four people. When the total number of students in the class is not a multiple of four, individual groups should be selected based on the specific situation. This should be accompanied by increasing the number of people, subsequently allowing groups of approximately six individuals. As the

Table 1.1 The organizational arrangement of collaborative problem solving teaching

Section	Durations	Teaching suggestions
Section 1.1	1 min	Introduction to Task 1 (Independent Thinking)
	10 min	Independent task Tasks A and B (two-sided A4 paper, one for each student), pen, ruler, calculator (two for each desk)
		In the end, instruct students to place their answer sheets in Envelope 1
Section 1.2	1 min	Introduction to Task 2 (Peer Collaboration)
	10 min	companion task
		Tasks C and D (A4 paper printed on both sides, one shared among peers), pen, ruler, calculator (two for each desk)
Section 1.3	1 min	Introduction to Task 3 (Group Collaboration)
	20 min	group task Tasks E and F (double-sided A4 paper, only one per group), pen, ruler, calculator (two per table), square block (six per table)
		In the end, students should return their answer sheets to Envelope 3, with brief discussions carried out on their respective solutions
Section 1.4	2 min	Teacher conclusions and comments

group size increases, the available resources also theoretically elevate to facilitate student achievement. However, the interaction between group members becomes more complex at this time, with students' collaborative skills being highly required. This leads to the continuous occurrence of problems based on a loss of responsibility and inadequate cohesion. As such, the group size should not be too large. Given the difficulty of the collaboration task in this study, groups of four to six people were appropriate.

1.4.2 Cognitive and Social Interaction During Collaborative Problem Solving in Mathematics Classrooms

We developed our current research on students' cognitive and social interaction during collaborative problem solving based on previous research. Based on Sun et al. (2020), in-depth research was conducted into the middle school knowledge construction process in collaborative mathematical problem solving, including their collaborative knowledge construction paths, characteristics, levels, and performances (Sun, 2020). The results stated that the students' overall participation in collaborative knowledge construction was good, due to its being the main focus of mathematical problem solving. They also experienced consensus formation, information sharing and understanding, disagreement discovery and clarification, content negotiation and co-construction, and verification and adjustment. Six stages were observed in the evolution of multiple perspectives, including integration, fission, mutation, etc.

Based on these results, a new perspective was provided to comprehend students' collaborative learning quality and improve the efficiency and quality of collaborative problem solving.

Core social interaction issues in students' collaboration process for mathematical problem solving were subsequently explored, such as (1) the characteristics of peer collaboration and interaction, (2) mathematical communication, (3) authoritative relationship, (4) the type and structure of conflict discourse, and (5) the process of learners' participation. These issues were based on producing in students' collaborative learning the elements and characteristics of social interaction needed to optimize the communication process and cultivate students' collaborative ability. Based on Zhang et al. (2021), a positioning theory was initially used to construct a framework for middle school students' participation in collaborative problem solving. A "negotiation event coding and chain analyses-interactive role position coding and change process-story line construction" was also developed as the main experimental path (Zhang et al., 2021). This explored group members' interaction roles and the evolution of the negotiation topic during the collaboration process based on micro-case research. The results showed that student's participation in the negotiation events included initiation, response, evaluation, non-interaction, and non-speaking, and that their role patterns changed. These were divided into three role models—single, combined, and transformed—that provided theoretical guidance for understanding students' social participation path within the collaborative problem solving process. Several experts have conducted empirical research on teachers' intervention strategies in students' collaboration process.

Optimizing middle school students' cognitive and social interaction has become a hot topic in collaborative learning research. This study, based on secondary school mathematics classrooms, emerged from the multidisciplinary cognitive and social psychology perspectives. The core topic of collaborative problem solving was evaluated using empirical methods, including (i) the mathematical collaborative teaching model, (ii) the path and performance of collaborative knowledge construction for middle school students, and (iii) the model and characteristics of social interactions. Moreover, an in-depth analysis of teachers' role in student collaboration was carried out to excavate interactive elements, help achieve efficient mathematical teaching, and improve middle school students' collaborative problem solving ability.

In the series of research on cognitive and social interaction, the authors focus on topics including collaborative knowledge building (Chap. 3), negotiation discourse (Chap. 4), social authority (Chap. 5), opportunity to learn (Chap. 6), mathematical communication (Chap. 7), conflict discourse (Chap. 8), and peer student interaction (Chap. 9).

1.4.3 Teacher Noticing and Guidance During Collaborative Problem Solving

The current research also focuses on the teacher's role and behaviour in collaborative problem solving, which has attracted broad interest (Webb, 2009; van Leeuwen and Janssen, 2019a, 2019b). Based on (Dong et al., 2013), a case study was conducted on teachers' intervention activities in secondary school mathematical collaborative learning classrooms. This depicted that Chinese mathematics teachers mainly judged the group process through observation, often aiming their intervention objects at individuals rather than the entire group. Their intervention content also lacked guidance and evaluation of collaboration and communication, which provide many inspirations for and reflections on collaborative teaching in Chinese mathematics classrooms. To have a deeper understanding of teachers' assessment of groups, (Li et al., 2021) used eye-tracking technology to analyse the attention of pre-service mathematics educators. The results confirmed that pre-service teachers focused on students' problem solving ideas and outcomes, due to their being attracted to learners who spoke most frequently. However, sufficient focus was not fixed on those with low participation. The analysis of mathematics teachers' attention to students' collaboration processes also helped effectively explain educators' behavioural intentions and their causes from a cognitive psychology perspective.

Experienced-pre-service comparison of teacher noticing during students' collaborative problem solving is done through eye-tracking in Chap. 10, where find that experienced teachers distribute their attention more evenly and notice more important facets of group teaching. Teacher intervention during collaborative problem solving is evaluated and analysed in Chap. 11, where find teacher intervention is mostly effective, but less effective for heterogeneous groups. Authors in Chap. 11 also discussed control and equality of teacher intervention and suggestions are also given for fostering teacher guidance during collaborative activities. In Chapter 12, the author evaluates students' ability to collaborative problem solving, finding that students lack a sense of collaboration in communicative dialogue, giving suggestions on training ability.

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Chapter 2
Examining Junior High School Students' Collaborative Knowledge Building: Based on the Comparison of High- & Low-Performance Groups' Mathematical Problem-Solving

Sun Binbo

2.1 Introduction

Nowadays, the global situation is undergoing profound and complex changes (Xi, 2022). To meet these challenges and seek further development, many countries/jurisdictions and international organisations have gradually attached importance to developing students' "21st-century skills" in the field of basic education, going beyond traditional subject knowledge and to focus on contemporary competencies and literacies for Education 4.0 (Griffin et al., 2012; World Economic Forum, 2020). As a future-oriented education reform, various countries/jurisdictions and organisations have highlighted the key capabilities and core competencies necessary for students' learning, working, and living in the twenty-first century, such as collaboration and problem-solving, in their educational objectives and learning contents for cultivating practical and innovative talents with global competitiveness (Deng and Peng, 2019; UNESCO, 2021). Integrating "21st-century skills" into the national education system and various learning areas, such as science, art, humanities, and mathematics, has become the main practice in many countries (OECD, 2018). Mathematics, the "queen of science," is the basis of science and an important part of school curricula. Realising the development of students' "21st-century skills" in mathematics learning and promoting practical and innovative talents have become the main trends in mathematics curriculum reform (Sun et al., 2019).

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School classrooms are the main place for talent training and curriculum implementation. Classroom learning is an important carrier for promoting students' development and achieving educational goals. The sample of Chinese 15-year-old students' average collaborative problem-solving (CPS) performance (OECD, 2017a), junior high school's role in connecting the preceding and the following, and junior high school students' unique physical and mental development create a singular learning environment. Focusing on junior middle school mathematics classroom teaching and learning, promoting the transformation and innovation of teaching methods, and carrying out mathematical problem-solving-based collaborative learning (MPSCL) meets the requirements of contemporary learning theory, creates an autonomous, cooperative, inquiry-based learning environment, and reflects current development trends. MPSCL provides a carrier for realising the above educational goals and has become an active exploration of how to strengthen the educational function of mathematics classroom learning and teaching in the new era and build a high-quality education system (Sun and Guo, 2020). However, MPSCL still faces many theoretical and practical difficulties. Changing teaching methods and improving learning quality are still in the exploratory stage. For example, the widely used collaborative learning approach is inefficient; the learning process is a mere formality, and students' poor collaborative learning performance leads to low teaching and learning quality (Li, 2019; Wang, 2019).

To fully understand students' MPSCL performance, the quality of student groups' problem-solving solutions was assessed based on the research project data and found wanting. High-performance groups completed the mathematical problem-solving task requirements and produced nice solutions, while low-performance groups completed less than half of the task requirements or provided solutions that were less than half as accurate. To some extent, this result confirms the ineffectiveness of collaborative learning in mathematics classrooms and students' poor collaborative problem-solving performance (Sun, 2004). Therefore, by examining and comparing high- and low-performance groups, it is necessary and important to study this team-based learning process further; in particular, how to optimise MPSCL and improve students' learning performance and learning quality have become important problems that need to be solved.

Learning science is closely related to curriculum, teaching, and learning. It guides curriculum, teaching, and learning development based on the latest research achievements and is the key to ensuring that curriculum reform is scientific, forward-looking, and effective (Peng and Liu, 2019; Chi et al., 1994). With the development of social constructivism and socio-cultural cognition, learning is the social negotiation of knowledge, and establishing a learning community to carry out collaborative knowledge building (CKB) has become a new learning metaphor and has attracted much attention (Ma, 2004). CKB provides a critical perspective for understanding problem-solving-based collaborative learning.

As a key term in constructivist learning theory, knowledge construction is the core concept of the constructivist knowledge view and epistemology. Knowledge construction theory holds that humans cannot directly understand objective reality as it is independent of how they perceive it. The perceptions and experiences from which humans construct their understanding of objective reality are mediated by their existing knowledge and experience. Learning is just the process through which humans adapt to their experience world. When human experience differs from cognition, the unbalanced result triggers the process of human adaptability (learning); reflection on successful adaptive operation produces new or revised concepts (Von Glasersfeld, 1982).

Since the rise of constructivist learning theory in the 1980s, educational theorists and practitioners' different perspectives on constructivist theory have led to diverse understandings of "knowledge construction." Although the concepts of "radical constructivism" and "social constructivism" provide some directions for understanding "knowledge construction," there are various views on knowledge construction in these categories. Therefore, knowledge construction has a diversified development orientation in educational research. Different perspectives have produced different emphases in knowledge construction research, leading to many different research aspects, such as mechanism, teaching, and evaluation.

Compared with traditional knowledge "transmission-reception" classroom teaching and learning, knowledge construction theory fundamentally guides education to reconstruct its teaching activities, promotes students to enter a cultural atmosphere of creating knowledge, and emphasises that knowledge construction is a process of information transmission and meaning understanding through language interaction, which needs to be realised in a specific learning environment. Therefore, a large amount of research in the field of Computer Supported Cooperative Learning (CSCL) has addressed knowledge construction themes, including knowledge construction principles, knowledge construction community, collaborative knowledge construction, and so on (Scardamalia and Bereiter, 1994). Thus, we will focus on individual knowledge acquisition in knowledge construction in a learning community. Stahl pointed out that individual independent learning does not necessarily result in a problem-solving task being properly solved. Jointly building knowledge and meaning through cooperation makes it possible to better deal with problem-solving tasks by making learning enter the socialisation process.

In a learning community, students must cooperate to complete problem-solving tasks jointly and create new intelligent products. In this process, group members need to share their personal views in the learning community, establish different perspectives on problem-solving based on their different personal backgrounds, promote mutual understanding and cooperation through mutual exchange of and consultation on knowledge and experience, promote problem-solving, and realise the continual improvement of small members' views and ideas, so as to create new public knowledge and improve and optimise each group member's personal knowledge structure. Therefore, CKB emphasises building knowledge in solving practical problems, pays attention to learners' creation and sublimation of knowledge through team interaction, and develops valuable views, ideas, strategies, and methods for the learning community.

As the main learning body, students establish various relationships through interactive behaviours, such as raising questions, expressing opinions, providing resources, etc. Educational data-mining technology, in-depth analysis of relationship networks and contents in interactive behaviour, and analyses of information flow and meaning sharing can help teachers, students, and other stakeholders more deeply understand the collaborative problem-solving learning process and promote knowledge discovery, knowledge production, knowledge sharing, and knowledge innovation. With the transformation and upgrading of the current global economy from traditional manufacturing to knowledge and scientific and technological innovation, school education and teaching should pay more attention to information interactions and how knowledge is acquired, generated, and innovated in the cooperative problem-solving process, promote the development of high-quality classroom teaching, and take this as the carrier to develop students' collaborative problem-solving ability and other necessary core competencies for citizens in the twenty-first century.

However, few studies focus on students' face-to-face group collaboration and problem-solving from this perspective, analyse or explain this learning process, or mine students' problem-solving-based collaborative learning data in a micro way to trace and compare high- and low-performance groups' CKB behaviour processes.

Therefore, this study aims to optimise students' MPSCL process and improve learning efficiency and quality. The following questions are addressed in this research:

1. What is the CKB process when junior high school students participate in MPSCL activities in mathematics classrooms?
2. Are there any differences in CKB between high- and low-performance groups that produce different quality mathematical problem-solving solutions?

This research used design-based principles, controlling for the influence of external macro factors and investigating the students' micro-level CKB process in comparing high- and low-performance groups' mathematics problem-solving to provide research support for improving classroom teaching and learning for talent training.

2.2 Methodology

Studying students' MPSCL from the CKB perspective is an effective way to understand team-based interactive processes and evaluate this learning quality to support academic analysis and teaching improvement. This section details the study's main concepts, research design, sample learning task, participants, research methods, and technical road.

2.2.1 The Main Concepts

1. Collaborative/Cooperative learning is a teaching theory and strategy based on the theories of psychology, sociology, and educational technology and aimed at promoting students' all-around development. It takes group activities as its basic organisational form, teacher–student and student–student classroom interactions as its driving force, and group performance as its basis for evaluating teachers' and students' goals, interactions, and roles (Wang, 2002). Collaborative learning emphasises that group members have shared task goals and realise joint learning through meaning negotiation, the division of labour, and collaboration.
2. MPSCL includes team collaboration and problem-solving processes (OECD, 2017b) taking place in school mathematics classrooms under specific space–time conditions, with group members (usually four-person groups) jointly participating in learning activities to solve a given problem (learning task) through meaning negotiation, the division of labour, and collaboration and complete its transformation from its real state to its ideal goal. Here, problems refer to open, comprehensive, and complex mathematical problems with realistic situations.
3. Knowledge construction, the core concept of constructivist learning theory, originates from the philosophical position that humans cannot directly understand objective reality and can only build their understanding of the world from their perceptions and experiences mediated by their previous knowledge. Therefore, learning is a process by which humans adapt their experience world (Jonassen & Kwon, 2001). Knowledge construction is a process and result based on junior high school students' existing knowledge and experience, relying on mathematical problem-solving, realising the improvement of old and new knowledge and experience, and creating new meaning.
4. Collaborative knowledge building. The research on the learning process has increasingly focused on the social essence of knowledge construction, emphasising that it is affected by the learning community (Hung and Der-Thanq, 2000). It focuses academic attention on the impacts of environment and society on individual cognition. Thus, different from individual knowledge construction, CKB emphasises the social process of learning and believes that learning is constructed by individuals' cognitive processes and meaning negotiations between individuals and groups. In this research, CKB reveals that against the background of MPSCL, students use personal cognition to interact with their learning community members to build public knowledge through meaning negotiation, and division of labour, which realises the deepening of group members' cognition and the development of high-order thinking. Mathematical problem-solving realises individuals' meaningful learning, and collaboration makes rich individual knowledge construction and reliable results.

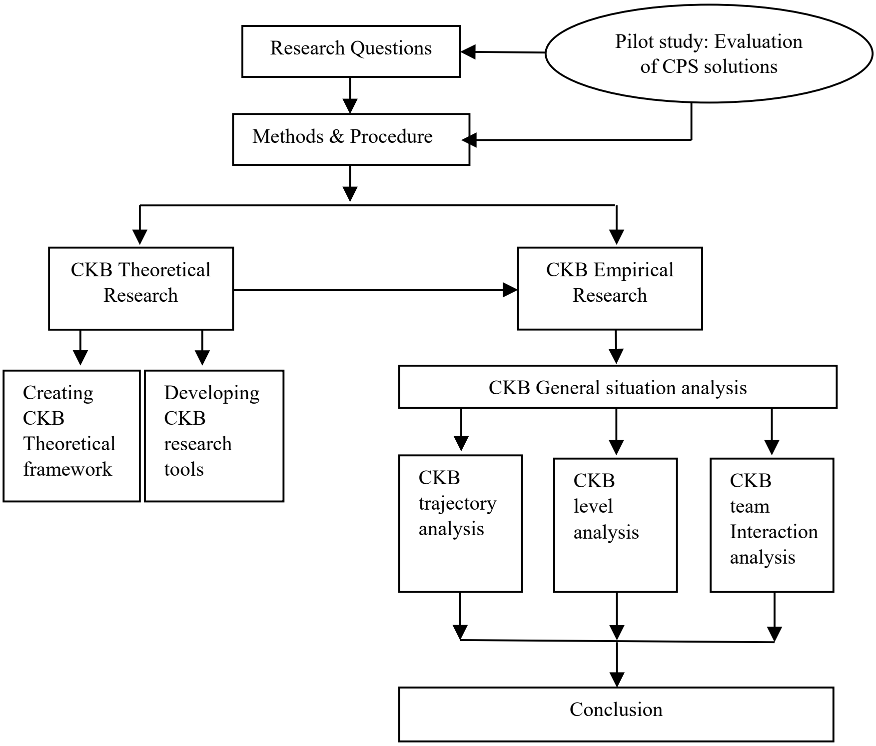


Fig. 2.1 The design map of CKB research

2.2.2 *Research Design*

This study's discussion of students' MPSCL from a CKB perspective began with a quality evaluation of groups' collaborative mathematical problem-solving results, selecting high- and low-performance groups as participants. Second, it developed an analytic framework and research tools. Finally, it traced students' MPSCL process from the micro level to analyse their collaborative knowledge building. The research design map is shown in Fig. 2.1.

2.2.3 *Sample Learning Task*

Design-based methods were used to conduct an experimental investigation, taking the mathematical problem "Xiao Ming's Apartment" as an example (see Appendix Task 1). The main reasons for choosing this problem as a learning task and conducting a case study are as follows:

First, this problem (the design of Xiao Ming's apartment) has a realistic situation. It requires students to draw a possible graphical representation of Xiaoming's apartment and mark each room's possible functions, length, and width. This situation is closely related to pupils' future-oriented social life and is rooted in each student's real-life experience. The concept of apartment exists in students' cognitive schema, and their pre-knowledge and experience can provide a carrier for group collaboration, mathematical problem-solving, and learning.

Second, it is an open-ended problem. Because it has no unique solution, any solution that meets the task requirements and conforms to scientific facts and mainstream values is correct and reasonable. Therefore, the problem provides enough space for group collaboration, mathematical problem-solving, and learning.

Third, the difficulty of this problem is in line with the cognitive abilities of and can be solved by junior high school students. The mathematical knowledge, activity experience, and mathematical competencies involved in solving this problem are within the scope of junior high school students' physical and mental development, in line with the characteristics of students' cognitive development at this stage. Students can successfully complete the mathematical problem-solving task.

Fourth, it is representative for students to experience a mathematical process—identify key mathematical concepts from a real-world situation, extract primary mathematical knowledge, and bring the results back to real life for evaluation and reflection after mathematical activities such as abstraction, modelling, and problem-solving.

2.2.4 Research Participants

1. Pilot study: Evaluation of students' collaborative problem-solving solutions. To understand the quality of collaborative mathematical problem-solving solutions, a two-dimensional evaluation framework for the completion and correctness of CPS solutions was constructed, and an evaluation table was developed, based on the existing literature on project-based learning, problem-based learning, and cooperative learning achievement evaluation (see Table 2.1). Through several rounds of discussion and revision with experts in the field of mathematics education and repeated coding verification combined with sample data, the reliability and validity of the evaluation framework and research tools were ensured to accurately evaluate the quality groups' CPS solutions.
2. Based on the pilot study "evaluation of students' collaborative problem-solving solutions," which provides the specific criteria, high- and low-performance student groups were selected as research participants. The researcher and a graduate student majoring in mathematics education independently rated the collaborative problem-solving solutions of 29 groups in three classes. In addition, where the scores of the two were inconsistent, a professor in mathematics education was invited to score the cases again to ensure quality evaluation results. On this basis, six high-performance and six low-performance groups were selected, as shown in Table 2.2.

Table 2.1 Solution quality evaluation form

Quality grade	Code	Description	An instance
Perfect	3	All tasks and requirements have been completed, and the solutions are all correct	As shown in Fig. 2.2, the team has completed all the tasks: draw a possible apartment and mark the functions, length, and width of each room. At the same time, the solutions meet the requirements with five rooms and a total area of 60 m^2, and the design is reasonable. Therefore, the solutions are all correct, and the comprehensive evaluation is perfect
Good	2	Although not all the task was completed or correct, half or more of the requirements were completed, and the solutions were correct	As shown in Fig. 2.3, the team completed the two requirements, drawing a possible diagram of Xiaoming's apartment and marking the possible functions of each room, but the team did not mark the length and width of each room. Therefore, this team completed most of the learning tasks. Although the team did not mark the size of each room, it met the learning task requirements of the apartment with five rooms and a total area of 60 m^2. This design was reasonable. Therefore, this group's solution is mostly correct. The comprehensive evaluation is good

(continued)

Table 2.1 (continued)

Quality grade	Code	Description	An instance
Not bad	1	Less than half of the task requirements have been completed, or more than half of the task requirements have been completed, but less than half of the problem-solving results are correct	As shown in Fig. 2.4, the team only completed the requirement of drawing a possible diagram of Xiaoming's apartment. The functions of each room were not marked, and the length and width of each room were not marked too. Therefore, this team completed a small part of the learning task. Due to the lack of information, it can only be determined that the possible diagram of the apartment is correct and meets the requirement of five rooms. Therefore, the results are few correct. The comprehensive evaluation is not bad
Not completed/unqualified	0	The activities are not carried out according to the learning task requirements, or the problem-solving results are incorrect	There are no problem-solving solutions or the solutions are incorrect

Here, the score refers to the latest competency-oriented mathematics test results of the sample students (Guo et al., 2015), which is essential evidence teachers use to group students based on the principle of "homogeneity between groups and heterogeneity within groups."

2.2.5 Research Method

This research adopted a mixed research paradigm. Literature review, video observation, content analysis, statistical analysis, social network analysis, and comparative study were comprehensively used to conduct this research.

1. Literature review. A literature review informed the whole research process, helping the researcher recognise the history and current situation of CKB research and other relevant research, clarify the academic positioning of this research, and learn from existing research ideas and methods to provide references for the research design. At the same time, the research's results were incorporated into existing academic contexts for understanding and interpretation, and its innovations, shortcomings, and future research direction were clarified.

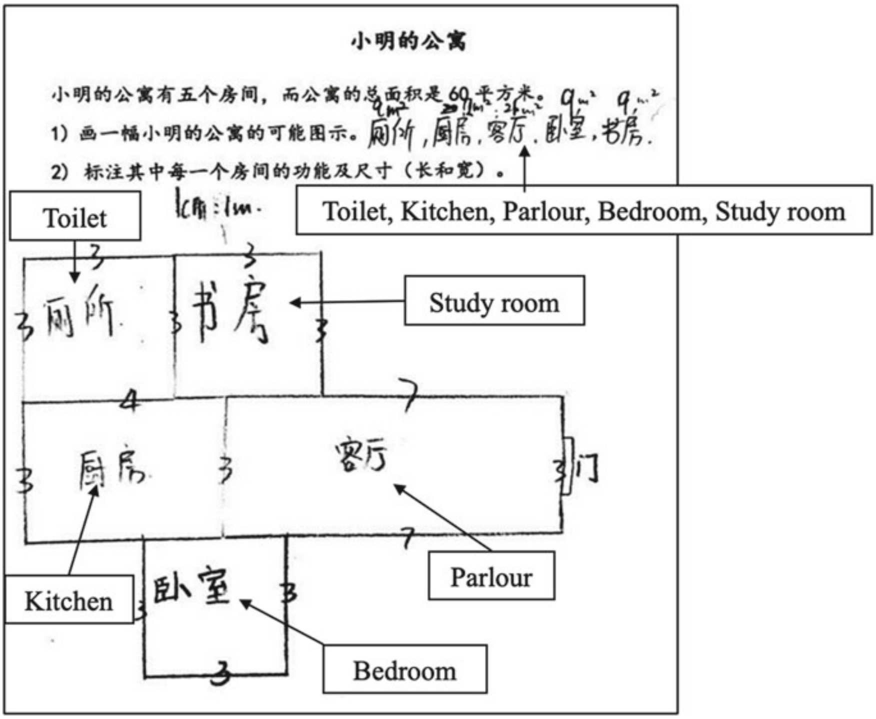


Fig. 2.2 An example of a high-quality solution

2. Video observation. Video observation of the 12 groups' collaborative learning processes was conducted to capture a realistic view of student group collaboration and mathematical problem-solving, understand the students' communication, discussion status, and interactive learning processes, and (combined with content analysis) make the interactive learning data coding more stable and objective (Man and Clarke 2019).
3. Content analysis. The coding unit was each student's interactive dialogue sentences in the 12 groups. Each student's interactive content was marked to indicate whether it conformed to and reflected the coding system's definition of the index to ensure a detailed and objective description of the process and mechanism of students' CKB.
4. Statistical analysis. Based on content analysis, the encoded data are further quantified to describe the junior high school students' CBK behaviour processes. The statistical analysis methods in this research mainly involved descriptive statistical analysis, with the targeted goal of describing the basic form of the research data.
5. Social network analysis (SNA). Social network analysis is mostly used in the CSCL field to study the connections between social actors and the structural functions of these connections (Zheng, 2010). The interaction processes of

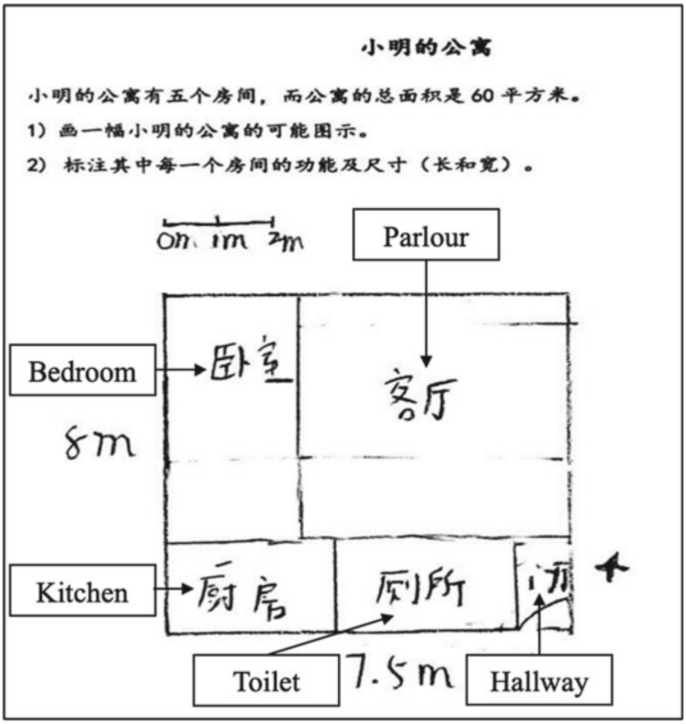


Fig. 2.3 An example of a medium-quality solution

the students in each group were analysed using SNA to understand the team interaction of students' CKB.

6. Comparative study. Comparative study was used to understand the similarities and differences in the high- and low-performance groups' CKB behaviour processes to further analyse students' MPSCL performance and characteristics and understand their CKB.

2.2.6 Analytical Framework

Based on Pólya's (1945) and Schoenfeld's (1992) mathematical problem-solving process models, and referring to Gunawardena et al.'s (1994) and Stahl's (2000) knowledge-building interaction models, a preliminary analysis framework was formed. The framework was modified by incorporating sample data and repeated observation and refinement to create a three- and six-stage coding system for junior high school students' CKB levels (see Table 2.3).

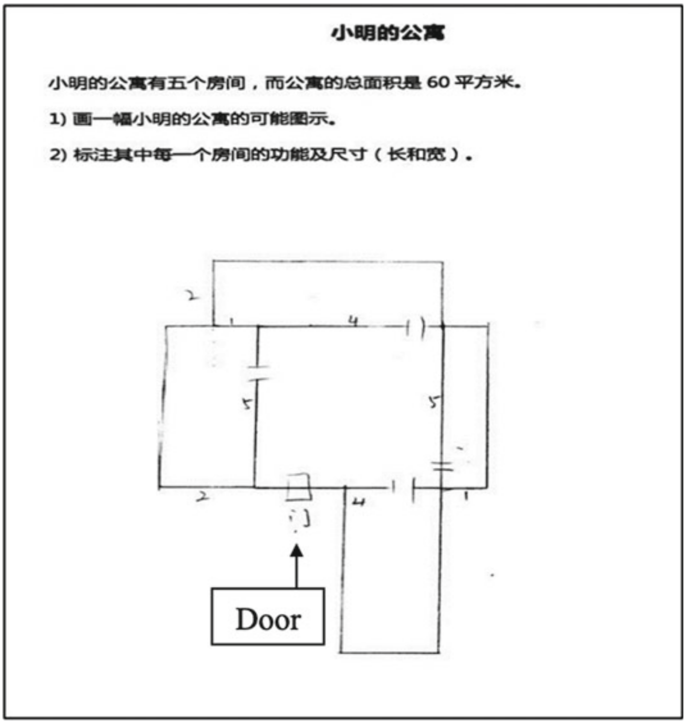


Fig. 2.4 An example of a low-quality solution

The CPS learning process involves knowledge transmission and acceptance and interactions between students for knowledge construction. Based on the mathematical problem-solving process model and referencing Jiang and colleagues' coding schema for evolutionary knowledge construction in an information technology environment (Jiang et al., 2019), the viewpoints coding system was further constructed by repeatedly observing the sample data, as shown in Table 2.4.

2.2.7 Research Implementation and Data Processing

Taking Grade 7 students in T School District of B city as an example, an MPSCL activity was carried out in six classes of three schools in the second semester of Grade 7. Students' process data were recorded and transcribed into text by means of classroom video recording and students' dialogue tape recording. Twelve groups of interactive text data were formed with a total of 3244 interactive dialogues and more than 90,000 words of transcripts for coding analysis. Based on the basic steps of learning analytics (Li et al., 2012) and CKB interactive analysis (Shaffer et al.,

Table 2.2 Research participants

High performance	Group structure			Low performance	Group structure		
Group No	Student No	Gender	Score	Group No	Student No	Gender	Score
01aS2	S1	F	88	01aS1	S1	M	95
	S2	M	79		S2	M	76
	S3	F	41		S3	M	92
	S4	M	70		S4	F	84
02aS1	S1	M	91	02aS2	S1	F	78
	S2	M	91		S2	F	73
	S3	F	70		S3	M	47
	S4	F	59		S4	M	100
02aS4	S1	M	91	02aS6	S1	M	91
	S2	M	95		S2	M	98
	S3	F	76		S3	M	73
	S4	F	58		S4	F	65
03aS4	S1	F	46	03aS2	S1	M	71
	S2	M	85		S2	F	70
	S3	F	87		S3	M	88
	S4	M	94		S4	F	92
03aS5	S1	M	97	03aS3	S1	F	85
	S2	F	89		S2	F	89
	S3	F	82		S3	F	66
	S4	M	86		S4	M	70
03aS7	S1	F	100	04aS4	S1	M	60
	S2	M	98		S2	M	75
	S3	F	79		S3	M	98
	S4	M	61		S4	M	89

2016), this study designed a process for analysing junior middle school students' CKB, as shown in Fig. 2.5.

2.2.8 Description of Reliability and Validity

The validity guarantee of this research is based on two aspects. First, it relied on research theories and models that have high international and domestic recognition and are based on a large body of previous research to form a preliminary framework for coding junior middle school students' CKB. Second, based on the repeated exploration of research data, the preliminarily determined coding system was modified and

Table 2.3 Coding system of CKB process

Level	Stage	Code	Description
Shallow CKB	Information sharing and understanding	I	Clarify the learning tasks of collaborative problem-solving in mathematics classroom, including clarifying the objectives of mathematical problem-solving, the rules of problem-solving learning activities, the definition of concepts related to specific mathematical problems and other detailed problems; expound the understanding of collaborative problem-solving learning task in mathematics classroom, including understanding mathematics problem situation, core concepts, and activity rules; Agree with the views, understandings or views expressed by the panellists; Confirm or agree with the reference cases provided by the team members; Group members ask questions or answer questions to clarify their understanding or views on collaborative problem-solving learning tasks; Clarify and understand the discussion of relevant topics caused by mathematical problem-solving tasks. Clarify the strengths and advantages of yourself or team members in the process of division of labour and cooperation
	Discovery and clarification of differences	II	Identify and clarify the differences among team members in the process of cooperative problem-solving; Group members ask or answer questions to each other to clarify the source and extent of differences; Adhere to or restate the team members' own positions and views, and provide evidence to support their views
Medium-level CKB	Content negotiation and co-construction	III	Negotiate or discuss divergent views, opinions, and contents; Establish a cognitive perspective, negotiate or discuss strategies and contents to promote mathematical problem-solving; Approve the opinions and contents put forward by the team members; Analyse the common points of views, ideas, or opinions among team members; and Propose or discuss new views on this basis to build knowledge
	Content verification and adjustment	IV	Test or adjust the contents, ideas, views, opinions, and jointly constructed knowledge contents of the discussion and consultation of the group members, including the scientificity, rationality, culture, artistry, and value of the discussion views and contents

(continued)

Table 2.3 (continued)

Level	Stage	Code	Description
Deep CKB	Consensus reaching and Application	V	The team members reach an agreement and apply the newly constructed knowledge to complete the mathematical problem-solving task; The panellists summarised the contents of the consensus
	Reflection and evaluation of achievements	VI	Related metacognition of group members, such as self-feeling, change, and improvement of group members after collaborative mathematical problem-solving activities. Team members check and evaluate the results of cooperative mathematical problem-solving after the application of new knowledge

improved, then submitted to senior experts and scholars in mathematics education for review and revision to obtain expert validity support.

In this research, repeated coding ensured coding consistency. Based on the initial coding, two high- and low-performance groups were randomly selected to verify the coding. Coding consistency between the groups was over 90%, indicating good coding stability.

2.3 Results

The following research results were formed through quantitative and qualitative analysis of the high- and low-performance groups' mathematical problem-solving data.

2.3.1 General CKB Situation

1. High-performance groups. The number of sentences and proportion distribution of high-performance groups is shown in Table 2.5. The CKB discussion refers to the substantive and constructive discussion carried out by students to solve the mathematical problems of "Xiaoming's Apartment." It includes 1408 interactive sentences, accounting for 89.33% of the total, indicating that the students in high-performance groups gave full play to their collective wisdom, contributed their personal views and knowledge when participating, negotiating, and discussing, and integrated their collective strength to promote solutions to mathematical problems. Further analysing the CKB process in each high-performance group revealed differences in the proportion between the number of sentences and irrelevant discussions. For example, Group 03aS5 students spoke the most in

Table 2.4 Coding system of CKB viewpoints

Primary classification	Secondary classification	Code	Description
Shared view	Q & A	A1	Seek information about mathematical problems, ask informational questions, and give detailed answers
	Explain	A2	Share your views on mathematical problems, explain your strengths or weaknesses, identify information, describe your views, explain your views, and put forward suggestions
Discussive view	Divergence	B1	Put forward different views, question or refute the views put forward by others
	Endorse	B2	Support, supplement, or improve the ideas put forward by others
	Defend	B3	Provide further explanations to defend your point of view
	Consensus	B4	Negotiate and reach an agreement on concepts, ideas, and strategies related to mathematical problem-solving
Sublimation view	Summary	C1	Summary of the discussion. The process of summarising and summarising the solutions and Strategies of mathematical problems
	Evaluate	C2	Value judgement or evaluation of other people's or their own views, mathematical problem solutions, etc
	Reflect	C3	Evaluation of collaborative mathematics problem-solving and its learning

discussions while Group 02aS1 students spoke the least, a difference of 203 sentences.

In addition, tracking the irrelevant discussions of each high-performance group identified four types of irrelevant discussions. The first was discussions about completing the formalised CPS learning task requirements. For example, the teacher asked students in each group to fill in their names on the task paper based on the grouping list, and the students said something about this. The second type was irrelevant discussions deriving from the CKB process (deviation topic discussions), which accounted for a large proportion of all discussions. The third type was irrelevant discussions caused by differences of opinion among team

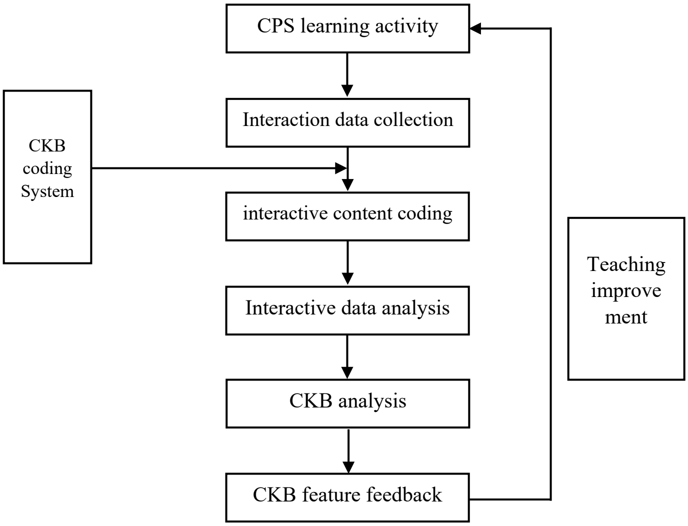


Fig. 2.5 CKB analysis process

Table 2.5 Number of sentences and proportion distribution of high-performance groups

Category	Groups						Total
	01aS2	02aS1	02aS4	03aS4	03aS5	03aS7	
Discussion of CKB	307 (93.88%)	156 (88.64%)	187 (78.24%)	256 (98.08%)	328 (86.54%)	174 (82.46%)	1408 (89.33%)
Irrelevant discussion	20 (6.12%)	20 (11.36%)	52 (21.76%)	5 (1.92%)	51 (13.46%)	37 (17.54%)	185 (10.80%)
Total	327	176	239	261	379	211	1593

members, while the fourth included irrelevant discussions while implementing the solution, such as discussing who drew well or asking others for a pencil, ruler, eraser, etc.

2. Low-performance groups. The low-performance groups' sentence and proportion distributions are shown in Table 2.6. There were 1358 CKB sentences in low-performance groups, accounting for 82.91% of the total, indicating that the students in low-performance groups were jointly building knowledge in the mathematical problem-solving process and integrated their collective strength to promote mathematical problem-solving. Further analysis showed differences in each low-performance group's number of CKB sentences and proportion of irrelevant discussions. For example, Group 03aS2 communicated and discussed the most (388 sentences), while 04aS4 had the fewest communications and discussions (166 sentences).

Table 2.6 Number of sentences and proportion distribution of low-performance groups

Category	Groups						Total
	01aS1	02aS2	02aS6	03aS2	03aS3	04aS4	
Discussion of CKB	340 (93.83%)	175 (78.13%)	189 (89.15%)	346 (89.18%)	201 (70.03%)	107 (64.46%)	1358 (82.91%)
Irrelevant discussion	33 (8.85%)	49 (21.88%)	23 (10.85%)	42 (10.82%)	86 (29.97%)	59 (35.54%)	292 (17.70%)
Total	373	224	212	388	287	166	1650

In addition, there were three types of irrelevant discussions in the low-performance groups. The first concerned completing the formal CPS task requirements, such as communications about controlling problem-solving time. The second group of irrelevant discussions derived from the CKB process, accounting for a large proportion. Finally, group members had some irrelevant discussions when implementing mathematical problem solutions, such as discussing whose academic performance was good, looking for pencils and rulers, etc.

3. Comparison of the groups' general CKB situations. The sentence and proportion distribution between the high- and low-performance groups is shown in Table 2.7. There were 1593 sentences CKB spoken by high-performance groups, only 57 fewer than were spoken by low-performance groups (1650). This shows there was little difference in the total amount of CKB communication and discussion between the high- and low-performance groups. The proportion of CKB discussions in the high-performance group was 6.42% higher than in the low-performance group, indicating the former had a relatively more CKB around the mathematical problems of "Xiaoming's Apartment" and a somewhat more efficient mathematical problem-solving process.

   In addition, the high- and low-performance groups had similar types of irrelevant discussion content, i.e., content related to completing the formalised CPS learning task requirements, topic deviations (e.g., when discussing the design scheme for "Xiaoming's apartment"), or implementing mathematical problem solutions.

Table 2.7 Number of sentences and proportion distribution of high- and low-performance groups

Category	Group category	
	High-performance groups	Low-performance groups
Discussion of CKB	1408 (88.39%)	1358 (82.30%)
Irrelevant discussion	185 (11.61%)	292 (17.70%)
Total	1593	1650

2.3.2 Trajectory of CKB

1. High-performance groups. In solving collaborative problems, the high-performance groups generated 1187 views, averaging 197.83 per group and 49.45 per capita. The high-performance groups' cumulative view distribution is shown in Table 2.8. Shared views accounted for the highest proportion, mainly because team members sought mathematical problem information, shared their views on the problem, and explained their views. The discussion view was mainly used to find and explore inconsistent views, concepts, or statements, discuss problems, clarify or distinguish meanings, similarities, and differences in terms and views, integrate views, and deepen understanding. The proportion of sublimated views was relatively low, mainly because the group members summarised and sublimated the fragmented and scattered views in the negotiation and discussion process, which changed their knowledge structure and thinking mode and became their final basis for forming and optimising mathematical problem solutions.
2. Low-performance groups. The low-performance group generated 1340 views, averaging 223.33 per group and 55.83 per capita. The distribution of the low-performance groups' views is shown in Table 2.9. First, shared views accounted for up to 55.83% of the total, highlighting that each student could freely advance opinions and suggestions and state their personal views and ideas. Second, the discussion view accounted for 36.34%, mainly involving the evolution of existing views, the integration of similar or related views, the demise of naive, wrong, or meaningless views, and the emergence of new views. Finally, the sublimation view accounted for 10.75%, which was relatively low.
3. The comparison of CKB trajectory. The number of views in different groups is shown in Table 2.10. Low-performance groups contributed more views than high-performance groups, showing a higher level of thinking divergence (to a certain extent). In detail, the percentage of shared views was 17.11% lower in the high-performance group than in the low-performance group, the discussion view was 5.23% lower, and the sublimation view was 19.43% higher. In addition,

Table 2.8 Number of views and the percentage of high-performance groups

Primary classification	Secondary classification	Number of "Views"	Percentage (%)
Shared view	Q & A	106	8.96
	Explain	352	29.75
Discussive view	Divergence	134	11.33
	Endorse	80	6.76
	Defend	80	6.76
	Consensus	74	6.26
Sublimation view	Summary	110	9.30
	Evaluate	86	7.27
	Reflect	161	13.61

Table 2.9 Number of views and the percentage of low-performance groups

Primary classification	Secondary classification	Number of "Views"	Percentage (%)
Shared view	Q & A	144	10.75
	Explain	565	42.16
Discussive view	Divergence	261	19.48
	Endorse	115	8.58
	Defend	59	4.40
	Consensus	52	3.88
Sublimation view	Summary	85	6.34
	Evaluate	35	2.61
	Reflect	24	1.79

Table 2.10 Number of views of high- and low-performance groups

Primary classification	Secondary classification	Group category			
		High-performance groups		Low-performance groups	
		Number of "Views"	Percentage (%)	Number of "Views"	Percentage (%)
Shared view	Q & A	106	8.96	144	10.75
	Explain	352	29.75	565	42.16
Discussive view	Divergence	134	11.33	261	19.48
	Endorse	80	6.76	115	8.58
	Defend	80	6.76	59	4.40
	Consensus	74	6.26	52	3.88
Sublimation view	Summary	110	9.30	85	6.34
	Evaluate	86	7.27	35	2.61
	Reflect	161	13.61	24	1.79

there were differences in the evolution of views between the high- and low-performance groups. The high-performance group showed a better spiral, while the low-performance group had a messy view of evolution.

2.3.3 Level of CKB

1. High-performance groups. CKB level is a macro description of the process quality for students' interactive learning and reflects the effect of constructing knowledge together in the learning community (Shi, 2011). Table 2.11 shows the number and proportion distribution of talking sentences from high-performance groups, divided by their CKB level. There were 260 deep-level sentences (16.32%), 604 middle-level sentences (37.92%), and 559 shallow sentences (35.09%).

Table 2.11 The number and proportion distribution of CKB level of high-performance groups

Level	Groups						Total
	01aS2	02aS1	02aS4	03aS4	03aS5	03aS7	
Shallow	84 (25.69%)	81 (46.02%)	78 (32.64%)	101 (38.70%)	98 (25.86%)	117 (55.45%)	559 (35.09%)
Medium	170 (51.99%)	54 (30.68%)	93 (38.91%)	99 (37.93%)	130 (34.30%)	43 (20.38%)	604 (37.92%)
Deep	53 (16.21%)	21 (11.93%)	16 (6.69%)	56 (21.46%)	100 (26.39%)	14 (6.64%)	260 (16.32%)

This shows that the high-performance groups could reach a deep-seated CKB level, showing that MPSCL had some effect. Each group had high-order interactive dialogues, promoted in-depth processing of views, and promoted the CKB process.

Detailed analysis of the above results revealed that although each high-performance group's CKB reached a deep level, the proportion of deep-level sentences was not high. Medium-level CKB dominated interactive dialogue, but some groups (e.g., 02aS1, 03aS4, and 03aS7) had mainly shallow-level interactions. The high-performance groups' interactive dialogue data were coded and counted based on the CKB stage, as shown in Table 2.12. The high-performance groups' CKB reached the deep-seated "reflection and evaluation of achievements" stage. For example, Group 03aS5 had the highest CKB and a higher proportion of deep-level CKB, indicating its CKB had a good effect.

2. Low-performance groups. The number and proportion distribution of the low-performance groups' CKB levels is shown in Table 2.13. On average, the groups had 74 deep-level CKB interactions (5.41%), 670 medium-level (48.98%), and 614 shallow-level (44.88%). This shows that the low-performance group also reached deep-level CKB, with high-order interactive dialogue in each group, indicating their collaborative learning on the topic of "Xiaoming's apartment" had an effect.

   Although the low-performance group reached deep-level CKB, the proportion was low; overall, medium-level CKB dominated, with some groups (e.g., 02aS2 and 04aS4) being dominated by shallow-level CKB. This shows that low-performance groups' deep-level CKB was not sufficient, and their collaborative learning process was not efficient. The number and proportion distributions for CKB levels at each stage are shown in Table 2.14. The low-performance groups' CKB reached the deep-seated "Reflection and evaluation of achievements" stage. For example, Group 03aS2 had the most dialogue sentences, but a relatively low proportion of deep-seated knowledge building.

3. Comparison of the groups' CKB levels. The proportion distribution of the different groups' CKB levels is shown in Table 2.15. Although all groups' CKB reached a deep level, medium-level CKB dominated overall, accounting for the highest proportion in both the high- and low-performance groups. At the same

Table 2.12 The number and proportion distribution of CKB level at each stage of high-performance groups

Level	Stage	Groups											
		01aS2		02aS1		02aS4		03aS4		03aS5		03aS7	
		Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)
Shallow	Information sharing and understanding	45	14.66	56	35.90	47	25.13	66	25.78	77	23.48	105	60.34
	Discovery and clarification of differences	39	12.70	25	16.03	31	16.58	35	13.67	21	6.40	12	6.90
Medium	Content negotiation and co-construction	90	29.32	39	25.00	68	36.36	68	26.56	83	25.30	33	18.97
	Content verification and adjustment	82	26.71	15	9.62	25	13.37	31	12.11	47	14.33	10	5.75
Deep	Consensus reaching and Application	31	10.10	6	3.85	5	2.67	32	12.50	56	17.07	7	4.02
	Reflection and evaluation of achievements	20	6.51	15	9.62	11	5.88	24	9.38	44	13.41	7	4.02

Table 2.13 The number and proportion distribution of CKB level of low-performance groups

Level	Groups						Total
	01aS1	02aS2	02aS6	03aS2	03aS3	04aS4	
Shallow	145 (42.65%)	104 (59.43%)	78 (41.27%)	153 (44.22%)	84 (41.79%)	50 (46.73%)	614 (44.88%)
Medium	164 (48.24%)	63 (36.00%)	101 (53.44%)	188 (54.34%)	107 (53.23%)	47 (43.93%)	670 (48.98%)
Deep	31 (9.12%)	8 (4.57%)	10 (5.29%)	5 (1.45%)	10 (4.98%)	10 (9.35%)	74 (5.41%)

time, the high-performance groups' had a much higher proportion of deep-seated CKB (10.91%), indicating that the low-performance groups' deep-seated CKB was insufficient.

Table 2.16 shows the number and proportion distribution of CKB levels at each stage. Although the students in the high- and low-performance groups reached the stage of "Reflection and evaluation of achievements," the low-performance groups' proportions of "content verification and adjustment," "consensus reaching and application," and "reflection and evaluation of achievements" stages were much lower than in the high-performance groups. At the same time, students in the low-performance groups had a greater proportion of different views (8.8% higher) than their peers in the high-performance group.

2.3.4 Team Interaction of CKB

1. High-performance groups. MPSCL in high-performance groups led to the emergence of interactive groups. Most group members spoke actively in the CKB process, promoting the circulation and generation of knowledge among groups. Group 01aS2's social network relationship is shown in Fig. 2.6, as an example. S1 and S4 continually communicated around the "Xiaoming's apartment" design task, negotiating views, exchanging meanings, and forming a subgroup of two people.

   Based on the students' speech frequency, high-performance group members showed better participation, with average CKB-related speech frequencies of 66.38 and 59.29. This interactive group had a variety of team interaction modes, including two-person and three-person interactive subgroups. The smaller the number of subgroups, the less balanced the group members' speeches. For example, the number of speeches in Group 01aS2 is shown in Fig. 2.7. The group featured a typical two-person interactive subgroup. S4 and S1 talked the most (140 and 119 sentences, respectively), while S3 spoke the least (11). Part of Student 3's speech had nothing to do with mathematical problem-solving, and the relevant content they did offer was at a shallow level, showing insufficient participation in interactive learning.

Table 2.14 The number and proportion distribution of CKB level at each stage of low-performance groups

Level	Stage	Groups											
		01aS1		02aS2		02aS6		03aS2		03aS3		04aS4	
		Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)	Number	Percentage (%)
Shallow	Information sharing and understanding	73	21.47	63	36.00	60	31.75	59	17.05	44	21.89	38	35.51
	Discovery and clarification of differences	72	21.18	41	23.43	18	9.52	94	27.17	40	19.90	12	11.21
Medium	Content negotiation and co-construction	110	32.35	48	27.43	90	47.62	163	47.11	92	45.77	35	32.71
	Content verification and adjustment	54	15.88	15	8.57	11	5.82	25	7.23	15	7.46	12	11.21
Deep	Consensus reaching and Application	12	3.53	2	1.14	3	1.59	4	1.16	3	1.49	1	0.93
	Reflection and evaluation of achievements	19	5.59	6	3.43	7	3.70	1	0.29	7	3.48	9	8.41

Table 2.15 The high- and low-performance groups' number and proportion distribution of CKB level

Level	High-performance groups		Low-performance groups	
	Average number	Percentage (%)	Average number	Percentage (%)
Shallow	559	35.09	614	44.88
Medium	604	37.92	670	48.98
Deep	260	16.32	74	5.41

Table 2.16 The high- and low-performance groups' number and proportion distribution of CKB level at each stage

Level	Stage	Groups			
		high-performance group		low-performance group	
		Average number	Percentage (%)	Average number	Percentage (%)
Shallow	Information sharing and understanding	396	27.83	337	24.63
	Discovery and clarification of differences	163	11.45	277	20.25
Medium	Content negotiation and co-construction	381	26.77	538	39.33
	Content verification and adjustment	210	14.76	132	9.65
Deep	Consensus reaching and Application	137	9.63	25	1.83
	Reflection and evaluation of achievements	121	8.50	49	3.58

2. Low-performance groups. Low-performance groups' CKB also promoted the emergence of interactive groups. Most group members could actively speak and contribute knowledge to the learning community. Group 01aS1's social network relationship is shown in Fig. 2.8, as an example. S1 and S3 constantly exchanged views and discussed the design of "XiaoMing's Apartment," forming a two-person subgroup.

   The students in the low-performance group also showed good participation with average CKB-related speaking frequencies of 68.75 and 57. There were diverse group interaction modes, including two-person and three-person interaction subgroups. For example, the number of speeches in Group 01aS1 is shown

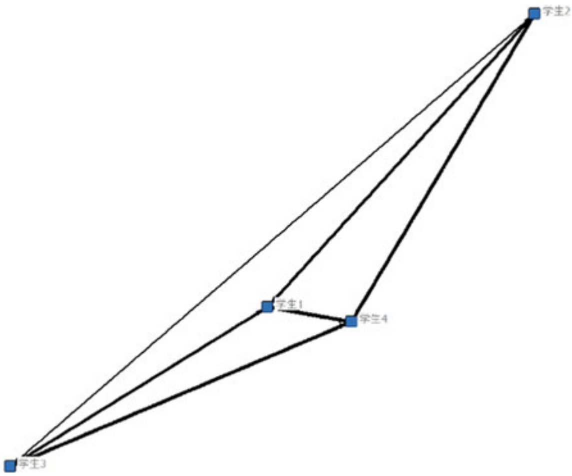


Fig. 2.6 01aS2 CKB social network relationship

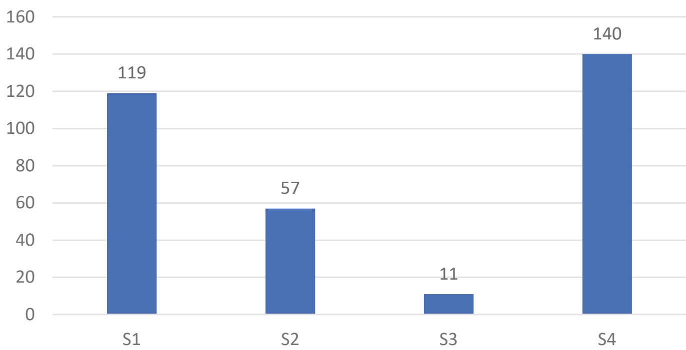
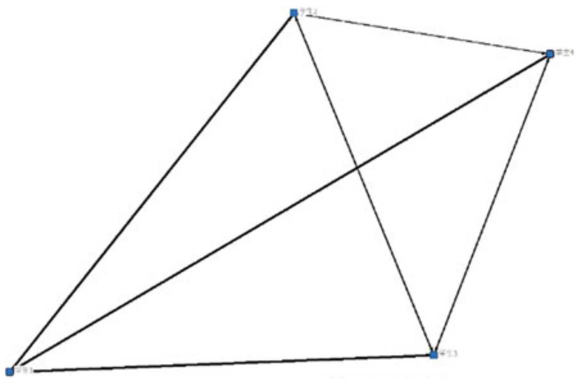


Fig. 2.7 Number of speeches of 01aS2

Fig. 2.8 01aS1 CKB social network relationship



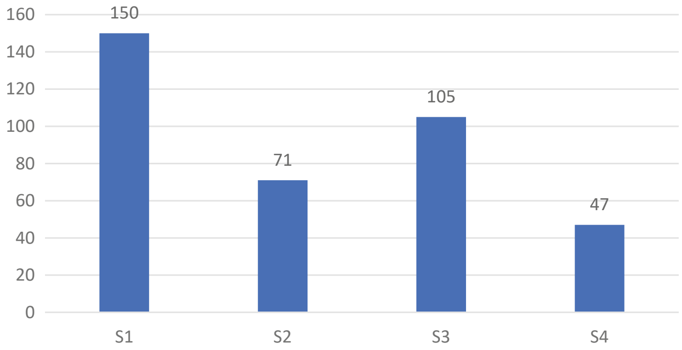


Fig. 2.9 Number of speeches of 01aS1

in Fig. 2.9. The group featured a typical two-person interactive subgroup, with S1 and S3 interacting a lot (150 and 105 sentences, respectively), while S4 spoke the least (47). Some of S4's speeches had nothing to do with mathematical problem-solving, and the CKB-related content they offered was at a shallow level, indicating insufficient participation in interactive learning.

3. Comparison of teams' CKB interactions. The data show that the high- and low-performance groups' CKB promoted the emergence of interactive groups. Most group members spoke actively, participated in exchanges, and contributed their personal knowledge to the learning community, promoting the generation and flow of information among group students. For example, S1 and S3 (in Group 01aS1) had continual communication and dialogue around mathematical problems, negotiating views, exchanging meanings, and forming a two-person subgroup.

   Comparative analyses of the speech frequencies in the high- and low-performance groups revealed that students' interaction groups in the high- and low-performance groups had diverse interaction modes—mainly two-person and three-person interaction subgroups. Some individual students rarely participated in MPSCL activities, rarely interacted with other group members, and played the role of listener. For example, Student 4 in Group 02aS6 almost never spoke in the group cooperative mathematics problem-solving process, showing insufficient participation in interactive learning.

2.4 Discussion

This research has investigated the similarities and differences in high- and low-performance junior high school student groups' CKB behaviour processes by assessing the groups' CPS quality to better understand the students' CKB. This section expounds on the main findings.

Existing studies have researched CKB from different perspectives. One is to explore students' CKB based on the problem-solving-based collaborative learning task, as different learning tasks affect group students' CKB differently. The second is to explore CKB from the perspective of learning resources and learning environment, as students' willingness and ability to propose personal thoughts and views depend on having good learning resources and learning environments. The third is to explore the CKB based on students' knowledge and experience. When dealing with new problems, students must establish a conceptual system to have an opportunity to succeed; this concept is not simply passed from teachers to students, but must be understood by students themselves. In addition, students' mathematical competencies, emotions, attitudes, values, and "21st-century skills" (represented by critical thinking, innovative thinking, and collaborative problem-solving) will affect the CKB process. The fourth perspective explores CKB based on each group member's active participation and meaningful interaction, as group power is the fundamental factor to accelerate the process of group CKB and promote the generation of intelligent products (Diez-Palomar et al., 2021).

Although the collaborative problem-solving-based learning task, learning resources and learning environment, students' knowledge and experience, and mathematical competencies, emotions, attitudes, values, and "21st-century skills" affect student groups' CKB. However, by examining and comparing high- and low-performance groups CKB, it is necessary to unlock the black box of this comprehensive and critical learning behaviour processes in determining whether a problem can be successfully solved. At present, few studies focus on students' face-to-face group collaboration and problem-solving in real classrooms from the CKB perspective, nor analyse and explain students' MPSCL from this perspective. Therefore, this research focused on a junior high school mathematics classroom, paying attention to students' collaborative learning processes and results and using design-based principles to investigate students' micro-level CKB behaviour processes by comparing high- and low-performance groups' mathematical problem-solving. Its findings are discussed below.

First, although there was a proportion of irrelevant discussion in both the high- and low-performance groups, CKB-related discussions predominated; that is, students could closely focus on the mathematical problems of "Xiaoming's apartment," giving full play to their collective wisdom, contributing their personal views and knowledge through participation, negotiation, and discussion, and integrating collective strength to promote the mathematical problem's solution. The high- and low-performance groups had similar types of irrelevant discussions, such as irrelevant discussions related to completing the formalised CPS task requirements, derived by group members in the CKB process (deviated topic discussions), or generated by group members when implementing the mathematical problem-solving solutions. In addition, there were some differences in the high- and low-performance groups' general CKB processes. For example, the high-performance groups had slightly fewer MPSCL information interactions (1593) than those in the low-performance group (1650). However, the high-performance groups had a higher proportion of CKB-related discussions than the low-performance groups (less than 6.42%), indicating a higher proportion of CKB related to "Xiaoming's apartment" and fewer irrelevant discussions, macroscopically reflecting the high-performance group's more efficient CKB.

Second, the junior high school students' CKB went through six stages: information sharing and understanding, discovery and clarification of differences, content negotiation and co-construction, content verification and adjustment, consensus achievement and application, and reflection on and evaluation of achievements. There were many kinds of view evolution paths and similarities in the view evolution paths of the high- and low-performance groups. In particular, shared views predominated in both groups, followed by discussion views and then sublimation views. Specifically, the high-performance groups' shared views accounted for 38.72% of the total, followed by discussion views (31.11%) and sublimation views (a relatively low 30.18%). The low-performance groups' shared views accounted for 55.83% of the total, followed by discussion views (36.34%) and sublimation views (10.75%). In addition, there were some differences in the two groups' CKB evolution paths. For example, the proportion of sublimated views in the high-performance group was much higher than in the low-performance group. Students in the high-performance group showed a better view evolution spiral, indicating the low-performance groups' deep-seated interactions were deficient, and their view evolution process lacked continuity.

Third, although the junior high school students' CKB reached a deep level and there was interactive discussion in the "reflection and evaluation of achievements" stage, most discussions were at a medium or shallow level, indicating that each group's collaborative learning in the "Xiaoming's apartment" theme had a certain effect, and their high-level thinking participation was insufficient. In addition, there were differences in the high- and low-performance groups' CKB levels. For example, students in high-performance groups had a much higher proportion of deep knowledge construction than students in low-performance groups (10.91%), especially in the "consensus reaching and application" and "achievement reflection and evaluation" stages. The proportion of high-performance group students in the "divergence and clarification of views" and "content negotiation and co-construction" stages is 8.8%, lower than in the low-performance group. This further proves the dilemma of insufficient in-depth interactive learning and the high proportion of different views among students in the low-performance group.

Fourth, generally, the junior high school students' participation in CKB was good. They could solve mathematical problems through active speech and communication to form interactive groups, generate information, and make it flow between different individuals, indicating that both the high- and low-performance groups carried out MPSCL. Most group members spoke actively, participated in communications, and contributed personal knowledge to the learning community, promoting the generation and flow of information among groups. At the same time, the interaction groups had a variety of interaction modes, mainly including two-person and three-person interaction subgroups. There was also a phenomenon in which individual students rarely participated or interacted with group members, such as in high-performance Group 01aS2 and low-performance Group 02aS6.

2.5 Conclusion

As the above outline indicated, this chapter has considered MPSCL's unique teaching value for integrating "21st-century skills" and promoting practical and innovative talents, aiming at investigating junior high school students' CKB process based on a comparison of high- and low-performance groups' mathematical problem-solving for promoting student groups learning efficiency and performance.

Junior high school students could carry out CKB with mathematical problem-solving as the core. CKB-related discussions accounted for over 80% of all discussions in both the high- and low-performance groups. Student groups' CKB went through six stages and there were many kinds of view evolution paths with shared views accounting for the highest proportion, followed by discussion views and sublimation views, respectively. CKB reached the deep level, but the proportion was low. The junior high school students mainly had medium- and shallow-level CKB discussions, accounting for over 35% of the total. Generally, the junior high school students' participation was good and had a variety of interactive modes, mainly including two-person and three-person interactive subgroups. Some students rarely participated in the intra-group collaborative knowledge building process or interacted with other group members.

However, there were some differences in the high- and low-performance groups' CKB processes. Specifically, high-performance groups had slightly fewer MPSCL information interactions, a higher proportion of CKB-related discussions, and fewer irrelevant discussions than low-performance groups. The high-performance groups had fewer views than the low-performance groups but a much higher proportion of sublimation views and showed a better view evolution spiral, while low-performance group students' view evolution was relatively chaotic. High-performance groups had a higher proportion of deep-level CKB, a higher proportion of students in the "information sharing and understanding" and "content verification and adjustment" stages, and a lower proportion of students in the "different and clear views" and "content negotiation and co-construction" stages.

Since the twenty-first century, achieving the integration and development of "twenty-first century skills", such as collaboration/cooperation and problem-solving in mathematics education, and promoting the cultivation of practical and innovative talents has become a global education concern(Cao and Sun, 2019; OECD, 2017a, 2017b). As practical and innovative talents, their significant characteristics are reflected in their specialised knowledge structure in specific fields, their capabilities to deeply represent real-world problems, identify problem meaning patterns, and design flexible problem-solving strategies, which require deep-level knowledge building(Sawyer, 2012; EU, 2008). An experimental survey of junior high school student groups' MPSCL and examining their CKB based on the comparison of high- & low-performance groups' mathematical problem-solving in the classroom, we had a certain understanding of students' CKB process. These CKB characteristics could provide evidence for enhancing MPSCL, especially for teachers' design of new learning task, guidance for students' collaboration/cooperation, and problem-solving. From this perspective, further research is needed to explore student groups CKB addressing different collaborative problem-solving tasks, expanding student groups, and understanding how to promote the occurrence of higher-order thinking during MPSCL.

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Chapter 3
How Did Students Solve Mathematics Tasks Collaboratively? An Investigation of Chinese Students' Participation in Groups

Shu Zhang and Yiming Cao

3.1 Introduction

Problem-solving has always been a central topic in mathematics education. In the literature, researchers' exploration of the problem-solving process is usually focused on the entire process of completing the task and solving the problem. Pólya (1994) first introduced the problem-solving process academically in his book, which attracted the attention of mathematics education researchers. Pólya's mathematical problem-solving model comprises four stages: understanding the problem, proposing a solution, implementing the solution, and checking whether the problem is solved (Pólya, 2011; Pólya, 1944). The process emphasises that learners must clarify the known conditions and the problem objectives, draw graphs, verify and perform other practical processes, and exercise their problem-solving thinking ability by repeating the process. At the end of the twentieth century, Alan Schoenfeld further developed the problem-solving model, proposing that it is not necessarily a smooth linear process, and learners may constantly need to adjust their problem-solving strategies and processes. In his model, problem-solving must incorporate the following processes: analysing and simplifying problems, clarifying problem-solving principles and mechanisms, designing problem-solving strategies, and constructing problem-solving methods from macro to micro perspectives. Difficulties, large or small, may be encountered in the second stage, so learners must go through further

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exploration and exploration, returning to the problem analysis stage to make adjustments, implement problem-solving strategies, obtain possible results, and verify their feasibility, further generalising feasible results (Schoenfeld, 1985). Therefore, in Schoenfeld's problem-solving model, the problem-solving process' analysis and design phases are cyclical processes, emphasising flexibility, and iterative construction.

However, the above models involve solving mathematical problems from the perspective of cognition and subject knowledge skills. Although such models emphasise the transformation and application of mathematical knowledge and skills in completing problem-solving tasks, they do not consider the emotional, psychological, or social processes that learners may experience. For example, it has not been considered whether the learner will be unable to complete the problem-solving process due to their inability to propose a reasonable problem-solving strategy, which affects their confidence in problem-solving. Different from a single mathematical problem-solving situation, multiple students usually participate in the open-ended mathematical problem involved in a collaborative mathematical problem-solving situation. In this process, group members must not only complete tasks to solve problems but also experience social processes such as group work, coordinating problem-solving subgoals with other members, and listening to others' opinions. Because of its complex social nature, it becomes difficult to establish a widely recognised collaborative mathematical problem-solving model in the current research field.

Chinese scholars Xu and colleagues proposed using collaborative literacy and its three elements (vision recognition, responsibility sharing, and negotiation and progress) to indicate whether students can work well in groups. Their model emphasises the procedural nature of collaboration and believes that in the process of completing collaborative tasks, learners need to mobilise their meta-cognitive strategies, adjust goals at different stages, make continuous corrections through immediate execution, reflection, and evaluation, and gradually realise the purpose of collaboration (Xu et al., 2020). The model not only emphasises the procedural nature of task completion in the context of collaboration but also macroscopically describes the social process that learners need to participate in the collaboration process. However, completing the task in a collaborative mathematical problem-solving situation requires considering not only the process but also the sociality in the collaborative situation. Therefore, it is necessary to promote more specific and related research rooted in the task completion process to explore the path of collaborative mathematical problem-solving. The negotiation process in group work may be seen as a starting point for related research.

3.2 Literature Review

3.2.1 Students' Negotiation Process in Groups

Students' involvement in collaborative group mathematics problem-solving involves not only participation but also negotiation. In a group, everyone may initially have a different understanding of the same problem but then, through discussion, gradually come to understand each other's views and perspectives. The researchers use the phrase "negotiation of meaning" to conceptualise the social negotiation process through which people negotiate to make themselves understood and understand others. Clarke (2001a, 2001b) described the negotiation process as "a cyclical process of interpretation, reflection, and representation based on shared goals" (p. 35). In the negotiation process, group members gradually understand others' words and thoughts. When viewed as a negotiation process, the content of group negotiation can illustrate the discussion's core and the participants' common goal (e.g., solving a problem).

The existing research usually adopts one of two ways to study the negotiation process. Some researchers have focused on exploring group influences in the negotiation process; that is, information shared, exchanged, and refined within the group in, for example, the discussion process. DeJarnette (2018) saw negotiation as a process in which group members exchange information and behaviour during turn-taking in group activities. On the other hand, some researchers, in addition to focusing on the information shared and exchanged within the group, also pay attention to the degree of individual negotiation in group activities; that is, how each member negotiates or compromises with themself through moment-to-moment discussion. For example, Engle et al. (2014) studied the extent to which an individual's multiple aspects (knowledge authority, freedom of dialogue, spatial priority) can be compromised in a social context. This study does not view these two perspectives on inquiry negotiation as dichotomous and considers both. By focusing on the negotiation process in collaborative mathematical problem-solving, researchers pay attention to the purpose and topics shared in group discussions and individuals' participation in and contribution to the group discussion process.

3.2.2 Research on Negotiation Discourse in Mathematics Classrooms

Although peer-led group collaborative learning has a long history in classroom practice, research systematically exploring whether mutual communication among students can promote mathematics learning and the communication mode therein dates from the 1980s (Webb, 1982). Based on 19 published relevant empirical studies, Webb (1989) summarised a simple model between peer communication and mathematics learning, arguing that in group discussions, issues such as high-level interpretation, member feedback quality, and whether to get feedback are all important group interaction factors affecting mathematics learning. Current research on mathematics education promotes the development of mathematical discourse activities by promoting students' in-depth communication activities in the classroom (Cobb and Bauersfeld, 1995; Inagaki et al., 1998; Moschkovich, 2002; Chan and Sfard, 2020). The dialogues and exchanges in group activities provide rich information for exploring the meaning behind interactive and discourse behaviours from students' perspectives.

Australian scholars David Clarke and Lihua Xu compared the speech of students in mathematics classrooms in Shanghai, Seoul, and Melbourne and deeply explored the role of language in mathematics classrooms in different cultural backgrounds, finding that whether students have a place to speak in classrooms is a culturally influenced behaviour. For example, in the Seoul classroom, students only spoke in groups and rarely voiced individual comments, whereas there was almost no group-speaking session in Melbourne classrooms. Additionally, the terminology used by teachers and students varied in different classrooms and teaching practices. Although teachers and students had frequent exchanges and conversations in Melbourne classrooms, mathematical terms were used relatively infrequently. In the group speeches in Shanghai and Seoul classrooms, there were two discourse patterns: teacher questioning—collective answering—teacher feedback, and teacher questioning—individual student answering—collective feedback. The mathematical content contained in these two modes of discourse was different, with the former containing less mathematical content and the latter usually including an understanding of basic problem-solving concepts or procedures. Students were given opportunities to strengthen their understanding of concepts and participate in the teaching process during group assessment. Thus, different discourse patterns create different student engagement and learning opportunities (Xu and Clarke, 2019).

Supported by the international Learner's Perspective Study (LPS) project, mathematics researchers from various countries have researched discourse manifestations in mathematics classrooms based on classroom teaching videos. Chinese scholars have also used videos to analyse the structure of teacher–student discourse and compare the discourse volume in Chinese mathematics classrooms. Early research on teacher–student discourse in Chinese classrooms is usually based on statistical research methods, comparing the number of teachers and students in the classroom and differences in practice. Cao and colleagues analysed the quantity and length of teacher–student discourse in a five-class mathematics classroom for four teachers and concluded that the proportion of teacher and student discourse varied greatly between different mathematics classrooms, although teachers' discourse in the classroom generally is far greater than students' (Cao et al., 2008). Dong et al. explored the influence of teacher–student interaction in the classroom and questioned the core of students' mathematics ability learning and development, taking the discourse interaction between teachers and students as the main research object and the teacher's questioning as the key starting point. Their study proposed that teachers' questions not only consciously included teachers' consideration for achieving curriculum goals but also unconsciously included how to develop students' thinking skills (Dong et al., 2019). Although the above similar research highlights the importance of teacher–student interaction discourse in the classroom, it also reveals that the teacher discourse's absolute power in the classroom is dominated by teaching and may ignore peers' discourses to an extent (Webb, 1989). In fact, students will establish a notable discourse order and culture in a specific classroom environment.

In the context of collaborative mathematics problem-solving, multiple foci occur in student interactions. Firstly, students need to negotiate the subject matter of mathematics, including the tools used in solving problems and the particular language of mathematics, such as facts, procedures, propositions, and so on (Schoenfeld, 1992; Steinbring, 1991). Secondly, students must negotiate norms of social behaviour suited to the mathematics classroom, including teachers' and students' rights and duties, to ensure the class' effective functioning (Brousseau, 1986, 2002). Yackel and Cobb (1996) advanced the concept of social norms and proposed the construct of sociomathematical norms, suggesting that some mathematics-specific norms exist in mathematics classrooms, including, for example, what is considered mathematically sophisticated or an acceptable mathematics answer. In a recent study, Zhang et al (2022) explored Chinese and Australian students' negotiative foci in terms of either facts and procedures, didactical norms, or social/interpersonal consideration, and found that compared with the Australian counterparts (8.95%), the Chinese pairs (4.84%) relatively less focus on negotiating didactical norms.

Since discourse is affected by the mathematics classroom's cultural environment, different discourse environments can result in different mathematics learning. As students follow and shape certain social norms in classroom dialogue, how and to what extent can these discourse characteristics be embodied in collaborative mathematical problem-solving contexts? In collaborative mathematics problem-solving, students gradually achieve the group's goal through communication and negotiation in the problem-solving process and complete or fail to complete the task. Therefore, examining the group's negotiation process is not only an in-depth excavation of its members' participation process but also a reflection of the cultural factors affecting it. Therefore, we examined five groups' negotiation processes in collaborative mathematics problem-solving to understand the characteristics of group interaction, further analyse group members' participation in tasks, and identify implications for future research and teaching practice. Specifically, this paper:

(1) analyses the negotiative event chain for all discussion groups' task completion processes, allowing researchers to more accurately understand and describe each group's task completion process; and
(2) codes and classifies each group's task completion process to analyse and explore each member's participation process in the different task completion processes.

The above exploration provides a foundation for understanding student participation in collaborative mathematics problem-solving, which may help students improve the quality of collaborative group discussion and teachers' instruction in practice.

3.3 Materials and Methods

3.3.1 Data Source

The data in this study were derived from the Australian ARC Project: Social Essentials of Learning. This study focused on the collaborative problem-solving process of five groups in a class (Class 1B) of a middle school (LHZX) in City B taught by math teacher Zhang (pseudonym). There are two main reasons for recruiting these five groups, first, the five groups in this class were taught by the same teacher, so we assume that certain characteristics in terms of classroom norms could be determined. Second, the five groups have different gender composition in terms of number of female students and male students, which allow us to observe how gender composition may influence students' collaborative work. In addition to student video data and classroom materials, after-school interviews with teachers were important in this study. During the interviews, teachers evaluated and explained group members' behaviours, which was a source for understanding students' participation behaviours. All five groups in this study completed the task to a certain extent, but each group's problem-solving process and work differed. The group works as a whole completed the basic requirements of the task within an acceptable range. The commonality among these groups was that each had relatively abundant verbal interactions in various forms and relatively clear problem-solving paths in the group problem-solving process. Each group included four students and worked on the following mathematical Task one (see Appendix).

3.3.2 Analytical Approach

Based on the above purposes, this study analyses the video data and the transcripts of five groups that collaboratively work on a mathematical task. First, through repetitive observation of the recorded video, the researchers get familiar with the groups as well as the context of their collaborative work. On this basis, the transcriptions are used as another primary data. The researchers divide the transcribed text into negotiative event, determine the discussion topics in each negotiative event according to the discussion content of the students in the group discussion process, and further arranges all the topics as a chain which describes the problem-solving process of each group.

After obtaining the negotiative event chain, based on the similarity between different topics or the similarities of the functions of the negotiative events, the problem-solving process' corresponding characteristics will be obtained. Combined with the recorded videos, the analysis of student participation during the problem-solving process will be given in terms of how students participate in each and a series of negotiative events.

Operationally, the following concepts need to be clarified.

(1) Negotiative Event (NE): An NE is defined in this study as "an utterance sequence constituting a social interaction with a single identifiable purpose" (Chan and Clarke, 2017). The group discussion topic is the "single identifiable purpose" in this study.
(2) Discourses: Discourses in this chapter refer to students' complete or incomplete sentences spoken to themselves and each other during the discussion, as transcribed from the video. In the transcript, each sentence transition counts as a sentence, regardless of whether the sentence constitutes a full and strict sentence in literature.
(3) Discourse volume: Discourse volume usually refers to the number of discourse sentences, as described above.

3.3.3 Validity

The validity of this study in defining negotiative events and dividing negotiative events mainly comes from the following guarantees. First, the definitions are clear and standardised. Clarke and Chan have deepened and consolidated the definition of negotiative events in the related literature and tested its validity in parallel Australian data (Chan and Clarke, 2008, 2019). Second, the coding process was examined by experts in education or mathematics education from our and other countries. After the definition and code were developed, they were reported to the International Classroom Teaching Research Center and the Mathematical Science Research Group of the University of Melbourne, Australia, respectively, until all experts recognised the final results.

In this study, the negotiative events in each group were coded by multiple people to determine their reliability. The initial coding time for all groups was from February 2019 to April 2019. In October 2019, the researchers re-coded and proofread all coding processes, with 94.7% consistency. Three groups were randomly selected for a third encoding in December 2019, reaching a 93.0% consistency. The error was controlled within five sentences. In the final stage, the researchers selected three groups to code independently with the other two researchers, discussed and negotiated uncertain divisions, and ensured that all three parties' coding results were close to the same.

3.4 Results

3.4.1 An Overview of a Negotiative Event Chain

Tracing the topics discussed during the task completion process can determine each group's task completion process. The following Table 3.1 lists the negotiative event chain of each group. These negotiative event chain topics could describe how each group worked on the task and solved the mathematical problem.

Based on Table 3.1, two main characteristics of the five groups in the negotiative event chain during collaborative problem-solving can be obtained.

(1) The number of negotiative events in each group varies

The above table shows that the number of negotiative events discussed by the five groups is different; 21, 21, and 20 negotiative events were determined by three groups of 1BG1, 1BG2, and 1BG4, respectively, a relatively similar amount, while the other two groups in 1B, 1BG3 and 1BG5, discussed 12 and nine topics, respectively. The number of negotiative events corresponds to the amount of discourse during the five group dialogues. Analysis of the discourse volume in the five group discussions shows that although the prescribed time to complete the task was the same, the discourse volume of each group in a given time differed. The Table 3.2 gives the five groups' total utterances during the discussion; that is, the total number of sentences for all students.

According to this table, the groups' discourse volume seems to correspond to the number of negotiative events determined. For example, groups 1BG3 and 1BG5 were very close regarding their number of negotiated events and total volume of speech, as were groups 1BG1, 1BG2, and 1BG4. We would wonder whether groups with similar numbers of negotiative events and discourse volume have similar characteristics in the problem-solving process. The next sections will provide a more in-depth analysis of the negotiation process.

(2) The content involved is different—the intersection of pure mathematics and contextual knowledge

Based on the content knowledge involved in each negotiative event, the researcher clarified three types of NEs, including mathematics and task-related (MT) NE, non-mathematics but task-related (NMT) NE, and off-task (OFF) NE. The first type means that during this NE, students' discussion focused on task-related mathematical knowledge, the second type means that students' discussion was about the task context and certain related mathematical knowledge, and the third type NE means students' discussion was off-task during these NEs. The following Table 3.3 summarises the distribution of different content negotiative events in each group during the entire discussion process.

It can be seen from the above table that almost all group discussions were task-related. The proportions of MT and NMT events in groups 1BG1 and 1BG3 differed.

Table 3.1 Negotiative event chain of the five groups

No	1BG1-3M1F	1BG2-2M2F	1BG3-4M0F	1BG4-1M3F	1BG5-0M4F
NE1	How to write your name in Chinese characters? [Name labeling]	How big is 60 square metres? How many rooms are in the apartment?	What rooms should be included?	[Casual talk]	What rooms should be included?。
NE2	What tools are needed for drawing?	Which types of rooms should be in an apartment?	Can we use scratch paper?	Can we draw a foursquare?	The arraignment of 60 square metre and rooms?
NE3	How to decide the length and width of the apartment?	[Casual talk]	The size of toilet and living room?	Can we use scratch paper and pencil to draw?	The size of toilet and other rooms?
NE4	Which one is better, foursquare or rectangle?	Whether 2 square metre is enough for a toilet?	How big is the classroom?	How many square metres of rooms?	What is the size of the other rooms?
NE5	What is the area in total? And what are the size of each room?	Whether 5 square metre is enough for a toilet? How big is for bricks?	What is the size of the balcony and kitchen and what is the size of remaining area?	Is it possible to tear the paper?	What is the size and location of the rooms except the bathroom?
NE6	Where should be the location of each room? What should be the direction of the apartment?	How long is one edge of a brick?	Where should be the location of kitchen and toilet?	What is the shape and size of the room?	Operative arrangement of who will write and who will draw the picture; size and location of the living room
NE7	Is the apartment one layer or two?	How big is one brick? How big is four bricks?	What rooms can be painted?	What is the size and number of rooms?	The size and location of the study and bedroom
NE8	[Causal talk]	Is 5 square metres enough for a bathroom?	What is the general size of a two-bedroom apartment and the size of Xiao Ming's apartment?	[Casual talk]	Check if there are other topics required

(continued)

Table 3.1 (continued)

No	1BG1-3M1F	1BG2-2M2F	1BG3-4M0F	1BG4-1M3F	1BG5-0M4F
NE9	Do we need to consider about the furniture?	Is 5 square metres enough for a bathroom?	The location of the balcony?	Whether it is necessary to have a balcony?	Overall design as well as other edge designs such as elevators, etc.
NE10	Size of rooms and location (specific to kitchen, bathroom, and bedroom)?	Where are the bathroom and toilet located in the apartment?	[Casual talk]	How many rooms do we have in total?	
NE11	Location of room in terms of kitchen and bathroom	How big do the other rooms need to be (bedroom and study)? How many square metres are left?	Whether it is needed to have a study room?	A study room is necessary? And the size of the balcony?	
NE12	How many rooms?	Should we draw the diagram first?	"You design is bad," quarrel in group	What is the function of living room?	
NE13	Location of toilet and its structure	Will a 6 × 10 rectangle work?		The size of the balcony?	
NE14	Whose work should be picked as the submitted group work?	[Pass the straightedge between students.]		The size of the balcony is compared with the size of the bathroom and the size of the living room	
NE15	How many steps should we take to draw?	[Pass a straightedge and eraser between students		Who will draw the diagram on the task sheet?	
NE16	Do we need to draw doors?	Is a scale needed? How big does the living room need to be?		Living room size? Kitchen size?	
NE17	Do we need to draw doors as holes?	Is it appropriate for the apartment to be rectangular in shape?		How are group drawing roles assigned?	

(continued)

Table 3.1 (continued)

No	1BG1-3M1F	1BG2-2M2F	1BG3-4M0F	1BG4-1M3F	1BG5-0M4F
NE18	Am I suitable for being a designer?	How many square metres are left? Is the remaining area OK for a study? Do I need to paint the door?		Balcony size and bathroom size	
NE19	Whether doors should be seen as rooms?	Whether it is enough for a 16 square metres of bedroom?		Is it possible to consider switching the bathroom and balcony??	
NE20	The location of rooms in the apartment and their connections?	What furniture is needed in the living room? How many square metres are left in the kitchen?		Overall feedback and discussion about the design	
NE21	Calculate the area of each room? And requirements for submitting the worksheet	Is this diagram realistic in the physical world? [Label the rooms and doors			

Table 3.2 The total discourse volume of five groups

	1BG1	1BG2	1BG3	1BG4	1BG5
Total discourse volume	223	228	110	231	112

Table 3.3 The number of different NE categories

Topic category	1BG1	1BG2	1BG3	1BG4	1BG5
MT	4	10	2	8	4
NMT	15	9	9	10	5
OFF	2	1	1	2	0

Both groups showed relatively more discussions about non-mathematics and task-related topics and relatively fewer pure mathematical topics. The proportion of MT and NMT events was similar throughout the discussion for the other three groups.

3.4.2 *Participation of Group Members in the Process of Task Completion*

The above provides a brief overview of how the groups complete the tasks, but the details in terms of how each group member participates in the negotiation process and how the students interact with each other are still not clear. Combined with transcript analysis, the following sections will provide a more detailed analysis of the negotiation process from multiple perspectives.

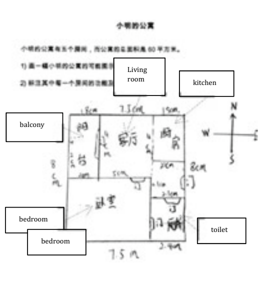
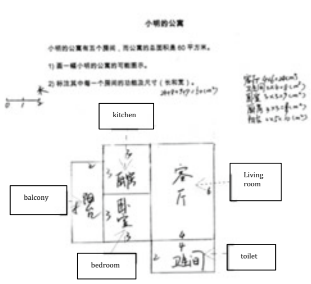
3.4.2.1 The Effect of Discussion Content on Student Participation

Analysing the negotiative event chain deepens our understanding of the negotiation content involved in the task completion process. In the group task completion process, the negotiation content involved has a non-negligible effect on students' participation in problem-solving. The number of MT and NMT events discussed by different groups differed throughout the discussion process, as did each member's participation process.

Among the five groups in class 1B, the proportions of MT and NMT events in the 1BG1 and 1BG3 groups were quite different, with more NMT topics than MT topics tending to be discussed. Groups 1BG1 and 1BG3 were composed of three boys and one girl, and four boys, respectively; that is, boys accounted for a larger proportion of each group. As seen from the transcript and video, it seems that group members rarely kept raising questions about the mathematical content during the group discussion process. Individual students in the group undertook the entire mathematical calculation process and mathematical problem-solving. The following Table 3.4 shows the task completion of the two groups, 1BG1 and 1BG3.

In fact, during the 1BG1 group's discussion, S1 took the main role in completing the task, including drawing the figure, doing all the mathematical calculations, and so on; some corresponding data mismatches appeared in the apartment map as drawn

Table 3.4 Worksheet of groups 1BG1 and 1BG3

1BG1	1BG3
	

(e.g., inconsistent width and length on both sides of the balcony, etc.). 1BG3's situation was slightly different. While the mathematical data in the 1BG3 team's task list was correct and met the requirements, almost all mathematical calculation tasks were undertaken and completed by S3 during the group's discussion. Although other students questioned S3's room design, layout, and other issues, S3 did not change the data and design and insisted on completing the task list according to his ideas.

On the other hand, for groups 1BG2, 1BG4, and 1BG5, the number of events for the two types of content was similar, with the discussion of MT and NMT events showing a certain regularity. For example, in the 1BG2 group, during the whole discussion process, MT events usually appeared in a series; that is, once the discussion of one MT event was initiated, the probability of the group continuing to discuss several MT events would be relatively high. The NMT events were similar, showing the phenomenon of getting together; that is, several associated events may be NMT events. A similar situation was also seen in 1BG5.

Undeniably, students must master mathematical knowledge and certain contextual knowledge about the task in the collaborative mathematics problem-solving process. The occurrence of a series of MT events had certain benefits for students to discuss and share mathematical knowledge fully. In the discussion process, students continued to extend their knowledge and realise or clarify their lack of understanding through group communication and discussion. The following are excerpts from group 1BG2 during NE6. In this segment, after S3 asked how many (square) metres were in 360 square centimetres, S1 and S2 estimated it as the area of a classroom floor tile and then, through this, estimated whether the apartment's individual rooms were reasonably sized. After this incident, a series of MT incidents assessing whether the size of the bathroom was appropriate appeared.

S3 (NE6.2): Wait, wait, how many square metres equal to 360 square centimetres?

S1 (NE6.3): That's too small.

S2 (NE6.4): I use a scratch paper.

S4 (NE6.5): She's talking about a brick.

S1 (NE6.6): That should be 3.6 square decimeters.

S3 (NE6.7): That is 3.6 square decimeters.

S1 (NE6.8): How is 60 times 60 to 360?

S3 (NE6.9): 3600. That is 36 square decimeters, and then, 0.36 square metres, not even one square metre, then you say 4 bricks.

S1 (NE6.10): Four bricks are bigger than one square metre.

S4 (NE6.11): Four bricks make one square metre? What is 0.36 times 4?

S2 (NE6.12): If it is 60 square centimetres, you need 6 cm across.

S4 (NE6.13): 4 blocks are 1.44 square metres.

Note: S1, S2, S3, and S4 represent the four students, respectively, while NE6 is the event number. All the sentences in this event are coded in order; for example, the first sentence is NE6.1, and the paragraph starts from NE6.2.

As seen from students' discussions, in the course of negotiation, whether the MT event string could be realised or caused depended on whether there was a group member who could ask and insist on the mathematical problem, such as, "how many square metres equal to 360 square centimetres?" In the above segment, S3 raised this issue for the second time in 1BG2's group discussion. When first raised, the other group members did not answer; when asked again, students started to provide feedback and calculations and finally decided on an area standard that could be used for further calculations. A similar problem also occurred in the 1BG3 group. When S4 raised the question, "how big is the classroom" during the discussion process, it was ignored by other group members; after S1 responded, "I don't know," S4 did not insist on re-proposing. The reason for S4's question in group 1BG3 should be the same as the reason for S3's question in 1BG2 (i.e., corresponding to the actual area size rather than the mathematical area size); however, during the group discussion, due to various members' participation, the interactive and dynamic nature of a certain problem cannot guarantee an immediate response or solution once a problem is raised. Therefore, the same question asked by different group members in different groups can also create different situations. Repeatedly asking the same questions, as S3 did in the 1BG2 group, can cause a string of MT topics.

3.4.2.2 Students' Volume of Discourses Are Different

Analysing the negotiative event chain provides a basis for exploring students' participation process in collaborative mathematics problem-solving and opens up a direction for analysing group members' discourse volume in the whole negotiation process. In group problem-solving, each group member's discourse volume can reflect their verbal participation in the negotiation process to a certain extent. By counting each student's utterances throughout the entire discussion and calculating the proportion of all utterances, we can see how many words each student contributed to their group's discussion.

Figure 3.1 shows the students' percentage of speech volume in the five groups during the discussion process. The number of speeches (utterances) a group member makes during the entire negotiation process is their volume of speech and reflects their discourse contribution to the problem-solving process, to a certain extent.

In the 1BG1 group, the discourse volumes for S1 and S2 were relatively large, accounting for 36% and 31% of all utterances, respectively, while S3's and S4's discourse volumes accounted for only 14% and 18%, respectively.

In the 1BG2 group, S3 had the highest speech volume (38%), while S1 had the lowest (14%). S4 and S2 accounted for roughly the same amount of discourse, 23% and 25%, respectively.

In the 1BG3 group, S1's and S3's proportions of speech volume were similar (29% and 27%), with S4 contributing 25% and S3 19%.

In the 1BG4 group, S4, S3, and S2 contributed 30%, 27%, and 25%, respectively, while S1's speech volume accounted for 18% of all utterances.

In the 1BG5 group, S1 spoke the least (6%), with S2, S3, and S4 accounting for 21%, 33%, and 39%, respectively.

To summarise, in the five groups of class 1B, except for the large difference in speech volume between S1 and other members in group 1BG5, the group members' speech volumes were not very different. Comparing each group member's discourse

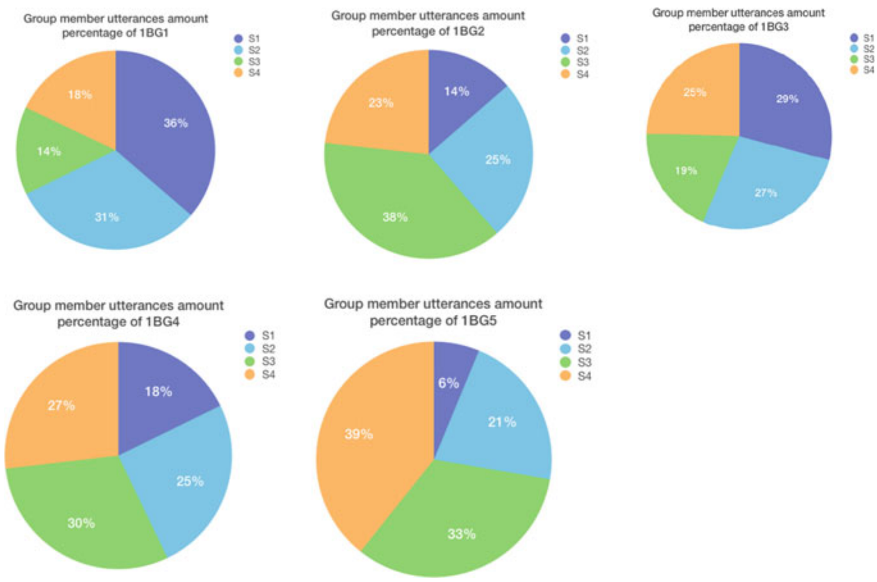


Fig. 3.1 The proportion of utterance amount in five groups

volume shows there were usually some members with more discourse volume and some others with less.

3.4.2.3 Students Have Different Levels of Control Over Tasks

Analysing the participation of group members from the perspective of topic content and discourse volume alone is not enough to describe the participation process because even if some members speak a lot or participate in specific topics, the participation process differs in other problem-solving stages. Further understanding group members' control power in the group problem-solving process helps to describe individual students' participation processes.

In the actual problem-solving process, the contextual task information that changes the discourse volume in each negotiative event differs in each group. For example, if the negotiative event with the most speech groups is excluded from the 20 negotiative events in the 1BG4 group (NE19; 39 utterances), the other events remain relatively stable. However, during the NE19 discussion, panellists experienced a relatively fraught row over whether the locations of the bathroom and study could be interchanged, which could reflect the relationship between group members to a certain extent.

The 1BG4 group comprises three girls (S1, S2, S3) and one boy (S4). In the NE19 discussion, G4S2, G4S3, and G4S4 were asked whether the positions of the bathroom and study room in the already formed draft need to be interchanged. Disputes arose, with S2 and S3 insisting they needed to be swapped and S4 disagreeing.

Symbol description: S1, S2, S3, and S4 represent the four students. NE19 is the event number. All event sentences are coded in sequence, e.g., the first sentence is NE19.1.

S2 (NE19.5): You draw, you draw. Why is this set of data so messy? What is 6 square metres? 6 square metres is a balcony, what is 5 square metres?

S3 (NE19.6): Toilet.

S4 (NE19.7): 5 square metres?

S2 (NE19.8): 5 square bathroom?

S3 (NE19.9): Can't we just change the balcony and bathroom?

S1 (NE19.10): No time.

S3 (NE19.11): No problem, just change its label.

S4 (NE19.12): Oh, you don't have to change it, you can just do it.

S3 (NE19.13): Have you ever seen a balcony bigger than a bathroom?

S2 (NE19.14): The bathroom is smaller than the balcony.

S4 (NE19.15): Huh?

S2 (NE19.16): The bathroom should be bigger than the balcony, otherwise how can you stay there?

S4 (NE19.17): My bathroom is smaller than the balcony.

S2 (NE19.18): My grandma's bathroom is too big.

S4 (NE19.19): That's it, no time.

S3 (NE19.20): Changed the label——

S2 (NE19.21): Change the label and the words.

S3 (NE19.22): The location does not need to be changed.

S4 (NE19.23): But you also changed the size…

S3 (NE19.24): Isn't that the end of the name change?

S4 (NE19.25): that data…

S3 (NE19.26): Our data is on scratch paper.

S4 (NE19.27): You should change if you like.

S3 (NE19.28): Changed the name, but the data remains the same.

S4 (NE19.29): Well, change if you like.

S3 (NE19.30): The balcony is still small, how can I get it?

S4 (NE19.31): Change it if you like.

S2 (NE19.32): Yes, then we will change it.

S4 (NE19.33): What else to say.

S2 (NE19.34): Yes, he was nagging there, nagging like an old woman.

…

(This paragraph shows that the disputing parties did not have a common understanding and did not seek the others' understanding.)

The above intercept shows the quarrel between the members of the 1BG4 group during the final stages of the group discussion. S2 first raised a question about matching the relevant data between the room and the room area, causing the rest of the group to address and clarify the size and function of the various rooms. S4 responded to S2's question and proposed that five square metres had been designated for the bathroom. S4 and S2 then tried to verify whether a five-square-metre bathroom was appropriate. S3 thought that the bathroom was too small and proposed changing the positions of the bathroom and balcony. At this point, S1 expressed concerns about the time to complete the task. S3 again proposed simply swapping the names of the two rooms on the task list. S4 also raised objections and questions at this time. To this point, both parties' intentions were relatively clear. S4 thought the bathroom should not be replaced, while S2 and S3 insisted on changing it. S1 raised concerns about time but did not explicitly oppose or approve the swap.

In the ensuing discussion, the two sides gave reasons for the exchange. S2 and S3 believed the bathroom should not be smaller than the balcony, which S4 did not recognise. S2 and S3 proposed changing the rooms' labels, stating there was no time to redraw the plans. S4 believed that when the rooms were relabelled, the data would also change accordingly, so changing the label was not just changing the name. S2 and S3 then proposed that the data were draft data and could be changed. In the end, S4 compromised and ended the dispute with, "change it if you like."

Analysis of the above dialogue shows that the process that caused and evolved the dispute was as follows. First, before the discussion began, the two parties had inconsistent understandings of whether the bathroom or balcony was larger. S2 and S3 believed the bathroom should be larger than the balcony, while S4 believed it could be smaller. During the discussion, although S2 and S3 suggested that it would be fine just to change the name, S4 believed that would lead to a change in the data (and thus necessitate recalculation). At this point, S2 and S3 might not have understood S4's thinking nor responded to S4's questions from the perspectives of data and mathematics, and the dispute reached its peak. In the end, S4 "reluctantly" accepted the proposals of S2 and S3, and S1 changed the label in the graphics. S4 used "you" several times in the final discussion to refer to and differentiate themself from S2 and S3, reflecting the separation of the whole and its parts.

Although the reason for the dispute started from a lack of consensus, neither party carefully considered and calculated the other's proposals during the discussion, so it was difficult to form a consensus. S4's grudging compromise and separation confirm the lack of true consensus and, to a certain extent, reflect that S4 was on the "weak" side of the group discussion.

Figure 3.2 illustrates the evolution of the group discussion's content and the group members' participation in discussing NE18, dividing it into three stages. Based on this process, the group discussion was affected by establishing consensus among subjects, the relationship between the whole and the individual, and (because S4 was the sole boy) possible gender differences. While these factors were reflected in a single negotiative event, they may exist in all of them. The negotiative event chain can also indicate the influence of different factors, which may intensify during the discussion. Therefore, team members' engagement throughout the task completion is closely related to the procedural changes in consultation on specific issues.

In the above discussion process, S2, S3, and S1 had relatively stronger control over the task than S4. Therefore, both from the negotiation result (the group did not accept S4's proposal) and the negotiation process (the group did not accept S4's proposal),

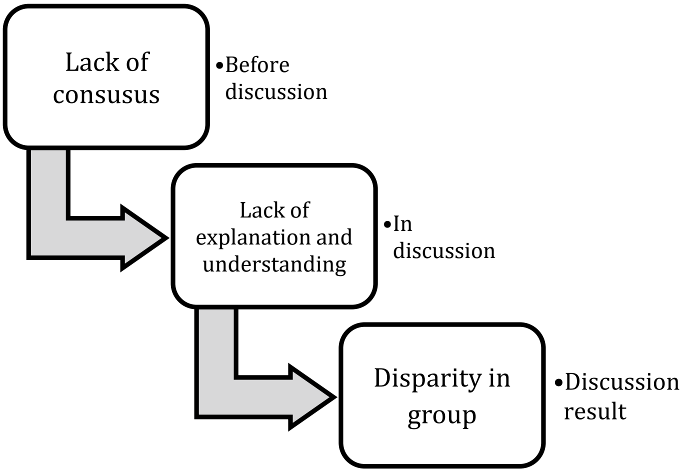


Fig. 3.2 The evolution model of NE18

a temporary intra-group separation occurs, and S4's control over the negotiation process continues to weaken, making their participation change.

The same phenomenon appeared in other groups, reflecting a gradual transfer of control over the problem-solving process. In the 1BG2 group, for example, although all group members initiated the task completion process, control gradually transferred to individual members as the discussion progressed. Since the negotiative event S2 initiated did not match the current task content, their participation in and control over the process gradually declined, weakening the entire participation process.

3.5 Summary

This chapter achieves two research goals. First, providing an overview of the negotiative event chain facilitates understanding the task completion process and how students negotiate on different topics to complete a task in each group. Second, based on the negotiative event chain, students' participation and negotiation details are interpreted from different perspectives regarding the discussion content, students' utterances amount, and group status in terms of collaboration or disparity in negotiation.

Corresponding to the problems and tasks in this study, the five groups reflected specific and individual characteristics. First, although the number of negotiative events related to the negotiative event chain differed in each group, most were related to the mathematical problem. Second, the amount of discourse about negotiative events in each group's negotiative event chain varied. Third, the content of negotiative events shows there were more MT events than NMT events, i.e., the students discussed mathematical content less than contextual content. Individual groups or classes also exhibited characteristics that provided new perspectives for further understanding group participation in problem-solving.

In addition to analysing each group's problem-solving process, the group members' participation process was explored from three perspectives—discussion content, student utterances, and group status in negotiation—with the first two providing a basis for exploring the last. The study found that (1) specific negotiation content probably impacted group members' participation in the task completion process; (2) group members' discourse volume proportion showed a degree of gender difference in each group; and (3) group members' responses to a particular sub-task may influence their participation in the overall task completion process.

The research in this study further consolidates the arguments already made in the previous research; that is, student participation in collaborative mathematics problem-solving needs to be investigated from various perspectives. The study provides certain empirical trials in terms of understanding how Chinese students work collaboratively in mathematical tasks. As seen from current literature, although the east–west comparison about mathematics teaching and learning has been widely conducted, it rarely reveals the characteristics of student interactions among Asian students. Most researchers who state that collaborative learning can be beneficial for students' mathematics learning are based on research conducted in Western culture (Xu and Clarke, 2019). To investigate how collaboration may happen in Asian student groups and whether such collaborative work contributes to students learning, it is essential to first understand how students interacted with each other in groups. Therefore, such trials can pave the way for readers to understand the negotiation process in each group's task completion process.

However, the analysis in current study is still at a macro level; more combined micro- and macro-level analyses are needed in future studies. On the one hand, more in-depth analysis about student interaction and their mathematics learning in Asian classrooms should be done so as to verify how collaborative learning actually plays its role in both students' mathematics learning and Asian mathematics classrooms. On the other, if that collaborative learning does contribute to students' learning, how should teachers and students make efforts on facilitating this benefits through teaching and learning, how can educators help teachers and students in practice should be further investigated.

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Chapter 4
Research on Individual Authority and Group Authority Relations in Collaborative Problem Solving in Middle School Mathematics

Jue Wu

4.1 Introduction

Classroom teaching practices have attracted the increasing attention of researchers (Li et al., 2022; Zhao et al., 2022). In teaching practice, front-line teachers are committed to promoting class management based on a fair and equitable system or the theory of teacher-centred authority in class management. Researchers have focused on examining teacher authority while paying insufficient attention to student authority. Teachers should not only pass knowledge to their students but also encourage them to think and learn by themselves, which is good for teaching and learning (Díez-Palomar et al., 2021). Good teacher-student relationships come from the integration of teacher authority and student authority. However, very little attention is paid to student authority in mathematics education. Collaborative problem solving (CPS) can be a good vehicle for exploring student authority. It provides a mathematical learning environment that involves non-teacher-led activities. In CPS, students use a range of mathematical and non-mathematical forms of language to gain authority, which influences the process and outcome of learning. For these reasons, the study of individual authority and group authority relations in CPS in mathematics is bound to become particularly important.

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4.1.1 Conceptual Framework

4.1.1.1 Authority

The term "authority" is widely used in social life and has different meanings in different cultures and contexts. In the Chinese dictionary (*Ci Hai*), it refers to two concepts: (1) power and prestige, and (2) a force of prestige and dominance developed in the course of human society. Also, there are different interpretations of authority in different disciplines. From a sociological perspective, authority is a force that convinces people without pure violence based on two elements: voluntary obedience and belief. In psychology, there are two manifestations of authority: formal and informal, which highlight the influence individuals and groups have on other people and groups.

There are also different interpretations of authority. Despite different disciplinary perspectives, all definitions reflect at least two characteristics. First, authority emphasises the relationship of obedience between authority objects and other authority subjects, based on value recognition. Second, the operation of authority produces a relationship between authority objects and other authority subjects in terms of influencing and being influenced. It is the recognition of authority objects to authority subjects in terms of ideology and obedience in terms of behaviour.

This study examines changes in the relationship between individual authority and group authority relations in CPS in mathematics. The definition of student authority used herein is based on the theory that authority emphasises the obedience of authority objects to authority subjects. This authority relationship between three and more people is the theoretical basis for the definition of group authority relations. Students' group authority relations change through interactions, and changes in authority are dynamic.

4.1.1.2 Student Authority in Mathematics Activities

This study examines the relationship between students' individual and group authority in CPS. In teacher-empowered student collaboration and management of student-led CPS activities, authority shifts from being unilaterally held by the teacher to being distributed between the teacher and the students. Mathematics activities research has begun to examine the authority relations between students.

In terms of research on authority in classrooms, several studies have pointed to definitions of authority in classrooms. Cohen (1994) defined authority as "an agreed-on rank order where it is generally felt to be better to be high than low rank." Status can be thought of as a relationship of power among peers. That power can be academic, as in status, derived from perceived smartness, or social, as in status derived from popularity. This somewhat vague positioning of authority highlights its intellectual and social categories. Ernest (2008) proposed that a teacher "has two overlapping roles—namely as director of the social organisation and interactions in the classroom (i.e., social controller) and as director of the mathematical tasks (i.e., task controller). This distinction corresponds to the traditional separation between being 'in authority' (social regulator) and being 'an authority' (knowledge expert)." Researchers defined authority as a resource of control associated with the right to lead and the obligation to follow (Amit & Fried, 2005; Ernest, 2008; Pace & Hemmings, 2007). Teachers can be both an intellectual authority based on their knowledge and a social authority based on their power to issue instructions to students and control their behaviour. Boaler and Greeno (2000) believed that social authority always operates in classrooms, occurring wherever humans interact. Intellectual authority relations are at play when individuals are engaged in intellectual work, defined in schools as engaging in academic tasks.

From the above scholars' explanations of authority in mathematics education, student authority relations in CPS in mathematics can be divided into social authority and intellectual authority. The ACT21S project "CPS" consists of two main components, "Collaboration" and "Problem solving." The problem solving component consists mainly of skills required to solve problems. Intellectual skills reflect the management of tasks and include task management, learning, and knowledge building. This paper combines this perspective to define individual authority and group authority relations in CPS.

4.1.1.3 Student's Individual Authority in CPS in Mathematics

The definition of individual authority is derived from Langer-Osuna (2016). Students' individual authority refers to the students' personal intellectual and social authority in CPS activities. Students' individual intellectual authority refers to the fact that students' individual behaviour is a useful source of information (or lacks such credibility). A student's individual social (directive) authority refers to the student being deemed to have (or not have) the right to issue directives to group members.

4.1.1.4 Group Authority Relations in CPS in Mathematics

The definition of group authority is derived from Langer-Osuna et al. (2020b). Group authority relations are formed as a result of the authority of three or more people operating in the group. Group authority relations refer to an intellectual or social relation of submission between authority subjects and authority objects, which is also a relationship of influencing and being influenced. In the current research, seven types of group authority relations can be formed in cooperative groups as intellectual and social authority compete and disperse. Group authority relations are reflected in students' utterances and behaviours in classroom communication and are dynamic in their changes.

4.1.1.5 Conceptual Framework for Individual and Group Collective Authority Relations

This section defines students' individual authority and group authority relations in CPS in mathematics. This diagram explains how individual authority forms group authority relations and the types of group authority relations.

Figure 4.1 explains the relationship between individual authority and group collective authority. (1) Individual authority is composed of intellectual and social authority. In a group of three or more, individual authority forms group authority relations; specifically, individual intellectual authority forms group intellectual authority relations and individual social authority group collective social authority relations. (2) Group intellectual authority relations include shared and concentrated intellectual authority relations; contested intellectual authority relations are formed through students' sharing of, concentration of, or competition for intellectual authority. (3) Group social authority relations include shared, concentrated, and contested social authority relations, as well as disbanded social authority relations due to the sharing, concentration, competition, or dissolution of students' social authority. (4) These seven different group authority relations have conceptual crossovers. Shared intellectual and shared social authority relations belong to shared authority relations. Concentrated intellectual and concentrated social authority relations belong to concentrated authority relations. Contested intellectual and contested social authority relations belong to contested authority relations.

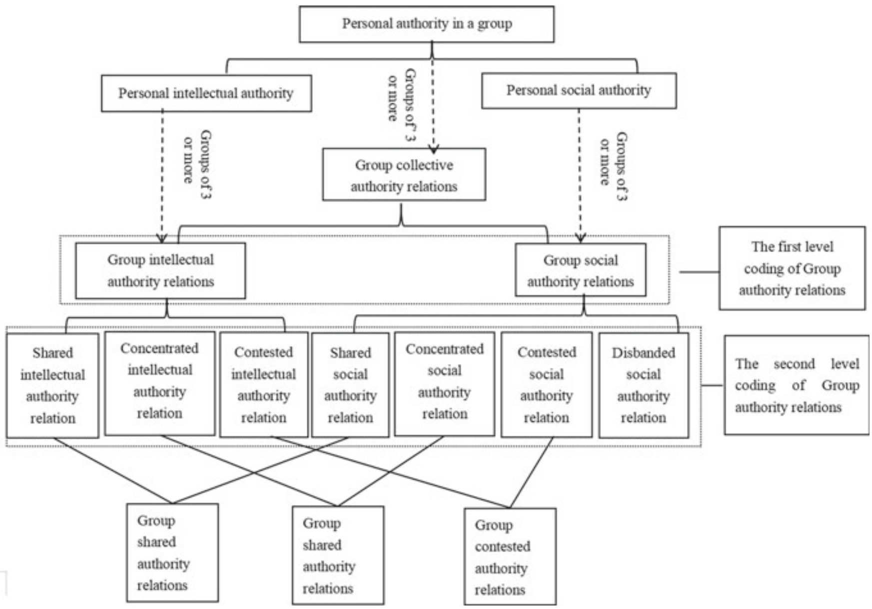

Fig. 4.1 Conceptual framework for individual authority and group authority relations

4.1.1.6 Research Questions

Through exploring the characteristics of authority within different structured groups, it is possible to explore the potential relationship between authority and performance. Further, the transformation of authority possibly influences students' activity during CPS (Langer-Osuna et al., 2020b). Based on a video analysis of CPS activities in a middle school, this study examined student authority relations in the CPS stage to answer two research sub-questions:

Question 1: What is students' individual authority in high- and low-scoring groups in CPS in middle school mathematics?

As this study explores individual and group authority relations, it first explores individual authority (individual intellectual authority and social authority) in high- and low-scoring groups in CPS in mathematics.

Question 2: How do students distribute and shift group authority relations in high- and low-scoring groups in CPS in middle school mathematics?

Individual authority interacts in groups of three or more to form group collective group authority relations. This question explores the specific distribution and variation of the seven different group authority relations.

4.2 Research Design

This study focused on the students' authority relations in CPS in middle school mathematics. Two sub-problems were used to explain the characteristics of individual and group authority in CPS. The overall idea of this research is shown in Fig. 4.2.

This research mentality diagram depicts the concepts: of (1) node (related group authority relations proposal negotiation unit); (2) coverage rate (the ratio of node dialogues generating group authority relations to all dialogues); and (3) individual authority rise and fall (the frequency with which an individual's proposal is accepted or rejected).

4.2.1 Data Sources

Purposeful sampling is applied for the current research. Six four-student groups were selected for the study. The six groups were drawn from Teacher A's classes (C01a and C01b) and Teacher B's classes (C02a and C02b) (see Table 4.1) within the same school. This study further divided the six groups into three high-scoring and three low-scoring groups for more pertinent analysis, based on the group score table for mathematical collaboration problem solving (Appendix 4.1). Students scoring above 6 were in the high-scoring group; students scoring below 6 were in the low-scoring

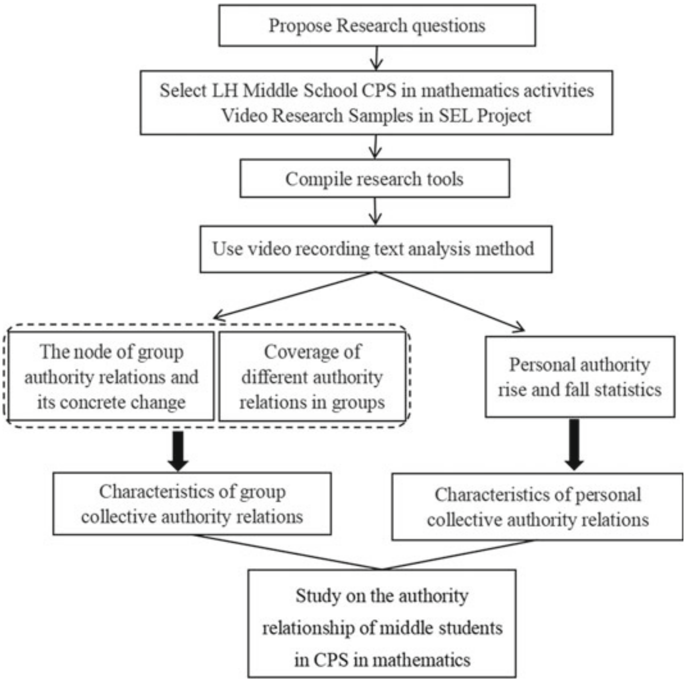

Fig. 4.2 The overall research roadmap of this study

Table 4.1 The attributes of the object group in this study

School	Group classification	Group name	Teacher	Score
LH middle school	High-scores groups	01b-02	Teacher A	10
		01a-02	Teacher A	7
		02a-04	Teacher B	9
	Low-scoring groups	01a-01	Teacher A	5
		01b-01	Teacher A	5
		02b-05	Teacher B	3

group. The three high-scoring groups received scores of 10, 9, and 7, respectively, while the three low-scoring groups received scores of 5, 5, and 3. The groups selected were those who spoke up most actively in classes.

4.2.2 Data Analysis

This study chose a qualitative research method based on video recordings. After transcribing the video dialogues, video text analysis methods were used to generate statistics on the rise and fall of individual students' authority in the high- and low-scoring groups. Student authority relations in CPS in mathematical activities were explored by counting group authority relation nodes and different group authority relations as a percentage of coverage (as a ratio of total discourse). This study was conducted using NVivo version 12.

4.2.3 Coding Scheme

This sub-section describes two research tools used to explore the individual authority and group authority relations in CPS in mathematics. There were several reasons for selecting and adapting these two research instruments. The first reason was the similarity of the two research samples. The original scales were initially used to study 10–11-year-old students' CPS in mathematics activities; this study involved Grade 7 students' CPS in mathematics activities. The second reason was the operability of the research codes. The research tools provide operational definitions that can be analysed in classroom videos, making both individual and group authority relations more visible. Third, both research tools have a strong theoretical basis, making them persuasive.

(1) An analytical tool for coding individual authority in CPS in middle school mathematics

This study drew on Langer-Osuna's (2016) findings on individual authority, adapting them accordingly to form an individual authority coding analysis tool in conjunction with this study (see Table 4.2). This study omitted the coding of individual authority statistics related to teachers assessing the quality of students' arguments and the merits of students' behaviour. The groups consisted of four people, two boys and two girls, coded as B1, B2, G1, and G2.

This study was conducted to represent the dynamics of students' individual authority and authoritative characteristics through their individual intellectual and social authority statistics. Individual authority rise and fall statistics refer to the frequency of each group member's successful and devalued bids for intellectual authority and social authority. Engle et al. (2014), in their work, *Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument*, referred to a proposal negotiation unit (PNU). A PNU is a set of interactions that begin with a discourse that makes a suggestion around a problem (for example, presenting an idea to be evaluated or giving an instruction). The data for this study were analysed at the event level of the PNU, where a PNU is a group authority relation node. Table 4.3 explains the methodology

Table 4.2 Coding of individual authority statistics for CPS in mathematics in this study

Coding	Definition	Sample
Impact on problem solving	The student's idea is positioned to be part of the completion of the problem solving pathway or final answer (or is rejected)	G2's contribution was written on a shared task list as part of the final answer "I think it should include the kitchen, the toilet, the two bedrooms, the balcony and the living room "Right, right, right" B1 and G2 started drawing the five rooms In the task list it is possible to see the results of their correspondence with the dialogue
Individual intellectual authority	Student proposals are used as a source of applicable information (or lack credibility) and have an impact on problem solving outcomes	"First you have to draw a good scale" "Right" (or someone else indicates the default) or "There should be another aisle drawn" "Just draw the room directly"
Individual social authority	Students are seen as having (or not having) the right to issue instructions to group members that have an impact on problem solving	Respond to an instruction: "First, make the picture bigger" G2 responds to related instructions and enlarges the picture

for the rise and fall statistics of individual authority in a given PNU. For example, a suggestion might include, "I know what to do, let's add numbers." This would be coded as a bid for intellectual authority; a group response such as "yes" after adding the numbers would be coded as a successful acceptance of the bid, positioning the first speaker's intellectual authority. Conversely, a response such as "no" would be coded as a rejection of the bid, thereby devaluing the first speaker's perceived authority. In this study, B1 represents the first male, G1 the first female, and so on.

Figure 4.3 shows the results of CPS in mathematics for the groups corresponding to the authority relation nodes in Tables 4.2 and 4.3. Students' individual authority statistics were based on students' conversations and the results of CPS in mathematics.

Students' interactions were qualitatively analysed after coding. The interplay of absorption frequency (represented by positive signs) and rejection frequency (represented by negative signs) were analysed through a specific CPS in mathematical videos.

Table 4.3 Schematic table of the way individual authority statistics for CPS in mathematics are presented

Contested authority PNU: Area of Apartment toilets, kitchens, living rooms	
Authoritative relation events	Changes in individual authority
B1: For example, the toilet and the kitchen are $20\ m^2$ in total G2: $20\ m^2$ in total? G1: $10\ m^2$, or else it's gone B1: So the kitchen is only $5\ m^2$ in total	The kitchen and toilet areas in the task list total $20\ m^2$ [B1 authority increased, recorded as B1 (+1)]
G1: The toilet is smaller G1: The bedroom is $10\ m^2$ and the toilet is 5, that's 15 in total, how many bedrooms? B1: That's only 1	The bedroom in the task list is $15\ m^2$ and the toilet is $10\ m^2$ [The authority of the G1 is reduced and is recorded as G1 (−1)]
B1: Living room assumed to be 15 G1: It's a bit small B1: That's all that's left, how much more, 60 m^2 in total, your living room takes up $50\ m^2$	The living room in the task list is not 15 [The authority of the B1 decreases, The authority of the G1 increases, note as B1(−1), G1(+1)]

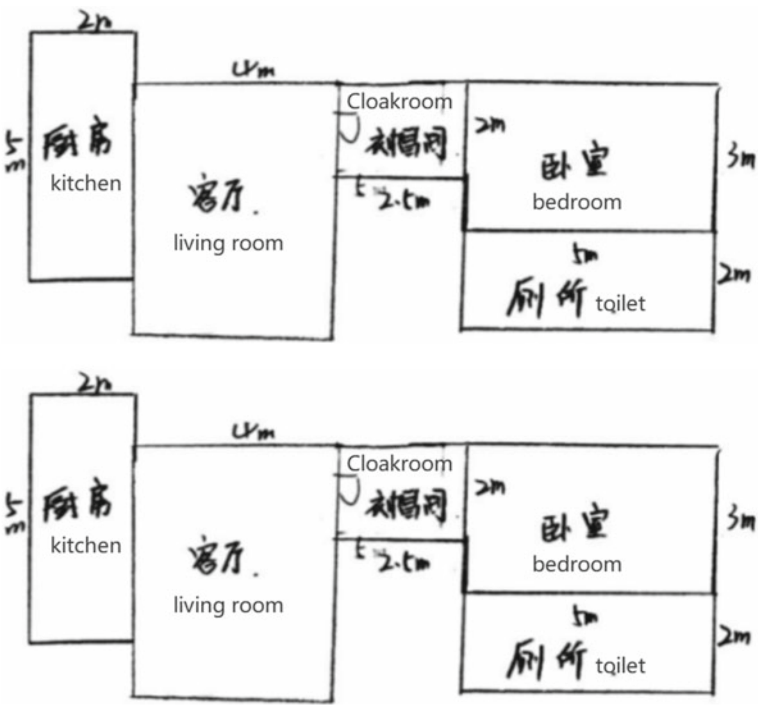

Fig. 4.3 Apartment layout task sheet in Table 4.3

(1) Tools for analysing group collective authority coding in CPS in middle school mathematics

Based on Langer-Osuna's (2020a) coding of group collaborative authority relations in CPS in mathematics, this study developed the group authority relations coding analysis tool. The individual authority mentioned above forms group authority relations in groups of three or more. Students' individual intellectual authority forms intellectual authority relations in groups of three or more. The group's intellectual and social authority can be transformed between shared authority, concentrated authority, contested authority, and disbanded authority. There are seven different group authority relations. This study examined the different social and intellectual authority relation nodes and coverage and their translation between collaborative group collectives. Table 4.4 shows an adaptation of Langer-Osuna's (2020a) coding of group authority relations to form an analysis tool for this study. The study was interpreted in the context of a CPS task in middle school mathematics called "Xiao Ming's Apartment".

This study first set intellectual and social authority relations as the primary codes. Shared, contested, and concentrated intellectual authority relations; shared, contested, and concentrated social authority relations; and disbanded social authority relations were set as secondary codes. Conceptual crossover occurred between the seven different group authority relations. Shared intellectual and social authority relations belong to shared authority relations, concentrated intellectual and social authority relations belong to concentrated authority relations, and contested intellectual and social authority relations belong to contested authority relations.

The data in this research were analysed at the PNU level. A PNU is a group authority relation node. For details, see the example in Table 4.3. This study counted the variation in social and intellectual authority relations across different groups and the nodes and continuity accounted for different types in the 15 min CPS video.

4.3 Results

4.3.1 Individual Authority Study Results of the High- and Low-Scoring Groups

This section explores statistics on the rise and fall of students' individual authority in CPS, referring to the frequency of each group member's successful and devalued bids for intellectual and social authority. B1 and G1 represent the first boy and first girl in the group, and so on. Specific statistical methods are explained in Table 4.3. For example, B1's suggestion, "I know what to do, let's add numbers," would be coded as a bid for intellectual authority; positive group responses (e.g., "Yes") after adding the numbers would be coded as a successful acceptance of the bid and recorded as B1 (+1), locating the first speaker's knowledge authority. Conversely, negative responses (e.g., "No") would be coded as a rejection of the bid and recorded as B1

Table 4.4 Coding of collective group authority relations in CPS in mathematics in this study

Distribution	Social	Intellectual
Shared	Multiple students' bids to manage their own and others' participation are taken up. This includes voiced negotiation of roles and distribution or management of tasks Example: the boys put the girls in charge of drawing pictures at the beginning of the task and the girls put the boys in charge of coming up with ideas	Multiple students' bids to contribute to the intellectual work are taken up. Disagreement about a mathematical idea or solution is in the service of reaching a consensus Example: a student is marking the area of each part of the kitchen and living room and discussing the agreement with the rest of the class to complete it
Concentrated	Bids to manage participation are taken up only in relation to one student. This includes instances where only one student successfully issues directives in the group Example: a student assigns roles to others in the group. One student instructs another student to write the name of the group and the student is instructed to write the name of the group This will increase the instructional authority of the instructing student as described in the table above	Bids to lead the intellectual work are only taken up in relation to one student. This includes instances where only one student's mathematical contributions are considered in the group Example: one student declared that the group would use blocks to solve. Other suggestions are rejected and the group continues with the discussion
Contested	Multiple bids to manage participation are rejected such that there is no settled authority Example: one student told another student to draw a toilet and a bedroom, as well as a bedroom with a toilet inside. The peer refuses and some other students suggest other options, some of which are accepted and some of which are ignored	Multiple bids to author ideas, offer help or lead the work are rejected such that there is no settled authority Example: one student told another student that he should draw the scale first and his companion refused. Other students suggested options such as drawing the outer frame of the plane first, some of which were accepted by their peers and some were ignored by them
Disbanded	N/a[1]	When the collaboration disbands into independent or off-task activity

(−1). The frequency of absorption (a positive sign) and rejection (a negative sign) are shown in Table 4.5.

In high-scoring group 01a-02, B1 had intellectual authority bids accepted six times, while G1 had intellectual authority bids accepted eight times. While the difference in intellectual authority between the two was not significant, their authority bids were accepted significantly more often than those of the other two students in their group. In groups 01b-02, B2's intellectual authority bid was accepted seven times and G2's social authority bid was accepted six times. Group 02a-04's B2 and G2's intellectual and social authority bids were also accepted more often than the

Table 4.5 Statistics on the rise and fall of individual authority in the high- and low-scoring groups

	Group	Member	Intellectual authority	Social authority		Group	Member	Intellectual authority	Social authority
High-scoring group	01a-02	B1	+6 (−2)	+3 (−2)	Low-scoring group	02b-05	B1	+1 (−3)	+1
		B2	+1	+2 (−3)			B2	+3 (−4)	+2
		G1	+8 (−4)	+3			B3	+2	−1
		G2	+4 (−2)	+1			B4	+4 (−3)	+2
	01b-02	B1	−1	0		02b-01	G1	+3 (−2)	+3 (−1)
		B2	+7 (−2)	−2			B1	+4 (−3)	−1
		G1	+5 (−1)	+2 (−2)			B2	+3 (−1)	+3 (−1)
		G2	+2 (−4)	+6 (−4)			B3	+1 (−2)	−2
	02a-04	B1	+1	−1		01a-01	G1	+4 (−3)	+2 (−2)
		B2	+6	+4 (−3)			G2	+4 (−2)	+1 (−1)
		G1	+2 (−1)	+1 (−1)			B1	+3 (−3)	−3
		G2	+4 (−3)	+3 (−2)			B2	+4 (−2)	+4 (−4)

other students in this group. Therefore, for the high-scoring groups sampled, group members' authority bids were accepted more often, with one or two students having significantly higher individual intellectual or social authority than the other students in their group.

In the low-scoring group, the frequency of authority bids being accepted and rejected authority bids was more balanced across group members. For example, in group 02b-05, B4 made four intellectual authority bids, but they were rejected three times in favour of other group members' authority bids. Based on Tables 4.5 and 4.6 offers a statistical summation for the individual social and intellectual authority for the high- and low-scoring groups.

Table 4.6 shows that there were 111 authority bids in the high-scoring group and 98 in the low-scoring group, indicating that individuals in the high-scoring group were more inclined to contribute to CPS in mathematics. There were more intellectual authority bids than social authority bids in both the high-scoring (64 vs 45) and low-scoring (64 vs 34) groups. The high-scoring group had a higher intellectual authority summation (26) than the low-scoring group (10). It also had a significantly higher total authority summation (31) than the low-scoring group (12). Thus, it can be concluded that students in the high-scoring group were able to have their proposals endorsed more often than the low-scoring group.

4.3.2 Group Collective Authority Study Results for the High- and Low-Scoring Groups

The first part analyses the intellectual and social authority relations in the high- and low-scoring groups under primary coding. The second part offers a comparative analysis of the different authority relations in the high- and low-scoring groups under secondary coding.

4.3.2.1 Nodal Analysis of Intellectual Authority Relations and Social Authority Relations in High- and Low-Scoring Groups

In this study, relevant authority relation nodes were counted and coded. Teacher intervention discourses were not included into coding. The coding process removed non-responsive self-monologues and conversations after members had stopped using their pencils for the task.

In this article, the layout of the apartment, dimensions of the rooms, and names of the rooms (kitchen, living room, bedroom) are in the category of intellectual authority relations. Specific functions (toilet shower, bedroom sleeping) belong to the category of social authority relations. The PNU mentioned above is a set of interactions that begins with a discourse making a proposal around a coded component (e.g., presenting an idea to be evaluated or giving an instruction). The table below

Table 4.6 Statistical table of individual authority results for high- and low-scoring groups

High-scoring group					Low-scoring group				
Group	Member	Intellectual authority	Social authority	Authority	group	Member	Intellectual authority	Social authority	Authority
01a-02	B1	+4	+1	+5	02b-05	B1	−2	+1	−1
	B2	+1	−1	0		B2	+1	+2	+3
	G1	+4	+3	+7		B3	+2	−1	+1
	G2	+2	+1	+3		B4	+1	+2	+3
	Summation	+11	+4	+15		Summation	+2	+4	+6
01b-02	B1	−1	0	−1	02b-01	G1	+1	+2	+3
	B2	+5	−2	−3		B1	+1	−1	0
	G1	+4	0	+4		B2	+2	+2	+4
	G2	−2	+2	0		B3	−1	−2	−3
	Sum	+6	0	+6		Sum	+3	+1	+4
02a-04	B1	+1	−1	0	02a-01	G1	+1	0	+1
	B2	+6	+1	+7		G2	+2	0	+2
	G1	+1	0	+1		B1	0	−3	−3
	G2	+1	+1	+2		B2	+2	0	+2
	Sum	+9	+1	+10		Sum	+5	−3	+2
High-scoring group authority bid summation (times)		66	45	111	Low-scoring group authority bid summation (times)		64	34	98
Total number of authorities in high-scoring group		+26	+5	+31	Total number of authorities in the low-scoring group		+10	+2	+12

shows an example of a contested authority negotiation unit, where a PNU represents an authority relation node.

Contested authority PNU: Area of apartment toilets, kitchens, living rooms
B1: Let's say the toilet and the kitchen are 20 m^2 in total G2: 20 m^2 in total? G1: 10 m^2, otherwise there would be no more space to use B1: So the kitchen is only 5 m^2 in total G1: The toilet is a bit smaller G1: The bedroom is 10 m^2 and the toilet is 5 m^2, that's a total of 15 m^2, how many bedrooms? B1: That's only 1, right? B1: Let's say the living room is 15 G1: It's a bit small B1: That's all that's left, how much more does the living room take up, 60 m^2 in total, your living room takes up 50 m^2

In the last authority relation event, it can be concluded that B1 and G1 created a relevant dispute over the size of the apartment and proposed separate solutions to the problem. Some were accepted and some were rejected. Events like these were defined as contested intellectual authority. Similarly, authority relation PNUs identified throughout the CPS in the mathematics process were noted as authority relation nodes (Table 4.7).

The high-scoring group had 38 authority relation nodes, compared to 57 for the low-scoring group. The high-scoring group had more intellectual authority relation nodes than social authority relation nodes, while the low-scoring group had the opposite. The difference in the number of intellectual and social authority relation nodes between the high- and low-scoring groups was not significant. Also, based on the specific video recording text analysis, this study found that in terms of intellectual authority relations, shared authority was the most frequent, and concentrated authority was the least.

Table 4.7 Nodal table of intellectual authority relation and social authority relation for high- and low-scoring groups

	Group	Total authority relations node	Intellectual authority relation nodes	Social authority relation nodes
High-scoring group	01b-02	12	8	4
	01a-02	16	8	8
	02a-04	10	6	4
	Sum	38	22	16
Low scores Group	01a-01	25	11	14
	01b-01	9	4	5
	02b-05	23	11	12
	Sum	57	26	31

Table 4.8 Table of coverage of intellectual authority relation and social authority relations for high- and low-scoring groups

	Group	Total authority relations Coverage (%)	Intellectual authority relations coverage (%)	Social authority relations coverage (%)
High-scoring group	01b-02	69.61	47.71	21.90
	01a-02	73.82	50.48	23.34
	02a-04	71.66	58.33	13.33
	Average	71.70	52.17	19.53
Low-scoring Group	01a-01	61.40	30.92	30.48
	01b-01	70.76	47.93	22.83
	02b-05	81.10	32.2	50.88
	Average	71.08	37.02	34.73

The volume of discourse was counted using NVivo application to determine related coverage, referring to the ratio of conversations corresponding to nodes of authority relations to total conversations. Table 4.8 counts the ratio of group social authority relations and social authority relations under the group authority relations level code.

The average authority relation coverage for the high-scoring group was 71.7%, roughly the same as for the low-scoring group (71.08%). Both the high- and low-scoring groups had more intellectual than social authority relations. In terms of average coverage, both groups had more intellectual than social authority relations. The high-scoring group's higher average intellectual authority coverage was more pronounced than its higher coverage of social authority relations.

4.3.2.2 Analysis of Specific Changes in Different Authority Relation Nodes in High- and Low-Scoring Groups

Relevant authority relation nodes were counted in this study. The high-scoring group had a total of 38 authority relation nodes, of which 22 were intellectual authority relation nodes and 16 were social authority relation nodes. The low-scoring group had 57 authority relation nodes, of which 26 were intellectual authority relation nodes and 31 were social authority relation nodes. This section lists all authority relation nodes by names in chronological order. A line graph is used to illustrate the changes in their specific authority relations. The horizontal axis has 25 authority relation nodes. The vertical axis represents the seven authority relations: shared, contested, and concentrated intellectual authority relations; shared, contested, and concentrated social authority relations; and disbanded social authority relations. This diagram facilitates observation of the specific changes and continuity of different group authority relations, based on Table 4.7 (Figs. 4.4 and 4.5).

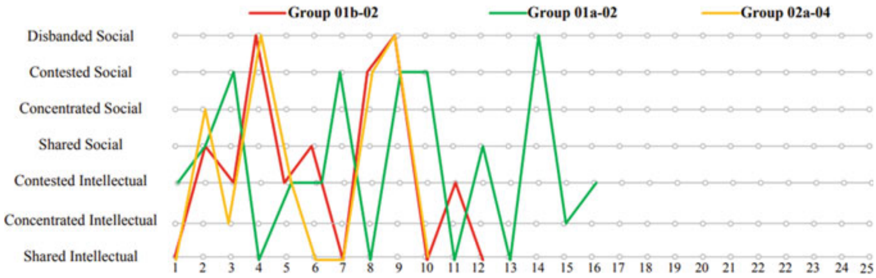

Fig. 4.4 Change in authority node for high-scoring group

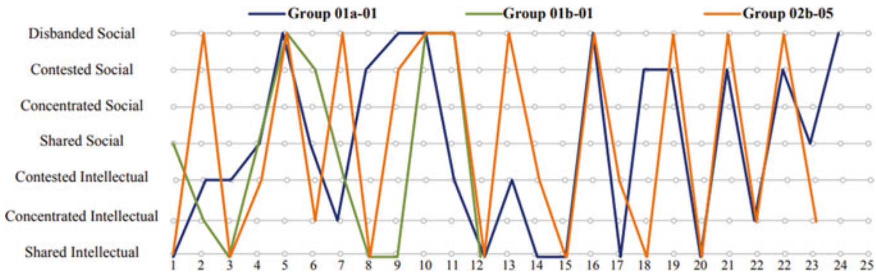

Fig. 4.5 Change in authority node for low-scoring group

From the previous section, it can be concluded that there were more intellectual than social authority relations in the high-scoring group. In terms of stability, the high-scoring group had less variation in authority relation nodes. In both the high- and low-scoring groups, shared intellectual authority nodes were the most common and concentrated authority was the least common throughout the 15-min mathematical CPS task.

In summation, there were 12 shared intellectual authority nodes in the high-scoring group and 17 in the low-scoring group. In terms of specific changes in group authority relations, the high-scoring group's 12 shared intellectual authority nodes produced contested and concentrated social authority relations without obvious tendency. However, the low-scoring group's shared intellectual authority relations were very likely to develop into disbanded social authority relations. In the 02b-05 group, there were five "shared intellectual authority relation → disbanded social authority relation" changes, compared to only two in the 01b-01 group with only seven authority relation changes. In terms of specific changes in their successive intellectual authority, contested authority and shared authority always alternated over time, with shared intellectual authority relations developing into contested intellectual authority relations and contested intellectual authority relations developing into shared intellectual and shared social authority relations. In terms of social authority relations, contested and disbanded social authority alternated. Contested social authority was prone to becoming disbanded social authority in the group.

4.3.2.3 Comparison of Shared, Contested, Concentrated, and Disbanded Authority Relations in the High- and Low-Scoring Groups

This study proposes a crossover between primary and secondary codes in the conceptual framework for group authority relations. There was conceptual crossover in the seven different group authority relations (shared intellectual and social authority relations belong to shared authority relations; concentrated intellectual and social authority relations belong to intellectual authority relations; and contested intellectual and social authority relations belong to contested authority relations), thereby forming four different group authority relations: shared authority relations, contested authority relations, concentrated authority relations, and disbanded authority relations (Fig. 4.6).

Shared authority was the highest of all authority relations for both the high- and low-scoring groups. Each group worked together to make intellectual or social contributions to solve the problem. Concentrated authority relation nodes were the least represented, accounting for only 4.22% of all nodes in the low-scoring group and reflecting that students shared authority during CPS in mathematics rather than clustering authority on the same person. The low-scoring group had a 23.27% rate of disbanded authority, second only to shared authority. The differences in the percentage of coverage of this component between the high- and low-scores groups were significant. Engaging in off-task activities significantly negatively influenced the low-scoring group's CPS outcomes.

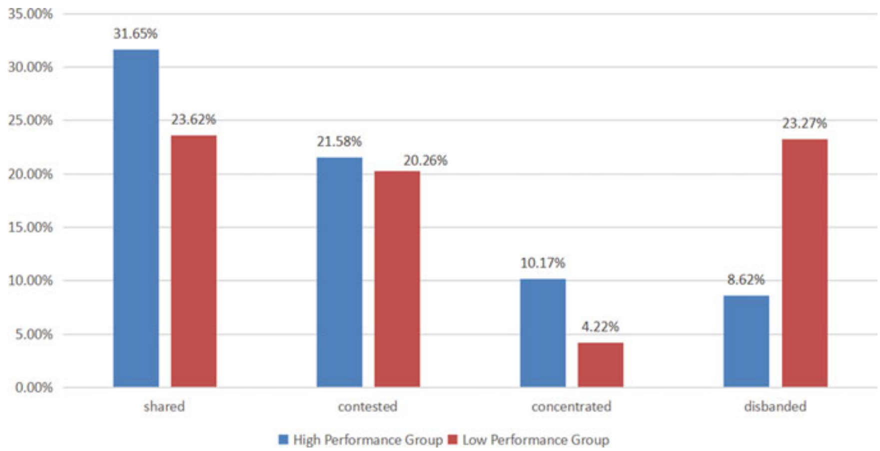

Fig. 4.6 Comparison of shared, contested, concentrated, and disbanded authority relations coverage for high- and low-scoring groups

4.3.2.4 Comparison of Shared, Contested, Concentrated, and Disbanded Authority Relations Under Intellectual and Social Authority in the High- and Low-Scoring Groups

Figure 4.7 shows a comparison of authority relation coverage for the seven different group authority relations under Level 2 coding.

After secondary coding, intellectual authority relations represented a very high proportion of shared authority relations, 28.08% in the high-scoring group and 17.79% in the low-scoring group. Shared, focused, and contested intellectual authority relations were higher in the high-scoring group than in the low-scoring group. There was little difference between the high- and low-scoring groups in contested intellectual authority. The low-scoring group had the highest disbanded social authority coverage. Disbanded authority relations are not conducive to high scores in CPS.

4.4 Conclusion

The study found that students in high-scoring groups had more individual authority bids and acceptances. The imbalances in individual authority predisposed to high scores in CPS outcomes.

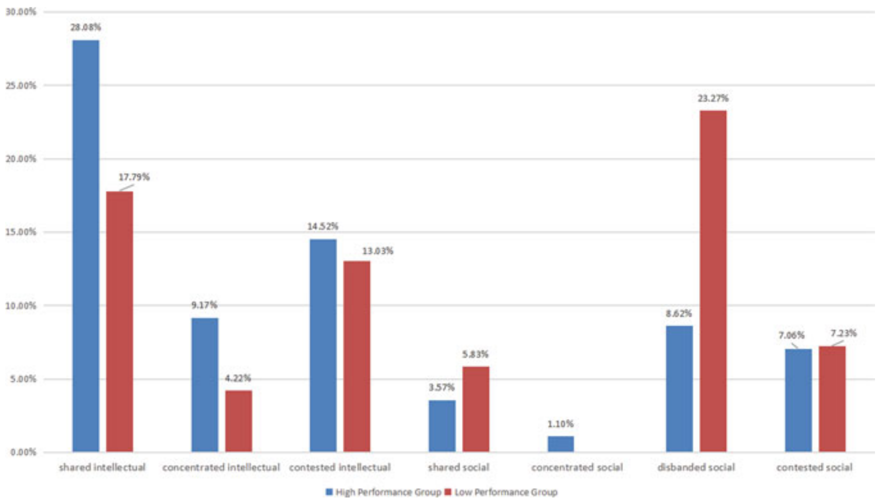

Fig. 4.7 Comparison of coverage of different authority relations under Level 2 coding for high- and low-scoring groups

4.4.1 Analysis of Individual Authority in CPS in Middle School Mathematics

4.4.1.1 More Individual Student Authority Bids and Acceptances in High-Scoring Groups Than in Low-Scoring Groups

The total number of bids was greater for the high-scoring group members, indicating they were more likely to contribute to CPS in mathematics than their low-scoring peers. Members made more intellectual than social authority bids in both groups. The authority summation was significantly higher in the high-scoring group, indicating its members' proposals were approved more often than in the low-scoring group.

Students in high-scoring groups were more likely to express and address task-related ideas, accept criticism, and respect other group members' opinions. The results indicated that timely feedback is an important feature of deep discussion. Group members' ability to give reasoned explanations for others' questions or timely feedback on others' suggestions facilitated deeper group discussion.

The unidirectional and bidirectional connections arising between the four factors are presented in Engle et al. (2014) influence model. The more verbal or eye contact is made, the more students enter the conversational layer and interaction space, making it easier to produce high-quality arguments and thereby increase authority. This is a cyclical process that ultimately affects CPS outcomes. In conjunction with this study, it can be concluded that a bold approach to the articulation of mathematical reasoning ideas and an openness to criticism are factors that promote good CPS outcomes in middle school mathematics.

4.4.1.2 Imbalance in Individual Authority Makes It Easy for CPS to Result in High Scores

Authority bids were accepted more often in the high-scoring group, with one or two students within each high-scoring group having significantly more individual intellectual or social authority than the other students. Authority bids were more balanced across group members in the low-scoring groups. For example, in the 02b-05 group, B4 made five intellectual authority bids but was rejected four times, while other group members made fewer authority bids. The low-scoring group was more balanced in terms of the frequency of authority bids being accepted and rejected. Other research on the role of social skills in middle school leadership in problem solving activities (Sun et al., 2017) indicates that groups that experience discussion achieve better problem solutions. In conjunction with the findings of this study, this shows that one to two leaders must be present in each group to lead other members through CPS and make it more effective.

4.4.2 Analysis of Group Authority Relations in CPS in Middle School Mathematics

The characteristics of group authority relations in CPS in mathematics in middle school were derived by counting the group authority relation nodes (authority nodes) and the discourse coverage of different authority relations in students' groups throughout the CPS, and then classified and interpreted in this study.

4.4.2.1 Shared Intellectual Authority Relations Are Most Conducive to Producing CPS in Mathematics Results

The specific variation in nodes showed that the number of shared intellectual authority nodes was highest across authority relations for both the high- and low-scoring groups. The frequency of intellectual authority relations was generally higher than for social authority relations in both the high- and low-scoring groups, so the group contributed intellectually or socially to problem solving. Video text analysis revealed that the shared intellectual authority relations period produced the most CPS results.

4.4.2.2 Least Occurrence of Concentrated Authority Relations in CPS in Middle School Mathematics

The lowest percentage of concentrated authority was found in the high-scoring group, with 1.1% of concentrated social authority. No concentrated social authority relations nodes appeared in the low-scoring group, reflecting that students shared authority during CPS in mathematics and teachers dispersed student authority during CPS.

4.4.2.3 Contested Authority as a Catalyst for Authority Change in CPS in Mathematics

Contested authority was second only to shared authority in both the high- and low-scoring groups, while contested intellectual authority was more predominant in terms of its secondary coding. This study has shown that contested and shared authority always alternated over time. Specifically, shared intellectual authority relations later developed into contested intellectual authority, while contested intellectual authority relations later developed into shared intellectual authority relations, facilitating CPS outcomes.

Cobb (1995) noted that argument for authority can be productive in the classroom, facilitating the more equitable distribution of authority and supporting different students' opportunities to learn. However, from an intellectual perspective, that can hinder the formation of CPS outcomes. Previous studies (Langer-Osuna, 2011; Langer-Osuna et al., 2020a) have shown that contested social authority may disrupt the process in ways that reshape the dynamics of cooperation. Taken together, the existing research and the present study's findings suggest that contested authority catalyses excellent CPS outcomes with contested intellectual authority and low CPS outcome scores with contested social authority under secondary coding.

4.4.2.4 More Stable Changes of Authority Relations Promote High Scores CPS in Mathematics Outcomes

The high-scoring group had more intellectual authority relation nodes than social authority relation nodes, while the opposite was true in the low-scoring group. The high-scoring group also had more stable authority relation performance, with more coverage of social authority relations than the low-scoring group. Shared intellectual authority relations were more unstable in the low-scoring group. The "shared intellectual authority relation → disbanded social authority relation" process changed several times during the task solving process. Non-engagement with on-task activities occurred after shared cooperation. Stable shared intellectual authority relations were most conducive to producing CPS in mathematics outcomes. As Langer-Osuna et al. (2020a) pointed out, shifts in social authority relations are more dynamic than shifts in perceived authority. Combined with the characteristics of group authority relations in this study, the high-scoring group had a higher proportion of intellectual authority relations.

4.4.2.5 Summary

The current research has revealed the characteristics of high/low-scoring groups. In the current research, more individual student authority bids and acceptances and more imbalance of authority within groups are found in the high-scoring groups, which insights teachers to consider the authority distributions within groups. Teachers can encourage students to more generate shared intellectual authority and contested authority in process of collaborative activities in classroom.

Appendix 4.1: Scores Rating Scale for CPS in Mathematics Outcomes in Middle School

Scoring dimensions	Scoring rules	Score (marks)
Overall requirements (2 marks)	The group agrees and submits a final copy of the problem solving results	2
	The group was not in agreement and there were several task list issue resolution results	0
If there are multiple task order resolution results, the first one (below the task order question) will be scored		
Apartment length and width (2 marks)	Complete labelling of the length and width of each room (with scale or side length units)	2
	Only the length and width of individual rooms are indicated or the length and width of each room are indicated in full but without a scale or side units	1
	No room lengths and widths are indicated or only the area of each room is indicated	0
Apartment size (2 marks)	Complete with the area or length and width of each room, the sum of the individual rooms is 60 m^2	2
	The area or length and width of each room are fully indicated and the sum of the individual rooms is not equal to 60 m^2	1
	Only individual room dimensions are indicated or no room dimensions are indicated to give a total area of 60 m^2	0
Number of rooms (1 mark)	It can be clearly seen that there are five rooms	1
	There are no 5 rooms or the picture is confusing so you can't tell there are 5 rooms	0
Apartment features (2 marks)	Complete labelling of room functions, e.g. kitchen, bathroom, etc	2
	Not fully labelled room features	1
	No room features marked	0
Apartment layout (1 mark)	Apartment layouts are sensible shapes: quadrilateral, triangular, circular, etc	1
	The layout of the Apartment is completely unreasonable and unrealistic and cannot be designed properly	0

Scoring criteria: The high- and low-scoring groups in this study were assessed in absolute terms. The current absolute curriculum assessment in our schools is a percentage system, with a passing mark of 60 (60% of the total score). This system has been used in China's schools for roughly a century, for several reasons. First, it was developed in connection with an educational reform movement that took place in China in the late nineteenth and early twentieth centuries. Through this reform movement, Chinese education began to move away from the shackles of feudal education and towards modern education, as a result of learning from European and American education. Second, it was a result of convention. Therefore, as the maximum total score in this study was 10, a score greater than 6 identified the high-scoring group and less than 6 the low-scoring group.

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Chapter 5
The Development and Use of Opportunity to Learn (OTL) in the Collaborative Problem Solving: Evidence from Chinese Secondary Mathematics Classroom

Yinan Sun and Boran Yu

5.1 Introduction

In the 1960s, American educational psychologist Carroll proposed the concept of Opportunity to Learn (OTL) which focuses on whether students can gain sufficient practice experience in their learning (Carroll, 1963). Since then, great efforts were paid to define what counts as OTL and study OTL as the unit of learning. Prior studies mostly concerned the classroom level and believed OTL results from teachers' behaviour (Elliott & Bartlett, 2016). The assessment of OTL in PISA 2012 was also based on teachers' instructions and support (OECD, 2014). Analysing OTL benefits teachers by improving their teaching practices (Stevens, 1993). In addition to teachers' support, OTL could appear from interactions among students. Through collaborative learning, students can learn by explaining their thinking, collectively reflecting on the solution, and learning from peers, which could be seen as OTL provided for students (DeJarnette, 2018).

Raising students' ability to collaboratively solve problems is necessary in the context of the latest curricular reform, which implies the great value in studying how OTL develops and is used by students in this context. Firstly, the new curriculum emphasised that students should be the centre of classroom teaching while teachers provide guidance and support for their learning. Prior studies suggested the complex mechanism of students' learning in the classroom (Langer-Osuna et al., 2020) and

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thus it is of great importance to study their classroom learning performance. Understanding how OTL develops and is used by students would maximise students' self-learning. Secondly, one of the important aims of classroom learning is to gain the knowledge and develop the key competencies necessary to suit their further development in society. This aim has made studying how students gain and use OTL imperative for supporting students' sustainable development. Finally, collaborative problem solving provides students with an open environment where they discuss the problem with each other to develop a solution on which everyone agrees through different phases (Salminen-Saari et al., 2021). Compared with traditional teacher-centred classroom teaching, collaborative problem-solving teaching allows students to express themselves more and thus would develop more OTL through student interactions.

This study investigated how OTL develops in the context of collaborative problem solving and how students use it. Firstly, we analysed the types of interactions students made and the discourse they used to figure out how OTL develops in collaborative problem-solving context. Two students failed to join in problem-solving tasks and their groups were analysed to figure out how they interacted with other group members how interactions could generate OTL and why they failed to seize the OTL developed from these interactions.

5.2 Literature Review

Since the concept of OTL was first proposed in the 1960s, scholars have tried to define it in different ways. Allwright divided OTL into three categories: opportunities to input, opportunities to exercise, and opportunities to manage (Allwright et al., 1991). This categorisation was accused to be oversimplified and it has been proposed that OTL should be divided into the opportunities to input, opportunities to output, opportunities to interact, opportunities to feedback, opportunities to practice repeatedly, opportunities to understand discourse, and opportunities to understand OTL (Mao, 2016).

Past studies have paid great attention to how students take advantage of OTL, how teachers could provide more OTL for students, and the assessment of OTL. A large proportion of studies of OTL in the classroom focus on OTL in online learning, the inequalities in OTL brought by racial or spatial differences, and the relationship between teachers' attitudes towards teaching and students' OTL. Additionally, several studies focus on the macro-level of students' OTL (e.g., tuition fees, access to education) (Wang, 2018). While these studies have explored how external factors affect students' OTL, less is known about how students themselves create and use OTL.

Studies evaluating OTL aim to delineate how teachers provide students with important learning resources. Scholars used different ways to assess OTL, such as discourse analysis, questionnaire, interview, classroom observation, and field notes. Reeves and colleagues used text analysis and classroom surveys to analyse

African mathematics classroom videos. They evaluated the extent to which students accomplished classroom tasks to compare OTL use in mathematics classrooms in different countries (Reeves et al., 2013). Wang followed the OTL evaluation framework proposed by Stevens and designed a questionnaire that surveyed the content coverage, content exposure, teaching emphasis, and instructional delivery quality. Based on questionnaire results, he further analysed the relationship between students' achievements and the OTL they received (Wang, 1998). Herman et al. (2000) developed an OTL evaluation framework consisting of four dimensions—classroom content, teaching strategies, teaching resources, and assessment—based on teacher and student questionnaires and interviews. Other domestic scholars evaluated OTL from ten dimensions (including aim, preparation, meaning, etc.) and found that secondary students could feel the opportunities in method and those in difference but found it difficult to feel opportunities in meaning and those in challenges (Yin, 2018). Hao used a questionnaire to study whether OTL appeared in classroom teaching equally (Hao & Hu, 2016).

OTL has been studied as a key variable predicting learning results. Goodlad proposed that it is necessary to consider OTL as an important variable if one links learning results and the OTL gained by them (Goodlad et al., 1979). Freud thought that differences in academic achievement could be seen as differences in OTL. Studies suggest that students' academic achievement relates to OTL after controlling for students' learning ability and socioeconomic status (Vernon, 1971). Similar studies suggest that OTL predicts students' academic achievement more than teachers' skills, teaching efficiency, and expectations.

Few domestic studies have studied OTL in Chinese classroom settings. Most studies, using data from large-scale international surveys like PISA, conclude that OTL may lead to differences in students' learning achievement in different contexts. PISA surveyed students' OTL in three dimensions: (1) learning formal mathematics; (2) learning textual mathematics problems; and (3) learning applied mathematics. A re-analysis of PISA 2021 data suggested that Chinese students' mathematics performance is closely related to OTL, and that they had more opportunities to learn formal mathematics but fewer to learn applied and textual mathematics (Teng, 2014). Several researchers have analysed teacher-student interactions to study students' OTL (Bai & Lin, 2016).

Other studies have found a weak association between OTL and secondary students' academic performance but a stronger association between OTL and their non-academic achievements. They point out that self-learning plays an important role in learning, but students find it hard to get used to OTL provided by others. To sum up, the use of OTL is a possible learning result; thus, OTL is unequal to learning results. Additionally, while OTL use is closely related to students, it is also influenced by other factors like economics, resources, etc.

In the context of collaborative problem solving, OTL can influence students' learning. Students can respond to others in the collaborative learning process, but whether they can understand each other is uncertain. However, most studies have focused on how teachers' intervention can influence students' OTL. This gap implies the necessity of understanding how students interact to attain and use OTL.

5.3 Theoretical Framework and Method

5.3.1 Commognitive Approach to Study Mathematics Learning

Commognitive approach was originated from a learning science stance that conceptualize learning as participationist rather than acquisitionist (Sfard, 2007). This approach conceptualise learning as a form of communication and thus suggests that learning can be achieved by revising and expanding one's discourse. In the commognitive approach, learning mathematics means "to individualise the historically established discourse known as mathematical" (Chan & Sfard, 2020). Here, "Individualise" refers to being the agentive participant of this discourse. Specifically, students can not only follow its rules but also transform it flexibly according to their will and use it to inform the next step. Highly individualised discourse would become the primary media for one's thinking. In this case, learning is conceptualised as one's interactions with oneself. When it comes to case with more than one individual, learning is tantamount to conversations which have multiple channels and models not limited to verbal discussion. When two students are in a discussion, the discussion flow has three different channels (as shown in Fig. 5.1). There existed inter-personal channels where two individuals are interacting with each other. Intra-personal channels, however, allows interactions with oneself to promote both individuals' learning. Among these channels, only the inter-personal channel is observable, while the intra-personal is hidden from observers. This commognitive approach has been widely used in empirical studies examining collaborative learning because of its emphasis on the dynamic interactions in learning. In this study, we believe that OTL is developed from these two forms of interactions and three different channels among group members in the four-student groups' discussions.

Fig. 5.1 Multiple channels in interactions

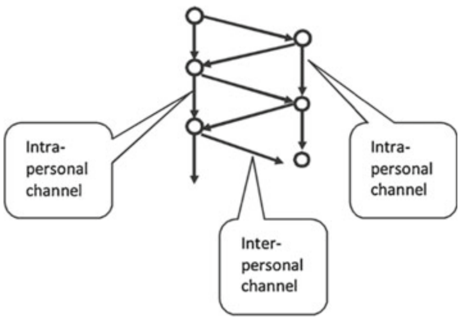

Table 5.1 Types of opportunity to learn (OTL)

Type of OTL	Feature	Circumstances	Discourse type
I	Take the initiative in discourse	Mathematical Context	Mathematizing or subjectifying
II	Change in the discourse itself	Asking mathematical problems Explaining to Partners Making cognitive conflicts	Mathematizing and subjectifying

5.3.2 Opportunity to Learn (OTL)

Chan and Sfard (2020) generalised what kinds of interactions could count as OTL and in what circumstances OTL will develop. They suggested that there are two types of OTL, one of which is generated from a change in the initiative of students' discourse. This change can give learners more opportunities to participate in interactions with mathematical discourse. This interaction, where learners talking about mathematical objects, could be called mathematizing. OTL will develop when learners interact and talk about mathematical objects or subjectify these objects. Here, subjectifying means students articultating their dispositions about mathematics. The development of the other type of OTL needs a change in the type of discourse itself rather than in the initiative of discourse. These OTL may appear in object-level or meta-level discourse or when learners meet problems that need to be verified. Additionally, to use these OTL, students must actively participate in group discussions by expressing themselves or listening to others' explanations. These OTL could also be developed when students get in touch with language at different levels and have conflicts with others. The Table 5.1 generalises the two types of OTL and their characteristics.

5.3.3 Sampling and Data Collection

We used videos of collaborative problem solving in a secondary mathematics class in Beijing from the Social Essential of Learning (SEL) project. In this class, the collaborative problem-solving task was Taks A, "Xiaoming's Apartment" (see Appendix for more details). A detailed investigation of interactions within the group is critical to understand how students can take advantage of OTL in collaborative learning activities. Rather than focusing on the successful learning cases, this study focused on students who failed to use OTL to reveal why students learn little in collaborative problem-solving. After carefully reviewing numerous collaborative problem-solving videos, two students and their groups were selected for further analysis to explore how OTL develops and benefits students through their interactions. Specifically, we found that some students got involved successfully initially but gradually got lost in the group discussion and further identified these groups for in-depth investigation.

Each group included four students (two boys and two girls), coded separately as S1, S2, S3, and S4. Two characteristics were identified in these two groups:

(1) Each student in the group spoke, so we could decide the role each played based on their discourse types.
(2) In each group, one member was identified as a "focus student" who failed to participate in the group discussion actively. These focus students were marginalised because they did not pay sufficient attention to what other group members said or because other group members always reject their opinions.

5.3.4 Data Analysis

Videos were transcribed for further analysis. Firstly, to explore the discourse used in collaborative problem solving, we divided students' discourse into two categories: mathematizing and subjectifying. There were two levels within the mathematical category: meta-level and object-level. The object-level concerns students' talking about mathematical objects while the meta-level concerns students' reflection on their talking about mathematical objects. Table 5.2 presents the coding scheme of students' discourse. We used different coloured circles with arrows to represent the type of discourse and direction of students' interactions. As shown in Fig. 5.2, black circles refer to students' discourse at the object level, meaning their discussions about mathematics itself. Grey circles refer to students' discourse at the meta level, meaning their thoughts or comments on some mathematical objects. White circles refer to students' discourse at the subject level, meaning their discussion about the learners themselves. Arrows show the types of interaction. Circles with arrows pointing to the bottom-left represent inter-person proactive interactions (students' initiatively discussing with others), while those with arrows pointing to the upper-left represent inter-personal reactive interactions (students' responding to others), and those with arrows pointing to the bottom represent intra-personal interactions (students' talking to themselves). After trial coding and a refinement of the coding scheme, two researchers independently coded the data, attaining an 86% consistency rate.

Additionally, we calculated the proportion of different types of discourse (Fig. 5.3) and the proportion of different types of interactions (Fig. 5.4). Considering there are four students within each group, we identified two different roles students are playing: leaders and followers. Leaders are charactersied with more mathematising discourse and more inter-personal interactions (e.g., S3 in the example). In this study, we focus on the interactions between focus students and leader students as such interactions could generate more OTL by getting focus students involved in collaborative learning. Although OTL appears in interactions indifferent forms and contents, we followed the definition by Chan and Sfard (2020) that captured OTL based on how discourse was developed and changed through students' interactions.

After coding the discourse and determining each student's role, we conducted a detailed analysis of the focus students' discourse to determine how they interacted

Table 5.2 Coding scheme of students' discourse

		Inter-personal		Intra-personal
		Proactive	Reactive	
Mathematizing	Object-level (OLM)			
	Meta-level (MLM)			
Subjectifying	Subject (S)			

Fig. 5.2 Examples of discourse analysis

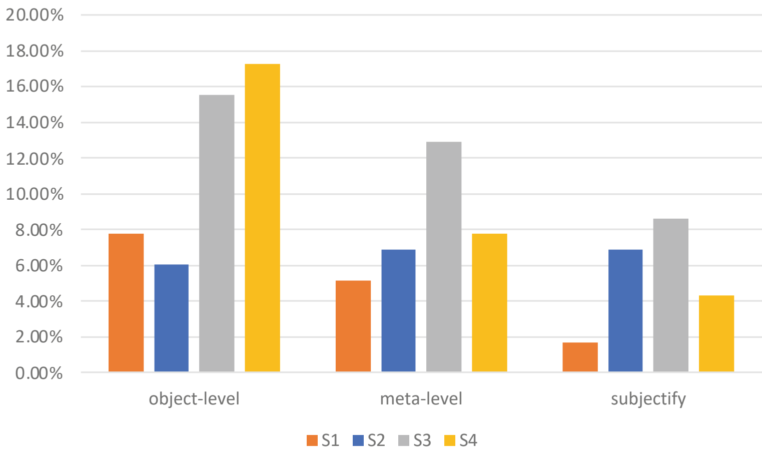

Fig. 5.3 The discourse types of students (example)

with other students and caught the OTL created from these interactions. The next section introduces each group's collaborative problem-solving process and discusses the development of OTL. Figure 5.5 outlines the data analysis process in this study.

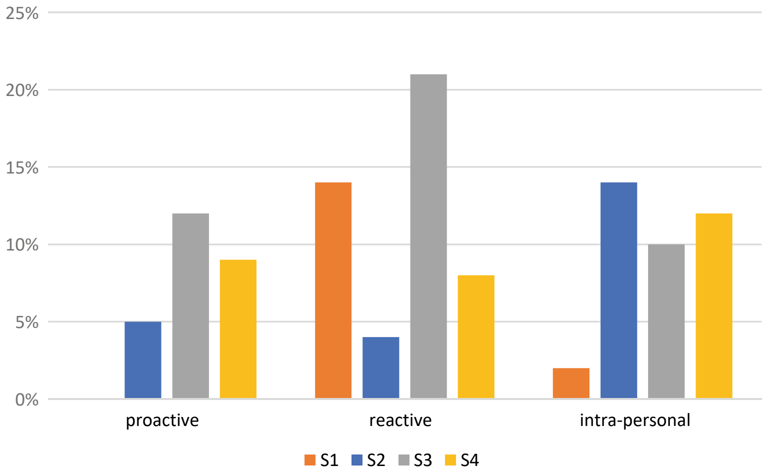

Fig. 5.4 The interaction types of students (example)

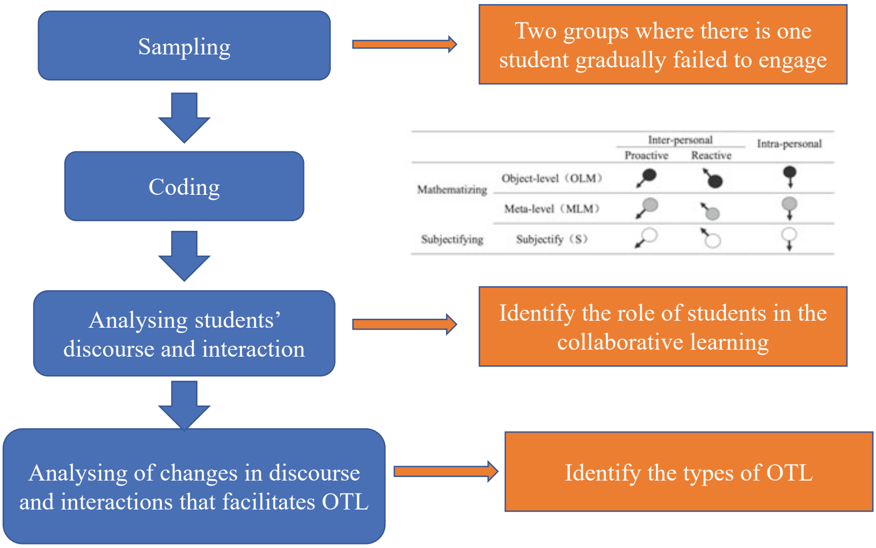

Fig. 5.5 Data analysis diagram of this study

5.4 Results

5.4.1 Group 1

Group 1 had two boys and two girls. S2 was the focus student and participated less actively through the collaborative problem-solving process. At the beginning of the discussion, S2 interacted with others very well. As the discussion continued, however, S2's opinions began to differ from other group members', leading to conflicts. As a result, S2 missed several OTL generated from interactions among other group members, making it harder for him to follow later parts of the discussion.

5.4.1.1 The Stages of Discussion

We divided the collaborative problem-solving process into three stages based on S2's performance and the group discussion content.

Stage 1: At this stage, group members mainly discussed the structure of the apartment and what one square metre looks like. However, S2 had vastly different opinions, and conflicts continually emerged. For example, when S4 proposed that the bathroom should be two square metres, S2 questioned her idea immediately, saying it was "impossible." When S4 amended her idea and said, "five square metres should be enough," S2 continued to reject it, declaring, "five square metres could never be enough." From what S2 said, it was apparent that he wanted to join the group discussion by rejecting others' views and gaining their attention. However, other students chose to ignore his rejections and went on with their original discussion. Finally, S2 stopped rejecting others (under the teacher's guidance), and the group entered the next stage of discussion.

Stage 2: At this stage, group members discussed the overall arrangement of the apartment, including what rooms the apartment had and how to furnish them. S2 did not keep pace with the others in. While the group discussed how to divide the apartment into different rooms, S2 considered how to draw the apartment or proposed irrelevant requests. For example, when S2 wanted more pieces of white paper to draw on, S3 refused his request, saying, "You do not have to draw all the time." Whenever S2 tried to start a topic, the others ignored or rejected him, leading to his failure to get involved in the group discussion.

Stage 3: At this stage, group members finished most of their prior discussions and started drawing the apartment's arrangement on paper. S2 barely had any chance to participate in the discussion and instead just talked to himself. When other students decided to use a scale of 1:100, S2 said "it is irrational to use a scale of 1:1," showing he had not paid attention to what the others had said. Later, when the group discussion focused on how to draw the kitchen, bathroom, and living room, S2 did not express an opinion; however, when the other members had almost finished the drawing, S2 began to question the result, again saying something like "this is impossible in real life."

Overall, the focus student's performance followed some patterns at different stages of the group discussion. The group discussion went on in an organised way. The group firstly focused on what an apartment should have (e.g., a kitchen, bathroom, and living room) and how large one square metre was, which was necessary preparation work for drawing the apartment's floor plan. Then they moved to the overall arrangement of the apartment, the size of each room, and finally finished the task. At the beginning of the group discussion, the focus student kept questioning and explaining his ideas. But as the discussion continued, the focus student did not keep pace with others; instead, he thought about drawing (a task that should be done later) while his peers are discussing the arrangement. It is apparent that he wanted to take the initiative in the discussion but failed due to a mismatch between his ideas and those shared by the group. Finally, he lost interests and missed many of the ongoing discussion's details, leading to a situation where he could only speak to himself.

5.4.1.2 Analysis of OTL

OTL developed in all three stages mentioned above through different types of discourse and interactions (see Table 5.3 for more details).

OTL in Stage 1: The first OTL appeared when the group was discussing the area of the bathroom. When S4 proposed that the bathroom could be two square metres, the focus student of this group (S2) questioned this idea, responding that it was "impossible." In this interaction, their type of discourse transformed from the object level to the meta level with S2's detailed explanation ("Because you need enough space to take a shower"). This explanation contributed to the development of the second OTL.

The third OTL was also developed while discussing the area of the bathroom. When S4 tried to adjust the bathroom to five square metres ("Actually, five square metres are enough"), S2 still rejected this new idea ("five square metres could never be enough"), which caused conflict between S2 and S4 again. In this conflicted interaction, the type of discourse transformed from the subject level to the meta level. The OTL did not work because S2 did not explain why he thought five square metres are not enough or suggest how many square metre should be enough.

The fourth OTL appeared from the discussion of the area of four bricks. When S3 suggested the area to be 36 square decimetres, S2 objected again, much as in the first OTL. The fifth OTL happened when the group was discussing the area of the bathroom under the teacher's guidance. When the teacher checked on how Group 1 was progressing, S3 mentioned that they could not confirm the area of the bathroom. S2 followed S3's question and asked, "How many square metres does a bathroom generally have?", which produced OTL through questions.

It is noticeable that the conflicts between S2 and the other students were partly caused by the difference in the type of their discourse. More specifically, S2's discourse was mostly at the meta level (e.g., "impossible"), while the students interacting with him turned the direction back to the object level. This could be seen as a latent conflict in the discourse structure.

Table 5.3 A summary of OTL in Group 1

Episode	OTL	Discourse type	Discourse type shift	Type of OTL
Ep.1	[36a]: Impossible	Mathematizing	Meta-level → Mate-level	II. Conflict
	[41a]: First of all you have to have a bath, a bathroom. You must have a bath [83c]: Impossible	Mathematizing Mathematizing	Subjectify → Mate-level Subjectify → Mate-level	II. Explain II. Conflict
	[45a]: 5 m^2 is really not enough	Mathematizing	Meta-level → Mate-level	II. Conflict
	[102]: How many are there?	Mathematizing	Meta-level → Mate-level	II. Ask
Ep.2	[124]: That's not even enough for you	Subjectifying	Subjectify → Subjectify	II. Conflict
	[135b]: Listen to me first, listen to me	Subjectifying	Subjectify → Subjectify	I. Initiative
	[153]: Narrow it down a bit, how much?	Mathematizing	Object-level → Subjectify	II. Ask
	[168]: You think, you have to have a writing desk, a ventilated area, and a bed	Mathematizing	Object-level → Subjectify	II. Explain
Ep.3	[206]: 1:1? Impossible	Mathematizing	Meta-level → Object-level	II. Conflict
	[257]: You can't be like that, can you?	Subjectifying	Object-level → Subjectify	II. Conflict

OTL in Stage 2: The sixth OTL appeared when the group discussed the apartment's overall structure. S2 thought it better to draw their ideas on the paper first, while S3 disagreed ("We can discuss first and then draw on the paper…. You do not have to draw now"). At this time, conflicts appeared, and the discourse turned from the object level to the subjectifying level.

The seventh OTL appeared when discussing whether a bathroom should have a washing machine, as S3 suggested. S2 rejected this idea and sought to regain the interaction initiative ("Could you please listen to me first?"), moving the content of their interaction from the object level to the subjectifying level.

The eighth OTL happened when the group discussed the kitchen area. When the group discussed whether the kitchen area could be eight square metres, S2 did not pay full attention but questioned whether it should be smaller, directing the discussion content from the object level to the meta level.

The ninth OTL appeared while discussing the area of the bedroom, which S2 thought should be less than ten square metres, while others disagreed.

At this stage, S2's discourse mainly aimed at returning to the group discussion by gaining the discourse initiative. However, his attempts did not always keep pace with the other members, leading them to ignore or disapprove of his ideas and causing his failure to participate in the group discussion. For example, when S2 said, "Could you please listen to me first?" S3 choose instead to discuss whether a bathroom should have a washing machine.

OTL in Stage 3: The tenth OTL appeared when choosing the scale. When S4 decided to use the scale of 1:100 ("We can use 1 cm to represent 1 m."), S2 misheard the scale to be 1:1 and rejected S4's idea again ("1:1 is impossible."). It is apparent that S2 did not pay full attention to the others, leading to conflicts and turning the discourse type from the object level to the meta level.

The eleventh OTL emerged at the end of the discussion. When the group finished drawing, the focus student almost gave up on joining the discussion. S2 still made several criticisms ("This design is impossible"), but his ideas were again ignored.

Generally, the group's discussion followed an organised order, proceeding from discussing the apartment's overall structure to allocating the rooms and drawing the floor plan. However, the focus student consistently failed to follow the pace taken by the group. For example, the focus student wanted to draw when others discussed each room's area. Additionally, the focus student's ideas were always rejected or ignored, making him miss OTL.

5.4.2 Group 2

5.4.2.1 The Stages of Discussion

Based on the performance of S2 and the group discussion content, we divide the collaborative problem-solving process into three stages.

Stage 1: At this stage, the group focused on the general arrangement of the apartment and details about the kitchen and bedroom. In this process, S2 took a very active part in group discussion and tried to express his own ideas (e.g., "I think the living room should be here, and there should be three bedrooms next to the living room," "I know what questions you have"). However, S2 did not get an active response from other students. For example, when S2 proposed the idea of three bedrooms, S3 rejected it immediately ("It is insane to have three bedrooms in 60-square-metres").

The group spent a relatively long time discussing the area of the living room and the bedroom. When S2 proposed that he knew the key to solving the question, other students refused to give him the discourse initiative ("Could you stop talking?"). In the later discussion, S2 tried to express himself and actively join the discussion, but the other students seldom actively responded to him.

Stage 2: At this stage, the group had a general discussion on the area of each room. After repeatedly receiving negative feedback from other members, the focus student gradually lost enthusiasm for joining the group discussion. When the group had a heated debate about the area of specific rooms, S2 just echoed what others said and did not express his own ideas.

Stage 3: The main task in this stage was diagramming the apartment they had designed. S2 tried to rejoin the group discussion by initiating more interactions. When the group was discussing how to draw the graph, S2 proposed his ideas ("I think we can…," "Listen to me, there is a simpler way that…"). However, the other students ignored him and discussed whether the bathroom and the toilet should be separate. When S2 asked, "Why do you split the bathroom and the toilet? They could be put together," other members did not explain their thoughts and passed over this question ("He splits the two rooms, not me.").

Although the focus student joined the group discussion more actively at this stage, he did not contribute any constructive ideas. In effect, the other students formed a "smaller sub-team" to continue their discussion, leading to S2's missing many OTL that emerged from their interactions.

The focus students in the two groups had similar characteristics in the extent to which they actively participated in their group's discussion. At the beginning of the group discussion, the focus student of Group 2 did not actively participate. It was apparent that he wanted to join the group learning at the second stage of discussion, but failed because he had paid little attention to other students' ideas, leading them to ignore his. Even in the last stage, other students paid no attention to what the focus student said, which led to his missing many OTL.

5.4.2.2 Analysis of OTL

OTL developed in all of the above-mentioned three stages through different types of discourse and interactions (see Table 5.4 for more details).

OTL in Stage 1: OTL appeared four times in the first stage. The first OTL appeared during the discussion of the position of the living room and the bedroom. When the focus student proposed his idea ("I think the living room should be here and there are three bedrooms around the living room, you can put it"), other members rejected it, believing it was impossible to accommodate three bedrooms in a 60-square-metre apartment. Although he later tried to explain why there should be three bedrooms, the other students did not give him the chance, which made him lose this OTL. In this OTL, the type of discourse stayed at the object level.

The second OTL appeared in the discussion of the area of the apartment. The focus student suggested that he had found the key to solving the problem, but the others chose to ignore him. This negative response made it difficult for S2 to seize OTL. The type of discourse in this OTL moved from the object level to the subjectifying level.

The third OTL appeared when they were discussing the number of bedrooms. When S3 stated it was impossible to have three bedrooms in the apartment, the focus student questioned this ("Having three bedrooms could be possible."); the other students did not follow their discussion. At this time, the type of discourse changed from the object level to the meta level.

Table 5.4 A summary of OTL in Group 2

Stage	OTL	Discourse type	Discourse typte shift	Type of OTL
Stage 1	[42]: I think this should be the living room and then of the living room there are three bedrooms next to it, you put this ……	Mathematizing	Object-level → Object-level	I. Initiative
	[52]: I get it, I know what your problem is	Subjectifying	Object-level → Subject	I. Initiative
	[67b]: There can be three bedrooms, right?	Mathematizing	Object-level → Meta-level	II. Ask
	[76]: The flats generally have three or four floors	Mathematizing	Meta-level → Object-level	II. Explain
Stage 2	[92]: Why does the kitchen have to have a door?	Mathematizing	Object-level → Object-level	II. Conflict
	[104]: Total area, why do you need to count them one by one when you just put a circle around the periphery?	Mathematizing	Object-level → Meta-level	II. Conflict
	[121]: You paint this half of your master。bedroom at most	Mathematizing	Object-level → Object-level	II. Conflict
Stage 2	[148]: The way you hear me is relatively simple: you put ……	Subjectifying	Object-level → Subject	I. Initiative
	[169]: Why don't you just work this out? Why do you have to calculate the inside? Why don't you just work this one out?	Subjectifying	Object-level → Subject	II. Ask

Table 5.5 A comparison of the discourse type of the follower and leader in Group 1

Stage	Discourse type	S2 (%)	S3 (%)
Stage 1	Mathematizing	12.93	28.45
	Subjectifying	6.90	8.62
Stage 2	Mathematizing	17.53	26.81
	Subjectifying	14.43	17.53
Stage 3	Mathematizing	16.38	27.59
		9.48	12.93

The fourth OTL was in their discussion of how many floors an apartment should have. While the other students thought an apartment should have one or two floors, S2 thought one should have three or four. However, S2's questioning was still ignored by the other members ("Could you stop talking for now?"), which led to his failure to seize this OTL.

Table 5.6 A comparison of the discourse type of the follower and leader in Group 2

Stage	Discourse type	S2 (%)	S4 (%)
Stage 1	Mathematizing	9.09	15.20
	Subjectifying	12.52	2.27
Stage 2	Mathematizing	15.69	14.92
	Subjectifying	6.82	10.23
Stage 3	Subjectifying	28.82	25.54
	Subjectifying	14.77	13.64

OTL in Stage 2: The next three OTL emerged in the second stage. The fifth OTL appeared when the group discussed "whether a kitchen should have a door?" The focus student believed a kitchen does not need a door ("Why does a kitchen have to have a door?"), but the other group members said the "door" was simply the room's exit (on the picture), not a real one. The type of discourse stayed at the object level in this discussion.

The sixth OTL appeared when the group discussed each room's area to calculate the apartment's overall area. S4 thought the bathroom was similar to a half-circle and thus proposed calculating the area of a half-circle first. However, the focus student diverged from others and urged calculating the overall area based on the apartment structure rather than the area of each room. He refuted S4's idea ("We could calculate the overall area by looking at the one enclosed by the edge. Why do we have to calculate each one?"). This mismatch led to the other students' ignoring the focus student's idea, following their shared one, and continuing to discuss the area of the bathroom. The type of discourse turned from the object level to the meta level.

The seventh OTL appeared when the focus student proposed an idea as the group discussed the area of the main bedroom ("The main bedroom should be at most half of the one you drew."). This led the other group members to end this part of the discussion and start discussing the other four rooms. Their discourse type stayed at the object level.

OTL in Stage 3: The eighth OTL appeared while the group worked on the apartment floor plan. When the focus student tried to share his thoughts to initiate the discussion ("I had a simpler way to draw it that…"), the other members paid no attention and focused on discussing whether the bathroom and toilet should be placed together. The discourse type in this discussion turned from the object level to the subjectifying level.

The ninth OTL appeared when the group calculated the area of the bathroom. The focus student proposed his own calculation method ("We can directly calculate the area of this") but was rejected by the other members ("It will not work"). The discourse type of this short discussion changed from the object level to the subjectifying level. Due to repeated negative feedback, the focus student gradually stopped sharing his ideas.

The focus student in this group had some specific characteristics. In the first stage, the focus student did not participate actively but still tried to join the others. However, the focus student failed to integrate into the group due to negative feedback, leading to an inactive participation. Although the focus student tried to rejoin the group in the third stage, he did not succeed until the end of the discussion.

5.5 Discussion

5.5.1 Type of Discourse

In the collaborative problem-solving process, the proportion of mathematizing discourse used by the focus student in Group 1 first increased then decreased later, while the proportion of his subjectifying language remained at a relatively high level. For the focus student of Group 2, the proportion of mathematizing discourse was low at first but increased later, while the proportion of his subjectifying discourse increased at first but decreased later.

The focus student of Group 1 actively participated in the group discussion in the first stage and constantly tried to initiate discussion in the second stage to get back to the group discussion. However, the other members rejected or ignored his ideas as he is not at the same page, leading to his decreased participation in the third stage. Therefore, while the focus student's involvement was first active, this activeness decreased later, making him miss many OTL. We also found that the focus student in group 1 used mathematizing discourse more at the beginning of each discussion stage and subjectifying discourse more at the end of each satge.

The focus student of Group 2 was similar to the one of Group 1. Both actively participated in the group discussion in the some stages and tried to initiate the discussion, while the other students rejected their ideas. The difference lies in that Group 2 focus student did not actively join the discussion at the first beginning but tried to rejoin in later stages. The negative feedback from others in Group 2 made focus student failed to get involved in the second stage. In the third stage, the focus student tried to rejoin the group discussion and became more active. His inactive involvement in the first stage and failure to re-involve in later stages in the group discussion made him miss more OTL. Like his counterpart in Group 1, Group 2's focus student used mathematizing discourse more at the beginning of each discussion stage and subjectifying discourse more at the end.

A detailed analysis of the discourse type suggests that the proportion of different types of discourses may influence students' involvement in collaborative problem solving and further their use of OTL. More specifically, the more mathematizing discourse and less subjectifying discourse students used, the more active they were in the group discussion and the more OTL they could gain. On the contrary, if students use more subjectifying discourse and less mathematizing discourse, they might find it hard to join the group discussion and miss several OTL.

Table 5.6 and Table 5.7 compare the proportion of different types of discourse used by the leader student and focus student (who played the follower role) in each group.

For Group 1, S2 was the focus student and follower, while S3 led the collaborative problem-solving process, using the largest proportion of mathematizing discourse in the group. As shown in Table 5.6, the follower student and leader student influenced the proportion of the two types of discourse each other used: when the leader student used less mathematizing discourse, the follower student may use more. The same pattern appeared in their use of subjectifying discourse. This pattern may suggest that a mismatch in the discourse types could contribute to a failure in getting involved in collaborative problem-solving.

For Group 2, S2 was the focus student and follower, while S4 was the leader of the collaborative problem-solving process. Table 5.6 shows that in the first two stages, the discourse used by the leader student and follower student also influenced each other. As the third stage lasted a rather long time, both used a large portion of mathematizing and subjectifying discourse; however, the leader student still used less subjectifying discourse. Since the goal of group collaboration is to solve a mathematics problem, a extremely large proportion of subjectifying discourse could distract the flow of group discussion.

Figure 5.6 shows the relationship between the students' different roles in collaborative problem-solving and their use of discourse in different types. The research results suggest the following two conclusions:

(1) The proportion of different types of discourse used by students may influence their involvement in the group discussion. Especially, students who used more mathematizing discourse could more actively collaborative with others in the solving math problems.

(2) The leader student and follower student could influence the proportion of different types of discourse that each other used. When the leader student used more mathematizing (or subjectifying) discourse, the follower student may use less.

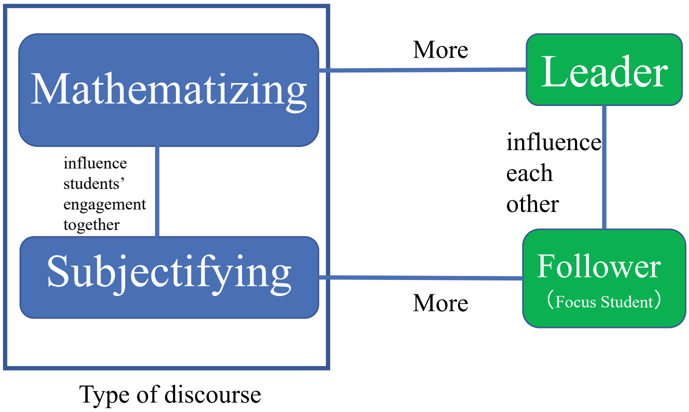

Fig. 5.6 The relationship between students' roles in the group and their discourse types

Several reasons for students' failure in participation were recognised from this perspective. Firstly, rather than trying to use more mathematizing discourse, the focus student used more subjectifying discourse to express himself, making it harder for him to communicate with others. Secondly, the focus student did not pay sufficient attention to the discourse used by the leader student or other students but tried to distract them, which prevented him from seizing and using OTL.

5.5.2 *Type of Interactions*

The type of interactions also changed in different stages (for details, see Group 1 in Table 5.7 and Group 2 in Table 5.8). For the focus student in Group 1, the proportion of intra-personal interactions decreased first and increased later, while the proportion of inter-personal interactions increased first and decreased later. In the first stage, the focus student interacted with other group members less and with himself more. In the second and third stages, these intra-personal interactions became less frequent and inter-personal interactions more frequent.

For the focus student in Group 2, the proportion of intra-personal interactions increased first and decreased later, while the proportion of inter-personal decreased first and increased later. In the first and the last stages, the proportions of these two types of interaction were essentially the same, while the focus student interacted with himself more and with others less in the second stage.

These changes suggest that the type of interaction may influence how students become involved in collaborative problem solving and whether they can use OTL. More specifically, the more students interacted with others, the more active they were in the group discussion, and the more OTL they took advantage of. In contrast,

Table 5.7 A comparison of the interaction type of the follower and leader in Group 1

Stage	Type of interaction		S2 (%)	S3 (%)
Stage 1	Inter-personal	Proactive	5	12
		Reactive	4	21
	Intra-personal		14	10
Stage 2	Inter-personal	Proactive	8	16
		Reactive	11	19
	Intra-personal		12	15
Stage 3	Inter-personal	Proactive	3	17
		Reactive	8	20
	Intra-personal		15	14

Table 5.8 A comparison of the interaction type of the follower and leader in Group 2

Stage	Type of interaction		S2 (%)	S4 (%)
Stage 1	Inter-personal	Proactive	7	6
		Reactive	13	15
	Intra-personal		10	9
Stage 2	Inter-personal	Proactive	2	2
		Reactive	11	15
	Intra-personal		13	13
Stage 3	Inter-personal	Proactive	7	6
		Reactive	9	13
	Intra-personal		11	15

the more students interacted with themselves, the less active they were in the group discussion and the more OTL they missed.

Tables 5.7 and 5.8 also compare the types of interactions by the leader student and focus student (a follower student) in Group 1 and Group 2. For Group 1, the proportion of proactive interactions S3 (the leader student) made continually increased, while S2 (the focus student and follower student) made more such interactions and less later. The frequency of S2's and S3's intra-personal interactions first increased and then decreased. Generally, the leader student made more inter-personal interactions (whether proactive or inactive), while the focus student made more intra-personal interactions. This could suggest that the leader student tended to interact with others, while the focus student (as the follower) tended to interact with himself.

For Group 2, the proportion of proactive interactions that S4 (the leader student) and S2 (the focus student and the follower student) made decreased first and increased later, while their proportions of inactive ones kept decreasing. S4 made more and more intra-personal interactions, while the frequency of these interactions S2 made increased first and decreased later. Similar to Group 1, the leader student interacted more with others while the focus student interacted more with himself.

Figure 5.7 shows the relationship between different roles in the group problem solving and the interaction type made by these students. The research results suggest the following two conclusions:

(1) The interaction type can influence the extent to which students participate in the group discussion and further their success in seizing OTL. Although intra-personal interactions can help students think independently, more inter-personal interactions can help involve them in collaborative problem solving.

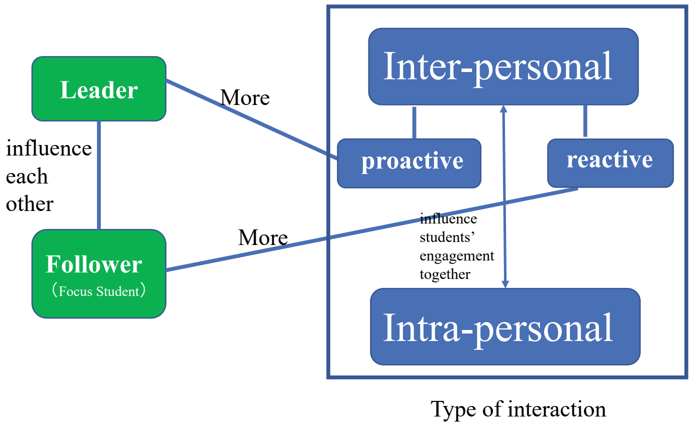

Fig. 5.7 The relationship between students' role in the group and the interaction type

(2) Students in different roles had different interaction type preferences. Leader students preferred interacting with others, while follower students preferred interacting with themselves.

Several reasons for students' failure in participation were recognised from this perspective. Firstly, the focus students made less interactions with others but always communicated with themselves. This excluded them from the group discussions and prevented them from integrating themselves into the group discussion. Secondly, the focus students did not follow the type of interactions made by the leader student as well as other members. When the leader students discussed ideas with other members, the focus students interacted with themselves. When the leader students and other members thought independently and interacted with themselves, the focus students tried to interact with others. However, by this time, the inter-personal interactions had nearly ended, leading to the focus students' keeping missing OTL.

5.5.3 *OTL and the Role of Student*

We identified the leader student, follower student (focus student), and other members in each collaborative problem-solving group. It is suggested that OTL development is closely related to leader students. In the collaborative work process, the follower students conflicted with the leader students by explaining, questioning, and taking the discussion initiative. While these conflicts can create OTL, the focus students missed the emerging OTL by being ignored and failed to participate in the group discussions. Figure 5.8 shows a more systematic representation of how students in different roles interacted to develop OTL. The analysis of discourse types and interaction types in the previous two sections suggests that the transformation between different discourse types and the new conversation contexts generated by the leader students could be

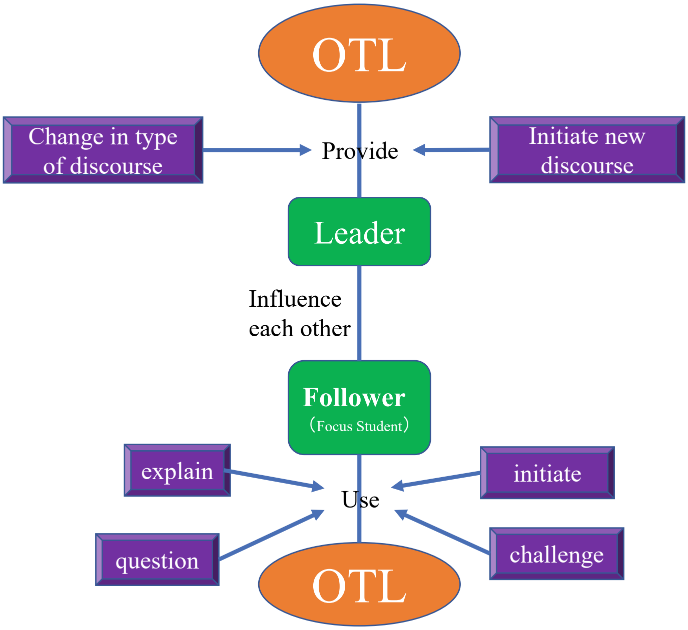

Fig. 5.8 The process of OTL development

the foundation for developing OTL. The follower students' explanations, questions, initiations, and conflicts further developed OTL. Several mismatches between focus students and leader students were summarised below from the perspective of OTL:

(1) The types of discourse used by the leader and focus students were always different. Specifically, when the leader students used mathematical language, the focus students used subject language.
(2) The focus students did not have a strong enthusiasm to join the group discussions. The conflicts they had with the leader students were not due to a mismatch in discourse type; they were a result of their blind refusal to participate. No matter what the leader students said, the focus students tended to focus on their own ideas rather than understanding the ideas the group has arrived upon.
(3) The focus students could not closely follow the pace of group discussion, which is usually controlled by leader students. For example, in Group 1, the group first discussed what an apartment should have and what one square metre was, then the overall structure of the apartment and the size of each room. Finally, they drew the plan of the apartment on paper. However, the follower student did not follow this shared.

5.5.4 A Summary of Factors that Influence the Development of OTL

This chapter presents a detailed discussion of the different discourse and interaction types among students in different group roles. Qualitative analysis suggests that students' discourse and interactions are related to OTL in a complex way, not a simple linear manner. More specifically, using mathematizing discourse and making inter-personal interactions promoted the extent to which students participated in the group discussion.

The discourse and interactions were also closely related to the leader students, focus students (usually a follower student who failed to engage in collaborative work), and other group members. This complex interactive relationship suggests that many factors influence OTL development. In the collaborative work on solving mathematics problem, discourse and interaction types differentiate students' roles in group discussions. Students who used more mathematizing discourse and made more proactive interactions were identified the leader students. We assume that these leader students are talking more about mathematics and communicating with others to promote the problem-solving. A change in the leader students' discourse type could facilitate OTL development. However, the alignment between leader students and focus students decided whether this OTL could be created successfully and further influenced whether the focus students could benefit from it.

In other words, the leader students provided OTL by changing the discourse type or initiating new discourse. Later, the followers could question or challenge leader students to create conflicts, which could further develop OTL. The followers could also develop OTL alone by explaining their ideas to others or initiating a discussion. Therefore, overuse of subjectifying discourse and frequently interacting with themselves were two main reasons the focus students (follower students) failed to participate in the group discussion. Figure 5.9 shows how different factors can influence OTL in collaborative problem solving.

5.6 Conclusion and Implications

This study has explored the development of OTL in collaborative problem solving in a secondary mathematics class. Instead of paying a close attention to successful cases, we focused on two groups where there is a focus students who failed to engage in group work and further to seize the OTL. By analysing students' discourse and interactions, we identified different roles students played and closely examined the interactions between leader students and focus students to study how OTL can be developed, seized, and used. We further conceptualised how these factors could influence the development and use of OTL as a supplement to the current theoretical conceptualisation of the mechanism of collaborative learning.

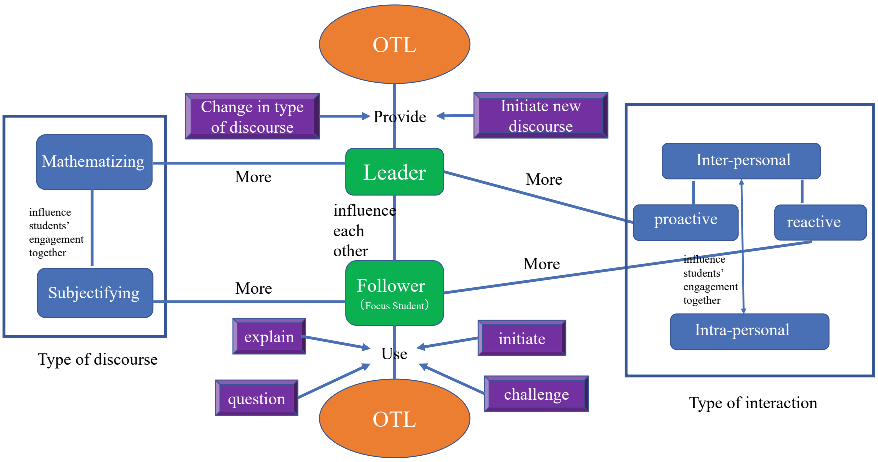

Fig. 5.9 A systematic review of how different factors influence the development and use of OTL

Results suggest that students' discourse and interactions are closely related to OTL development, as they can influence the extent to which students actively participate in group discussions and further their learning results. This is aligned with the nature of collaborative learning as social interactions that are highlighted by past studies (Palincsar & Herrenkohl, 2002). Specially, we found that students with less mathematizing discourse, more subjectifying discourse, and more intra-personal interactions are hard to participate in collaboration and thus fail to take advantage of OTL. According to Smith and MacGregor (1992), peer discussion and peer tutoring are the most common forms of collaborative learning, which requires an in-depth involvement in interactions. Despite the social nature of collaborative learning, Laal and Ghodsi (2012) also emphasised the academic nature of collaborative learning. This aligns with our findings that students used less mathematizing discourse would fail in get involved in group discussion to learn from their peers.

This study has several practical implications for teachers in the teaching of collaborative problem solving. Collaborative problem-solving teaching provides an open atmosphere for students' learning, one that allows them to communicate with each other freely and elaborate on their advantages. Recent studies have revealed multiple factors that may influence such socialization culture (Dong & Kang, 2022), for example. Students are the agents of this sociocultural learning, while teachers are facilitators who provide guidance for students' learning. However, as teachers do not assign each student a discussion role but let them take on different roles naturally, leader students may not realise their responsibility and thus may make other group members miss many OTL. Therefore, teachers could assign different group discussion roles to students so that they can discuss ideas in a more organised way and effectively use more OTL.

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Chapter 6
The Characteristics of Mathematical Communication in Secondary School Students' Collaborative Problem Solving

Jie Yang

6.1 Introduction

Researchers and teachers have gradually realised mathematical communication is an indispensable part of mathematics classroom teaching and learning. Through mathematical communication, teachers share mathematical knowledge and methods, while students improve their understanding of mathematical concepts. In 1989, mathematical communication ability was mentioned in *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989); since then, different countries have added mathematical communication to their curriculum standards and syllabi (Sun, 2003). Communication has been a key ability in the PISA 2000–2012 assessment frameworks, with the PISA2021 assessment framework identifying it as an important twenty-first-century skill (Cao & Zhu, 2019). Mathematical communication has become the focus of mathematics classes (Neria & Amit, 2004).

Chinese researchers and teachers have focused on mathematical communication in classes for rather a long time. Chen (1990) introduced American mathematics curriculum standards, which included the requirement that students "learn to communicate in mathematical language." Shen et al. (1991) published relevant literature on the development of mathematical communication ability.

Since the beginning of the twenty-first century, China has gradually added mathematical expression and mathematics communication requirements to mathematics curriculum standards for compulsory education and senior middle school. Both the *Mathematics Curriculum Standard for Compulsory Education (2011)* (Ministry of Education of the People's Republic of China, 2012) and *Mathematics Curriculum Standard for Compulsory Education (2017)* (Ministry of Education of the People's Republic of China, 2018) repeatedly emphasised the importance of mathematical

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expression and mathematical communication and highlighted the significant value of increasing mathematical communication in classroom teaching. Although research on mathematical communication in social and cognitive aspects has been conducted (e.g. Langer-Osuna et al., 2020), mathematical communication has become a key issue in Chinese mathematics education contexts.

Open-ended tasks could provide affordances for communication (Chan & Clarke, 2017). They can stimulate students to engage in activities in classroom and encourage students to explore and investigate (Osana et al., 2006). Teacher using open-ended tasks and asking students to explain their process and reasoning are beneficial to mathematical communication (Cai et al., 1996). Mathematical communication has attracted considerable attention from scholars worldwide, and studies suggest that Chinese teachers and students have started to pay attention to mathematical communication in their classes (Xu & Wang, 2017). However, various factors may interfere with the quality of students' mathematical communications in mathematics classroom. For example, students with poor communication skills would find it difficult to communicate with each other and express their opinions clearly, might not be patient enough to communicate, etc. (Su, 2003). Chinese students' mathematical communication ability still needs to be improved (Shen, 2014). Since the current situation, systematic research on students' mathematical communication in case of collaboration on open-ended tasks is necessary.

This study focused on the characteristics of mathematical communication in secondary school students' collaborative problem solving to address the following research questions:

(1) What are the elements of students' mathematical communication in open-ended collaborative problem solving?
(2) What are the characteristics of students' mathematical communication in open-ended collaborative problem solving? Is there any model of students' mathematical communication?
(3) What are the characteristics of mathematical communication between high-scoring and low-scoring student groups in open-ended collaborative problem solving? Is there any difference in the number of communication elements used in high-scoring and low-scoring student groups? Are there any differences in the roles of students' mathematical communication in high-scoring and low-scoring groups?

6.2 Research Methods

6.2.1 Data Selection

The data for this study were collected from eight Grade 7 classes from two junior high schools, LH and YC, in Beijing by the research team during the 2017–2018 school year. Each class had six to eight valid groups, with videos and task sheets being collected from 56 groups in total.

The collaborative problem-solving task used in the study is shown in Appendix Task (1) To quantitatively evaluate the results of students' collaborative problem solving, the task scoring criteria were formulated as shown in Table 6.1.

According to the scoring criteria determined in Table 6.1, the results of 56 teams' collaborative problem solving were scored. The scoring results are shown in Fig. 6.1.

Table 6.1 Scoring criteria

Basic requirements	Total area of the apartment (1 point)	The total area of the apartment is 60 m^2	1 point
		The total area of the apartment is not 60 m^2	0 point
	Number of rooms (1 point)	The number of rooms is 5	1 point
		The number of rooms is not 5	0 point
	Room size (length and width) (1 point)	Fully mark the length and width of 5 rooms	1 point
		Only the area of 5 rooms are marked, the length and width of rooms are not marked	0.5 point
		Only parts of the room size are marked	0.5 point
		The room size does not accord with the reality	0.5 point
		The room size is not marked	0 point
	Function of the room (1 point)	Fully mark the function of the 5 rooms	1 point
		The functions of the rooms do not accord with the reality	0.5 point
		Only mark the functions of some rooms	0.5 point
		The functions of the rooms are not marked	0 point
Total	4 points		
Further request	Show the calculation process of room sizes		0.5 point
	The relevant situation is described in detail		0.5 point
	The scale of the graph is marked		0.5 point
Total	1.5 points		
Total score	5.5 points		

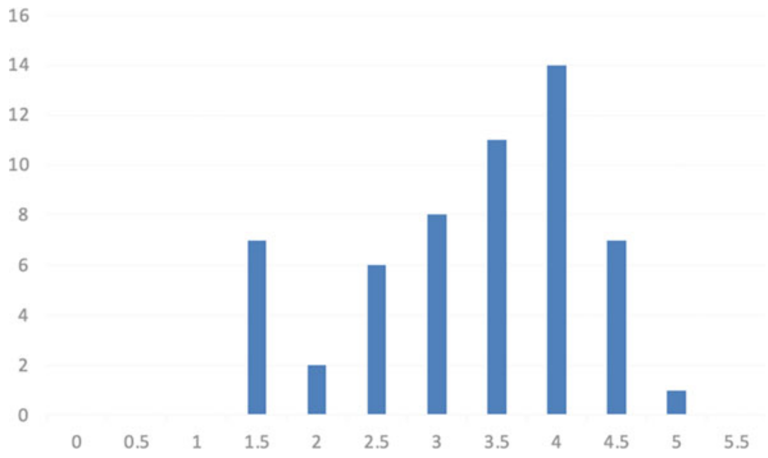

Fig. 6.1 Scoring results

Eight groups were selected as research subjects in this study to analyse students' mathematical communication in a focused way, as shown in Table 6.2. Specifically, this study plans to select a group with a score higher than 3.5 and a group with a score lower than 2.5 in each teacher's class. Groups with a score higher than 3.5 were named high-scoring groups, and those with a score lower than 2.5 points low-scoring groups. As all the groups in Teacher A's classes scored 2.5 points or more, two high-scoring and two low-scoring groups were selected from the two classes taught by Teacher B (in the same school as Teacher A).

This selection method minimised the influence of schools and teachers on the coding results. In similar external environments, group scores are significantly different. Therefore, the differences in high-scoring and low-scoring groups' mathematical communication characteristics could be better compared by analysing these groups.

Table 6.2 Participations

Group	School	Teacher	Class	Group number	Points
High-scoring Group 1	YC	Teacher C	Class 5	Group 1	4.5
High-scoring Group 2	LH	Teacher B	Class 3	Group 4	4.5
High-scoring Group 3	YC	Teacher D	Class 7	Group 6	4
High-scoring Group 4	LH	Teacher B	Class 4	Group 3	4
Low-scoring Group 1	YC	Teacher C	Class 5	Group 2	1.5
Low-scoring Group 2	LH	Teacher B	Class 3	Group 6	1.5
Low-scoring Group 3	YC	Teacher D	Class 7	Group 4	1.5
Low-scoring Group 4	LH	Teacher B	Class 3	Group 3	2

6.2.2 Data Analysis

After data collection, this study constructed the sets of elements in students' mathematical communication process based on existing literature. The following formula.

$$A_1 = \{E_1, E_2, \ldots, E_{n1}\}, \quad A_2 = \{E_{n1+1}, E_{n1+2}, \ldots, E_{n2}\},$$
$$A_3 = \{E_{n2+1}, E_{n2+2}, \ldots, E_{n3}\}$$

yielded the union of the above sets.

$$A = \{E_1, E_2, \ldots, E_n\}.$$

The videos of two groups were then selected for precoding, based on which we refined the element set mentioned above and deleted low-frequency elements from the coding results to construct an initial set of elements for mathematical communication in collaborative problem solving:

$$A' = \{E_1, E_2, \ldots, E_m\}.$$

To solve the problem of communication elements not belonging to the same level or overlapping each other, we established a more concise and efficient multi-level coding framework by seeking the patterns in students' mathematical communication and combining some elements into the same module.

Based on the established coding framework, this study coded students' mathematical communication processes in open-ended collaborative problem solving. Then, the researchers analysing the coded data and further summarised and constructed the main characteristics and process of secondary school students' mathematical communication in collaborative problem solving.

Additionally, we compared the differences between the characteristics of high-scoring groups' and low-scoring groups' mathematical communication from two aspects. The first one is the number of communication elements. The second one is students' roles in mathematical communication, identifying each group's leaders, supervisors, marginalised students, and blockers.

6.3 Study Results

6.3.1 Elements of Mathematical Communication in Secondary School Students' Collaborative Problems Solving

The set of elements of mathematical communication was constructed based on the relevant literature. This study reviewed recent literature on collaborative problem solving and mathematical communication and summarised communication elements from them, as shown in Table 6.3.

As shown in Table 6.3, some communication elements did not belong to the same level, and there were obvious overlaps and repetitions. Therefore, these elements were re-sorted by disassembling some elements and merging similar elements for a more effective coding system. The communication elements were divided into two main parts, the cognitive dimension and the collaborative dimension, as shown in Table 6.4.

A total of 54 communication elements were sorted. However, not all belonged to the same level or overlapped each other, and so did not fully cover all the dialogues in the students' mathematical communication process. Thus, students' mathematical communication could not be effectively coded by these elements.

To solve the above issues, two videos were selected from the collected data to further revise the elements. The two videos were of Teacher A's Class 1 Group 1 from LH middle school and Teacher C's Class 5 Group 1 from YC middle school. In precoding these two groups, we tried to code every conversation between the students based on the existing elements. When it was found that some students' conversations could not be classified using any of the above 54 elements, new elements were added to the set. The precoding results are shown in Table 6.5.

Two new communication elements were added, namely "Refute" and "Propose a plan." Many elements did not appear in the precoding process. These elements were deleted to make the element set more suitable for the selected subjects. The improved element set contained 19 communication elements, as shown in Table 6.6.

The mathematical communication element set displayed in Table 6.6 was taken as the first-level coding framework for mathematical communication in this study.

The students' dialogue in the mathematical communication process had certain patterns and thus could be divided into certain communication modules. Sometimes these modules could be clearly divided, and sometimes there was no obvious boundary. Dialogues between students in different modules could occur simultaneously. For example, while S1 and S3 may have a dialogue on one topic and S2 and S4 may have a dialogue on another, both dialogues can still be roughly divided based on the students' communication topics. Thus, the first-level coding framework given in Table 6.6 can be merged into ten communication modules. The explanations for these ten communication modules and their patterns based on the first-level coding are shown in Table 6.7.

Table 6.3 Elements of mathematical communication

Documents	Elements of mathematical communication
How to solve it (Polya, 1945)	"Understand the problem," "Devise a plan," "Carry out the plan" and "Look back"
Cooperative learning process model for mathematical problem solving and its application (Cao & Bai, 2018)	Analyse the problem; Locate possibly useful knowledge; Explore the path from the existing knowledge to the desired goal; Carry out a plan; Review and reflection
	Express views; Listen; Pose a question; Question; Respond; Urge; Encourage; Help; Argue; Reach an agreement; Reflect; Evaluate; Approve
The essences of cooperative learning (Zeng, 2000)	Assign tasks; Provide materials (information); Pose a question; Test; Summarise; Analyse the problem
The process-oriented shift of cooperative learning strategies in classroom (Cao, 2018)	Discuss topics together; form views; Criticise; Examine and discuss; Question each other; Comprehensive promotion; Improve views
	Express views; Share and exchange ideas; Listen; Discussion; Argumentation and reasoning; Clarify the meaning; Share understanding; Negotiate views; Collaboration; Formalisation; Artifact
	Problem representation; Exploration of evidence; Negotiation and reconstruction; Communication and explanation; Adjustment and reflection
	Individual problems; Analyse and define problems; Group discussion; Position and obtain solutions; Generalise; Presentation and internalisation
	Focus on the goal; understand the task; Learn and recall; Check the result; Evaluate
Assessment of collaborative problem-solving ability in international education assessment programme: index framework, assessment standard and technical analysis (Tan et al., 2018)	Analyse the problem; Assign tasks; Provide information; Analyse views; Seek opinions; Make a suggestion; Solve conflict; Monitor progress
A case study of teacher interventions in mathematics cooperative-learning classrooms in junior high-school (Dong et al., 2013)	Answer; Request for explanation; Supplementary explanation; Provide supporting problems; Provide deep-level problems; monitor; Evaluate (praise\criticise)
Lexicon project (Cao & Yu, 2017)	Analyse the problem; Pose a question; Answer; Correct errors; Comb knowledge; Generalise; Check; Encourage; Pose a rhetorical question; Pose a further question; Explain; Evaluate; Praise; Criticise; Summarise

Table 6.4 Elements of mathematical communication (improved)

Express views	Listen	Pose a question	Question
Cognitive dimension			
Respond	Argue	Reach agreement	Reflect
Provide information	Test	Summarise	Form views
Criticise	Inspect	Reason	comprehensive promotion
Improve views	Inference	Clarify the meaning	Share understanding
Negotiate views	Formalization	Objectivization	Artifact
Explore evidence	Negotiation and reconstruction	Explain	Adjustment and reflection
Analyse	Learn and recall	Check the result	Evaluate
Analyse views	Seek opinions	Make a suggestion	Solve conflict
Answer	Request for explanation	Supplementary explanation	Provide supporting problems
Provide deep-level problems	Correct errors	Generalise	Check
Pose a rhetorical question	Pose a further question	Analyse the problem	
Collaborative dimension			
Urge	Encourage	Help	Assign tasks
Monitor progress	Evaluate (praise \criticize)	Approve	

Note The "Artifact" refers to the specification process of students' thinking, including their experimenting by hand or writing answers down, while the meaning of this concept differs in different tasks

There were some similarities between the "negotiate," "argue" and "quarrel" modules, but also some obvious differences. Students would discuss an issue in the "negotiate" module, reach a consensus and promote the problem-solving process. For example:

> Student 1: You don't have to draw anything special, just mark each room. What's it for? Five rooms, three bedrooms? Two bedrooms, two bedrooms, one living room, one kitchen and one bathroom.
>
> Student 3: Two bedrooms and one living room, one kitchen and one bathroom.
>
> Student 1: Just right, two bedrooms, one living room, one kitchen and one bathroom, just right.
>
> Student 3: Two bedrooms, one living room, one kitchen and one bathroom, roughly the same.

In the above dialogue, Student 1 proposed that the five rooms should include two bedrooms, one living room, one kitchen and one bathroom. Student 3 responded to Student 1 and the two reached an agreement, promoting the problem-solving process.

Table 6.5 Precoding results

Elements	Numbers (Class 1)	Numbers (Class 5)	Total
Express views	45	40	85
Pose a question	20	42	62
Question	62	45	107
Respond	46	42	88
Provide information	16	25	41
Improve views	12	2	14
Explain	22	0	22
Answer	23	33	56
Correct errors	1	2	3
Refute	30	15	45
Pose a rhetorical question	6	1	7
Urge	11	31	42
Assign tasks	4	4	8
Evaluate	12	20	32
Approve	31	17	48
Propose a plan	2	4	6
Generalize	1	3	4
Summarize	0	2	2
Artifact	0	21	21
/	39	79	118
Total	383	428	811

Note "/" refers to the conversations that cannot be classified, including teachers' instructions or students' talking about irrelevant topics

Table 6.6 Elements of mathematical communication (Improved)

Express views	Pose a question	Question	Improve views
Cognitive dimension			
Respond	Artifact	Explain	Refute
Provide information	Correct errors	Summarise	Answer
Pose a rhetorical question	Propose a plan	Generalise	
Collaborative dimension			
Urge	Evaluate (praise /criticize)	Approve	Assign tasks

In the "argue" module, students criticise, refute and question each other on specific issues before finally reaching a relatively unified consensus, which promotes the problem-solving process. For example:

Student 1: Just draw it like this. Draw a big square.

Table 6.7 Modules of mathematical communication

Modules	Explanation	Pattern
Analyse the problem	Students analyse the information and requirements given in the problem	Some students provide material or information about the problem, and the other students respond
Make a plan	Students make plans to solve the problem and assign tasks to group members	It contains a large number of elements including "assign tasks" and "propose a plan."
Provide information	Students provide other resources or information to help solve the problem	Some students provide additional resources or information, while others respond
Negotiate	Students discuss a specific issue and finally reach an agreement	Some students express their views or pose questions, and the rest students' respond, answer or improve their views. This process will be repeated until the "approve" or "generalise" elements appear
Argue	Students criticise, refute and question each other on a specific issue, and finally reach a general agreement	Some students express their views or pose questions, others question or refute, and then some students explain or respond. The above process is repeated until the "approve" or "generalise" elements appear
Quarrel	(1) Students have repeated discussions on a certain issue and finally fail to agree with each other (2) Students can't focus on one topic because of the quick change among the topics they discuss. The topic of discussion changes without giving timely feedback after expressing their views or posing questions	Some students express their views or pose questions, and the others question or refute. The above process repeats, but there are no "approve" or "generalise" elements at the end
Off-task discussion	Students have meaningless discussions on issues unrelated to the task	A large number of "/" elements appear, that is, dialogues unrelated to the topic
Calculate	Students perform mathematical calculations	Students perform mathematical calculations
Artifact	Students present the results of the discussion on the task sheet	There are a lot of "artifact" elements (especially writing)
Evaluate	Students evaluate the process, methods and results of problem solving	There are a lot of "generalise," "summarise" or "evaluate" elements

Student 3: A square.

Student 3: The room can’t be square.

Student 1: Right, rectangle, rectangle.

In the above dialogue, Student 1 proposed drawing a large square to represent the apartment, but Student 3 refuted Student 1’s view, pointing out that the room could not be square. Student 1 agreed and they reached a relatively unified consensus, which promoted the problem-solving process.

Students also express their views, pose questions, refute and respond in “quarrel” modules. However, unlike in the “negotiate” and “argue” modules, they fail to reach an agreement. For example:

Student 4: Two bedrooms.

Student 2: But he lives alone.

Student 3: Why can’t there be two bedrooms in one’s apartment?

Student 2: Why don’t you set a grocery store? It could be small.

Student 4: Xiao Ming’s apartment.

Student 2: Then why don’t you have a utility room? Isn’t there usually a utility room at home?

Student 1: No one has a utility room. The utility room is combined with the toilet.

Student 4: Let’s first determine what the five rooms are.

In the above dialogue, Student 4 proposed having two bedrooms, while Students 2 and 3 refuted Student 4. When no consensus was reached, Student 2 shifted the topic and suggested adding a utility room, at which point Student 1 refuted Student 2. However, the group then changed the topic, leaving the problem unresolved.

The ten mathematical communication modules displayed in Table 6.8 were taken as the second-level coding framework for mathematical communication, based on which the two groups of students’ mathematical communication were coded. The results of the coding are as follows.

By integrating the analysis of the ten communication modules with Polya’s four-stage problem-solving process theory, these modules can be further divided into four main parts: “Analyse the problem,” “Make a plan,” “Carry out the plan” and “Evaluate.” The “Carry out a plan” part includes seven communication modules: “Provide information,” “Negotiate,” “Argue,” “Quarrel,” “Off-task discussion,” “Calculate” and “Artifact.”

Based on the postcoding process, a systematic and hierarchical three-level coding framework was finally formed, as shown in Table 6.9.

(I) The characteristics of mathematical communication in secondary school students’ collaborative problem solving

The dialogues of the eight groups were coded based on the three-level coding framework given in Table 6.9. The first-level coding results are shown in Table 6.10. The numbers in Table 6.10 represent the frequency with which each element appeared in each group’s mathematical communication.

Table 6.8 Postcoding results

Modules	Numbers (Class 1)	Numbers (Class 2)	Total
Analyse the problem	0	1	1
Make a plan	1	1	2
Provide information	1	1	2
Negotiate	5	5	10
Argue	10	7	17
Quarrel	12	5	17
Off-task discussion	0	3	3
Calculate	1	1	2
Artifact	1	4	5
Evaluate	1	2	3
Total	32	30	62

Table 6.9 Three-level coding framework

Coding level	Description
First-level coding	Try to code each conversation of students according to the 19 communication elements
Second-level coding	Code students' mathematical communication according to 10 communication modules
Third-level coding	Divide students' mathematical communication into four main parts

As shown in Table 6.10, the "express views" element was the most frequent (300 times), indicating that students were willing to state their ideas in the communication process. It was followed by "pose a question," "question," "respond," "answer" and "artifact" elements, all appearing more than 150 times. The elements "provide information," "refute," "urge," "evaluate" and "approve" appeared more than 50 times, while "improve views," "explain," "correct errors," "pose a rhetorical question," "assign tasks," "propose a plan," "generalise" and "summarise" appeared fewer than 50 times.

The second-level coding results for the eight groups are shown in Table 6.11. The numbers in the table represent the frequency with which each module appeared in each group's mathematical communication process.

As shown in Table 6.11, the "quarrel" module was the most frequent (46 times), indicating it was often difficult for students to reach an agreement in the discussion process. The elements "argue," "artifact" and "negotiate" appeared 43, 22 and 20 times, respectively, while "off-task discussion" made ten appearances. "Analysing the problem," "make a plan," "provide information," "calculate" and "evaluate" each appeared less than ten times.

Thus, the main characteristics of mathematical communication in secondary school students' collaborative problem solving can be summarised as follows:

Table 6.10 First-level coding results

Elements	HS Group 1	HS Group 2	HS Group 3	HS Group 4	LS Group 1	LS Group 2	LS Group 3	LS Group 4	Total
Express views	40	29	36	43	58	45	26	23	300
Pose a question	42	21	29	11	63	22	17	14	219
Question	45	14	18	11	36	21	12	23	180
Respond	42	20	21	27	38	17	13	12	190
Provide information	25	4	6	2	34	7	5	8	91
Improve views	2	2	1	0	10	3	1	2	21
Explain	0	6	4	2	4	2	1	1	20
Answer	33	21	32	7	47	16	14	12	182
Correct errors	2	1	0	0	0	1	1	0	5
Refute	15	10	17	8	22	4	6	9	91
Pose a rhetorical question	1	1	3	0	4	0	0	1	10
Urge	31	10	2	8	4	1	2	2	60
Assign tasks	4	10	4	9	7	6	1	4	45
Evaluate	20	4	15	11	2	7	8	1	68
Approve	17	16	25	10	39	9	5	2	123
Propose a plan	4	1	1	5	0	3	1	0	15
Generalise	3	1	2	5	2	4	0	0	17
Summarise	2	0	2	2	0	2	0	2	10
Artifact	21	12	56	15	0	23	21	2	150
/	79	60	15	23	25	24	41	66	333
Total	428	243	289	199	395	217	175	184	2130

Note HS means high-scoring and LS means low-scoring

(1) **"Negotiate," "argue," "quarrel" and "artifact" were the basic communication modules in mathematical communication**

The coding framework for this study contained ten communication modules. However, this study found that not every communication module appeared in each group's mathematical communication process. Only four modules—"negotiate," "argue," "quarrel" and "artifact"—appeared in all groups' discussions; the other six communication modules did not. Moreover, "negotiate," "argue," "quarrel" and

Table 6.11 Second-level coding results

Modules	HS Group 1	HS Group 2	HS Group 3	HS Group 4	LS Group 1	LS Group 2	LS Group 3	LS Group 4	Total
Analyse the problem	1	0	1	0	0	0	0	0	2
Devise a plan	1	1	1	1	1	0	1	0	6
Provide information	1	0	0	0	2	0	0	0	3
Negotiate	5	3	5	2	3	1	0	1	20
Argue	7	3	7	4	8	5	3	6	43
Quarrel	5	2	2	4	14	7	6	6	46
Off-task discussion	3	1	0	0	0	2	2	2	10
Calculate	1	0	2	0	0	0	0	0	3
Artifact	4	1	3	5	1	3	4	1	22
Evaluate	2	1	1	2	0	1	1	0	8
Total	30	12	22	18	29	19	17	16	163

"artifact" were the most frequent modules in the mathematical communication process.

"Negotiate" and "argue" were the most common, with each group experiencing five to 15 rounds of "negotiate" and "argue" modules. "Negotiate" and "argue" were the two most important communication modules for promoting the problem-solving process.

"Argue" and "quarrel" modules both contained "express views," "refute" and "question" elements. However, unlike in the "argue" module, students in the "quarrel" module either could not reach a consistent conclusion or changed the topic before doing so, despite still discussing the problem to some extent. Although the students could not reach an agreement, the "quarrel" module promoted the problem-solving process to an extent and was undeniably an important basic communication module in the problem-solving process.

"Artifact" was an important module, enabling students to present discussion results on a task sheet and visualise the problem-solving results. Therefore, "artifact" was also a basic communication module in the problem-solving process.

(2) **Students were willing to express their views and criticise each other in the mathematical communication process**

Based on the data, "express views" and "pose a question" were the most frequent communication elements, appearing 300 and 219 times, respectively. This shows that students actively expressed themselves and raised some questions in the mathematical communication process. There was no situation where they had no idea what to do.

In addition, the frequencies of the "question" (180) and "refute" (91) elements were also considerable, indicating that students dared to question and refute others' opinions in the mathematical communication process.

One possible reason for this is that the students in each group were all from the same class and familiar with each other. Therefore, silence was rare, and the students were willing to engage in communication.

(3) **The students' mathematical communication process was always incomplete**

The data revealed that many groups lacked "analyse the problem," "make a plan," and "evaluate" modules in their mathematical communication process, with "analyse the problem" being the most conspicuous absence. Only two of the eight groups analysed the problem, while two groups did not make plans or evaluate.

Among the eight groups, only two experienced all four parts of mathematical communication, indicating the students were unfamiliar with the collaborative problem-solving process. As Chinese teachers pay little attention to the collaborative problem-solving process in traditional mathematics classes, students rarely have opportunities to use it and are often unclear about the steps involved. When the students in this study faced this unfamiliar teaching method, their teachers did not guide them to experience the necessary problem-solving parts.

(4) **There were many cases in which students failed to reach an agreement in the mathematical communication process**

The data revealed the "quarrel" module appeared most frequently (46 times) in the eight groups, while the "argue" (43) and "negotiate" (20) modules were relatively less frequent.

This suggests that the students dared to refute and criticise other students' views, but had difficulty reaching a consensus. Students often changed the topic before reaching an agreement and left many issues unsolved in the problem-solving process. These issues usually need to be discussed later, thus wasting a lot of time.

(5) **Discussions unrelated to the problem were common in the students' mathematical communication process**

The data revealed that off-task discussions inevitably occurred in the students' mathematics communication. In the eight groups, students had a total of ten unrelated discussions, indicating they were easily distracted during the problem-solving process. There were too many distractions in their complex classroom environment; students were easily disturbed by other groups, the recording equipment or the situation of the problem. For example, in Low-scoring Group 3, students had an off-task discussion:

Student 2: Your family is in the real estate business.

Student 1: No.

Student 2: Look over there. There's a camera.

Student 3: Zhong, are you in the real estate business now? I mean now.

Student 3: Actually, I really want to get into the real estate business.

In this dialogue, the students were influenced by the recording device and the problem situation, leading to off-task discussions.

(6) **There were fewer conversations in the collaborative dimension than in the cognitive dimension**

Data analysis revealed that students' dialogues involved the collaborative elements "urge," "evaluate," "approve" and "assign tasks" 60, 68, 123 and 45 times, respectively, for an average of 74. Several student dialogues involved all four collaboration elements, indicating they were concerned about group collaboration. However, on average, student dialogues involved each cognitive element more than 100 times, indicating that they paid more attention to promoting the problem-solving process cognitively than through group collaboration.

However, students also had a certain amount of dialogues on the four elements of the collaboration dimension, which also showed students' concern for group collaboration.

By combining the three-level coding framework for mathematical communication with the main characteristics of mathematical communication, this study established a model of students' mathematical communication process, as shown in Fig. 6.2.

(II) **The differences of mathematical communication characteristics between high-scoring and low-scoring groups**

As shown in Fig. 6.3, there were similarities between high-scoring and low-scoring groups in first-level elements, but also great differences in certain elements.

The number of dialogues involving the "respond" element was significantly higher in the high-scoring groups than in the low-scoring groups. This indicates that students with high scores were more active in responding to other students' opinions and expressing their own.

An obvious difference between the high-scoring and low-scoring groups lay in the frequency of the "urge" element. This disparity suggests that students with high scores paid more attention to group discussion progress and actively promoted the problem-solving process, while students with low scores lacked awareness of their task progress, often leading to uneven time allocation and further incomplete discussion.

In addition, obvious differences existed between high-scoring and low-scoring groups in the "evaluate" element of their discussions. Students with high scores evaluated their problem-solving process and results more frequently and could summarise their current task progress, enabling them to identify shortcomings in their problem-solving process and solve the problem better.

The "artifact" element appeared in high-scoring groups' discussions an average of 104 times, compared to 46 times in low-scoring groups, indicating that high-scoring groups were more active in presenting discussion results on task sheets. Although the low-scoring groups drew some valuable conclusions in their discussions, they spent little time writing them down, which led to their low scores.

Overall, there were significant differences between high-scoring and low-scoring groups in several elements, including "respond," "urge," evaluate" and "artifact."

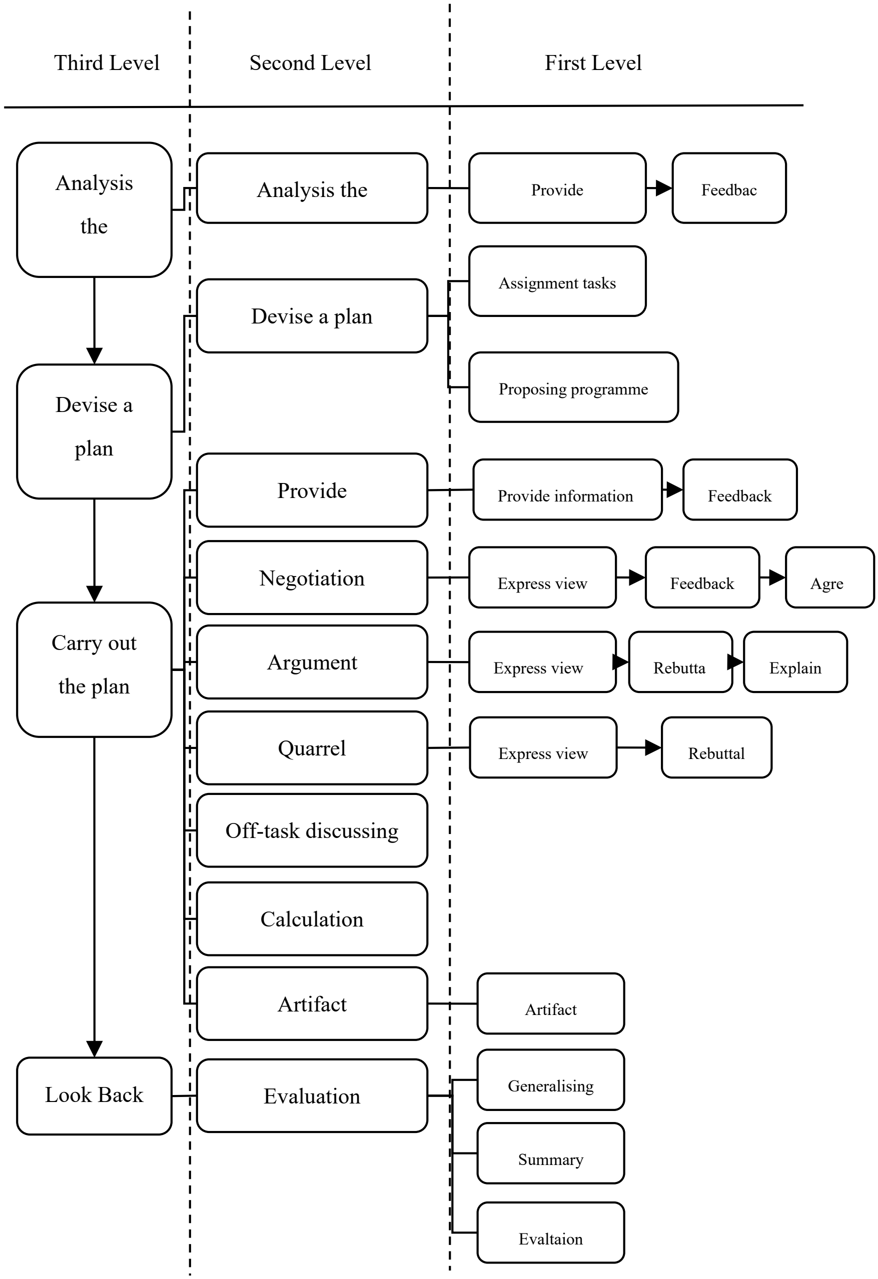

Fig. 6.2 Process model of students' mathematical communication

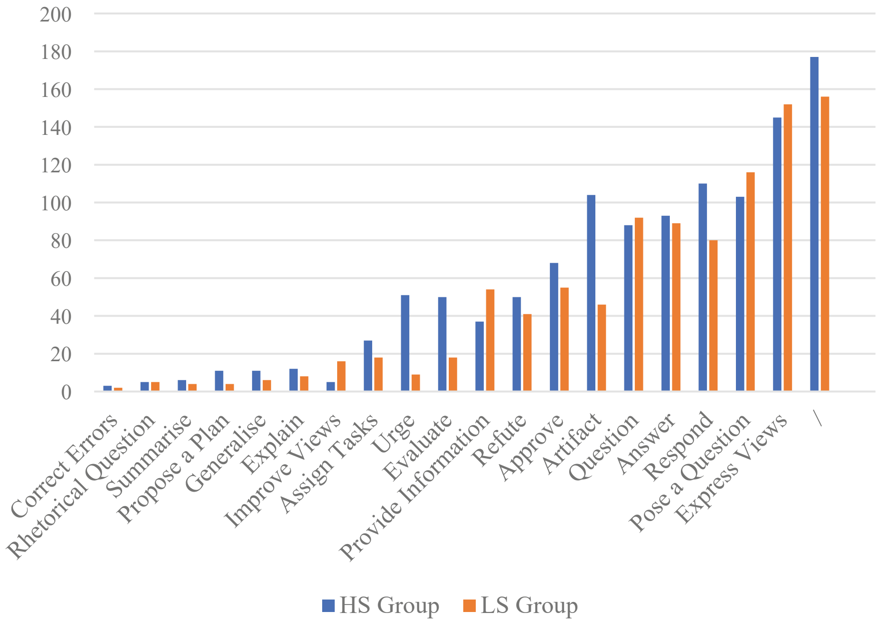

Fig. 6.3 First-level coding results

Figure 6.4 shows the similarities and differences between high-scoring and low-scoring groups in second-level coding results.

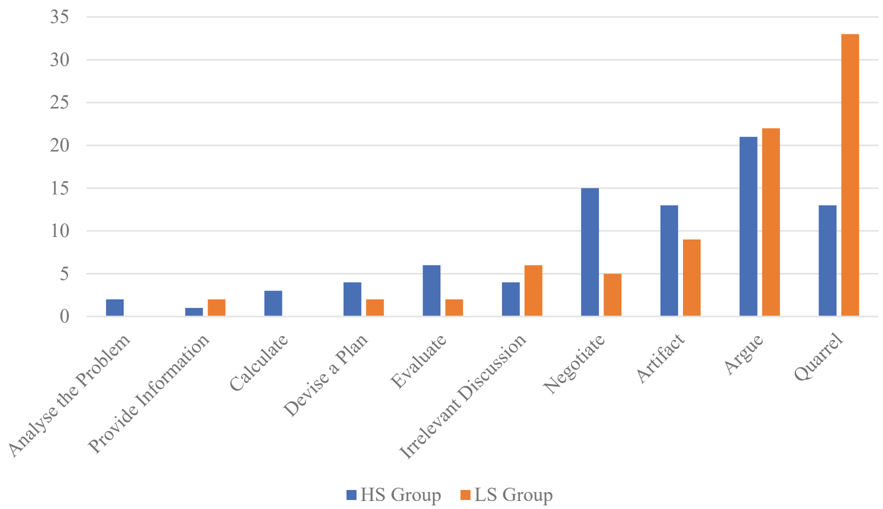

Fig. 6.4 Second-level coding results

There were significant differences between high-scoring and low-scoring groups in the "negotiate" and "quarrel" modules. There were, on average, 15 "negotiate" modules in the high-scoring groups and only three in low-scoring ones; in contrast, the mean number of "quarrel" modules in low-scoring groups reached 33, compared to 13 in high-scoring groups. This indicates that high-scoring groups could often reach a consensus, while low-scoring groups found doing so difficult because rapid topic changes and disruptions prevented thorough discussion of a question.

In total, the high-scoring groups experienced 13 "artifact" modules, while the low-scoring groups experienced nine, in part because low-scoring groups lacked awareness of the need to present discussion results on task sheets. Some low-scoring groups made some achievements in their discussions but did not write down their answers, which had a powerful impact on their final scores.

The "evaluate" module was very important in the problem-solving process. As shown in Fig. 6.4, high-scoring groups experienced six "evaluate" modules, while low-scoring groups only experienced two. Two low-scoring groups did not complete the final "evaluate" module, largely because their problematic time allocation prevented them from completing their discussion within the required time. This indicates that students with high scores paid more attention to summarising and evaluating their results after completing the task, while students with low scores often had neither the time nor consciousness to make a final summary and evaluation. There were also some differences in "analyse the problem," "make a plan," "off-task discussion" and "calculate" modules.

Thus, the mathematical communication characteristics of high-scoring and low-scoring groups can be summarised as follows:

(1) **Students in high-scoring groups more actively responded to other students' views**.
High-scoring groups had far more "respond" elements than low-scoring groups, indicating they were more active in responding to other students' views. They were not only concerned about their questions and opinions, they were also willing to discuss other students'.

(2) **Students with high scores paid more attention to group collaboration**.
The high-scoring groups had significantly more dialogues with "urge" and "evaluate" elements than low-scoring groups. This indicates that high-scoring groups not only paid attention to the group's problem-solving process, they also focused on the group's collaboration and more actively supported the progress of its discussions.

(3) **Students with high scores paid more attention to preserving written results**.
High-scoring groups had many more "artifact" elements and modules than low-scoring groups, indicating they paid more attention to the discussion process and to presenting results in written form. They were more active in showing discussion results on task sheets.

However, there were exceptions. As shown in Table 6.10, Low-scoring Groups 2 and 3 experienced three and four "artifact" modules, respectively. However, after further examination of these two groups' dialogues, we found their “artifact” modules all appeared at the early stage of their discussions. Low-scoring groups began to fill out their task sheets before reaching an agreement and, therefore, usually needed to spend a significant amount of time modifying or rewriting their answer later.

In addition, high-scoring groups were better at presenting the results of their discussions on task sheets, actively adopting various methods to optimise their written presentation of task results. For example, in High-scoring Group 1, Student 2 proposed writing drafts in pens and drawing the finalised design in pencil. Student 3 proposed marking all possible rooms first, then ticking off the rooms they chose to use. There were many other proposals and dialogues like these, as students with high scores generally spent more time presenting their results on task sheets.

(4) **Students with high scores were more likely to reach an agreement in their discussions**.
High-scoring groups had more “negotiate” modules and fewer “quarrel” modules than low-scoring groups. There was no significant difference in the number of “argue” modules between high-scoring and low-scoring groups. This indicates that students in low-scoring groups were willing to question and refute others’ arguments in the mathematical communication process, but had difficulty reaching agreements.

(5) **High-scoring groups experienced a more complete mathematical communication process**.
Among the four high-scoring groups, two experienced all four aspects of the problem-solving process—analysing the problem, making a plan, practicing the plan and summarising and evaluating. Low-scoring groups’ mathematics communication process was less complete than high-scoring groups’, with none of them completing all four steps. For exampe, Low-scoring Group 4 only experienced one part of problem solving, missing the other three.

(III) **There were differences in students’ roles in mathematical communication between the high-scoring and low-scoring groups**.
Each student plays a different role in the collaborative problem-solving process and contributes differently to solving the problem.

(1) **Leaders**

We calculated the percentage of each student’s dialogues to examine the different contributions they made, as presented in Table 6.12.

Table 6.12 shows that the percentage of each student’s dialogues in each group was unbalanced. Students who spoke the most accounted for over 40% of all group dialogues, while students who spoke the least accounted for less than 10%, especially in Low-scoring Group 2, where Student 4 contributed 0% of all dialogues. This phenomenon was common in both high-scoring and low-scoring groups, indicating that dialogue imbalance was not the reason for low scores.

Table 6.12 Percentage of dialogues within the group

Group	Student 1 (%)	Student 2 (%)	Student 3 (%)	Student 4 (%)
High-scoring Group 1	35	29	21	12
High-scoring Group 2	27	35	28	8
High-scoring Group 3	35	41	17	6
High-scoring Group 4	11	40	15	33
Low-scoring Group 1	17	35	32	13
Low-scoring Group 2	23	37	38	0
Low-scoring Group 3	28	18	36	13
Low-scoring Group 4	27	33	3	35

In each group, one or two students had the largest number of dialogues. However, it is not reasonable to determine group leaders based only on the number of dialogues. In High-scoring Group 1, there were eight "assign tasks" and "propose a plan" elements, five of which were put forward by Student 2, for example:

Student 2: No problem, I'll take the carbon pen first. The carbon pen first, and then the pencil.

Student 2: Draw a draft last. Take a pencil first.

In addition, Students 1 and 2 often played the role of topic proposers in the group discussion, proposing 16 of the group's 44 "express views" and "pose a question" elements. Students 1 and 2 generally controlled the direction of the discussion and led the group to solve the problem:

Student 1:60 m^2. An apartment may have a bedroom, a living room.

Student 1: But the question is how to calculate area?

Student 1: What's the living room for?

Student 2: Ok, it's time to write how many square meters each room occupies. The bedroom must be 25 m^2, 25.

Student 2: Agreed, and let's start drawing. We still have to draw the function of the room.

Therefore, this study calculated the percentage of "express views" and "pose a question" elements proposed by each student to decide the leaders of groups. Students who accounted for more than 30% of dialogues in these two elements were considered group leaders (see Table 6.13).

Based on Table 6.13, the eight groups can be roughly classified into three categories: no-leader groups, single-leader groups and double-leader groups. Low-scoring Group 3 was a no-leader group. High-scoring Group 4, Low-scoring Group 1 and Low-scoring Group 2 were single-leader groups. High-scoring Group 1, High-scoring Group 2, High-scoring Group 3 and Low-scoring Group 4 were double-leader groups.

The proportion of double-leader groups was higher in high-scoring groups. Discourse hegemony did not appear in these groups, meaning every student had the opportunity to speak, and the groups were more likely to have better problem-solving results.

Table 6.13 Percentage of "express views" and "pose a question" elements of each student

Group	Student 1	Student 2	Student 3	Student 4
High-scoring Group 1	16	16	10	2
	36%	36%	23%	5%
High-scoring Group 2	10	19	16	5
	20%	38%	32%	10%
High-scoring Group 3	25	26	9	5
	38%	40%	14%	8%
High-scoring Group 4	5	29	9	11
	9%	54%	17%	20%
Low-scoring Group 1	19	55	31	16
	16%	45%	26%	13%
Low-scoring Group 2	12	37	18	0
	18%	55%	27%	0%
Low-scoring Group 3	12	11	10	10
	28%	26%	23%	23%
Low-scoring Group 4	16	7	2	12
	43%	19%	5%	32%

(2) **Supervisors**

Supervisors of groups urged the group discussion's progress and promoted the problem-solving process. The role of supervisors in groups was identified by analysing the number of "urge" elements in students' dialogues, as shown in Table 6.14.

As shown in Table 6.14, there was no obvious supervisor in most groups. Only Student 1 in High-scoring Group 1, Student 2 in High-scoring Group 2 and Student 4 in High-scoring Group 4 showed supervisor characteristics, especially Student 1

Table 6.14 Number of "urge" elements

Group	Student 1	Student 2	Student 3	Student 4
High-scoring Group 1	19	5	2	5
High-scoring Group 2	2	5	3	0
High-scoring Group 3	1	1	0	0
High-scoring Group 4	1	2	1	4
Low-scoring Group 1	0	1	2	1
Low-scoring Group 2	0	1	0	0
Low-scoring Group 3	0	0	2	0
Low-scoring Group 4	0	0	0	2

in Group 1. There were 31 dialogues with "urge" elements in High-scoring Group 1, 19 of which were from Student 1. For example:

Student 1: Calm down, calm down, now we are going to solve this problem, not biology.

Student 1: Quiet, we're not here to talk, we're here to solve the problem, ok?

Student 1: Quiet, don't worry about that. Let's start to draw.

Student 1: Okay, shut up, both of you. Enough is enough, enough is enough ah, enough is enough.

This is an important reason why High-scoring Group 1 could successfully complete its task after three unrelated discussions. The supervisor in the group could shut down unrelated topics quickly and focus the discussion on the problem itself.

It is clear from Table 6.14 that high-scoring groups had a higher proportion of supervisors. Supervisors monitored the groups' problem-solving process and were an important factor in guaranteeing the groups completed their tasks in time. When the direction of the discussion deviated in a no-supervisor group, no student could stop it in time, leaving insufficient time for a complete and thorough discussion.

(3) **Marginalised students**

As can be seen in Table 6.12, marginalised students—such as Student 4 in High-scoring Group 1, Student 4 in High-scoring Group 2, Student 4 in High-scoring Group 3, Student 1 in High-scoring Group 4, Student 4 in Low-scoring Group 1, Student 4 in Low-scoring Group 2, Student 4 in Low-scoring Group 3 and Student 3 in Low-scoring Group 4—provided <15% of the dialogues in both high-scoring and low-scoring groups. Some of these students chose not to participate in the group discussions, while others were excluded and marginalised by other students.

The existence of marginalised students did not affect the discussion results. However, marginalised students will undoubtedly gain much less from the whole collaborative problem-solving process than other students. Teachers should pay more attention to these students by intervening in groups' collaborative problem solving and encouraging them to participate actively in the mathematical communication process.

(4) **Blockers**

In some groups, some students blocked the progress of group discussion. For example, in Low-scoring Group 3, Student 3 kept talking about his home and starting unrelated discussions, such as:

Student 3: Where do you live?

Student 3: How luxurious my home is, that's it.

These students blocked the group problem-solving process and led the discussion off-topic in an unrelated direction. Blockers were identified by calculating the proportion of each student's dialogues that were in the "unrelated discussion" module, as displayed in Table 6.15.

However, the high proportion of some students' dialogues in the "unrelated discussion" module was probably due to the large total number of their dialogues.

Table 6.15 Percentage of students' dialogues in "unrelated discussion" module

Group	Student 1 (%)	Student 2 (%)	Student 3 (%)	Student 4 (%)
High-scoring Group 1	38	22	24	16
High-scoring Group 2	26	38	33	2
High-scoring Group 3	–	–	–	–
High-scoring Group 4	–	–	–	–
Low-scoring Group 1	–	–	–	–
Low-scoring Group 2	38	13	50	0
Low-scoring Group 3	29	21	36	14
Low-scoring Group 4	56	44	0	0

Therefore, to locate blockers in groups more accurately, we subtracted the proportion of students' dialogues from the proportion of students' dialogues in "unrelated discussion" module. The results are shown in Fig. 6.5.

A positive number indicates a higher proportion of student dialogues in the "off-task discussion" module, and a negative number indicates a lower proportion.

Figure 6.5 suggests that the proportion of dialogues in the "off-task discussion" module was significantly lower for some students, including Student 2 in High-scoring Group 1, Student 4 in High-scoring Group 2, Student 2 in Low-scoring Group 2 and Student 4 in Low-scoring Group 4. In particular, Student 2 in Low-scoring Group 2 and Student 4 in Low-scoring Group 4 scored lower than 20%, indicating they paid more attention to problem-related topics and seldom participated in off-task discussions.

However, blockers were also found in these groups, such as Students 1 and 3 in Low-scoring Group 2 and Students 1 and 2 in Low-scoring Group 4. Blockers were mainly found in low-scoring groups, as they blocked the progress of group discussion and made it difficult for the group to focus on the problem.

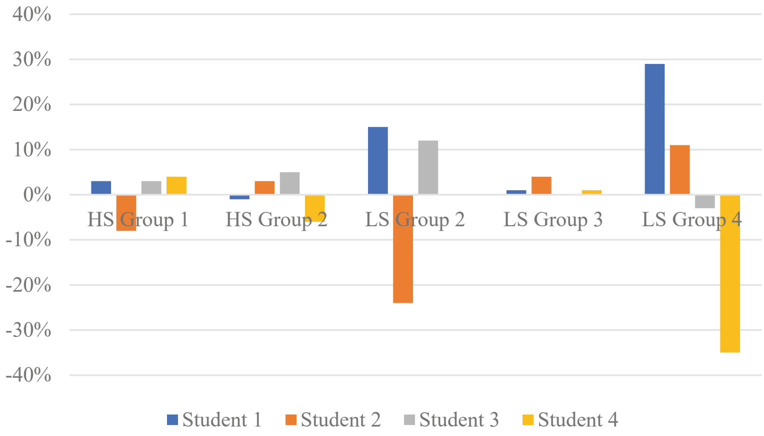

Fig. 6.5 Changes in the percentage of students' dialogues

6.4 Discussion and Conclusion

(I) Discussion

(1) Respond actively to other students' views

Constructivist learning theory believes that knowledge construction occurs in the interaction between learners; thus, it places great emphasis on "conversation" and "collaboration" (Chen & Zhao, 2012). Li (2001) pointed out that from this perspective, the mathematical communication between students should be active, and students should fully participate in the discussion. She further emphasised that understanding of problems is gradually deepened through the process of students' questioning, responding, reflecting and generalising. Thus, a positive response is of great significance in the mathematical communication process. Students promote the problem-solving process through questioning and responding to others' views, discussing the problem in great depth, and solving the problem.

Comparing the "respond" element between high-scoring and low-scoring groups reveals that high-scoring groups responded to others' opinions, questions, doubts and refutations more frequently and positively. In contrast, low-scoring groups were more likely to give no response to others. For example, in Low-scoring Group 2:

Student 3: Or there is no room here.

Student 2: You can draw a balcony and an open-style kitchen.

Student 3: Draw another bedroom.

Student 1: Let's draw a restaurant.

Student 3: It's impossible that he has a wife and children.

Student 2: Why is your door different from the others?

Student 2: Have you calculated how many square meters these rooms can have together? This is 6, it's only 6 m^2, 6 times 10, it's only 6 m, you draw 3 and 2 m.

Student 3: Forget it, I quit, you can draw it.

Student 1: What is this? You know, there's no problem if we expand it.

In this dialogue, Students 1–3 put forward many views and questions (draw a balcony, open-style kitchen, bedroom, etc.). However, these views and questions did not receive positive feedback, making the discussion topic shift too fast. The discussion of the room's function had not yet ended when the topic shifted to the room's size.

Therefore, students should pay attention to others' views and questions in the discussion process and respond actively to ensure the group discussion remains focused on one topic. In the intervention of students' mathematical communication, teachers should not only pay attention to the results of problem solving, but also to students' discussions and remind them to respond to others' opinions.

(2) **Communication consistency**

Coding students' mathematical communication revealed that "negotiate," "argue," and "quarrel" were the basic communication modules of collaborative problem-solving process. They were also the three critical communication modules in promoting the problem-solving process. Therefore, students should pay full attention to these three communication modules.

There were obvious differences in "negotiate" and "quarrel" modules between high-scoring and low-scoring groups. Whether a group can reach an agreement during its discussion is particularly important in the problem-solving process.

Previous research found that after making some aspects of tasks public, there would more likely be a shift to cognitive authority (Langer-Osuna et al., 2020). However, in the current research, when discussing a given issue, students commonly had different opinions and questioned each other. This cognitive conflict is constructive in open-ended collaborative problem solving. However, if students ignored this conflict and started to discuss other issues, they would need to discuss this topic again later. This will waste time and affect the atmosphere of group communication. Therefore, students should pay attention to communication consistency.

In the intervention process, teachers should pay full attention to conflicts among students and encourage students to resolve them through group discussion. However, if teachers find that students have failed to resolve the conflicts despite sufficient discussion, they should intervene (Dong et al., 2013).

(3) **Presentation of results**

The "artifact" module is often overlooked in the collaborative problem-solving process. However, the differences between high-scoring and low-scoring groups showed the "artifact" module's indispensable role in the problem-solving process. After completing their discussion, students should present the discussion results on the task sheets in detail. Especially in open-ended collaborative problem solving, teachers and students should pay more attention to the answer's completeness. In the task given by this research, students could also do extra exploration in addition to the basic requirements, such as providing a variety of house designs, describing the situation in detail and so on.

However, writing down answers at the beginning of group discussions without reaching an agreement is inadvisable, as students will often need to make numerous modifications to these answers during the discussion process, thus wasting time.

(4) **Integrity of the mathematical communication process**

Polya (1945) divided the problem-solving process into four stages: "Understand the problem," "Devise a plan," "Carry out the plan" and "Look back." He stressed that each stage is important and cannot be replaced.

The coding results for the students' mathematical communication models suggest that students and teachers did not do enough to "analyse the problem," "make a plan" and "evaluate" parts of the mathematical communication process. Teachers should let students go through the complete collaborative problem-solving process to ensure the completeness of the mathematical communication process. Students should start with practical problems and then summarise the mathematical problems through analysis. Teachers should not do this for the students but may give them some hints. In addition, teachers should remind students to make a plan to solve the problem, rather than directly starting to discuss some of its details. Teachers could also organise students to report or adopt other means to summarise and evaluate the problem-solving process and results. Students could also independently carry out the summary and evaluation work in the mathematical communication process, which is an important module for helping students find mistakes and adjust the problem-solving direction. Students get the most from problem solving and develop their mathematical communication ability best when they undergo the whole mathematical communication process.

(5) **Students' participation in mathematical communication**

Through observation, Wang (2002a, 2002b) found that imbalance is very common in students' participation in collaborative learning. In this case, students with higher participation levels tended to gain more than those with lower participation levels (Wang, 2002a, 2002b). This study also found a similar phenomenon by analysing students' contributions to dialogues. In both high-scoring and low-scoring groups, marginalised students contributed to only a small number of dialogues. While this did not significantly impact the groups' problem-solving results, the marginalised students will undoubtedly gain less from the collaborative problem-solving process than other students. Therefore, teachers should pay special attention to these students during an intervention, analyse why they are marginalised and encourage them to participate actively in group discussions.

Our review of group discussion content revealed that some teachers paid attention to marginalised students in some groups. For example, in High-scoring Group 4, the teacher tried to let Students 1 and 3 participate in the discussion between Students 2 and 4, as shown below:

Teacher: Did you split up again?

Student 2: Based on Feng Shui, the house must be square.

Teacher: I want four of you to discuss, you are now stuck at two, right?

Student 2: And you can't slam the door.

Student 2: What do you think if we put a toilet next to the bedroom?

Student 4: If theirs doesn't work, try ours.

Student 2: Let's combine the bathroom with the bedroom, and the dining room, all in a small space.

However, it can be seen from the above dialogue that students did not listen to the teacher's arrangement. Students 2 and 4 continued their discussion, and Students 1 and 3 were not involved and eventually asked their peers, "Are you unhappy to have stupid teammates like us?" In the after-class interview, the teacher revealed that the four students in this group were not partners in regular math classes and seldom discussed issues; thus, in this lesson, these students might not have known each other well and may even have hated each other. This is an important reason why this group had difficulty conducting mathematical communication. This situation suggests that a long time is needed to form groups for collaborative mathematics problem solving.

Although some studies and theories support that heterogeneous groups perform better than homogeneous groups in collaborative learning (Bowers et al., 2000; Wang, 2002a, 2002b), Wang and Chen (2008)'s experimental study found that, although heterogeneous groups are more suitable for solving open-ended collaborative problems, the differences between heterogeneous groups will also lead to communication barriers that hinder the problem-solving process. It has not yet been agreed upon whether heterogeneous or homogeneous groups are more conducive to students' collaborative problem solving.

Therefore, while teachers can consider the principle of "homogeneity between groups and heterogeneity within groups" when grouping students, they should also fully consider students' personality characteristics. Teachers should not assign students with conflicts to the same group and should not regroup existing regular in-class discussion groups without considering students' opinions.

(6) **Students' roles**

There were significant differences in students' roles in the problem-solving process between high-scoring groups and low-scoring groups, especially for supervisors. Although teachers play a supervisory role in the collaborative problem-solving process, it is difficult for teachers to pay attention to every group's discussion or monitor their problem-solving process. There are usually five to eight groups in every mathematics class in China. Teachers could set up supervisors within groups who would be mainly responsible for the following group tasks: (1) supervising the problem-solving process and reminding the group it is behind schedule; (2) stopping group members from discussing topics unrelated to the problem and maintaining the group's discussion discipline; (3) encouraging marginalised students to participate more in group discussion; and (4) maintaining a good group discussion atmosphere, identifying problems that cannot be solved through group discussion and seeking help from teachers when groups have conflicts and cannot reach an agreement.

(II) **Conclusion**

This study has constructed a set of elements of students' mathematical communication in collaborative problem solving and established a three-level coding framework. By coding the mathematical communication of eight selected groups, we have summarised secondary school students' mathematical communication characteristics.

Students promoted the problem-solving process through "negotiate," "argue" and "quarrel" modules and presented their discussion results on task sheets in "artifact" modules. While students were not afraid to express their opinions and refute other students' opinions, there were various problems in the mathematical communication process, including having difficulty reaching agreement, an incomplete mathematical communication process, etc.

We compared the differences in high-scoring and low-scoring groups' mathematical communication characteristics from two aspects—the number of communication elements and students' roles in mathematical communication. The study results showed that high-scoring groups had advantages in various aspects. High-scoring groups were more active in responding others' views, reached agreement more frequently, were better at presenting results on task sheets, followed a more complete collaborative problem-solving process and paid more attention to the progress of group discussion. There also existed differences in students' roles between high-scoring and low-scoring groups. For example, supervisors were more common in high-scoring groups, and blockers were more common in low-scoring groups.

(III) **Inspiration**

Since the new curriculum reform, the "student-centred" principle has entered Chinese mathematics classes, and students' participation in mathematics classes is becoming wider and deeper (Zhao et al., 2019), while the international situation is similar (Wang et al., 2013). Collaboration is receiving increasing attention in mathematical problem solving research (Liljedahl & Cai, 2021), and collaborative problem solving has also attracted extensive attention from researchers and teachers. Our analysis of mathematical communication in collaborative problem solving revealed that students are not adaptable to this teaching method. This can mainly be seen in the incomplete process of students' collaborative problem solving; students do not know what to do when given an open-ended collaborative problem-solving task.

In addition, students also frequently asked teachers for help during the collaborative problem-solving process, asking questions (e.g., "Does the balcony count as a room?" "Does the toilet count as a room?" "Does the corridor count in the total area of the room?") to which there was no specific answer. This shows that students were more accustomed to traditional teacher-led teaching than student-led teaching based on problem solving. Even though student-centred instruction has been widely proven more effective than teacher-centred instruction (Granger et al., 2012), the teacher-centred instruction has enjoyed prominence for decades in mathematics education (Stephan, 2020) which possibly set barriers for students' collaboration on the open-ended tasks. Students tended to ask teachers for help rather than discuss issues with their peers. Restricted by classroom time limits and heavy teaching loads, it is difficult for teachers to incorporate problem solving into their daily classroom teaching. Consequently, students have long been unable to develop their collaborative problem-solving and mathematical communication abilities.

Teachers could explore integrating problem solving into their daily teaching further. For example, in some parts of a lesson (e.g., introducing new lessons or exercises), teachers could let students discuss or debate issues to improve their collaborative problem-solving and mathematical communication skills.

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Chapter 7
A Study of Conflict Discourse in Mathematical Collaborative Problem Solving

Jingbo Zhao

7.1 Research Background

Conflicts exist in all aspects of society. The process of communication between people is a collision of ideas and opinions. In the state of collaborative learning, the exchange and discussion of issues between group members is always accompanied by conflict and negotiation (Supena et al., 2021). Lewis Coser, the representative of social conflict theory in the 1960s, pointed out that conflicts contribute to identification and unity within a group. Differences in thinking and methods lead to conflicts. However, the ultimate point of group cooperation is to solve problems together. In the state of cooperation, verbal conflict is the communication process that can help group members achieve unity of thought and approach, which is conducive to group harmony and solidarity.

In the process of collaborative learning, the final objective is to arrive a consensus. During the process, conflict is a prelude to collaboration, and any type of conflict ultimately leads to collaboration (Rahim, 1989).

The earliest research on conflict discourse can be found in Brennis, Lein, and Boggs' late-1970s study of children's controversial discourse and its structural forms. Conflict discourse is a term first proposed by Grimshaw (1990) in his book (Conflict talk: Sociolinguistic investigation in Conversations). There are many similar terms in English for conflicting language in communication, including conflict episodes (Eisenberg & Garvey, 1981), aggravated disagreement (Goodwin, 1983), conflict talk (Grimshaw, 1990), disorderly discourse (Briggs, 1996), verbal opposition (Kakava, 2002), etc. There are also different discourse terms in Chinese, such as conflicting discourses, contrastive discourses, oppositional discourses, etc. In addition, Greatbatch, Hutchby, Kakava, Kotthoff, Myers, and many other scholars, aided by the

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concepts and tools of conversation analysis, have offered many discussions and analyses of discourse interruptions, discourse overlap, silence, confrontational discourse, topic switching, and divergent discourse markings, greatly enriching the content and significance of conflict discourse research.

Domestic scholar Yingling Zhao's (2004) work, *Conflict Discourse Analysis*, was the prelude to studying conflict discourse in China. In subsequent research, various scholars have studied conflict discourse from different perspectives. Libin Ruan (2018) pointed out that the current research on conflict discourse in China can be divided into five categories: the application of pragmatic theory to conflict discourse research, the study of conflict discourse structure, the study of conflict discourse communication strategies, the study of practical effects of conflict discourse, and research into the generation mechanism of conflict discourse. For example, Yongping Ran (2010) gave an overview of the study of conflict discourse within the scope of pragmatics, summarising the definition of conflict discourse and its positive and negative characteristics and exploring the scope and main topics of conflict discourse. Yingling Zhao (2004) used structural analysis to explore the structural features of conflict discourse, while Liying Zhou (2009) divided conflicting discourse into command-refuse, questioning-resist, and challenge-confrontation categories, pointing out that speech-space contradiction would lead to conflicting discourse. Ping Liu (2010), Shuangping Gong (2011), and other scholars have analysed the communicative strategies of conflict discourse from different perspectives, achieving fruitful research results and showing the comprehensiveness of China's research on conflict discourse and the diversity of its research angles.

Students will have disagreements, arguments, refutations, quarrels, and other verbal behaviours in the collaborative problem-solving process, leading to the emergence of conflict. In the process of conflicts, a consensus is finally reached after active negotiations. The collaborative problem-solving process is "conflict-negotiation-conflict-...-problems are solved," and conflict and negotiation are the main content of discourse analysis in collaborative problem solving.

Conflict in the collaborative state helps to establish the identity of the group. Group collaboration is a process in which all members participate, with each having a different position and status. Harré and Langenhove (1998) believed that participants would actively seek or be assigned a certain role. Once their role is fixed, participants may tacitly approve, argue, or overthrow the rules, so discourse conflicts can also promote the establishment of new norms within the group.

There is more and more attention to the complexity of collaborative problem-solving research (Langer-Osuna et al., 2020; Salminen-Saari et al., 2021). As a way of classroom learning, collaborative problem solving has strong social attributes. Group members will inevitably have conflicts in the interactive, collaborative problem-solving process, making exploring conflicts in the process of collaborative problem solving from the perspective of discourse worthwhile.

In collaborative problem solving, various disputes will arise in interaction due to group members' different cultural backgrounds and individual differences. Therefore, the research on conflict discourse in collaborative problem solving is based on this interactive dilemma.

Based on the analysis of the existing studies, relevant analysis theory is used to deeply study the interactive process of conflict in mathematics collaborative problem solving. The core of the research is several aspects of qualitative analysis, including conflict discourse formation contexts in collaborative problem solving, the type and structure of conflict discourse, the conflict conversation differences in different performance groups of conflict language characteristics, the individual factors that produce conflict discourse, etc.

This research aims to provide a comprehensive analysis of the types and structures of conflict discourses and the characteristics of conflicting language in the process of mathematics collaborative problem solving based on mathematical problems, and explores individual factors affecting conflict discourse through comparative analysis of different performance groups to understand group members' participation better.

7.1.1 Method

This section first determines that the research object is a four-person group activity, and that the theme task of the group is "Xiao Ming's Apartment." After the classroom group activities were recorded, invalid samples were removed, and the video samples were transcribed to allow deep processing of the text data. First, the conflict events were divided, and then the Python programme was used to process the data statistically. Next, high- and low-performance group objects were selected based on the quality of group results. Finally, the individual conflict styles of group members were analysed.

7.1.2 Data Collection

7.1.2.1 Participant

This study mainly selected the four-person group's collaborative problem-solving classroom videos. In a group of four, members' participation in the collaborative problem-solving process is relatively high, and conflict discourse phenomenon is relatively prominent. This study selected groups with no or little teacher intervention as the research objects as the verbal interaction between group members is free and real in this state.

Bruxelles and Kerbrat-Orecchioni (2004) pointed out that the notable feature of multi-person conversations is "alliance," meaning people with the same opinions will unite to refute another person's point of view, which is more likely to cause conflicts. The theme task of the four-person group was "Xiao Ming's Apartment" (See appendix for details).

7.1.2.2 Data Collection Techniques and Analysis

Purposeful sampling is applied for the current research. The participants from eight classes in LH Junior Middle School and YC Junior Middle School in the City of B. After removing videos with blurry images, unclear sounds, and groups of more than four persons, we took 35 four-student groups as research samples. A video sample was deemed suitable for this research if it was problem-oriented, purposeful, and had clear steps that facilitated the research's smooth progress. All participants were working on the same task without any guidance by teachers.

7.1.2.3 Instruments

To fully reveal the conflict and negotiation process in collaborative mathematical problem solving, this research conducted in-depth processing of the collected text data, constructed conflict events, and used computer coding technology to perform word segmentation and produce language statistics.

(1) The division system of conflict events

When categorising conflict events, we started with task conflict and relationship conflict. Three aspects were mainly considered. First, if the group members had conflicting views during the discussion; second, if the dialogue between group members was confrontational; and third, if the group members had strong emotional expressions when conflicts occurred. Conflict events mainly have three links: the conflict's trigger, the conflict's intensification, and the conflict's end. When dividing conflict events, we should pay attention to whether these three links are complete. In particular, the forms conflict endings take in the third link are relatively diverse, including stalemate, compromise, concession, successful negotiation, and transfer. In the high- and low-performance groups, different ending methods had different effects, and the quality of the final group plan was different (Fig. 7.1 and Table 7.1).

(2) The linguistic feature system of conflict discourse

This chapter mainly combined the retrieval function from the Antconc corpus retrieval software and the retrieval and screening function in Excel to analyse the text data and collect and sort the conflicting words' language features. In Python, Jieba is used for word segmentation in corpus, and Antconc software is used for word frequency statistics. The Python programme is as follows: (Fig. 7.1)

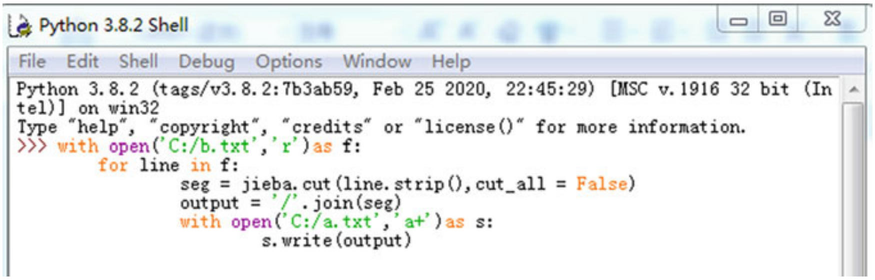

Fig. 7.1 Statistical processing of data using the Python programme

(3) Selection of Research sample for Different Performance Groups

Collaborative problem solving is an important learning method. The quality of a collaboration's results is a significant indicator of its learning effect. The collaborative group finally reaches an agreement through good communication and collaboration to complete the group task. The final result is a solution, or the product is displayed and then evaluated by the teacher.

The evaluation of collaborative mathematics problem solving in this research mainly refers to the evaluation criteria established by Dr. Binbo Sun in his doctoral dissertation. In establishing this standard, we participated in revising and demonstrating the framework to better understand this evaluation standard's quality, reliability, and validity. This evaluation standard views completion and correctness as a two-dimensional framework, as shown in the following table (Table 7.2):

Based on the collaborative performance evaluation criteria and project data, four graduate students were divided into two groups to evaluate the quality of all 48 collaborative task list data groups. Finally, the graduates and I discussed the disputed group to determine its collaborative performance. On this basis, we selected the high-performance group (excellent) and low-performance group (general) as research data for comparative analysis.

(4) Personal conflict style system in group members

We encoded the five conflict styles (competitive, collaborative, compromise, avoidance, and compliant) in Thomas's (1974) model and the gender of the group members to explore the individual impact factors of collaborative performance (Table 7.3).

7.2 Results

This research used 35 groups as a research sample to study the types and structures of conflict discourses, language features, and individual factors affecting students' collaborative problem resolution in conflict resolution in collaborative mathematical problem solving. The following conclusions were obtained.

Table 7.1 Conflict events and codes

Conflict event process	Student	Conversation	Code
Trigger of conflict	S2	Which number multiply by which number equals 60?	M1
	S3	Noted	M2
	S1	Think about it first. Think width first. Think about the possibilities of length and width first, think about that!	M3
Intensification of conflict	S2	2, 3	M4
	S1	1 multiply by 60 is impossible. 2 m wide is impossible. The width must be at least 5 m or 10 m, right? How long is the length if the width of 5 m, it is 10, and the length is 12	M5
	S3	There are five rooms	M6
	S1	5 rooms, yes, it must be calculated like this first. If he wants to..., the length is calculated based on the integer solution, then if it is 10, if the width is 6, the length is 10, right? If the width is 8, is it divisible?	M7
	S2	Yes, it is	M8
	S1	If the width is 8, 8/50, 60, 60/8	M9
	S4	Don't you think you design it too squarely?	M10
	S1	15.5	M11
End of conflict	S2	In fact, the square is better. let's use 10, 10 and 6 is better, the smaller difference between two numbers is better	M12
	S1	Yes, the bigger difference between two numbers, the bigger you think the room is	M13
	S2	True!	M14
	S1	But there can't be too big difference, a thin strip is not good	M15
	S2	Let's try 6 and 10	M16
	S4	6 and 10, 8 and 7.5 is the best	M17
	S2	The width is 6.	M18
	S1	Ok, you draw 6 and 10, then you two draw 6 and 10, and we draw 7 and 15	M19

7.2.1 Analysis of Conflict Discourse Types in Mathematical Collaborative Problem Solving

The type of Group Conflict discourse directly affected group collaboration performance. Analysing collaborative group performance based on group conflict has always been a research hot spot for scholars at home and abroad. Determining the type

Table 7.2 Evaluation form for the quality of collaborative mathematics problem-solving results for junior high school students

Grade	Score	Characteristic	Example
Excellent	3	Completed all collaborative problem-solving task requirements and the problem-solving results were all correct (draw five room icons, mark room functions, and mark room dimensions)	As shown in Fig. 7.2, the group completed all the collaborative problem-solving tasks of "Xiao Ming's Apartment." Draw possible functions of Xiao Ming's apartment and mark the function and size (length and width) of each room. When the group met the learning task requirements of the apartment with 5 rooms and a total area of 60 m^2, and the design was reasonable, the group were all correct. Comprehensive evaluation is excellent
Good	2	Half or more of the learning task requirements were completed, and the problem-solving results were half or more correct	As shown in Fig. 7.3, the group completed the two requirements of "drawing a possible diagram of Xiao Ming's apartment and marking the possible functions of each room," but the group did not mark each room. The size, namely length and width, the group completed most of the learning tasks of collaborative problem solving. Therefore, most of the team's problem-solving results were correct, and the overall evaluation was good
Fair	1	Completed less than half of the learning task requirements, or completed more than half of the learning task requirements, but the problem-solving results were less than half correct	As shown in Fig. 7.4, the group only completed the 1 requirement of "drawing a possible diagram of Xiao Ming's apartment." Neither the function of each room was marked, nor the dimensions (length and width) of each room were marked. Therefore, the group completed a small part of the learning task of collaborative problem solving. Therefore, the is a small number of correct, and the comprehensive evaluation is fair
Poor	0	The activities are not carried out following the requirements of the learning task, or the problem-solving results are incorrect	There is no problem-solving result, or the problem-solving result is poor

of group conflict is a continuous development and improvement process, and there are different classifications of conflict types. Initially, Guetzkow divided group conflicts into task conflict and relationship conflict, while Jehn established and defined a three-dimensional model involving group task conflict, process conflict, and relationship conflict. Process conflict is a conflict caused by inconsistent task resolution methods or procedures. However, Passos and Caetano and others believed that task conflicts include process conflict. As such, the two dimensions of task conflict and relationship conflict proposed by Jehn are generally used, and have different effects on group performance.

Classifying group conflicts in Chinese culture can effectively stimulate cognitive conflicts and control emotional conflicts when constructing collaborative groups. The

Table 7.3 Coding table of conflict-handling style in student collaborative problem solving

Type of conflict	Code	Description
Competitive	1	Competitive individuals chase their attention at the cost of others. This is a mode of power positioning. People use all seemingly appropriate means to be advantageous: the ability to argue, titles, or economic sanctions. Competitiveness may mean defending their power, defending what they believe is correct, or simply trying to win
Collaborative	2	Contrary to avoidance. The collaborative type involves an effort that attempts to cooperate with others to find a solution that fully meets the concerns of both parties. This means studying the problem in-depth to identify the potential concerns of the two and finding ways to satisfy the concerns of both parties. The collaborative between two people may take the form of exploring differences to learn from each other's insights and resolve a situation unanimously. Otherwise, it will cause them to compete for resources or conflict; forms also include trying to find creative solutions to interpersonal problems
Compromise	3	The purpose is to find a mutually beneficial, mutually acceptable solution that can partially satisfy both parties. It sits between the competitive and compliant types. The compromise type gives up more than the competition type but less than the compliant type. Similarly, it deals with problems more directly than the avoidance type, but not as deep as the collaborative type. Compromise type may mean compromise, compromise with each other or find a quick, middle ground
Avoidance	4	Individuals do not directly pursue the attention of themselves or others. They don't focus on resolving conflicts. The avoidance type may take the form of avoiding the problem diplomatically, postponing the matter until a more suitable time, or simply withdrawing from a threatening situation
Compliant	5	Contrast with competitive type. When conforming to others, individuals ignore their interests in order to satisfy the concerns of others. This model contains elements of self-sacrifice. Conformance may take the form of selfless generosity or charitable acts, and the ability to obey others' orders or succumb to others' opinions when they are reluctant

same two-dimensional classification of group conflicts is also applicable to traditional Chinese cultural backgrounds. The group collaborative problem-solving process is also a group work process. This study mainly uses Jehn's classification of group conflicts to analyse the types and characteristics of task conflicts and relationship conflicts in the students' collaborative problem-solving environment and lay the foundation for subsequent research.

7.2.1.1 Task Conflict Based on Mathematical Problems

The group discuss a task, with members disagreeing about its issues, ideas, and judgements; these differences of opinion cause task conflict (Jehn, 1995). Cosier and Dalton (1990) believed that the task conflict encourages group members to make their voices heard when formulating a plan. Different voices help ensure members have a deep and complete understanding of the work task and may increase the plan's options. Amason (1996) showed that members who have experienced group task conflict are more inclined to respect, cherish, and fulfil group discussion results. Task conflict is one of the main conflicts in collaborative group conflict and significantly impacts the effectiveness of collaboration.

Mathematical problem solving is the main task of collaborative problem solving, and group collaboration is the learning method. The process of group members' reviewing problems, thinking about problem-solving methods, forming problem-solving ideas, and finally completing the task is a discussion process. Task conflicts based on different understandings of tasks are more prominent, such as (Table 7.4):

The dialogue above involved group members discussing how to determine the length and width of a 60 m^2 room. In this dialogue, task conflict was obvious. When group member T1 asked, "How much is the length and width of the room?" group

Table 7.4 Example of task conflict

Task	Student	Conversation
Discuss the length and width of the room with an area of 60 m^2	S1	Think about it first. Think width first. Think about the possibilities of length and width first, think about that!
	S2	2 and 3
	S1	1 multiply by 60 is impossible. 2 m wide is impossible. The width must be at least 5 m or 10 m, right? How long is the length if the width of 5 m, it is 10, and the length is 12
	S3	There are five rooms
	S1	5 rooms, yes, it must be calculated like this first. If the length is calculated based on the integer solution, if the width is 6, the length is 10, right? If the width is 8, is it divisible?
	S2	Yes, it is
	S1	If the width is 8, 8/50, 60, 60/8
	S4	Don't you think you design it too squarely?
	S1	15.5
	....	
	S2	Let's try 6 and 10
	S4	6 and 10, 8 and 7.5 it's the best
	S2	The width is 6
	S1	Perfect, You draw 6 and 10, then you two draw 6 and 10, and we draw 7 and 15
	S4	I think the figure is so square
	S2	Yes, let's see who is better, and then design our own

members T2, T3, and T4 put forward different ideas based on their understanding. Although they finally reached a consensus after negotiation ("You two draw 6 and 10, and we draw 7 and 15"), there were contradictions and conflicts in the task development in this dialogue.

Task conflicts in collaborative problem solving are mainly based on mathematical task solving, which has unique language characteristics. Task conflicts can be divided

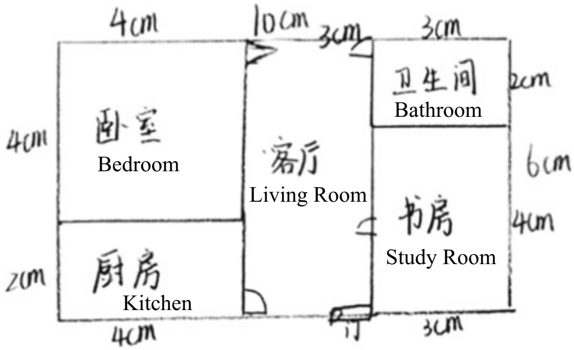

Fig. 7.2 Examples of quality achievements in collaborative mathematics problem solving (excellent)

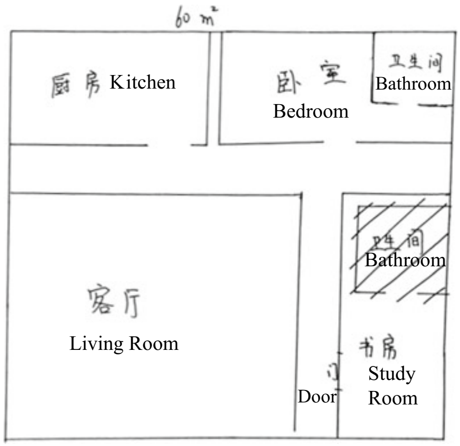

Fig. 7.3 Examples of quality collaborative results mathematics problem solving (good)

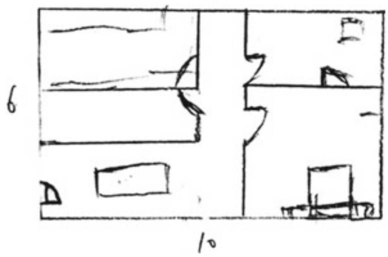

Fig. 7.4 Examples of quality collaborative results mathematics problem solving (general)

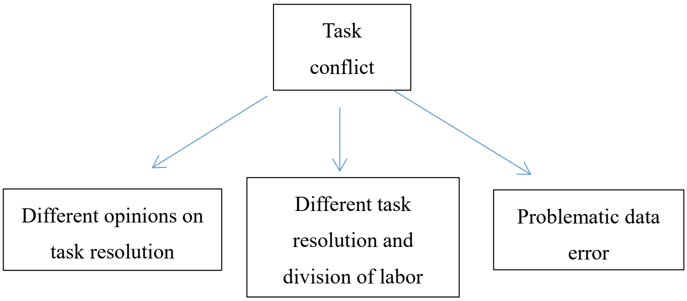

Fig. 7.5 Type of task conflict

into three types: different opinions, a different division of tasks, and conflicts caused by data errors (Fig. 7.5).

7.2.1.2 Relationship Conflict

Relationship conflicts are contradictions between people, characterised by dislike or personal attacks and accompanied by negative emotions, such as frustration, anger, annoyance, etc. The conflict in the problem solving of group collaboration is caused by the disharmony of the interpersonal relationship of the group members, and it will be accompanied by other emotional characteristics when the conflict occurs.

The junior high school stage is a period where individual physical and mental contradictions are concentrated and prominent. It is also a critical period for individual psychological and academic development. A large number of empirical studies in China and abroad have proved that the imbalance and tension of interpersonal relationships can have a destructive effect on group function, resource allocation, or trust relationship, which will cause anxiety among group members and affect their cognitive process of the problem, and then affect team performance, and eventually reduce the work efficiency of the organisation.

Interpersonal conflicts are divided into explicit conflicts and implicit conflicts. Explicit conflicts are manifested as more serious disputes and differences, with obvious characteristics of interpersonal contradictions, while implicit conflicts are manifested as mild opposition and non-collaborative. The relationship conflicts of students in the process of solving collaborative problems are more obvious. The following cases are a set of more typical explicit conflicts in relationship conflicts.

In Table 7.5, students S1 and S2 had a fierce conflicting dialogue. These conflicting dialogues were not aimed at the problem. S1 adopted the words "Don't speak," which was an expression of dissatisfaction with S2. S2 responded. "What is wrong? Otherwise I will tear the paper." The words between the two seem to have exceeded normal mathematical problem communication scope, so the two are in a relationship conflict.

Table 7.5 Example for relationship conflict

Student	Conversation	Conflict process
S2	The closestool in his house is installed in the toilet	
S1	**Don't speak, ok?**	Initiation of relationship conflict
S2	**What's wrong? Otherwise I will tear this paper**	Conflict process
S1	**Tear it off. Take another one**	
....		
S2	His closestool is installed in the rice bowl and he defecates in the bedroom	
S2	Settle on the bed	
S1	**Don't speak**	

In the process of relationship conflicts, especially explicit conflicts, have their unique language characteristics: intrigue, teasing, sarcasm, quarrel, complaining conflict, fighting, complaining, obstruction, personal, personality, pressure issues, relationship, society, trouble, etc.

In addition to common explicit conflicts, invisible conflicts in relationship conflicts cannot be ignored. Due to the hidden nature of hidden conflicts may not be directly observed, and its potential harm may be more significant. The following is an invisible conflict.

"S2 I think they watch too many thrillers.

....

S4 Same as looking for an ancient tomb. "

When the group members discussed "the issue of the orientation of Xiao ming's apartment," different opinions arose, especially members S2 said "I think they watch too many thrillers," and S4 said "Same as looking for an ancient tomb." These two sentences do not seem to be overly related to the issue under discussion, but in fact they are S2 and S4's rebuttal to the solution of the room designed by S1 and S3. It may not seem to be related to the discussion, but in fact the communicating parties will implicitly convey their real thoughts and opinions to each other, thus non-publicly threatening the other party's face.

In short, the interpersonal conflict in collaborative learning, whether explicit or implicit, greatly influences collaborative learning. It will affect the realisation of collaborative learning goals and the growth of group collective wisdom. It will also reduce students' trust in collaborative learning and affect healthy personality development in the long run.

7.2.2 *An Analysis of the Overall Structure of Conflict Discourse in the Problem Solving of Group Collaborative*

As a kind of speech analysis, conflict discourse analysis is composed of speech acts and is accompanied by verbal interaction processes such as arguments, quarrels, and objections in conflict. The formation process of conflict discourse generally includes the initial stage, the conflict stage, and the end stage of the conflict. The ultimate goal of group collaborative is to reach a consensus through discussion and analysis of members. Therefore, there must be a process of negotiation if there is conflict. Conflict and negotiation are alternate processes in the whole process. You can not only consider the conflicting words but also pay attention to the negotiation speech act. The following is the discourse analysis of conflict and negotiation on the collected corpus, mainly analysing the three conflict processes from the three aspects of the problem task and the final solution reached through negotiation.

7.2.2.1 Conflict and Negotiation Caused When Discussing the Area of the Whole Room

The group's first task to solve the whole problem is to have a whole concept of the total area of 60 m^2 of the room. How big are 60 m^2? The group members express their opinions through their own life experiences. At this stage, for the grasp of the overall area, the group members unified their opinions relatively quickly, and did not form a strong verbal conflict, after several rounds of verbal interaction quickly reached a consensus of opinion, so there is no step-by-step process of exploring conflict and negotiation.

(1) The beginning of the conflict

S2 Which number multiply by which number equals 60?

S3 Noted.

S1 Think about it first. Think width first. Think about the possibilities of length and width first, think about that!

In the initial stage of discussing the total area, S1 raised a question that would cause everyone to conflict: "Think about how many possibilities are there for length and width?" This provides the starting discourse for the subsequent conflicts. This is a question type of questioning, and the answer is generated by the questioning.

(2) Conflict stage

When there is a problem, there are disputes among group members. This kind of dispute is the main stage of the conflict. With this argument process, when one student puts forward a point of view, the group members refute from their perspective and refute others. Expressing one's own opinions is also a manifestation of conflicting discourse.

S2 2, 3

S1 1 multiply by 60 is impossible. 2 m wide is impossible. The width must be at least 5 m or 10 m, right? How long is the length if the width of 5 m, it is 10, and the length is 12.

S3 There are five rooms.

S1 5 rooms, yes, it must be calculated like this first. If the length is calculated based on the integer solution, if the width is 6, the length is 10, right? If the width is 8, is it divisible?

S2 Yes, it is.

S1 If the width is 8, 8/50, 60, 60/8.

S4 Don't you think you design it too squarely.

S1 15.5.

It can be seen from the above process that the four group members participated in the design of the overall area. They had a heated debate on the length and width, refuted other people's suggestions, and expressed their opinions.

(3) Closing stage—the result of the negotiation

After several rounds of arguments, one party's point of view was adopted by the other students, and finally accepted or obeyed the other's point of view, finally reached a consensus, and the conflicting discourse ended. This process reflects the post-conflict negotiation process.

S2 In fact, the square is better. let's use the 10, 10 and 6 is better, the smaller difference between two numbers is better.

S1 Yes, the bigger difference between two numbers, the bigger you think the room is.

S2 true.

S1 But there can't be too big difference, a thin strip is not good.

S2 Let's try 6 and 10.

S4 6 and 10, 8 and 7.5 is the best.

S2 The width is 6.

S1 OK, you draw 6 and 10, then you two draw 6 and 10, and we draw 7 and 15.

The above verbal interaction process well reflects the post-conflict negotiation process. When S2 put forward the "10 and 6" proposal, both S1 and S4 agreed and finally reached a consensus. The entire conflict process ended with a final consensus.

7.2.2.2 Conflict and Negotiation Caused When Discussing the Layout and Function of the Room

The layout and function of the room are the focus of the group's conflict. During the whole discussion, the group members conducted different rounds of debates around the functional design of each room and finally formed a solution.

(1) The beginning of the conflict

S2 It's the size of each room, does that door count as size? The door is still count.

S1 Don't count the door, you count the door facing North East, the head of the bed faces north, right? Then the bed is facing south.

S2 Walls don't need to be measured anymore.

………………………………………

The group members' discussion on the layout of each room started with a debate about doors and walls. Are doors and walls counted in the total area? Does it count as the room layout? Lay the groundwork for the following controversy.

(2) Conflict stage

The collaborative group has many discourse conflicts in arguing about the layout and function of the room, involving disputes about doors, furniture, etc., that have little to do with problem resolution, as well as conflicts in the kitchen, bedroom, bathroom, etc.

S4 Wouldn't it be weird if I put the kitchen next to the toilet?

S2 Not weird, It's not your own home anyway.

S1 No, the kitchen and the toilet can't be side by side, the kitchen should be next to the bedroom, and the living room should be next to the kitchen.

In the dialogue process above, S4 used a question to describe whether the kitchen and the toilet were weird if side by side? S2 stated that the attitude was not weird, while S1 believed that the kitchen and the toilet could not be together, which led to a conflict of words between the two parties.

(3) Closing stage—the result of the negotiation

After a series of conflicts and constant negotiation, a consensus was finally reached. The following dialogue process:

S2 Draw it, draw quickly.

S1 Can't be so quickly. Follow whose drawing?

S2 I haven't finished mine yet, based on your drawing.

S1 Follow my drawing, I am going to draw two toilets.

S2 Don't draw this locker, just five rooms.

S1 I Know.

S4 You don't need to mark the lockers

S1 One, two, three, I can only mark a toilet and then mark a kitchen, the living room is in the middle, no need to draw, the living room does not account for the number of rooms, five.

S4 the living room must be occupied too.

S1 OK, then the living room occupy, the living room occupy.

S3 that's enough.

S1 Enough for the living room, right?

After several rounds of conflict and negotiation, the room layout designed by S1 was finally adopted.

7.2.2.3 Discuss the Conflict Caused by the Size of the Room

This group designed the room by dividing the room in the overall rectangular area. After the room layout is determined, the size is a matter of measurement. Therefore, there are not many conflicting words in this part, but there have been disputes about the size of the room when determining the layout, The following conversation process:

S2 If you draw the bedroom like this, it's too small. Even if you want to paint the bathroom, you can't paint it.

S3 Our bedroom must be small.

S2 You see how small my bedroom is.

S3 Yours is too small.

Finalise the size of the room:

S1 2.1, this is correct, 2.1, 2.1, 1, 2, 3, 4, 5, 6 are still worse than this one, this one is 2, so it is 4.6.

S3 Still wrong, you turn.

S4 The picture is reversed.

S1 4.2 cm, 1.8 cm, then here is 1.9 cm

As long as the room layout is determined, the size of each small room is a matter of measurement. This is not a significant objection for the group members. It will be faster to reach a consensus and solve the whole problem.

Conflict discourse is a harmonious-oriented argumentation process. When designing the overall area, the room layout, and the area of each part, group members' understanding and resolution of problems began with conflict. Through the discourse conflicts, they formed subgroups with other members who recognised or accepted their views, tried to maintain their rationality in a conflicting way, and refuted the other side's views before finally reaching a full-group consensus. Thus, conflicting discourse led to more active collaboration.

7.2.3 The Overall Linguistic Features of Conflict Discourse in Collaborative Problem Solving

Verbal conflict is the main form of conflict in the collaborative problem-resolution process. This paper used the collected four-person groups' collaborative problem solving as a corpus database to analyse the verbal characteristics of the conflicting language in the collaborative problem-solving process. This paper's classification of discourse features is based on Scott's (2002) classification, which is divided into 12 types. On this basis, this chapter combined the language characteristics of middle school students in the group collaboration process to study the discourse characteristics and distribution trends of conflict discourse in the collaborative mathematical problem-solving process among secondary school students in a classroom environment. It further analysed the effects of different characteristics of discourse and distribution trends on the effectiveness of collaborative problem solving.

7.2.3.1 Classification of Conflict Language Features

Scott (2002) classified the verbal characteristics of conflicting discourse into twelve categories—absolutes, negation, discourse markers, emphatics, floor bids, flow, indexical second-person pronouns, modals, repetition, questions, turn length, and uptake avoidance—based on the prominence of the conflict (see Table 7.6).

This research combined Scott's classification of linguistic features in conflicting speech with middle school students' verbal features in collaborative mathematics problem solving to classify the verbal features of the conflicting utterances in this research, which were mainly reflected in absolutes, negation, discourse markers, emphatics, floor bids, flow, indexical second-person pronouns, modals, repetition, questions, turn length, uptake avoidance. In this study, the conflict fragments in the corpus were combined to determine the analysis units, identify similar patterns in the use of linguistic features in the problem task context, and analyse their meanings and main feature words.

Table 7.6 Classification of conflict discourse language features

Type	Expressions of conflicting discourse
Absolutes	All, anyone, anything, anywhere, all, every, every person, everything, no matter where, never, no one, no, no place, impossible, must, absolute, sure, certain, definite, only
Negation	No, wrong, false, can't, no way, not at all
Discourse markers	But, now, ok, then
Emphatics	Many, added, most, real, for example, in case, if
Floor bids	Let me come to him or her + Verb (example: tell, say), please wait
Flow	
Indexical second-person pronouns	You, all of you, yourself, your selves
Modals	Possibility: be able to, can, maybe, probably
	Necessity: must, should
	Predictive: shall
	Semi-modal words: Have to do something
Repetition	Restatement of words and sentences
Questions	Interrogative sentence
Turn length	In number of words per turn
Uptake avoidance	Avoidance of previous topic

7.2.3.2 The Overall Distribution Trend of Language Features of Conflict Discourse

Based on a statistical analysis of the selected data, the linguistic feature statistics for the vocabulary level of conflict discourse in the groups' collaborative problem solving are shown in Table 7.7.

The above data show that, at the word level, second-person pronouns (you, all of you, etc.) were used most in the interaction process, followed by negative form words. The application of the negative form was mainly to express dissent and deny the other party's point of view. Modal words (can, may, must, should, will, etc.) were most likely to cause conflicts between group members, as their application reflected the group members' relative negotiating significance in the problem-solving process. Application analysis of discourse marker pairs (then, but, etc.) found that turn-taking pragmatics were the least common; however, in actual classrooms, some students compete for the right to speak and do not necessarily use turn-taking words.

Figure 7.6 shows the overall distribution trend of the language features of conflicting words in the students' collaborative mathematics problem solving.

As the above trend chart (Fig. 7.6) shows, second-person pronouns are the most often used in collaborative classroom teaching in China. These words are used to show that the individual has a clear direction, indicate the opposition and distance between oneself and the other party, and accuse the other party (Connor-Linton, 1989). Applying words such as "no, wrong" in negative form negates the other side's viewpoint or argument and is the beginning of the controversy. In this study, the frequency of extreme generalisation and emphasis was not very high, but it could play a role in strengthening a student's rebuttal and tone. The turn-taking contest had the lowest frequency, showing that participants would interrupt the other party's discussion to express their opinions directly. This phenomenon was verified by reviewing the whole-class video.

In summary, the overall trend of language features in conflict discourse is a more general overview of language features in group collaboration. However, it can also reflect some students' language preferences in this process, which has specific reference significance for teachers intervening in collaborative classroom teaching.

Table 7.7 Statistical table of linguistic features at the lexical level of conflict discourse

Word-level speech feature	Total
Extreme generalisation	483
Negative form	886
Discourse markers	529
Emphasis	68
Turn-taking	16
Second-person pronouns	1154
Modal words	622

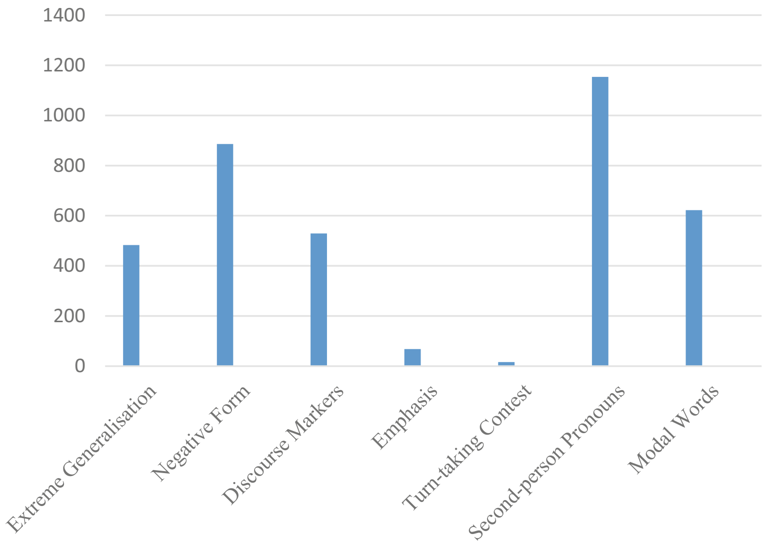

Fig. 7.6 Trend map of the overall distribution of language features of conflict discourse

Speech act theory was first proposed by the British philosopher J. L. Austin in the 1950s. Building on this theory, Yingling Zhao proposed that conflict discourse comprises three steps—the initial speech step, conflict speech step, and end speech step. The following will analyse the participants' language characteristics from these three aspects.

7.2.4 Analysis of Individual Factors that Affect Students' Collaborative Problem Resolution Conflict Discourse

7.2.4.1 Factors of Students' Individual Conflict Processing Styles During Group Conflict Discourse

Different conflict-handling styles will affect the generation and development of conflict discourse and group collaboration performance. In student collaborative problem solving, the group members' personal conflict-handling style impacts the conflict in the collaborative process. Based on Thomas' (1974) model, this study divided conflict management styles into five types: competitive, collaborative, compromise, avoidance, and compliant. The Thomas model classifies conflict management styles in two dimensions—one's concern for oneself and one's concern for others or the relationship—representing different directions.

Combining students with different conflict-handling styles impacts student interaction and collaboration greatly. There have been some research results exploring group conflict and group performance against the background of Chinese culture. The interaction of conflict management and conflict significantly impacts group performance, and relationship conflict negatively impacts group performance and learning. There is a significant positive correlation between task conflict and group performance. Competitive conflict style leads to increased relationship conflict, leading to decreased group performance; avoidance conflict style and relationship conflict are significantly negatively correlated.

This study analyses the conflict-handling style of 20 selected high- and low-performance groups. Each group member was divided into different conflict styles based on their group's interactive collaboration performance. Data coding statistics are as follows (Table 7.8).

Of the 40 students in the 20 groups selected, most had a collaborative conflict management style, echoing Shichang Fu (2013), who concluded that the most preferred style was collaborative, followed by avoidance, compromise, competitive, and compliant. However, Tang Suping and Wang Jing (2006), in a sample survey of mainland university students, found the most frequently used conflict-handling style was compromise, followed by collaborative, competitive, and compliant. The least

Table 7.8 Each group member conducted a conflicting style coding table

		S1	S2	S3	S4
High-performance groups	1aG2	2	3	4	1
	1bG1	2	2	4	3
	1bG6	2	2	2	2
	2aG1	2	2	3	2
	2bG2	2	3	2	2
	2bG5	2	4	2	2
	3aG5	3	2	1	2
	3aG7	2	2	2	2
	3bG3	2	2	2	5
	4bG3	2	2	3	2
Low-performance groups	1aG1	2	4	2	5
	2aG2	1	3	2	2
	2aG6	4	2	2	2
	2aG7	2	2	2	2
	3aG2	5	2	1	3
	3bG2	2	1	5	2
	4aG1	2	3	2	5
	4aG4	4	4	4	4
	4bG1	2	2	4	5
	4bG2	2	2	1	4

Note Competitive (1), Collaborative (2), Compromise (3), Avoidance (4), Compliant (5)

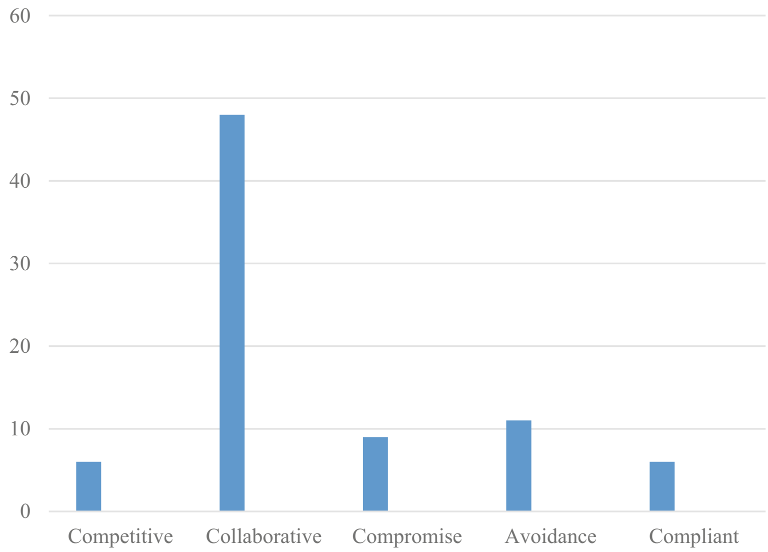

Fig. 7.7 Conflict style statistics chart of group members

used is the avoidance style. Their conclusions were drawn from different samples of different sizes; the sample size needs to be expanded for further sampling verification (Fig. 7.7).

The table shows that the occurrence of competitive conflict management style in the low-performance group was 10%, higher than in the high-performance group. Group members' competitive style will increase team conflict and negatively affect team performance, thereby reducing teamwork efficiency, in line with Chen's conclusion that competitive conflict leads to increased relational conflict, leading to decreased team performance.

The high-performance group's collaborative conflict style adoption rate was 70%, higher than 50% reported in the low-performance group. The 1bG6 and 2aG7 groups displayed collaborative conflict management styles; however, 1bG6 was a high-performance group, but 2aG7's group work efficiency was low. This shows that while the collaborative conflict style effectively mitigates team conflict and improves work efficiency, resolving relational conflict consumes much time and energy and distracts team members from their tasks, thus reducing team performance.

The number of members with a compromised conflict style was twice as high in the high-performance group than in the low-performance group, indicating that the style positively alleviates conflicts and improves group work efficiency.

The low-performance group's avoidance conflict management style adoption rate was 20 percentage points higher than the high-performance group's (7.5%), indicating the low-performance group used avoidance conflict management far more frequently. All four members of the 4aG4 group adopted the avoidance conflict management style, the efficiency of their group collaboration was low. Although avoidance conflict management delayed their relationship conflicts, it caused group members to withdraw from threatening situations to alleviate conflicts, putting the group's tasks on hold and reducing group performance, consistent with the conclusion drawn by Shichang Fu (2013).

The compliant conflict management style appeared only once in the high-performance group and five times in the low-performance group. The compliant management style involves sacrificing one's ideas to ease conflict and accommodate others. It can ease relationship conflict, but the group members' views and ideas are too singular and have no significant impact on improving group performance.

7.2.4.2 Factors in Member Participation Roles in Group Conflict Discourse Process

Group members' conflict discourse roles significantly impact the entire conflict process, with different interactive role models having different collaborative effects on the discourse conflict process. Analysing group members' participation roles throughout the discourse conflict facilitates an in-depth understanding of the collaborative mathematical problem-solving process.

The basic sequence structure of conflict events is unchanged, reflecting the structural characteristics of the discourse itself. Conflict discourse research based on the analysis of relevant corpora (e.g., Gruber, 2001; Nguyen, 2011) found that the basic sequence of multi-person conflict discourse comprises three discourse steps. The steps may be continuous or discontinuous, and others may be inserted. This study called for the problem to be solved in groups of four. While most of the sequence structure in a conflict event was interspersed with several steps, this does not affect the overall situation of the participant's role.

Analysing conflict incidents in the groups' collaborative mathematics problem solving revealed that participants often played different roles in the conflict incident process. Based on the sequence structure of conflict incidents, these roles can be divided into opinion presenter, expression of dissent, refutation, evaluator, non-interactive, and other participating roles (Table 7.9).

The following describes a conversation case of a participant (Table 7.10).

Students' roles in the events mentioned above were a dynamic interaction process that ran throughout the conflict and were not static. First, S4, as a point of view presenter, put forward the idea of having "a bedroom, a living room, a bathroom, and a kitchen." S2 raised an objection to this, saying, "There must be a kitchen and a living room." S4 put forward a rebuttal ("I said the living room"), after which S3 commented, "two rooms and one living room." At the same time, S1 asserted that the apartment "must have a balcony" as a point of view presenter during the balcony design discussion. At this time, S4 raised another objection: "Why draw the balcony separately? I painted it together with the bedroom." In the bathroom design, S3, as the point of view presenter, suggested "two toilets." S2 objected, saying, "Why draw two bathrooms?" leading S3 to rebut with "My house has two bathrooms." S1 and S2 evaluated the functions of the two toilets.

Table 7.9 Classification table

Role	Description
Opinion presenter	Play the role of the initiator when discussing a topic, mainly to put forward your own opinions and opinions on a certain issue
Expression of dissent	Expressing dissent, for or against the issue raised by the initiator, leading to conflict
Refutation	Mainly a rebuttal to the dissident, it may be the statement of the point of view, or it may be other members of the group
Evaluator	Group members who comment on the content of the current discussion or the speakers of the group
Non-interactive	In a conflict event, the content of the group members' speeches did not produce activities with other group members, which was a self-talking activity, and even did not participate in the interactive process during the entire interactive process

Table 7.10 Participating role case

Student	Statement	Participating role
S4	One bedroom, one living room, one bathroom, one kitchen	Opinion presenter
S2	Which apartment does not have a kitchen? A living room	Expression of dissent
S4	I said the living room	Refutation
S3	Two rooms and one living room	Evaluator
S4	Five rooms, why a study room? Xiao Ming maybe a student	Evaluator
S2	My grandma's house is also $60m^2$ and has five rooms	Evaluator
S1	The apartment needs a balcony	Opinion presenter
S4	Balcony? Draw it together with the bedroom	Expression of dissent
....		
S3	Two bathrooms	Opinion presenter
S2	Why draw two bathrooms?	Expression of dissent
S3	I have two bathrooms in my house	Refutation
S1	My house also has two bathrooms. One for the toilet, the other one for bathing and washing clothes	Evaluator
S2	I'm not. The two bathrooms in my house are one for washing hands and the other for bathing	Evaluator

Throughout the conflict process, group members switched roles, and a consensus was established through interactive conversion.

7.2.4.3 Gender Factors of Individual Members in the Process of Group Conflict Discourse

Gender differences in conflict discourse in group collaborative problem solving is another research focus. American scholar Goodwin (1990) conducted many empirical studies on the differences between boys' and girls' conversational styles in debates. Boys use conflicting discourses to "maintain their status" (Emihovich, 1986), while girls use negotiation to avoid conflicts and maintain their collaborative relationships (Eckert, 1990). Australian scholar Mary Barnes's study of participation patterns in group collaborative situations found that girls are often in a state of being helped and their mathematical problem-solving tasks less involved (Mary Barnes, 2003).

To study the conflict and negotiation process under different gender combinations, a group was added to the original 20 groups; thus, the sample in this section is 21 groups, and the gender distribution of the members is as follows (Table 7.11).

In this study, boys and girls showed differences in their conflict discourse structure and speech types in the groups' collaborative problem solving because of their different possessive and controlling desires for the "right of speech." Among the 21 groups, 14 had the same ratio of boys and girls; of the remaining groups, two were all boys, one was all girls, and four had various male-to-female ratios. Different conflict processes were more obvious in different gender groups.

7.3 Discussion and Findings

This research has discussed the conflict and negotiation process in collaborative group mathematics problem solving through video and text analyses based on observed classroom teaching. It has analysed the types of conflict discourse, the structure of conflict discourse, and the linguistic characteristics of conflict in the collaborative problem-solving process and discussed the factors affecting the conflict process and the effect of cooperation.

Table 7.11 Gender distribution table of high and low-performance group members

		S1	S2	S3	S4
High-performance group	1aG2	0	1	0	1
	1bG1	1	1	1	0
	1bG6	0	0	0	1
	2aG1	1	0	1	0
	2bG2	1	0	0	1
	2bG5	1	1	1	1
	3aG5	1	0	0	1
	3aG7	0	1	0	1
	3bG3	0	0	1	1
	4bG3	0	1	0	1
Low-performance group	1aG1	1	1	1	0
	2aG2	0	1	0	1
	2aG6	1	1	1	2
	2aG7	0	1	0	1
	3aG2	0	1	0	1
	3bG2	0	1	1	0
	4aG1	0	1	1	0
	4aG4	1	1	1	1
	4bG1	1	0	1	0
	4bG2	1	0	0	1

7.3.1 Discussion

7.3.1.1 Build an Analytical Framework for Conflict Discourse in the Context of Collaborative Mathematical Problem Solving

Conflict discourse exists in all aspects of daily life in society. A review of the literature found that there have been many studies on conflict discourse at home and abroad, but relatively few have used conflict studies to analyse classroom teaching, especially in the context of collaborative learning. This research has drawn on the existing results of conflict discourse research in other disciplines, combined with case studies and other empirical research methods, to analyse conflict discourse in students' collaborative mathematics problem-solving process from a micro perspective and establish the conflict context and conflict types in the collaborative learning process. Its research framework was based on conflict discourse's structure and speech characteristics and other research frameworks, starting with micro-level video of specific groups' collaborative learning to reproduce the group members' conflict process in collaborative learning, analyse conflict discourse theory, and explore the reasons for the conflict. The process is shown in Fig. 7.8.

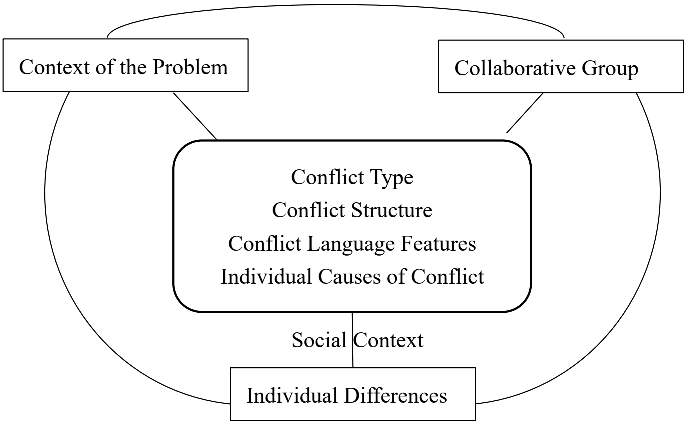

Fig. 7.8 Framework for analysing conflict discourse in the context of collaborative mathematical problem situations

7.3.1.2 Analysis of Conflict Discourse Types in Collaborative mathematics problem solving

In traditional Chinese culture, there are two types of conflict discourse in collaborative mathematics problem solving: task conflicts based on mathematical problems and relationship conflicts. Task conflicts based on mathematical problems manifest in three ways: different opinions on solving mathematical tasks, different divisions of mathematical tasks, and disputes over data in solving mathematical problems. Relationship conflicts are contradictions between people.

7.3.1.3 An Analysis of the Whole Structure of Conflict Speech Events

The problem of "Xiao Ming's apartment" was mainly divided into four steps: determination of the overall room area, determination of the room function layout, deciding the area of each room, and designing the final room layout. These four steps were reflected in most collaborative groups' problem-solving processes. Based on these four structural steps, further analyses of conflicting discourse were made.

Based on the first stage, the verbal dialogue in each step was divided into conflicting utterance events. "Conflict" is a factor of great concern in this study. Arguing between four people for a certain point of view was necessary when classifying conflicting speech events. They could only be classified as an analysis unit when they had a clearer attitude. Conflict and negotiation arising from discussion of the overall room size, conflict and negotiation arising from discussion of the room layout and function, and conflict arising from discussions about room dimensions were divided into beginning, conflict, and end stages.

Analysing the conflict discourse process showed that conflict and negotiation coexisted during the collaborative problem-solving process. To achieve the ultimate problem-solving goal, group members should focus on establishing a collaborative awareness of problem solving in the collaboration process. The degree of concentration of attention at the critical moment of problem solving, the contribution of group members in problem solving, and the achievement of these three process forms directly affect the final collaborative effect.

7.3.1.4 Ten Language Features of Conflict Discourse in Collaborative Mathematics Problem Solving

This research was based on Scott's classification of conflicting language features and the language features of middle school students in collaborative mathematics problem solving. It divided the conflicting language into absolutes, negation, discourse markers, emphatics, floor bids, flow, indexical second-person pronouns, modals, repetition, questions, turn length, and uptake avoidance. Among these, the frequency of indexical second-person pronouns was the highest, while negation, absolutes, discourse markers, and modals were in the middle (with little difference among them), and the frequencies of emphatics and floor bids words were the lowest. The frequency of language features at the lexical level reflects the language features of conflicting discourse in collaborative mathematics problem solving. However, many factors affect frequency, including social factors (e.g., participants' status, role, and scene) and discourse structure factors (e.g., discourse type). These deep-seated reasons will affect participants' choice of language forms like negation, modal words, discourse markers, and interrogative sentences.

7.3.1.5 Analysis of Individual Factors Affecting Students' Conflict Discourse in Collaborative Problem Solving

Analysis revealed that group members' various conflict management styles greatly influence the conflict and negotiation process in group collaborative mathematics problem solving, thereby affecting the effect of the collaboration. Competitive conflict style will negatively impact group performance, leading to a decrease in collaborative performance. While the collaborative conflict style is effective in mitigating team conflict and improving work efficiency, resolving relational conflicts consumes much time and energy and distracts team members from their tasks, thus reducing team performance. Compromise conflict management style reduces conflict and improves team efficiency. Although avoidance conflict management can postpone relationship conflicts, group members' withdrawing from threatening situations to alleviate conflicts puts group tasks on hold and reduces group performance. In compliant management style one sacrifices one's ideas to ease conflict and accommodate others. However, while it can ease relationship conflict, the group members' views and ideas are too singular and have no significant impact on improving group performance.

In a conflict event, participants often play different conflict and negotiation roles based on the sequence structure of conflict events, including opinion presenter, expression of dissent, refutation, evaluator, non-interactive, etc. These different roles are a dynamic interaction process running throughout the conflict process, and are not static. In the role change process, a consensus is established, and agreement is reached.

Analysis of the amount of discourse revealed a situation in which boys and girls were controlled separately when their four-person group had an equal number of boys and girls. In most cases, one boy and one girl student would play a dominant role. When the ratio of boys and girls was different, the discourse among members was concentrated on the side with the larger number of people. The atmosphere in male-majority groups was more active, but there were different collaborative effects.

7.3.2 Conclusion

7.3.2.1 Conflict Discourse Is a Harmony-oriented Argumentation Process that Will Lead to Active Collaboration

Conflict discourse is a harmonious-oriented argumentation process. When determining the apartment's overall area, room layout, and designing the area of each part, the group members' understanding and resolution of problems start with conflict. This led them to form subgroups through discourse conflicts with members who recognised or accepted their views, tried to maintain their rationality in a conflicting way, and refuted the other side's views before finally reaching a group consensus. Thus, conflicting discourse led to more active collaboration.

Group members' sense of collaborative problem solving was the basis and first condition for completing the problem-solving task. All interactions served to solve the problem together. After the teacher assigned the task, student 2 in the first group said: "Let's think of a solution together," which set the tone for this activity and reminded other students to be aware of solving problems together. While group members could propose solutions and often responded to and expanded on others' ideas, each felt the need to collaborate in solving problems.

7.3.2.2 Reasonable Negotiation among Group Members Helps Solving Problems

The ultimate goal of student collaborative problem solving is to solve problems. In the collaborative process, the resolution of conflicting words is based on rational negotiation in an appropriate discourse and a sincere interactive attitude. Previous research have illustrated polite disagreements are beneficial for correct ideas while rude disagreements are negative (Chiu, 2008). The group members reached a consensus based on their own experiences, with rational negotiation playing a very important role.

Cohesion is the most direct factor affecting group performance in composition, structure, and process. The group members' concentration had a lot to do with their awareness of collaboration. In the groups, members were occasionally distracted. For example, one student in the second group doodled on the task list while another student wrote on his paper. These four people spent most of their time drawing pictures and had to hurriedly regather themselves after the teacher's intervention. Concentration was easily re-established in such situations, given teachers' or group members' timely intervention. This was particularly prominent during generating and recording solutions. When a group was committed to the solution, the task list became the centre of its students' conflict and negotiation. They focused on the task unit to monitor the completion of the solution.

7.3.2.3 Group Members Should Have Their own Contributions in the Negotiation Process

Collaborations with more interactive dialogues bring better learning outcomes than those with fewer interactive dialogues. Students make their contributions through equal participation and hope others will recognise and adopt the solutions they propose. Group members collaborate for mutual benefit and, ultimately, to solve the problem. Students' participation including explaining their own idea and engaging others' idea is beneficial for students' mathematical learning (Webb et al., 2021). Mutuality is reflected in the nature of their dialogues, especially when handling partners' contributions. When one student proposes an idea, other students can participate. These exchanges may be marked by conflict and may involve another student's ideas. In this study, the dialogue was coded based on the response type the partner used when proposed a solution to the problem. After the partner proposed the solution, all responses from the two groups were coded as either acceptance, explanation, elaboration, rejection, or no response, meaning there were two situations of confrontation and negotiation. Be it through confrontation or negotiation, each member contributed to the final formation of the problem's solution.

7.4 Implication and Contribution

This chapter drew its research sample based on classroom videos of group collaborative problem solving for mathematics tasks, reproduced the conflict process through multi-angle text analysis, explored the cause of conflict, and provided a reference for teachers to understand and intervene in the process of student conflict. Collaborative mathematical problem solving combines collaborative and mathematical problem solving. From a sociological perspective, this teaching process can have positive significance for curriculum design, teacher's classroom teaching, student learning, and evaluation of student's problem-solving ability.

7.4.1 The Research's Implications for Social Interaction in Mathematical Collaborative Problem Solving

This research provides a research perspective for students in collaborative group learning, especially collaborative mathematics problem solving. Conflict and negotiation are common phenomena in collaborative group learning. Exploring the connotation and appearance of conflict in a problem-solving scenario is a less-studied area in China. This study examined the interaction process of group members in collaborative learning from the perspective of conflict and negotiation, recreated the picture of conflict in the problem-solving process, applied the theory of discourse analysis to actual classroom teaching research, focused on the relationship between the quality of collaborative effects and the process of conflict, and provided a good research perspective for classroom teaching research and a key to open the dark box of discourse interaction in collaborative problem solving.

7.4.2 Implications for the Reasonable Establishment of Collaborative Groups

Collaborative groups are central to the problem-solving context, and the creation of reasonable groups is the first condition for successful problem solving. Teachers should build reasonable, open, and inclusive study groups that are based on students' actual situations, including their ability levels, conflict style characteristics, gender, and other aspects. For example, students' different conflict styles will affect the group's collaborative effect. Previous analyses have concluded that the competitive conflict style will negatively impact group performance, resulting in increased relationship conflicts and, in turn, decreased collaborative performance. While the collaborative conflict style is effective in mitigating team conflict, thereby improving work efficiency, resolving relational conflict consumes much time and energy and distracts team members from their tasks, thus reducing team performance. Compromise conflict management style has a positive effect on reducing conflict and improving team efficiency. Thus, a reasonable allocation of students with different styles is especially important when creating collaborative groups.

7.4.3 Implications for Setting Scientific Mathematical Tasks Themes

Collaborative learning content is critical when implementing collaborative learning and teaching in classrooms. Problem-based task design determines the quality of group interaction, making it necessary to understand students' academic conditions and formulate learning goals. Thus, teachers should design problems and tasks that combine challenges, openness, and hierarchy, as well as preset, adjust, and dynamic generation (Guo, 2009). Through designing challenging, open, and hierarchical questions, every group member can effect benign communication and collaboration, avoid unnecessary conflicts, and strengthen their negotiation and communication, so every student has a suitable role, ensuring that group members participate together.

7.4.4 Implications of Teachers' Teaching Intervention in the Process of Collaborative Problem Solving

In the group collaboration and communication process, the dialogue between teachers and group members is a concrete manifestation of teacher intervention. The collaborative groups selected in this study went through their problem-solving processes with no (or minimal) teacher intervention. Thus, their conflict and negotiation process reflected their most natural state, providing a specific reference for teacher intervention in collaborative learning in regular classrooms. Existing studies have found that teachers lack guidance and fail to evaluate the group's collaboration communication before intervening; mainly, they intervene with individuals to solve cognitive problems, while largely ignoring students' interactive performance.

The content and method of teacher intervention should be diverse, not limited to one kind. Teachers' first intervene should involve grouping students based on students' knowledge, ability, and personal style. Second, they can various forms of procedural guidance to monitor the group's interactive process and give the group and its members time and space in their collaborative problem-solving process to make and correct mistakes (within a certain range). Finally, the teacher should give a sufficient evaluation of a groups' collaborative effect, so each member can experience their value in the group interaction.

7.4.5 Implications of Students' Participation in Mathematics Collaborative Problem Solving

Student participation in problem solving in collaborative learning is a process of problem solving through negotiation. In this process, students' conflicting discourse situation reflects their participation. During the conflict process, group members will have different, conflicting discourses based on their different roles, and will cause conflicts based on their different participating roles. Our analysis of conflict events in collaborative mathematics problem solving revealed that participants often played different roles in a conflict event process, based on the sequence structure of conflict events, which can be divided into such participating roles as opinion presenter, expression of dissent, refutation, evaluator, non-interaction, etc.

Group members' participation in the collaborative learning process can be divided into two basic states: marginal participation and core participation. Through the analysis of conflict and negotiation discourse, members' participation in the whole group collaborative learning process can be understood, and members' marginal participation transformed to core participation to achieve efficient cooperation.

In the upsurge of education reform emphasising group collaborative problem solving, micro-research is getting more and more attention. It is hoped that the conversational analysis of conflict and negotiation in collaborative problem solving in this paper will become the starting point for further research, so as to better understand students' cooperation and improve their ability to participate effectively in collaborative problem solving, which is important both educationally and socially.

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Chapter 8
Research on Student Interaction in Peer Collaborative Problem Solving in Mathematics

Zhengyi Zhang

8.1 Introduction

The consensus in the academic community is that collaborative learning plays a critical role in promoting students' learning and building social relations among students. Therefore, analysing students' performance in collaborative problem solving is of great significance for understanding students' collaborative process and improving teachers' understanding of collaborative problem solving. In current practice, the most common way of collaboration in primary and secondary schools is collaboration between two students. Therefore, this article will focus on the interaction of two-student collaboration pairs to analyse their interactions during collaborative problems solving (CPS) in mathematics. The study aims to gain insights into the performance of peer collaboration in the CPS process and provide implications for teachers for instruction when teaching CPS in mathematics.

Although many studies pointed out that collaborative learning had a positive effect on developing students' abilities, the internal structure of multi-person groups had some drawbacks. On the one hand, some students did not actively participate in collaborative discussions; on the other hand, it was difficult for teachers to pay attention to each student's performance, as doing so might cause some distractions from the given tasks (Sun & Wen, 2004). Zuo and Huang (2010) concluded that it was easier to build a consciousness to collaborate between pairs and that their independence could be stronger than in multi-person groups. Fleming and Alexander (2001) set up peer-collaboration treatment conditions and individual tasks (control conditions) to investigate whether the observed benefits of peer collaboration lasted for a certain time. Their study showed that students in the treatment condition did

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more outstanding performance than those in the control condition in strategy use, metacognitive understanding of strategy chosen, and recall gain. Compared with independent learning, peer collaborative learning could help to develop students' creative problem posing (Han et al., 2013), which further affected their ability to solve mathematical problems (Zhang et al., 2019).

One issue concerning how teachers organise collaborative groups in teaching practice is how to categorise students into groups or pairs, usually referring to whether groups or pairs should include students with similar or different traits. Students in pairs can have different characteristics, and different combinations of students may influence their collaborations. Students' ability levels can be a key factor affecting their interaction during collaborations to complete mathematical tasks (He, 2005). Most researchers favoured heterogeneous groups, but others disputed their effectiveness.

One aspect that some studies have confirmed the effectiveness of heterogeneous grouping in promoting students' abilities. Fantuzzo et al. (1992) paired 80 students with learning difficulties with students with higher academic performance to help the former improve their academic performance. Although the scholar did not explicitly mention that the groups' heterogeneous structure helped improve the students' academic performance, the matching method advocated heterogeneous groups or pairs. Zhang et al. (2019) matched 72 seventh grade students in pairs according to their final math scores and gave them a pencil-and-paper test, and found that peer collaboration with heterogeneous structure had the greatest impact on the students' ability to solve mathematical problems. Other researchers amplified the effectiveness of heterogeneous group structure from a theoretical perspective. From the perspective of group socialisation theory, one of the most effective group structures was heterogeneous (He, 2005). The classic "zone of proximal development" concept pays more attention to the differences in peers' abilities. It holds that teachers can only promote growth by pairing students with partners with stronger abilities. The learning condition is that the two peers' ability performance is inconsistent, because their different abilities afford them different views on problem solving. With collaboration, peers reached a consensus through interaction and reflection to make progress (Sun & Wang, 2009). Within the heterogeneous group, the authority distributions vary betweem students (Langer-Osuna et al., 2020). In the heterogeneous group, the high ability students are not affected by the grouping; i.e., the high ability students' academic performance was relatively independent of the group structure, while low ability students can learn more knowledge in a heterogeneous group (Saleh, 2005).

The other aspect that some researchers are uncertain about the effectiveness of heterogeneous groups. Mortweet and Utley (1999) reviewed a learning model, also called the whole-class paired learning model, one of the most in-depth peer collaborative learning research methods. The model is based on a peer matching form with little difference in ability, so students can get more effective feedback from each other. In this model, most students' scores could increase between 20 and 70% (Delquadri et al., 1986). Wang and Chen (2008) divided collaborative groups into homogeneous and heterogeneous groups from a cognitive style perspective, and tested them to research the differences in their problem-solving levels. They found that individual differences in heterogeneous groups would become obstacles to effective communication. Bowers et al. (2000) made a meta-analysis of 57 effective duos in collaborative groups to determine whether there was a difference in performance between the heterogeneous and homogeneous groups. The study found that although single studies had shown that heterogeneous groups perform better than homogeneous groups, the overall differences between the two were not significant. Some studies also set up reading tasks for collaborative learning and found that gifted students in homogeneous groups improved more than those in heterogeneous groups (Melser, 1999).

In addition to the above, there is a dispute regarding the structure of collaborative groups. For one thing, an individual's performance depends on the type of task, so it is difficult to explore which kind of collaboration is better without considering the task type (Bowers et al., 2000). For another, many studies judge the effectiveness of heterogeneous groups based on final test results, so different test methods and tasks may lead to fluctuations in the results. Research based on process performance within a group may result in higher stability than research based on test results. Moreover, there is less research on interaction performance within heterogeneous groups, and more on whole-group rather than individual performance, so it is difficult to see the specific performance characteristics of each student in heterogeneous groups. This study focuses on heterogeneous groups and explores the performance of each student in peer collaboration groups in mathematical problem-solving to determine whether students benefit, which is of great significance for helping teachers organise collaborative groups.

To sum up, pair collaborative problem solving is currently the most widely-used and effective form of collaboration. However, as there are differences in students' academic performance, we inevitably face the choice of group structure. This paper researches junior high school students' interaction performance in peer collaboration groups and investigates the efficiency and quality of their interaction processes and characteristics in collaborative mathematical problem solving. There are two main research questions.

1. How do students in heterogeneous pairs perform differently in peer interactions in collaborative mathematics problem solving?
2. What differences are there in peer interactions among students in heterogeneous peer collaboration groups, based on collaborative mathematics problem solving?

8.2 Research Method

8.2.1 Participants

In the current research, purposeful sampling is selected for investgating the situation of heterogeneous groups. The influence of students' academic performance on collaborative effect is obvious, and most researchers divide group members based thereon. Meij (1988) found that academic performance affected the number and type of questions students asked their partners. In researching students' interaction in pair collaboration, two concepts seem particularly useful: student interaction and pair collaboration. Student interaction refers to reciprocal communications, either verbal or nonverbal, among students. Pair collaboration refers to two students sitting together to complete a task through interaction. The research uses "Xiao Ming's apartment" as the task material for pair groups to solve and selects videos from the problem-solving processes of 106 seventh-grade students (54 peers) from three classes in a district of Beijing. This research defines heterogeneous groups by students' academic divergence within a pair group. Students' mid-term test scores (unified district examination, consistent scoring criteria) are taken as their academic performance. The gap in academic scores between the two pair members is arranged in descending order; students in the paired collaborative group with relatively high scores are defined as high academic performance students (HS), while those with relatively low scores are defined as low academic performance students (LS). To ensure that two peers could complete the task, each HS had to score at least 80 of a possible 100 points. The researcher selected the nine paired collaborative groups with most academic gap as research objects. Among them, the terms high and low academic performance are relative and do not refer to an absolute level of performance.

Because collaborative groups are generally two-student groups in daily teaching, and teachers cannot notice and guide each group, pair collaboration was carried out without teacher intervention in this study; teachers did not guide students on how to complete the task. The choice of research samples was in line with our daily teaching situation.

8.2.2 Encoding Frame

To deepen the research, researchers have gradually shifted attention from the effect of collaborative learning and influencing factors to the collaborative process. In the pair collaboration process, interaction cannot be avoided, and the interaction between two peers can help us understand and analyse students' interactive performance in the collaboration process (Wang, 2004).

Saleh et al. (2007), noting that students with general ability often did not make full use of heterogeneous group learning, proposed structured collaboration to help students overcome this participatory inequality. His study divided verbal interaction into 11 indicators: statement, argument, evaluation, question, request, proposal, confirmation, negation, repetition, order, and off-task. The scholar's coding of verbal interaction is more detailed, covering most dialogue behaviours and clearly describing each index.

As Saleh's coding framework marks most utterances as "argument," this study combined it with Gillies' (2003) and Guiller's et al. (2008) frameworks to encode the interaction process in pair collaboration more carefully. Saleh's "argument" dialogue in the coding framework was divided into two categories: "giving a point of view" and “explanation,” and the indicator of “checking” was added. After generating the draft coding framework, part of the data is coded to revise the framework. However, it was found that the “checking” dialogue did not appear in each group, so this category was deleted. Because the research object of this study is paired collaborative group heterogeneous paired collaborative groups, the academic performances of the two students in the group are quite different. Watching the video revealed that they often did not communicate; therefore, the researcher added the index “no communication” to the coding frame to further the research. In this study, four students shared a large desk, with adjacent students acting as a paired collaborative group to complete peer tasks. As each paired collaborative group’s interactions could be subject to external influence, the “out of group discussion” index was added to the coding framework. The final coding framework was based on this study’s research content, complemented by multiple viewings and analyses of specific videos by the researcher, as shown in the following Table 8.1. As the framework is conducted by previous research, the validity is ensured. After coding, the researcher conduct a re-code to ensure the reliability of the coding result.

8.3 Results

Per the research scheme, this section further explained and refined the research process, and classified and compared the paired collaborative groups based on their interaction performance. The change in task set location was an odd aspect of this study. Students’ verbal interactions were first analysed, followed by the relationship between task set location and the two peers’ academic performance gap.

In this study, NVivo12 was used to split the coding of nine pair groups of videos and calculate the length ratio of each interaction category. The coding results were analysed in R3.6.3 and SPSLS6.

8.3.1 Verbal Interaction

In this study, the video was analysed as a continuous sample, and discourse meaning was used as a coding unit to count the proportion of each interaction category rather than just recording utterance frequency, for more accurate analysis results. This study encoded and analysed videos of nine pair groups, counted the proportion and total proportion of HS and LS in 14 behaviours (13 verbal interaction and one “no interaction” categories), and calculated the corresponding average values, as shown in the following Table 8.2.

As shown in the Table 8.2, the high academic performance students’ proportion of verbal interaction time (47.36%) was significantly higher than that of low academic performance students (22.28%), to some extent indicating that LS could not fully participate in the discussion, while HS dominated the conversation. In addition, “no

Table 8.1 Coding framework

		Description
Task set location		1: On the desk of HS
		0: On the desk of LS
		−1: Between two students
Behaviour	Interaction category (speaker)	1: Statement: New content-related information that does not build on previous utterances
		2: Repetition: Quoting or relaying previous words without adding new information
		3: Evaluation: Personal opinion or judgment related to the task
		4: Explanaton: Give an analysis of the point of view
		5: Request: Request for the opinion or judgment of others
		6: Giving a point of view: Express opinions, make a statement on an issue, express their own ideas, relatively brief, basically no explanation
		7: Confirmation: Positive response to a previous utterance without further explanation
		8: Question: Request for content-related help or information
		9: Proposal: Suggestion for a joint activity or task division
		10: Order: Instructions and directions
		11: Negation: Negative response to a previous utterance without further explanation
		12: Off-task: Information not related to the task
		13: Out of group discussion: task-related discussion with non-peer relationship members
	No interaction	14: No communication: no communication between peers, and at least one person has task participation performance

communication" accounted for 30.31%, close to one-third of the total task duration, which reflects that communication in the two students' interactions was slightly less active and unequal in the nine pair groups.

The Table 8.2 shows a significant difference in the duration proportion of each indicator between HS and LS. To further illustrate this point, the researchers made a two-factor repeated measurement analysis of variance on the proportion of the two students' dialogues corresponding to the nine video groups for each indicator. The difference in the data corresponding to the dialogue categories was statistically significant ($p < 0.01$).

The two-factor repeated analysis of variance results (Fig. 8.1) show significant differences between HS and LS in repetition, evaluation, explanation, order, and negation. The researchers focused on the data for each indicator to further understand the conversational differences between peers in the collaboration process. "Evaluation," "explanation," "giving a point of view," "order," and "negation" of these dialogues were significantly higher in HS than in LS. Dialogue representation with critical

Table 8.2 Overall interaction proportion between HS and LS

	Index	Proportion of HS (%)	Proportion of LS (%)	Total proportion (%)
Verbal interaction	1: Statement	2.04	1.38	3.41
	2: Repetition	0.04	0.64	0.67
	3: Evaluation	0.84	0.07	0.91
	4: Explanation	27.97	7.72	35.69
	5: Request	0.36	0.47	0.83
	6: Giving a point of view	8.64	7.06	15.69
	7: Confirmation	0.48	0.45	0.92
	8: Question	1.39	1.49	2.88
	9: Proposal	0.56	1.01	1.58
	10: Order	1.72	0.52	2.24
	11: Negtion	0.87	0.19	1.05
	12: Off-task	0.19	0.76	1.95
	13: Out of group discussion	1.27	0.53	1.80
	Total	47.36	22.28	69.69
No interaction	14: No communication	0	0	30.31

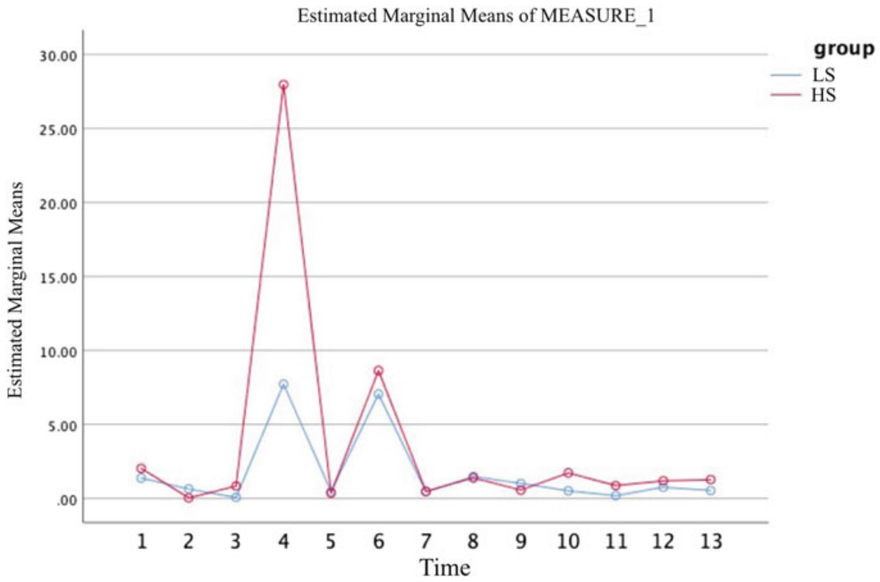
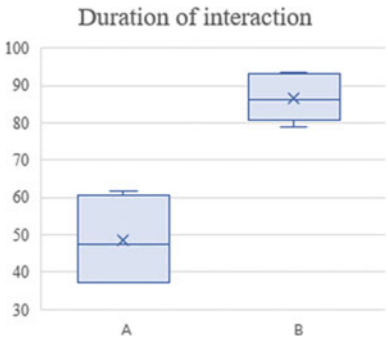


Fig. 8.1 Two-factor repeated ANOVA results

Fig. 8.2 Box plot of group dialogue duration



meaning, such as "evaluation," "order," and "negation," was higher in HS than in LS; LS's utterance proportion for these three indicators was close to 0, similar to the results of Saleh's study of students' verbal interaction (Saleh et al., 2007). In addition, LS were higher than HS in repetition and suggestion.

Certain differences were found in the dialogue duration between groups. Systematic clustering of the data on the two students' dialogue proportion duration corresponding to each video analysis result divided the nine groups into two categories, with the first to the fourth groups in one category and the remaining five in the other. The box diagrams in the following Fig. 8.2 clearly depict this classification difference in the two groups' conversation duration data.

A is the first to fourth groups and B is the fifth to ninth groups. There is a clear stratification in the dialogue duration between the two groups, with A's dialogue duration being lower than B's. The gap in A's two-student midterm test scores was over 40 points, while B's was between 10 and 40. Based on this finding, the following group analysis is divided into two categories, Category A and Category B.

8.3.1.1 Verbal Interaction of Category A

There were large gaps between the two students in each Category A. The average dialogue duration in the task-solving process was 48%, meaning over half of each group's time passed in a state of non-communication. The proportion and total proportion of HS and LS for the 14 indicators are shown in Table 8.3.

HS and LS showed more significant differences in dialogue categories in Category A. As in the previous part, HS were significantly higher than those with LS in terms of "order," "negation," "explanation," and other indicators. The differences in dialogue between HS and LS in Category A exceeded the nine groups' average performance. Even in the "out of group discussion" indicator, HS in Category A showed a significantly increased proportion, indicating more communication and discussion with non-group members.

While the overall communication time between the two students was less, HS spoke longer than LS, and the LS expressed fewer opinions. LS lacked confidence

Table 8.3 Interaction between the HS and LS in category A

	Index	Proportion of HS (%)	Proportion of LS (%)	Total proportion (%)
Verbal interaction	1: Statement	1.76	1.05	2.81
	2: Repetition	0	0.70	0.70
	3: Evaluation	0.59	0.16	0.75
	4: Explanation	17.69	1.68	19.37
	5: Request	0	0.21	0.21
	6: Giving a point of view	8.43	4.1	12.53
	7: Confirmation	0.41	0.22	0.63
	8: Question	1.53	1.13	2.67
	9: Proposal	0	1.27	1.27
	10: Order	2.27	0.20	2.47
	11: Negtion	1.23	0.19	1.42
	12: Off-task	0.31	0.88	1.19
	13: Out of group discussion	2.29	0.07	2.36
	Total	36.50	11.86	48.47
No interaction	14: No communication	0	0	51.53

and dared not express themselves, were suppressed or ignored by HS, and were not recognised by their peers, resulting in less interaction. This phenomenon was an inherent risk when using heterogeneous learning teams (Slavin, 1991).

[Episode 1]

LS: Is it possible that he has one student, father and mother, grandparents?

HS: Don't look at them, we can count it ourselves.

[Episode 2]

LS: In case, it is impossible for his grandfather and grandmother to have more than fifty, or more than sixty, or more than seventy, plus more than 140, more than 130, and more than 120, but he has only one.

HS: Come on, let's be 13, let's be 13.

LS: Figure out four possibilities. show him.

HS: does not respond to this, the two are in a state of non-communication.

These two small peer interaction episodes are from Group 1. In the first episode, LS cast an idea, but HS ignored it and did not respond positively. Similarly, in the second episode, LS analysed the problem, but HS's response was not related to LS's analysis and instead directly advanced their views. At this time, LS did not adhere to their views and compromised, suggesting that four possibilities should be calculated to show the teacher; HS did not respond to this, and the communication between the two was interrupted. Analysis of the two episodes showthat neither communication formed a complete collaboration process, and LS could not get a response of feedback after expressing their views. A complete collaboration should include an interactive process, expression, listening, questioning, etc. These behaviours collide in collaboration and communication to promote the problem's solution (Cao & Bai, 2018). However, the above episodes had no interactive process, leading to the collaboration's failure.

[Episode 3]

HS: If 30, 30–10 is 20. 32, let's get it 32.

No response.

HS: 32 + 32 + 12 =

No response.

HS: 64, 64, 76

No response.

HS: Well, it's 25. If it happens to be 25, his brother and sister are 25 years old. Brother, then it's 13. he was 13 years old.

No response.

HS: 13 * 2, 26.

No response.

HS: Then make them 15 years old!

HS: If they were 15, 15, one more, 15

Episode 3 is a dialogue episode from Group 4, located in the middle and late stages of the whole discussion process. No response meaned no one spoke at all during this period. In this episode, HS constantly expressed his opinion, but LS did not speak a word and had no response to HS's utterances. This completely non-responsive state (often found in Category As) was not related to HS's interruptions or disregard but to LS's failure to take the initiative to respond.

This may have been because LS was aware of their inability to participate in the collaboration and consciously gave up on the task. In other words, they were not listening to HS. It is also possible that LS's academic performance could not reach a certain level, making it difficult for them to express their ideas even though they were listening carefully to the HS. No matter the reason, the episode showed that a large enough gap between the two students' scores leads to a gap in their exchanges, making it difficult for each to supplement the other's views and preventing them from reaching the "thinking collision" state needed to promote constant problem solving.

[Episode 4]

LS: is it all equal to this?

HS: you don't move!

…

LS: poop

HS: tell you, don't mess up!

LS: hum

HS: I didn't want to be in a group with you. Don't mess up.

On the one hand, Episode 4 responded to the above-mentioned differences in dialogue categories between HS and LS. On the other hand, it also showed that HS resisted LS when they expressed their opinions or took some actions, preventing in-depth collaboration. To a certain extent, this situation explained why the dialogue duration between the two students was less than half of the total task time—they could not collaborate or communicate peer-to-peer, rarely exchanged views, and seldom thought deeply, eventually resulting in HS solving the problem by themselves. It was not an ideal peer collaboration.

In peer collaboration based on mathematical problem solving, a large gap between the two partners' ability to understand and solve problems makes their effective collaboration difficult. The two have few dialogues on tasks. LS were likely to be excluded or dominated by HS, and lacked the confidence to express their opinions. In other words, individuals in collaborative groups need to be recognised by other members. When the performance gap between the two is large enough, it is difficult for LS to obtain sufficient HS identity, leading to their marginalisation in or even separation from the collaborative peer group.

8.3.1.2 Verbal Interaction of Category B

The academic performance gap between the two students in each Category B was narrower, and the average group dialogue duration was 86.67%, much higher than in Category As. The communication between the peers was in a positive dialogue state. This study counted the proportion and total proportion of HS and LS for the 13 specific dialogue categories, and the average proportion of non-communicative behaviour in Category Bs, as shown in the Table 8.4.

Category B HS's discourse proportion was 56.05%, while LS's was 30.61%, both higher than in Category As. In addition, the differences in dialogue between HS and LS were significantly reduced, particularly in the indicator of "explanation," as were the differences in other indicators.

The Fig. 8.3 shows the performance of HS in Category A and Bs in various dialogues categories. A- and B-group HS's performances were largely consistent for such indicators as "statement," "repetition," and "request," while there were differences in Category A-HS's performance in such indicators as "explanation," "proposal," "order," "negation," "off-task," and "out of group discussion." HS's proportions of "order," "negation," and other indicators with strong words were significantly reduced, showing that HS recognised LS's peer role to a certain extent, regarded it as the object of equal communication, and were more willing to explain their ideas to them. The communication state between the peers was more positive, more coherent, and smoother, complementing each other. The LS gradually began to fight for the right to speak for themselves in the discussion process, which was no longer HS's solo play.

[Episode 1]

LS: let's reason

Table 8.4 Interaction between the HS and LS in Category B

	Index	Proportion of HS (%)	Proportion of LS (%)	Total proportion (%)
Verbal interaction	1: Statement	2.26	1.65	3.91
	2: Repetition	0.07	0.58	0.65
	3: Evaluation	1.04	0	1.04
	4: Explanation	36.19	12.56	48.75
	5: Request	0.65	0.68	1.32
	6: Giving a point of view	8.8	9.4	18.2
	7: Confirmation	0.53	0.63	1.16
	8: Question	1.28	1.77	3.05
	9: Proposal	1.02	0.81	1.82
	10: Order	1.29	0.77	2.06
	11: Negtion	0.58	0.18	0.76
	12: Off-task	1.89	0.66	2.55
	13: Out of group discussion	0.46	0.9	1.36
	Total	56.05	30.61	86.67
No interaction	14: No communication	0	0	13.33

HS: her parents must have…

LS: Mom and dad must be over 70

HS: Mom and Dad, oh, this is the sum, the sum

LS: almost 70 years old, all in all

HS: that's for sure, or a person is 70 years old, ha-ha-ha

LS: her parents must be over 70

HS: but, 42

LS: a person is 35. for example, over 70 years old, the average age is 70,

HS: she has a big sister and a little brother

The LS in the above episode constantly gave opinions, leading the dialogue in this small segment. At the same time, the HS was constantly echoing and thinking. The two peers were giving each other ideas and opinions so the direction of the discussion can be carried out smoothly. This kind of dialogue mode was much better than that in Category As; the two students complemented each other's views, communicated as equals, and expressed their views and thoughts to a greater extent.

[Episode 2]

LS: one of them is 28 years old

HS: how do you know?

LS: there is a middle age. You can either add one at the front or subtract one at the back

HS: maybe none of them is 28 years old. The average family is 28 years old,

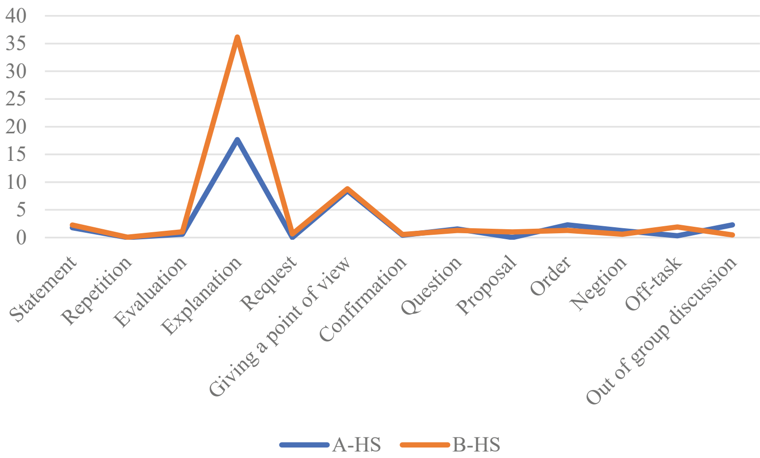


Fig. 8.3 The duration of dialogue categories of HS in A&B

LS: it's possible.

HS: they are all 28 years old, and there are four of them, 31, all of them…

LS: you see, you've got it. You minus 13 years old, that's 112

HS: then I want four numbers, no 28. Then you… What should you do then?

LS: wait a minute. I see. You take 112 minus 28, you calculate it.

HS: why do you do with 112 minus 28?

LS: as soon as you subtract 28, you will find the age of the second person, the first

HS: isn't that the age of the second person

LS: maybe it is!

HS: minus 28, divided by 3, equals 28

LS: why? You can also subtract 28 first

HS: minus 28, and then?

LS: subtract 28 and divide by 3

HS: it's still 28

In Episode 2, after the two students agreed that the age of the seventh-grade student was 13 years old and the cumulative age of the remaining four students was 112 years, they discussed whether the average age of the four students was 28 years old. Their discussion was in a relatively positive state throughout. The LS constantly raised questions. The HS's performance differed from Category A-HS's, offering a positive response instead of negating or interrupting the LS's questions and opinions. The two students constantly gave opinions and digest each other's opinions until a consistent conclusion is reached.

However, there are also problems in the dialogue between the peers because the process concerned discussing the students' average age, and the LS did not understand the concept deeply enough. If one was 28 years old, the average age of the remaining three must also be 28 years old, but the LS did not realise this key point and needed to discuss it with the HS. This discussion had two effects. On the one hand, LS's views were inefficient and did not promote problem solving; the HS had no problem understanding the average age but did not understand the LS's purpose, and so always followed their ideas, leading to low efficiency. On the other hand, it greatly promoted the LS's knowledge understanding because the LS guided the discussion, and the peers constantly exchanged views through equal communication.

[Episode 3]

HS: 28 years old. What if they have a student younger than the seventh grader?

LS: definitely, maybe mom and Dad, and then…

HS: it's impossible, impossible

LS: even up to mom and Dad, there are brothers and sisters

HS: two people, two

LS: Mom and Dad, brother and sister

No communication

LS: at the age of 28. the seventh grader. 112. Besides the seventh grader, there are four, four.

HS: then, if you look at it, we'll do it by hand

HS: so, what is 125 minus 13? it is 112, 112. If Mom. If Dad is 38 and mom is 35, it will be over 17. 28 minus 17 is 11.

LS: 11 years old, that's impossible. He cannot…

HS: there are only four people, and who is a 28 years old relative? Then if this is a second child, this second child will be eight years old and his father will be 40. Dad, mom.

Episode 3 is a group discussion after agreeing that the average age of the remaining four was 28. In this clip, the interaction between the peers is mutual. They first consider that the five people were a family, and then discussed the possible ages and identitied of the family members. When the discussion was is interrupted, the HS offered an idea, suggesing that the father was 38 years old and the mother was 35 years old. According to the average characteristics, 28 should be subtracted from the father's and mother's average ages to get the other member's age. This idea shows that the HS had a good understanding of the concept, was flexible, and constantly pushed the task forward, while the LS constantly collaborated and gave ideas.

While the LS helped identify the five households as one family, the HS promoted the age of each household, because the LS might have difficulty calculating the average but can provide other ideas. This shows that academic performance impacted students' mathematical problem solving to a certain extent. The communication between the peers shows mutual respect and equality. They threw ideas at each other and absorbed feedback to achieve the purpose of collaboration; it was not a one-person show.

In Episodes 1–3, the performance of Category B-LS differs from that of Category A-LS. The former are more active, have more dialogue behaviours, express themselves actively, and provide feedback to HS. The line chart below compares the discourse performance of LS in Category A and Bs (Fig. 8.4).

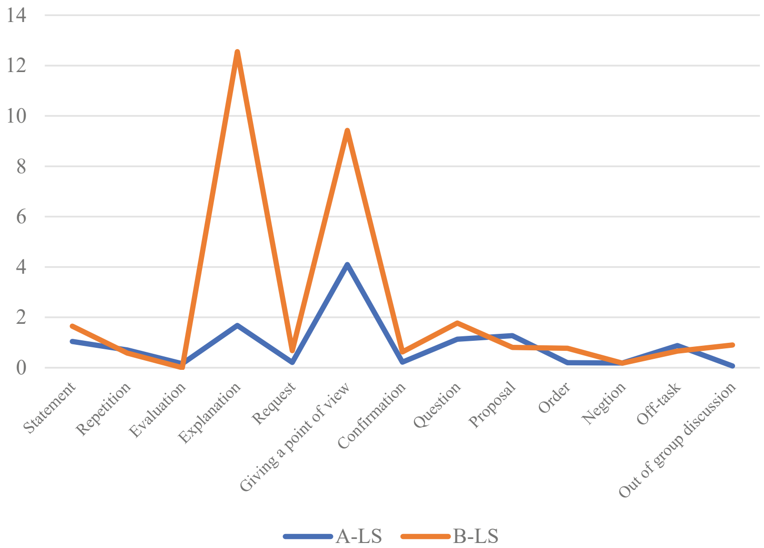


Fig. 8.4 The duration of dialogue categories of LS in A&B

In Category A and B, the performance of the LS is obviously different. Firstly, the frequency with which LS actively gave opinions was significantly higher in Category Bs, even slightly higher than that of HS in some groups. Additionally, the duration of their "explanation" and "giving a point of view" utterances was significantly longer. Secondly, both LS and HS used words of "order" in Category Bs, and so were more able to stick to their own opinions, consistent with the above episode. LS in Category Bs were no longer "marginal people" but participated in the collaboration and contributed to the pair collaboration in solving problems.

Based on the above analysis, students' interactions will be more active and equal when solving mathematical problems in paired collaborative groups if the performance gap between the peers is reasonable.

Category B-HS had different attitudes toward their peers than Category A-HS and communicated more with others in the same position. Category B-HS listened to their LS peers' opinions and gave appropriate feedback. Correspondingly, LS made great progress, actively expressing their views rather than passively receiving information in a negative position.

8.3.2 Task Set Location

As Category A-LS performance differed from Category B-LS performance, this study generated performance statistics on LS in CPS. The results showed that Category B-LS's participation degree was high, but Category A-LS's performance was inconsistent. The HS in Groups 1, 3, and 4 had significantly weaker participation than the HS in Group 2, leading the researcher to review the interactions in Group 2 carefully. The following two episodes show Group 2's dialogue.

[Episode 1]

HS: did you see the task?… Look the task again.

LS: five households in total.

HS: well.

LS: the average age of them is 25.

HS: well.

LS: one of them is a seventh grader.

HS: well

reflection

HS: how old is the seventh grade?

LS: 12 years old.

[Episode 2]

HS: change these two.

LS: can it be like this

HS: don't move!

HS: 38, this is 18. It's easy to give identity to the four people. The younger one is the aunt, this is his mother, this is his brother, this is his little brother.

LS: what about his father?

HS: cannot his father die?

LS: do you think his father cannot go on a business trip?

HS: come on, make up a paragraph

Episode 1 is a short interaction at the beginning of the task. The HS asks the LS to read the task again and approves what the LS said, ensuring they both (especially the LS) understand the task before starting the discussion. Group 2's HS behad differently than the HS in the other three groups, giving the LS time and opportunity to reflect and speak. However, the HS still dominated the interaction between the peers, who were not completely equal. In fragment 2, the LS was interrupted when trying to share views and denied by the HS, who showed impatience. Although the LS participated in the problem solving and interaction to a greater extent in Group 2 than in other groups, the interaction between the peers was not always active and equal, and there were some problems.

The position of the task set in Group 2 also differed. When solving problems, the task set was mostly placed between the peers in Group 2, while it was almost always placed on the HS's desk in the other three groups.

Based on the video of the nine groups' pair collaboration, the following conjectures are offered: when the task set was placed between two peers, the LS was more likely to participate in the interaction; when the task set was placed on the HS's desk, the LS was more likely to be dissociated or participate in other groups' discussions.

The researcher analysed that when the task set was placed in the HS's position, the LS could not get task-related text information and sometimes did not take the initiative to participate. In cases with no interaction, the LS could not get task-related information (text or verbal) and were less likely to participate in the task.

This study counted the interaction duration in the nine groups and found task set location may impact pair collaboration. The task set locations were divided into three categories: on the HS's side, on the LS's side, and between the two. Statistics on the proportion of these three categories help us find the relationship between task set location and LS performance. The proportion of HS-side task set location is shown as a line chart.

The Fig. 8.5 shows that the task unit was set on the HS side in Group 2 far less often than in Groups 1, 3, and 4. The task set in Group 2 was between the two students almost 85% of the time, allowing the LS to obtain task information effectively and participate in CPS. The HS did not control the task set location. At the beginning of the task, the HS chose to put the task set between the peers and made reasonable use of the blank paper given by the researchers.

In the other three Category As, task placement was generally on the HS's side, and LS did not participate actively, leading to the following reasonable conclusions. First, task set location impacts peers' interaction enthusiasm; when LS had more access to task information, they could participate in the discussion more actively. Second, when the academic performance gap between the peers was large, HS were likely to take absolute control of task set location, viewing the task set as their own and ignoring pair collaboration. Task set position directly reflected HS's attitude toward pair collaboration.

In Group 5, the side task set appeared less often on the HS side as the academic performance gap between HS and LS narrowed, appearing more often on the LS side or between HS and LS. This study found that in almost all groups, either HS

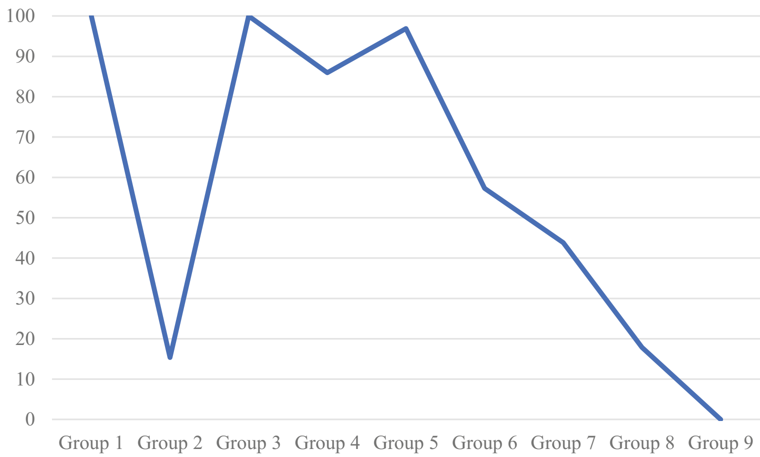


Fig. 8.5 The proportion of task set on the side of HS

Fig. 8.6 The picture of Group 1

had great decision-making power over the task set location or LS had little desire to control the task set. As the performance gap narrowed, the HS respected the LS significantly more and became more concerned about the LS's task understanding.

Task set location was mainly controlled by the HS. When the task set was always on the HS's side, the LS could not gain task initiative and behaved correspondingly. As the performance gap between the peers narrowed, the LS followed one of three paths: never winning the task list and being controlled by the HS; fighting for the task set and then giving up; or striving for the task set and ultimately succeeding.

In the first three groups' CPS processes, the LS did not show dissatisfaction with the task set location. The HS decided the task set position and the LS neither tried to change the task unit nor actively proposed writing answers on the task set. In Group 2, the task set was not placed on the LS's desk but between the two on the HS's initiative. HS generally controlled the task set; when the academic performance gap between the two was very large, LS could not control the task set (Figs. 8.6 and 8.7).

The episode above depicts the task set in Group 4. The girl was the HS, while the boy was the LS. After two minutes of discussion, the boy snatched the task set but it was immediately taken back by the girl.

LS: my God! This is your painting! Not even legs! I'll give the painting some more (the LS tries to take the task set)

HS: don't move! This is my own creation!

LS: what's the creation? (the LS take the task set successfully)

HS: what are you doing? It cannot be destroyed!

LS: This is the father of the seventh grader, so I'll draw all of them for you. Damn it, no one can get the task set. (The HS tries to get the task set back)

HS: let's all add feet to the painting. (The HS gets the task set successfully)

For about 33% of the time, the LS fought for the task set, adopting a tough attitude and saying, "Either we are half [or] alone." The HS compromised and guided the LS

Fig. 8.7 The picture of Group 3. *Note* The task set is on the desktop for HS

about what he should write ("Here you write, one is brother, one is sister"). However, the boy had few task-solving views and ideas and asked the girl, "a brother, and then what?." After the girl answered, the boy said nothing that promoted task development. After 30 s, the girl took back the task set and held it until the end of the task (Fig. 8.8).

Group 5's task set was almost always on the HS's side, but the two peers' interaction was slightly more positive than in the previous four groups and their dialogue lasted longer, accounting for about 80% of their time. During the discussion, the LS actively looked at the task to get task information (Fig. 8.9).

In Group 6, the task set was on the HS's desk for the first six minutes. Later, the LS began to fight for the task set and succeeded in getting it. When the HS proposed modifying the answer, he was opposed by the LS and did not resist. The LS held the task list for the remaining time. Their interaction was positive, with both peers voicing their views and refuting those with which they disagreed (Fig. 8.10).

The HS no longer insisted on holding the task set in the last three groups. In groups 8 and 9, the HS took the initiative to put the task set on the LS desktop or

Fig. 8.8 The picture of Group 4

Fig. 8.9 The picture of Group 5

Fig. 8.10 The picture of Group 6. *Note* In this picture, the front student is the LS, and the back student is the HS

between the two. In Group 8, the LS expressed that he wanted to write the answer on the set and strove for the initiative to complete the task. Although the LS failed, their behaviour did not appear in the first seven groups. In Group 9, the HS showed great respect for the LS, actively suggesting that the LS write the task answer.

The two peers' performances were significantly different in Category A and Bs, indicating that when the academic performance gap between peers was controlled within a reasonable range, HS paid more attention to the LS's views, and both had equal opportunities to get the task set, giving LS a stronger sense of participation in the CPS process.

8.4 Discussion

8.4.1 Discussion

1. Students in heterogeneous pairs perform differently in peer interactions in collaborative mathematics problem solving.

HS and LS had different mathematics problem solving performances in different pairing modes, mainly manifesting in two aspects: the amount of utterance between them and their interaction performance indicators. HS had a relatively large amount of utterances than LS in the task completion process and more frequently used the interaction indicators "order," "negation," and "evaluation." LS's had fewer utterances than HS and more frequently used "repetition" and "proposal" interaction indicators. In most cases, the HS controlled the discussion throughout the interaction; the LS, having neither the opportunity nor the courage to express their views, gradually deviated from the group discussion and became silent, consistent with Hou's (2017) findings.

Some scholars divide help-seeking into two types: execution requests and tool requests. Execution requests are oriented by dependence, while tool requests are oriented by mastery. The students with strong mathematical ability tend to offer instrumental help, while those with low mathematical ability offer executive help. This conclusion explains the differences in dialogue indicators between HS and LS. Nelson Legal and colleagues suggest that low-level students may not be effective helpers because they may not only lack the skills to recognise their need for help, but may also be unable to seek the best learning help.

2. Based on this study's collaborative mathematics problem solving exercise, the interaction in heterogeneous peer collaboration groups is affected by the academic performance gap between peers.

In collaboration, intellectual authority is more stable while social authority is more dynamic (Langer-Osuna et al., 2020). In the peer collaboration groups, the difference in academic level between HS and LS affected the group members' interaction performance. The nine peer groups were divided into two categories based on interaction time. Category A, which included Groups 1–4, had the least interaction time; Category B (Groups 5–9) had the most. There were differences in interaction performance between Categories A and B.

(1) Peer collaboration groups with a large academic performance gap rarely interacted.

In Category A groups, peer interaction was less, and LS seldom participated. The LS used significantly more "repetition," "proposal," and "off-task" interaction indicators than HS, who accounted for a higher proportion of "order" and "negation" indicators. In Category A groups, the task set was most often located on the HS's side, and sometimes the LS could not see the task information. In Category A groups, LS were students with learning difficulties.

According to the existing research, in collaboration groups, students with learning disabilities hold a negative attitude towards other group members in terms of willpower and behaviour characteristics, which seriously affects their communication in the group (Li, 2015). Other studies have also pointed out that the communication level between students with learning difficulties and their peers is relatively low (An et al., 2017). In addition, in terms of their psychological characteristics, students with learning difficulties mainly manifest hostility, interpersonal sensitivity, and so on. Low-level students have difficulty actively integrating themselves into collaborative mathematical problem-solving discussions without their peers' support and help, which hinders the formation of an active peer group. Case interviews in other studies revealed that many HS are unwilling to communicate with LS. They lack the psychological preparedness and skills to communicate with LS, and they worry that LS will negatively impact their learning and task completion (Jiang, 2007).

If the performance gap between peers is too large when collaborating to solve mathematical problems, the HS will put forward more views and opinions. LS have difficulty understanding and accepting new concepts and cannot keep up with the progress, so they passively or actively give up the opportunity to participate in the discussion (Zeng & Zhang, 2016), reducing peer interaction and collaboration quality.

(2) When the academic performance gap is reasonable, group interaction performance is better.

In Category B groups, the LS's academic performance was slightly higher than in Category A groups, and the academic performance gap between the peers was relatively small, leading to the following observations. First, the number of utterances between the peers increased significantly, and the interaction was more positive. Second, from a dialogue indicator perspective, HS used far fewer "order" and "negation" indicators; LS had more space to express their opinions, significantly improved their views and explanations, used more "order" indicators, and could even lead the interaction. Third, from a task set location perspective, the task set no longer appeared only on the HS's desktop but spent more time between peers or on the LS's desktop.

Some scholars divide students into excellent, middle, and poor categories and set control and experimental groups to explore the effects of collaborative learning on the three types. Research has found that excellent students have the highest acceptance level in peer relationships. Although there is no significant difference between the control and experimental groups, students with learning difficulties had lower acceptance levels than the control group in the collaborative learning process, while middle students' acceptance level was higher (Jiang & Tan, 2011), which can explain the difference in Category B and A students' interaction performance to a certain extent.

3. In pair CPS in mathematics, task set location affects peer collaboration.

In Category A, Group 2 showed relatively abnormal performance, with high dialogue time and high LS participation. The video revealed that the task set was placed between the peers in Group 2, and the LS had more opportunities to understand

and think about the task. This study also found that in Category B groups, the task set was less often placed on the HS's desktop than the LS's, and the interaction between them was more positive. LS gradually strove to control the task set. As mentioned above, LS who could not see the task information found it difficult to participate in the CPS. When the peers were in a "no communication" state, the LS could not get the task information and had even more difficulty concentrating on it. The widely-used Team Games Tournament collaborative learning method points out that every member needs to know the task materials to contribute to higher team scores. Therefore, task set location is extremely important and must ensure that both students can see and understand the task information to collaborate better.

8.5 Conclusions and Implications

This study has mainly focused on student interaction in pair collaboration. It found that LS did not find it easy to participate in cooperative discussions and that the discussion between the peers was sometimes unequal. The following implications were obtained.

1. Cultivate students' sense of collaboration.

A good collaborative group can improve students' performance because group members can encourage and help each other in peer learning (Slavin, 1991). HS can dominate the collaboration process, and their attitudes and expressions affect, to some extent, how well LS can participate. Therefore, before collaboration, students should be trained to realise that collaborative problem solving requires every student's participation and is not a one-person show.

Some studies point out that collaborative groups should establish a positive goal of interdependence through mutual learning goals, including learning materials, to ensure all group members can learn and understand the specified materials (Johnson & Johnson, 1999). Students need a positive sense of collaboration to ensure fairness in the collaborative problem solving process, so everyone can learn and understand the task materials.

If LS raise questions or suggestions in the peer collaboration process, HS should give timely feedback and reflection. Webb (1991) pointed out that how students' requests for help are responded to is more important than the kind of help they get and that accepted help is only effective when applied to solve problems.

2. Teachers should make appropriate interventions.

In the "no intervention" state, some groups cannot cooperate well and lack collaboration consciousness, requiring teachers to make timely adjustments. Students with learning disabilities' low-level information dependence makes it difficult for them to participate actively, and external intervention is needed (Chen, 2020; Zhang et al., 2021). Johnson pointed out that teachers should supervise students' learning and intervene appropriately to develop students' interpersonal communication and group collaboration skills (Johnson & Johnson, 1999). Students do not interact spontaneously in classroom groups (Fuchs et al., 1994) and must be trained before collaboration or given proper teacher guidance during it to improve the enthusiasm and quality of their discussion.

Besides intervening in organisational collaboration, teachers should also consider students' evaluation methods. In peer groups with a large gap in academic performance, one of the reasons for LS's low collaborative behaviour efficiency is that they are not recognised; a single evaluation based on academic performance will aggravate this phenomenon. Evaluations should be diverse and consider students' behaviour and performance from many aspects and angles, not only academic performance.

3. A collaboration group's organisation shall meet the reasonable matching mode.

Task type, group composition, and teacher support are the main reasons informing the effectiveness or ineffectiveness of collaborative learning (Kahilainen et al., 2007). Most studies advocate intra-group heterogeneity. Vygotsky's zone of proximal development theory advocates that students collaborating with more capable students benefits both. However, in pair collaboration to solve mathematical problems, if the academic performance gap between the peers is too large, the communication between them will be unequal, the LS's expression will be unrecognised or interrupted, and the LS's thoughts cannot keep up with HS's, making it difficult to continue the interaction and achieve the benefits of collaboration. When the academic performance gap between peers narrows, their interaction is relatively positive, and their utterances can collide, enabling them to exchange views and agree on goals through mutual debate and compromise and allowing all students, especially those with low knowledge levels, to benefit from collaboration.

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Chapter 9
Differences Between Experienced and Preservice Teachers in Noticing Students' Collaborative Problem-Solving Processes

Rangmei Li

9.1 Introduction

Collaborative problem-solving (CPS) has become a critical competency for teenagers' future lives (OECD, 2017). Currently, K-12 mathematics education attaches more importance to cultivating students' CPS skills through group teaching. However, some teachers do not know how to guide the collaborative process and facilitate student interaction (Le et al., 2018), especially in developing countries that rarely adopt group teaching methods. Inefficient collaboration places greater demands on mathematics teachers' professional group teaching abilities. Many previous studies have shown that teachers' effective guidance can promote the development of group activities (Van Leeuwen & Janssen, 2019). There is a close connection between teachers' noticing and classroom teaching behaviors (Blömeke et al., 2022). To respond to students' performance appropriately, teachers should attend to their behavior first and then interpret the meaning based on prior experiences (Jacobs et al., 2010).

Teachers' noticing, a critical component of teaching expertise, is the process of how teachers manage the vast amounts of sensory information they face in the classroom (Sherin et al., 2011). The prior educational studies show that teachers' noticing involves two main facets: attending to important classroom events and making sense of these events in an instructional setting (Santagata et al., 2021). In teacher education, teachers' noticing ability plays an important mediating role

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between teachers' resources (such as their knowledge and beliefs) and their teaching performance (Blömeke et al., 2015). Early research on teachers' noticing suggests that experienced teachers have stronger perceptions of classroom activities and can selectively allocate attentional resources to important events (König et al., 2022). However, to our knowledge, the informative results of past studies mainly focus on the traditional teaching classroom; teachers' noticing of students' group collaboration in China remains unexplored. Studying what they notice during students' collaborative process helps uncover the cognitive processes behind teachers' instructional strategies and behaviors. Thus, this study aims to investigate what teachers tend to notice during students' CPS processes and further explore the differences between what experienced and preservice teachers notice.

One of the most common methods of studying teachers' noticing is the video-based interview, which asks participants to describe and explain what they noticed after viewing their own or other teachers' instruction videos. It is well known that one's fixation is closely related to their cognitive process (Just & Carpenter, 1980). Using eye-tracking technology, we can obtain precise information on moment-by-moment teachers' visual fixation (i.e., where and when they fixed their eyes) and further explore their cognitive processes without interrupting their activities. Therefore, this study adopts both eye-tracking technology and video-based interviews to investigate the noticing differences between experienced and preservice teachers when viewing CPS processes. The research questions are as follows: What do teachers notice in collaborative problem-solving processes? What are the differences in teachers' noticing between experienced and preservice teachers?

Next, we review existing eye-tracking studies on collaborative problem-solving and summarize some key conclusions regarding the characteristics of teachers' noticing.

9.1.1 Factors Influencing the Quality of Collaboration

The collaborative problem-solving process involves two important elements requiring team members' attention: collaboration and problem-solving. The former emphasizes social interactions between group members; the latter highlights the cognitive interactions in the task itself, including extracting information, exploring strategies, and executing plans. During the collaborative process, group members must establish mutual understanding, maintain team organization, and reach a consensus by communicating with others. We can see that people's gaze plays a central role in the CPS process. Eye-tracking technology provides researchers with an unprecedented opportunity to obtain participants' eye-movement information in their natural environment, including where they focus and for how long.

Many researchers have used eye-tracking technology to study the key factors in collaboration quality. In addition to group relationships and prior knowledge of other group members (Sangin et al., 2008; Villamor & Rodrigo, 2017), the visual synchronization of participants is also critical to collaborate effectively (Salminen-Saari et al., 2021). Cherubini et al. (2008) found that students tend to point to an object when discussing it, to draw their partners' attention at the same time. If group members' gaze points draw closer, misunderstandings will be decreased, and collaborative activity will be more efficient. Thus, team members need to achieve joint attention through collaboration, and there is a close relationship between the fixation overlap rate and the quality of interactions (Jermann & Nüssli, 2012; Jermann et al., 2011). Some studies have even found that higher levels of visual synchronization are positively associated with students' collaboration quality (Cakir & Uzunosmanoğlu, 2014; Schneider & Pea, 2013; Uzunosmanoğlu & Çakir, 2014).

Accordingly, researchers have tried to promote group members' joint attention and mutual understanding by altering their noticing, such as by sharing text selections, providing partners' real-time gaze behaviors, and designing an online collaborative environment called the Virtual Math Team (Jermann & Nüssli, 2012; Schneider & Pea, 2013; Uzunosmanoğlu & Çakir, 2014). In addition to offering visual information on members, teachers' guidance could also facilitate members' joint attention externally and improve group collaboration quality. However, the studies applying eye trackers to the CPS process have mainly focused on problem solvers and rarely focused on teachers. Researchers have also focused on teachers' roles in supporting group problem-solving. For instance, Haataja et al. (2019) found that teachers' scaffolding intentions significantly affect their gaze behaviors; specifically, gazing at students' faces tends to be normal, regardless of what kind of scaffolding intentions. This impressive finding inspired us to further study teachers' cognitive focus with eye-tracking technology.

Teachers play an important role in group collaboration, as their guidance can promote the development of group activities. Van Leeuwen and Janssen (2019) synthesized 66 quantitative and qualitative studies on collaborative learning and found that teachers paid careful attention to giving feedback on students' problem-solving strategies, helping students organize tasks, and coordinating group participation, which positively influenced students' collaboration. To provide practical guidance, teachers should pay attention to students' performance. Wells (2017) explored how focusing on students' conversations and gestures affects teachers' interventions during the group problem-solving process, and found that dialogues emerged as students progressed on a problem and students' gestures became more pronounced with increased confidence. In addition, students participating in discussions often unconsciously imitated other group members' body postures. Teachers can better understand students' performance and make more meaningful decisions by focusing on these behaviors. Therefore, it is necessary to study teachers' noticing in the collaborative problem-solving process to improve their professional noticing skills.

9.1.2 Characteristics of Teachers' Noticing in Classroom Settings

Teachers' noticing plays a crucial role in classroom teaching, and a better understanding of teachers' noticing can thus help to improve mathematics teaching and learning. In the classroom, teachers often need to distribute many cognitive resources to process different kinds of behavioral information, adjust teaching schedules and achieve effective classroom management (Berliner, 1986; Hogan et al., 2003). Two definitions of teachers' noticing have been most frequently used in previous studies. In the first, Van Es and Sherin's (2002) learning to notice framework, noticing has three components, including "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader teaching principles; and (c) using what one knows about the context to reason about classroom interactions." The second definition concerns the professional noticing of children's mathematical thinking, which involves "attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (Jacobs et al., 2010). Combining these two definitions, this study defines teachers' noticing as identifying classroom activities and then understanding or interpreting these activities.

Many recent studies have suggested that noticing patterns are highly influenced by teachers' prior teaching experience. There are considerable differences between what experienced and novice teachers notice (Erickson, 2011). For example, experienced teachers distribute their attention more evenly across the classroom than preservice teachers (Van den Bogert et al., 2013). Experienced teachers' attention to critical objects occurs earlier and lasts longer (Miller, 2011). Thus, experienced teachers make more comprehensive observations and are skilled at distinguishing between important and unimportant information in complex situations. In addition, teachers with different levels of teaching experience also differ in how they interpret what they notice. In contrast to the simple descriptions provided by novice teachers, experienced teachers can better monitor, understand, and interpret events in more detail and make inferences or provide suggestions about what they see (Carter et al., 1988; Sabers et al., 1991).

Experienced and preservice teachers also differ in their reactions to students with problematic behaviors, with more experienced teachers using more modes to handle misbehaviors. Sabers et al. (1991) explored the noticing differences between seven experienced, four novice, and five preservice teachers while observing teaching videos presented on three computers simultaneously. The participants were asked to pay attention to specific facets of classroom management. The results showed that the experienced teachers paid attention to the inappropriate behaviors of some students, objectively inferred the possible reasons for those behaviors, and put forward specific ways to correct them. In contrast, the novice and preservice teachers only expressed their dissatisfaction and criticized the students' problematic behavior. Similarly, other studies have reported that experienced teachers can address such situations properly without dedicating too much attention to them in some cases (Ding et al., 2008; Wang et al., 2013). Learning to distinguish between important and unimportant information is thus a critical professional teaching skill in complex teaching situations (Berliner, 2001).

In summary, the rapid development of science and technology has given researchers more reliable and accurate tools to deeply analyze the collaboration process. It is essential to investigate what teachers tend to notice during CPS processes and determine the differences between what experienced and preservice teachers notice to help preservice teachers better understand classroom information and facilitate group collaboration.

9.2 Method

9.2.1 Data Collection

9.2.1.1 Participants

We hypothesize that teachers' ability to notice is closely related to their teaching experience and that experienced teachers could recognize important information under complex circumstances. We recruited 18 experienced teachers (77.78% female) with more than 15 years of secondary school mathematics teaching experience and 28 preservice teachers (67.86% female) who specialized in mathematics education but had no mathematics classroom teaching experience in China. All teachers participated in this study voluntarily. Finally, 13 experienced and 15 preservice teachers were selected for eye-tracking data analysis, who met the requirements that the eye-movement data had an accuracy or precision level of less than 0.5° and a sampling rate of more than 70%.

9.2.1.2 Materials

The research materials used include a video of the students' group collaboration and an outline of the structured interviews. This video was selected from the data from the project, "The Social Essentials of Learning: An experimental investigation of collaborative problem solving and knowledge construction in mathematics classrooms in Australia and China." The short video (10 min and 30 s) features four students collaborating on a mathematical task titled "The Tower Problem," which explicitly uses graphical elements (see Appendix (Task 3) for detailed information). As the students had just finished primary school and most had difficulties drawing solid figures individually, group collaboration was an appropriate way to solve the problem by providing the students with six blocks to imagine potential answers. The efficacy of such a task in stimulating student collaborative work has been tested by Clarke and Chan (2015).

It is essential to illustrate why we chose this group in this study. The group was heterogeneous, including two boys and two girls, and showed great individual differences between group members, including their mathematics performance and participation in group activities. While the students did not find the right answer, they had clear ways of thinking about the solution. In addition, the group collaboration video recording was of high quality, with clear imagery and audio and no one obstructing the camera. Therefore, this video met the research requirements.

To understand the participants' feelings after watching the video, they were asked four questions: (1) What do you think of the group collaboration in the video? (2) Did you see any inattentive or distracted students during the discussion? If so, what is your opinion about this? (3) Were you impressed by someone or something? Why? 4) Do you have any other thoughts or personal feelings about this group?

9.2.1.3 Apparatus and Procedure

A Tobii Pro X3-120 eye tracker with a sample rate of 120 Hz was used to record participants' eye-movement information as they watched the video. The eye tracker is compact and light (324 mm long and 118 g) and uses pupil-centered corneal reflection, a technique combining dark and bright pupil tracking. It was installed at the bottom of the 15.6-inch screen on the laptop used to play the video. Tobii Pro Lab was used as supporting data analysis software. In addition, a digital voice recorder was used to record the interviews. This study chose the "Tobii I-VT (Fixation)" option and kept the default 30°/s threshold. Data points with angular velocity below this threshold value are classified as part of a fixation; data points above are classified as part of a saccade.

Data were collected from each participant for approximately 30 min. Before starting, the participants were informed of the experimental process. First, each participant completed the group task individually to become familiar with it. Then, the participants were asked to sit on a chair positioned approximately 60 cm from the recording laptop, and the eye tracker was calibrated to their eyes. Once calibration was complete, the participants watched the video and then were interviewed. The participants were allowed to turn their heads freely while watching and were encouraged to share their thoughts during the interviews. In addition, to ensure the participants were not misled or interrupted, the interviewer did not express any views or opinions. Instead, the interviewer asked them repeatedly if they had any thoughts on the question posed.

9.2.2 Data Analysis

9.2.2.1 Analysis of Eye-Tracking Data

The CPS process was divided into two stages—a task-analyzing process (where all members identified the problem) and a problem-solving process (where all members focused on solving it). Due to considerable differences in the students' performance during collaboration, we selected three different group types (four students, three students, and two students) to identify the subtle differences between experienced and preservice teachers as they faced students of varied performance. Nonattentive or uncollaborative students, such as those joking with others or playing with personal belongings, were recognized as not involved in the group activity. Hence, there were four video segments in total, including one segment analyzing the task and three segments solving the problem, as shown in Table 9.1. For our analysis, each student in the segment was coded according to their participation based on the following three roles: speaking student (S), listening student (L), and misbehaving student (M). While each student could play different roles in different segments, the sequence number remained unchanged.

Before analyzing the eye-movement data from the different segments, four areas of interest of the same oval size were defined; each area represented a different student in the group, as shown in Fig. 9.1. We applied an index of the total fixation duration of the four areas of interest, namely the sum of all durations of the fixation points. The standard deviation of the fixation duration was used to evaluate uniformity in the distribution of visual attention; a higher value indicated less uniformity. Statistical significance was tested by using an independent sample *t*-test of the uniform distribution of attention between two types of teachers and applying a variance analysis

Table 9.1 Four segments

Segment	Time	Student performance
Segment 1: four students analyzing the task	0:58–1:40 (42 s)	S1, S2, and S4 discussed the problem and tried to understand it; L3 did not participate in the discussion
Segment 2: four students participating in problem-solving	7:04–7:39 (35 s)	All of the students discussed the condition involving more than three painted sides; S1 and S4 had a verbal interaction; L3 came up with her own ideas; S2 whispered to himself
Segment 3: three students participating in problem-solving	8:07–8:35 (28 s)	S1 and S4 had a heated discussion about one side being painted; L3 observed S1 and S4; M2 kept his head down and played with his calculator on his desk
Segment 4: two students participating in problem-solving	3:44–3:56 (12 s)	S4 described her thought process allowed while arranging blocks; L3 observed S4; M1 and M2 played with a recording device on the desk

Fig. 9.1 Division of four interest areas

of the fixation durations of different areas of interest. Due to our small sample size, we also applied the effect size. Cohen's d was selected to measure actual differences between the two types of teachers (Coe, 2002).

9.2.2.2 Analysis of Interview Data

To study teachers' noticing of students' mathematical thinking, Van Es and Sherin (2008) asked teachers to watch three video clips of mathematics classroom teaching and comment on what they noticed before and after receiving video training. They used the following codes to analyze the interview data: actor, topic, stance, level of specificity, and video focus. Two of these dimensions, topic and stance, were used to analyze our data and examine what the teachers attended to and how they analyzed this information. According to previous research, teachers focus on the fundamental elements of the mathematics teaching process, such as pedagogy, mathematical content, management, classroom environment, etc. (Frederiksen et al., 1998; Star & Strickland, 2008; Van Es & Sherin, 2008).

However, group teaching involves a more complex teaching process, with some unique features relative to traditional mathematics teaching. Teachers need to do more to prepare for group teaching activities, such as setting group teaching goals, establishing appropriate groups based on students' characteristics, designing tasks for group collaboration, etc. During group collaboration, teachers must support the problem-solving process and serve as facilitators by, for example, dealing with conflicts between group members and helping students who are not participating to focus on the group activity. In addition, there are more opportunities for students to express their opinions; in addition to cognitive engagement, students can be involved in social interactions with other members. Hence, we divided the topic of teachers' noticing into three categories: preparation for the group activity, support for collaborative learning, and group members' performance. Each category covers specific facets, as shown in Table 9.2.

Furthermore, regarding teachers' comments on the presented video, some research has focused on the level of description, interpretation, evaluation, etc. (Sabers et al., 1991; Seidel & Stürmer, 2014; Van Es & Sherin, 2008). We added two levels based on our interview results: prediction and suggestion. All levels of teachers' noticing examined are detailed in Table 9.3.

Therefore, we analyzed the interview data from two dimensions: what teachers noticed and how they commented on what they noticed. The analysis proceeded by recording the stance each teacher adopted when referring to the given topic and then measuring the corresponding percentage. Teachers using multiple stances to analyze the same topic were assigned the highest values. The transcripts were coded blindly, and any differences were discussed until the two coders reached a consensus.

Table 9.2 Three categories of teachers' noticing

Category	Definition
Preparation for group activity	Includes teaching goal, task design, and group construction (e.g., the specific goals of a class, mathematical problems to be solved, and group identification)
Support for collaborative learning	Includes learning resources, teacher guidance, and disciplinary management (e.g., materials and instruments available, teachers' intervention and measures taken, and concerns regarding misbehaving students)
Group member performance	Includes mathematical thinking, problem-solving approaches, and group interactions (e.g., the abstract mathematical thinking used, the specific operation of the problem-solving method, and verbal interactions between group members)

Table 9.3 Five levels of topic analysis

Level	Definition	Examples
Description	Describe what has happened	The teacher walks around and reminds the students to write down their group numbers
Explanation	Make inferences or speculations about what they see	A student does not focus on the problem due to her disinterest or misunderstanding of what other group members are discussing
Evaluation	Make a judgment	The two other students' spatial imagination abilities are stronger; however, this does not mean they learn mathematics well, as this is difficult to measure
Prediction	Anticipate future developments	If continued, the group activity is anticipated to create polarization in the classroom because the identified student is not benefitting
Suggestion	Propose appropriate solutions	The teacher tries to present real problems relevant to the students' real lives

9.3 Results

We recruited 18 experienced teachers and 28 preservice teachers to watch a video of four students collaborating on a task, and then interviewed them individually. Eye-movement data with an accuracy or precision level of more than 0.5° and a sampling rate of less than 70% were eliminated to ensure data quality. Then 13 experienced teachers (ETs) and 15 preservice teachers (PTs) were selected for analysis. The main results are shown below.

9.3.1 Differences in the Distribution of Visual Attention

Based on the mean standard deviation of fixation duration, shown in Table 9.4, the degree of uniformity in the distribution of visual attention of experienced teachers was smaller than that of the preservice teachers in all four segments.

The *t*-test statistical results revealed a significant marginal difference in attention distribution between experienced and preservice teachers ($p = 0.096$) for segment 1 (four students' task analysis stage). The *d* value indicated a relatively large difference ($d = -0.685$), meaning experienced teachers' attention was more evenly distributed than preservice teachers' attention. No statistically significant difference was found between the two groups in segments 2 and 3 (four and three students participating in problem-solving, respectively). However, when only two students participated, experienced teachers distributed their attention significantly more evenly than preservice teachers ($p = 0.002$, $d = -1.269$).

Variance analysis comparing these two groups' fixation duration for the four students in each segment showed a significant main effect between the subjects (F (1, 26) $= 3.783$, $p = 0.063$), and longer fixation durations for preservice than experienced teachers. The fixation durations of ETs and PTs for the four areas of interest in each segment are shown in Fig. 9.2. The differences between them are discussed below.

In segment 1, speakers 1, 2, and 4 gladly expressed their personal views and participated actively in the group discussion, while listener 3 mainly read the questions by herself or listened to others' ideas. We find a marginally significant difference

Table 9.4 Degree of uniformity in the distribution of visual attention

Segments	ETs	PTs	ETs-PTs	
	Mean (standard deviation)		*p* value	*d* value
Segment 1	1.705 (0.673)	2.141 (0.614)	0.096	−0.685
Segment 2	2.256 (0.803)	2.325 (1.091)	0.856	−0.070
Segment 3	2.825 (1.344)	2.948 (1.351)	0.818	−0.091
Segment 4	0.360 (0.149)	1.089 (0.717)	0.002	−1.269

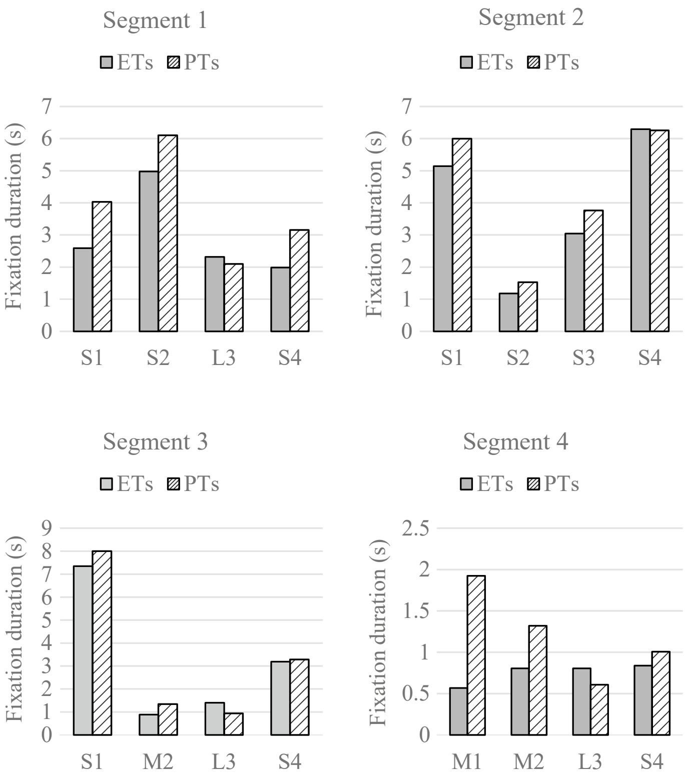

Fig. 9.2 Fixation durations of ETs and PTs for four areas of interest

between experienced and preservice teachers in noticing speaker 1 ($p = 0.078$, $d = -0.721$), with preservice teachers paying more attention than experienced teachers, as shown in Fig. 9.2. While there was no significant difference regarding the others, the difference in fixation duration between speakers 2 and 4 was still large ($d = -0.569$, $d = -0.639$). A larger sample should be used to test these statistical results further. In sum, it appears the preservice teachers paid more attention to speakers than experienced teachers.

In segment 2, speaker 3 suddenly said, "If there are more than three sides of the cubes painted, four sides are painted," then immediately noted, "The cubes at the top are impossible." Before this, she had hardly said a word. No significant differences in noticing for each student were found, but the difference for speaker 3 was larger, $d = -0.530$. The preservice teachers showed longer fixation durations for speaker 3 than the experienced teachers. Preservice teachers seemed more likely to view the students' discussions as a series of unrelated events occurring in chronological order. Therefore, when one student suddenly engaged in abnormal behaviors, the preservice teachers quickly focused on her. In contrast, the experienced teachers observed more systematically and were not distracted by speaker 3's new behavior.

In segment 3, speakers 1 and 4 had a heated discussion; listener 3 focused on them but said little. However, student 2, who was misbehaving, began to play with the calculator on the desk and paid no attention to the group activity. The results showed no significant differences between experienced and preservice teachers; still, the actual difference found for misbehaving student 2 was large ($d = -0.531$), showing that the preservice teachers paid more attention to misbehaving students. Moreover, the difference for listener 3 was also large ($d = 0.532$), indicating that experienced teachers looked at listener 3 longer than preservice teachers. In other words, the preservice teachers paid more attention to students with problematic behaviors for longer periods, and less attention to students who did not speak.

In segment 4, two students exhibited obvious problematic behaviors while the others focused on problem-solving. The misbehaving students played with the microphone on the desk, with student 1 mumbling some strange, amusing words into the equipment. The results showed that the preservice teachers focused significantly longer on misbehaving student 1 than the experienced teachers ($p < 0.05$, $d = -0.806$), and the two groups of teachers differed considerably in how long they watched misbehaving student 2 ($d = -0.596$). Specifically, the preservice teachers paid more attention to students exhibiting inappropriate behaviors than the experienced teachers.

9.3.2 Differences in the Features of Collaborative Problem-Solving Noticed

As shown in Table 9.5, almost all experienced teachers paid attention to all three categories, including preparation for the group activity, support for collaborative learning, and group member performance. They were more inclined to put forward suggestions than preservice teachers. However, preservice teachers focused primarily on group member performance, and many made evaluations or gave advice. Only half of the preservice teachers focused on preparation for the group activity, and nearly one-third made recommendations regarding teacher support for the collaborative problem-solving process. Thus, the more experienced teachers made more comprehensive observations and offered more practical suggestions. In addition, preservice teachers paid less attention to teaching support and were more likely to comment on participating members' performance. The specific content noticed in each category was analyzed next; the main results are below, with specific examples for the comment level in each category.

For the first focus (group activity preparation) with examples in Table 9.6, a higher percentage of experienced teachers than preservice teachers focused on the three sub-themes, as shown in Table 9.7. Preservice teachers tended to evaluate and offer suggestions about group construction and rarely thought about teaching goals. Only one preservice teacher made a judgment regarding teaching goals; he thought the group collaboration had low inefficiency and students failed to achieve the goal of

Table 9.5 Percentage levels for teachers during topic analysis

	Preparation for group activity		Support for collaborative learning		Group member performance	
	ETs (%)	PTs (%)	ETs (%)	PTs (%)	ETs (%)	PTs (%)
Description	5.56	0.00	5.56	10.71	0.00	0.00
Explanation	0.00	3.57	0.00	7.14	0.00	7.14
Evaluation	0.00	32.14	11.11	10.71	11.11	46.43
Prediction	0.00	0.00	0.00	0.00	5.56	3.57
Suggestion	88.89	17.86	77.78	35.71	83.33	42.86
Total	94.44	53.57	94.44	64.29	100.00	100.00

Table 9.6 Five levels of teachers' comments on the preparation for group activity

Level	Examples
Description	Group work is designed to stimulate active student participation in the classroom
Explanation	This may cause students to become too familiar with each other to focus on the task if working in the group by front and back tables
Evaluation	The mathematics problem designed has similarities with students' life experiences and will arouse their interest
Prediction	If two students' participation is low in each group, nearly half of the students in a class would gain very little and not achieve the teaching goals
Suggestion	Teachers should try to create meaningful, authentic problem situations where students can find value in studying the problem

Table 9.7 Focuses in preparation for group activity

	Teaching goal		Task design		Group construction	
	ETs (%)	PTs (%)	ETs (%)	PTs (%)	ETs (%)	PTs (%)
Description	16.67	0.00	0.00	0.00	0.00	0.00
Explanation	0.00	0.00	0.00	0.00	0.00	3.57
Evaluation	11.11	3.57	11.11	28.57	0.00	14.29
Prediction	5.56	0.00	0.00	0.00	0.00	0.00
Suggestion	16.67	0.00	38.89	0.00	77.78	17.86
Total	50.01	3.57	50.00	28.57	77.78	35.72

collaboration. While problems were generally solved faster when addressed collaboratively, the group could not identify the correct answer. In contrast, half of the experienced teachers mentioned teaching goals, including short-term course goals and long-term mathematics teaching goals. Nearly 80% of the experienced teachers made suggestions about grouping or dividing work among members, such as the factors to be considered, whether to assign group leaders, etc. Furthermore, experienced teachers tended to provide ideas on designing appropriate tasks to promote

Table 9.8 Five commenting levels of support for collaborative learning

Level	Examples
Description	The teacher just walked around and reminded the students to write the group number on the paper and didn't say anything else
Explanation	Although the teacher emphasized writing the group number in the upper left corner, students always missed some points, such as writing it elsewhere
Evaluation	I think it is appropriate to give six small blocks to students, not all of them, so there is still a process of abstraction
Prediction	She could say some ideas if we give her more blocks. She didn't gain anything in this lesson
Suggestion	As a teacher, it is not only the students who are actively performing that can be observed, but also the students not concentrating on the learning who need to be carefully observed for what they are doing and why they are not participating

Table 9.9 Focuses of support for collaborative learning

	Learning material		Teacher guidance		Discipline management	
	ETs (%)	PTs (%)	ETs (%)	PTs (%)	ETs (%)	PTs (%)
Description	11.11	14.29	0.00	10.71	5.56	10.71
Explanation	0.00	0.00	0.00	3.57	0.00	3.57
Evaluation	22.22	17.86	5.56	3.57	16.67	14.29
Prediction	0.00	0.00	0.00	0.00	0.00	0.00
Suggestion	16.67	7.14	66.67	32.14	0.00	0.00
Total	50.00	39.29	72.23	49.99	22.23	28.57

group collaboration, while preservice teachers tended only to evaluate whether the designed task was reasonable.

In terms of support for collaborative learning with examples in Table 9.8, the results show that more experienced teachers than preservice teachers noticed learning materials and teaching guidance, while preservice teachers were more concerned with disciplinary management (see Table 9.9). Regarding learning resources, preservice teachers were inclined to simply state that there were six blocks on the table or evaluate whether the blocks provided were appropriate; fewer of these teachers provided suggestions. Regarding teaching guidance, Approximately 70% of experienced teachers provided specific teaching methods or strategies, while a few preservice teachers only briefly described the classroom teachers' specific behaviors. In addition, 32.14% of the preservice teachers proposed possible solutions. Thus, compared to experienced teachers, preservice teachers paid less attention to learning materials and teacher guidance and were more concerned with discipline management. They tended to make superficial narratives or rough evaluations and could give some suggestions about instructional strategies.

Finally, regarding group member performance (see examples in Table 9.10), preservice teachers paid more attention to possible problem-solving approaches

Table 9.10 Five commenting levels of performance of group members

Level	Examples
Description	The girl asked whether the middle of the tower's top was also painted. Then the boy immediately said, "No, the tower is a length of 3, a width of 2, and there is no middle place. Fortunately, I play a Rubik's cube."
Explanation	Although the teacher emphasizes writing the group number, maybe she is not interested in the problem and does not have a strong desire to find the answer. Perhaps she does not understand the problem, so she could only listen to what they say
Evaluation	The two have relatively good spatial imagination, but that doesn't necessarily mean they have good grades. Maybe the other two kids have strong algebra skills too
Prediction	Continuing this kind of group collaboration will polarize the class, the good one is better and the weak one weaker. Everyone gains differently from this activity, and she will always be the one with the least
Suggestion	When solving mathematical problems, if you could also resort to some diagrams, charts, and other direct tools, you would find that the problem is getting easier

Table 9.11 Focuses on group member performance

	Mathematical thinking		Problem-solving approach		Group interaction	
	ETs (%)	PTs (%)	ETs (%)	PTs (%)	ETs (%)	PTs (%)
Description	0.00	0.00	0.00	21.43	0.00	0.00
Explanation	0.00	0.00	0.00	0.00	5.56	25.00
Evaluation	61.11	35.71	38.89	57.14	16.67	39.29
Prediction	0.00	0.00	5.56	3.57	11.10	0.00
Suggestion	16.67	0.00	27.78	7.14	66.67	35.71
Total	77.78	35.71	72.23	89.28	100.00	100.00

than mathematical thinking, such as spatial imagination and abstract reasoning. More than twice as many experienced teachers were concerned with mathematical thinking, and most offered remarks provided suggestions on cultivating students' mathematical thinking. As shown in Table 9.11, approximately 90% of preservice teachers mentioned problem-solving approaches, and 57.14% evaluated idea clarity. All teachers mentioned group interactions, but the experienced teachers were more skilled at making suggestions to improve students' collaboration skills and provide students equal opportunities to participate. Nearly 40% of preservice teachers only evaluated group collaboration, and some explained some students' lack of participation from various perspectives (e.g., students' personalities, interests, or mathematical abilities).

9.4 Discussion

This study has explored the differences between experienced and preservice teachers in noticing the collaborative problem-solving process, based mainly on teachers' visual attention distribution and specific CPS content. Unlike many previous studies on teachers' noticing, this study combined eye-tracking and interviews to collect data from several teachers watching a group problem-solving video. It used eye-tracking technology to obtain moment-to-moment gaze information and traditional structured interviews for supplementary explanations of teachers' thought processes. The results suggest that teachers with different teaching experience levels have different focuses when viewing group work videos. Experienced teachers distributed their attention more evenly than preservice teachers and noticed more important features of the CPS process. Next, we conduct a detailed discussion of three categories of group collaboration noticed by experienced and preservice teachers—group performance, support for collaborative learning, and preparation for group activity. For each facet, we summarize one bullet point.

9.4.1 Preservice Teachers Attend to More Superficial Information

First, regarding group member performance, preservice teachers focused significantly more on students who spoke frequently, such as speaker 1 in the segment where four students analyzed the task. These teachers paid less attention to students who spoke little, such as listener 3 in the segment where three students participated in problem-solving. Preservice teachers appeared more interested in speakers with salient behavioral features, ignoring those who, while not exhibiting explicit behaviors, might be involved in implicit thinking activities. This suggests preservice teachers' attention was easily drawn to students who spoke, and they paid more attention to the superficial and formal aspects of the discussion process (Star & Strickland, 2008). The preservice teachers lacked the requisite classroom experience to realize there was much to notice in the classroom and could not use other key information to understand the students synthetically.

Further analysis of this segment revealed that the preservice teachers spent significantly more looking at the four students than the experienced teachers, $p = 0.037$, while the experienced teachers spent more time observing the students placing the blocks ($M = 13.615$) than preservice teachers ($M = 11.582$). Teachers could clearly hear what students were saying when watching the group collaboration video. Experienced teachers focused more on substantively problem-solving rather than simply watching whoever spoke, showing that teachers' expertise in explaining students' understanding grows as their teaching experience increases (Jacobs et al., 2010). Thus, compared to preservice teachers, experienced teachers have a more detailed, comprehensive, and richer understanding of students' mathematical thinking.

The interviews also showed that preservice teachers were more concerned with superficial information. Preservice teachers tended to describe or evaluate specific problem-solving approaches, and the percentage of experienced teachers mentioning profound mathematical thinking was far greater than that of preservice teachers. Moreover, some experienced teachers even proposed specific means to cultivate students' mathematical skills. For example, one experienced teacher said, "We could try to use all 48 blocks first to help students whose spatial imagination abilities are poor and make them experience the mathematical process from the concrete to the abstract." In contrast, most preservice teachers were inclined to recall detailed solutions (e.g., "They discussed the blocks painted on more than three sides first and then discussed the others") or simply repeat what the students said. Some tended to comment only on whether the group understood the problem or solution (Carter et al., 1988).

Experienced teachers reflected on the students' overall performance rather than simply criticizing them based on what happened; they acknowledged that the group's idea, while slightly confusing, was common, and the right answer gradually emerged through heated discussion and repeated correction. As Sabers et al. (1991) noted, experienced teachers will provide strategies to increase student engagement and help students solve problems efficiently based on their performance, such as analyzing the problems slowly, thinking independently before discussing, and drawing diagrams to solve mathematical problems.

In addition, almost all teachers noticed considerable differences in the students' participation, with some students hardly participating in the group discussion. In response, some experienced teachers predicted the whole class' learning results based on this one group's collaboration and offered many impressive ways to encourage students to participate, such as by reminding students that they could use drawings to facilitate communication and asking group members to speak in turn. However, nearly half of the preservice teachers only complained about the students' poor participation and explained the group's performance based on their experiences with similar situations.

This may have occurred because preservice teachers are used to acting as students being taught rather than as mathematics teachers. Limited by this perspective, they try to understand student performance as much as possible but cannot think deeply about the teaching and learning process. Specifically, preservice teachers relate group performance to their own learning experiences and focus more on salient features that people easily attend to. In this study, when teachers watched the group collaboration video, they should have been more concerned with the collaborative process' implicit and substantive features and discussed important things like mathematical thinking, student participation, and teaching strategies.

9.4.2 *Preservice Teachers Are More Concerned with Students' Misbehaviors*

Second, preservice teachers with no teaching experience were inclined to be distracted by students' exhibiting problematic behaviors, such as the misbehaving students in the segments involving three and two students participating in problem-solving. This echoes existing studies on preservice teachers' noticing, which found they focus more on students' classroom misbehavior in traditional classrooms (Erickson, 1984; Sabers et al., 1991; Star & Strickland, 2008). It means preservice teachers are more concerned with disciplinary management and regard classroom management as their primary task.

Shen et al. (2009) investigated 527 elementary school teachers' perceptions of classroom problem behaviors in five provinces in China and found significant differences in the amount of time spent on classroom management and that the time spent decreased as teaching experience increased, indicating that more classroom experience broadens teachers' possible solutions to students' behavioral problems. Preservice teachers, lacking classroom teaching experience, find it harder to deal with classroom behavior problems, are unable to find appropriate ways to deal with them quickly, and may even worry about losing control of the classroom. As such, preservice teachers spend more time focusing on students' problematic classroom behaviors. Experienced teachers, on the other hand, are more concerned about students' mathematical thinking when group members discuss problems, and do not disrupt the group's problem-solving because of individual students' misbehaviors. They are more conscious of what to pay attention to and what to ignore in specific situations (Erickson, 1984). This echoes Miller's (2011) observation that experienced teachers do not pay attention to everything they see; instead, they quickly adapt their attention to suit the situation and actively ignore, to a certain extent, less important things.

Notably, the interview results showed that a higher proportion of preservice than experienced teachers mentioned disciplinary management, which is consistent with the results of our eye-tracking data analysis. We therefore conducted a supplementary analysis to investigate the misbehaviors recalled by the two types of teachers. Nearly 43% of the preservice teachers talked at length about how student 1 had said strange, funny words into the recording equipment, while only one experienced teacher mentioned it. This suggests that preservice teachers' inexperience leads them to be distracted by problematic student behaviors and more sensitive to classroom discipline.

In addition to classroom management, providing useful learning materials and teaching guidance is essential for supporting collaborative learning. Interestingly, fewer preservice teachers focused on learning resources and teaching guidance. Only half paid some attention to the teacher, who barely instructed the students beyond reminding them to write down their group numbers. Most instead tended to describe or evaluate the blocks on the desk; for example, "Not all 48 wooden blocks were given, and the rest should be imagined by themselves, so that students' geometric intuition could be developed" and "The blocks are rather strange and will interfere with the students' thinking, because they will keep placing the blocks instead of thinking about drawing."

The experienced teachers' perceptions were more comprehensive. They made specific recommendations on how many blocks to provide to improve the students' involvement and remarked on the advantages of using white paper for drawing. This corresponds to Star's (2008) finding that only 44% of preservice teachers correctly answered questions about the classroom environment before training, and generally did not notice important classroom environment features or feel these features were worth attention. This shows that preservice teachers do not pay enough attention to key details that facilitate collaborative group learning.

In addition, approximately three-quarters of the experienced teachers offered suggestions based on their teaching experience, such as teaching strategies for group collaboration. Greatly influenced by their accumulated classroom teaching experience, they could associate the group's performance with relevant general teaching principles or strategies.

9.4.3 Preservice Teachers Focus Little on the Teaching Goal of Group Collaboration

Third, among the three categories noticed by teachers, preservice teachers gave the least attention to preparing for the group activity and only one mentioned the teaching goal. In contrast, some experienced teachers provided advice on collaborative learning and mathematics activity goals. For example, one experienced teacher said, "If I were the teacher in the class, I would reflect on what I really wanted. If I wanted this group to give the right answer quickly, my attention would be on the most active students; but if I wanted to make all students improve, I would focus more on low-involvement students." This reflects this experienced teacher's in-depth thinking about effective teaching. Another ET noted, "Our students should be able to face their future life and interact positively with society, not only by solving problems but also by knowing how to communicate with others." This reflects the PISA 2021 mathematics framework, which identifies communication as one of eight important twenty-first-century skills (OECD, 2018).

Preservice teachers mentioned task design and group construction more often than teaching goals. They pointed out whether the group task was reasonable and provided specific advice, such as "When creating groups, it is important for the groups to be equal in terms of students' mathematical skills and personalities." However, the experienced teachers put forward more impressive and practical suggestions: "I prefer grouping students with various skill levels so they can help each other. However, if they need to solve problems, students with similar skills should be in the same group to easily exchange their thoughts and ideas. Therefore, I design tasks with different levels of difficulty for different groups, according to each group's skill level."

As we can see from the results, preservice teachers with a collaborative learning experience in middle schools could evaluate whether the task designed was appropriate for the seventh-grade students and roughly suggest how to effectively construct groups. However, it was difficult for them to determine the lesson's teaching goal from a teaching perspective. The experienced teachers not only offered specific advice based on their teaching experience, but they also provided ideas on mathematics teaching that were consistent with modern educational ideas. While the preservice teachers may have had some professional knowledge about teaching, they did not yet know how to apply abstract theories to actual teaching skills and effectively practice them. Therefore, when watching the video, the preservice teachers could not connect specific events to broader teaching principles; instead, they only attended to what was occurring on the surface.

Doyle (1986) noted that classrooms have three main properties: simultaneity, multidimensionality, and immediacy. Based on our analysis of a four-student collaborative activity, we can see that the students' participation varied greatly, and key changes occurred at the same time. Therefore, it is necessary for teachers to continually monitor several simultaneous events. Teachers must not only listen to speakers; they must also pay close attention to whether other students are understanding or keeping up with the activity. However, preservice teachers, who lack teaching experience, may focus primarily on more salient behaviors and ignore other implicit features of CPS, as they are used to observing the teaching process as learners. Teachers should give students showing little or no participation extra attention when teaching. Moreover, they must selectively perceive more significant features of specific situations and constantly strive to understand or analyze them. They should consider what is happening as a continuous whole and quickly make important decisions to respond to patterns that need to be changed.

9.5 Conclusions and Implications

Based on the discussion above, teachers' noticing ability is closely related to their classroom teaching experience. As shown by prior studies, experienced teachers' perceptions of group collaboration are more detailed, systematic, and comprehensive. Experienced teachers think more deeply about what they attend to and more often propose practical teaching strategies than preservice teachers. Although it is difficult to precisely describe what is noteworthy during CPS activities, it is appropriate to notice important objects and logically and reasonably observe the fundamental elements of the CPS teaching process, such as teaching goals, group construction, problem-solving, group interaction, and teachers' guidance. The experienced teachers in our study showed roughly these characteristics.

This study also presents some limitations. First, some relevant factors that may affect teachers' noticing were not considered in this study, such as teachers' beliefs and educational backgrounds (Jacobs et al., 2010). Second, as with many other studies, the sample size in this study was small, with no significant differences in participant characteristics. Increasing the number of participants could further verify the results. In addition, viewing a video of group collaboration in a laboratory setting differs from facilitating multiple groups in an authentic classroom context, which may limit our conclusions' generalizability. Next, we will investigate the differences in professional noticing between teachers with different experience levels in an authentic group teaching classroom situation and further analyze the possible reasons underlying their behaviors. We also intend to determine whether teachers' noticing changes group members' visual attention, leads to students' joint attention, or improves students' collaborative quality in other ways, as described in the literature. Such areas remain unexplored and require more in-depth exploration. Reflecting on teachers' noticing about group collaboration using innovative technology can help improve teachers' classroom practices and students' collaborative learning.

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Chapter 10
Teacher Intervention in Collaborative Mathematics Problem Solving in Secondary School

Yixuan Liu and Hang Wei

10.1 Introduction

Since the beginning of the twenty-first century, international organisations such as the United Nations Educational, Scientific, and Cultural Organization (UNESCO) and the European Union and countries such as the United States and Australia have put forward twenty-first-century competency frameworks to develop the knowledge, abilities, and attitudes expected of citizens, including their capacity for collaboration and problem solving (Peng & Deng, 2017). Similarly, China's Mathematics Curriculum Standards for Compulsory Education (Ministry of Education of the People's Republic of China, 2012) emphasises developing students' collaborative communication and problem solving capacities, indicating it is beginning to attach importance to collaborative problem solving (CPS) in mathematics and advocate problem solving orientation and problem-based and collaborative learning (Yu & Cao, 2017). Many large international assessment programmes such as PISA and ATCS21S are beginning to focus on measuring students' collaborative problem solving capacity.

Both PISA 2015 and ATCS21S deconstruct collaborative problem solving capacity as collaboration (social skills) and problem solving (cognitive skills); from these two concepts, the origins of collaborative problem solving are long-standing. The effectiveness of collaborative/cooperative learning has been of interest to academics since the concept's inception. Through a review of hundreds of studies, Bossert (1988) found that students in collaborative learning classrooms performed

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academically at least as well as and often better than those in traditional classrooms. In recent years, some meta-analyses have found that students achieve higher grades by working in small groups than in individual learning—e.g., in computer-supported collaborative learning (Chen et al., 2018) or face-to-face collaborative learning (Kyndt et al., 2013). It is widely agreed that collaborative activities can facilitate student learning, and the collaborative group learning model is becoming common in classrooms worldwide.

The mathematics classroom plays an irreplaceable role as a key site for developing students' collaborative problem solving skills. In mathematics classroom, there was a transfer from individual problem solving to collaborative problem solving (Pruner & Liljedahl, 2021). Collaborative problem solving capacity cannot be developed through a specialised subject and is not a separate discipline taught on a case-by-case basis through specific teaching content (Bai & Lin, 2016). Xu et al. (2020) suggested that subject-based and interdisciplinary curricula are important basics for developing collaboration literacy. Furthermore, due to the nature of the mathematics curriculum, CPS is required in mathematics classrooms, and mathematics tasks can be provided to foster it (Kong & Zhao, 2017). Collaborative mathematics problem solving can facilitate the development of related competencies.

CPS in mathematics could develop students' relevant skills, such as collaboration, problem solving, etc. It has been well noted that teachers play critical roles in such processes. Teachers act as designers and organisers of collaborative activities and are, for students, academic experts and managers in mathematics classrooms (Davidson, 1990). However, many researchers argue that simply grouping students does not guarantee that collaborative learning or collaborative problem solving will occur. More effort is needed from teachers to enhance further students' collaborative and problem solving capacity and provide them with adequate learning opportunities (Chen & Zhang, 2014; Hou, 2017). In group work, teachers also need to act as facilitators and guide students' collaboration through diagnosing and intervening, involving each student in the group's task to enhance the quality of their discussions and further the teaching objectives.

Collaborative learning was introduced later in Chinese classrooms, around the beginning of this century, and experienced difficulties initially. Group learning was not developed sufficiently (Wang & Wu, 2012), and teacher-group interactions were sorely lacking (Cao & He, 2009). In a 2013 survey of teachers from 13 provinces in China, 48.9% said they often carried out "collaborative learning in groups under the guidance of teachers" and 60.3% said they often organised discussion activities for students (Shi & Wang, 2018). With the popularisation of collaboration, the proportion of teacher–group interactions in China's mathematics classrooms has increased significantly in recent years; however, some studies have found that teacher intervention needs to be improved. A lack of interactions between teachers and individual students or groups has been noted (Yu & Cao, 2018), and guidance and evaluation of group communication are relatively lacking (Dong et al., 2013). In addition, there have been few empirical studies on collaborative mathematics learning in China (Wu et al., 2020), and there is still a lack of research on teacher intervention in collaborative learning, especially on the effect of teacher intervention on students' classroom performance.

Open-ended mathematics tasks are chosen in this study to investigate the impact of teacher intervention in collaborative problem solving in mathematics. Research has found that open-ended problems are suitable for collaborative learning (Wang, 2016), facilitate students' reasoning arguments and mathematical communication (Kosyvas, 2016), and help develop students' mathematical thinking and interest in learning.

Teaching and learning enhancement should be promoted to foster students' collaborative problem solving skills. This chapter analyses the current state of the situation and problems in a targeted manner based on an understanding of the current situation of teacher intervention in collaborative problem solving classrooms in mathematics. The study digs deeper into the situation's causes and is conducive to promoting collaborative problem solving activities in a scientific and rational direction. This chapter investigates how teacher interventions affect students' collaborative problem solving through experiments. It then selects typical cases to discover Chinese mathematics teachers' advantages and weaknesses in collaborative problem solving, mainly using coding frameworks such as teacher intervention focus and means.

10.2 Literature Review

10.2.1 Definition of Teacher Intervention in CPS

Webb (2009) classified the teacher's role in collaborative problem solving into several dimensions: preparing students for collaborative work, forming groups, structuring the group work task, and influencing student interaction through teachers' discourse with small groups and the class.

Previous research has shown that the effectiveness of collaborative learning depends largely on the quality of student interaction. Based on a literature review, Kaendler et al. (2015) compiled a framework for Implementing Collaborative Learning in the Classroom (ICLC). The framework clearly describes the teacher's role in facilitating student collaboration, dividing teacher–student interaction into two dimensions: the student and teacher levels. The student collaboration level includes three phases—pre-active, inter-active, and post-active—while the teacher level describes five competencies across all implementation phases. The framework in Fig. 10.1 argues that the various teacher competencies are based on teachers' professional knowledge and teacher beliefs.

In earlier studies, the definition of teacher interventions did not go beyond teacher-initiated activities. However, more recent researchers have included student-initiated interventions (e.g., Kajamaa et al., 2020).

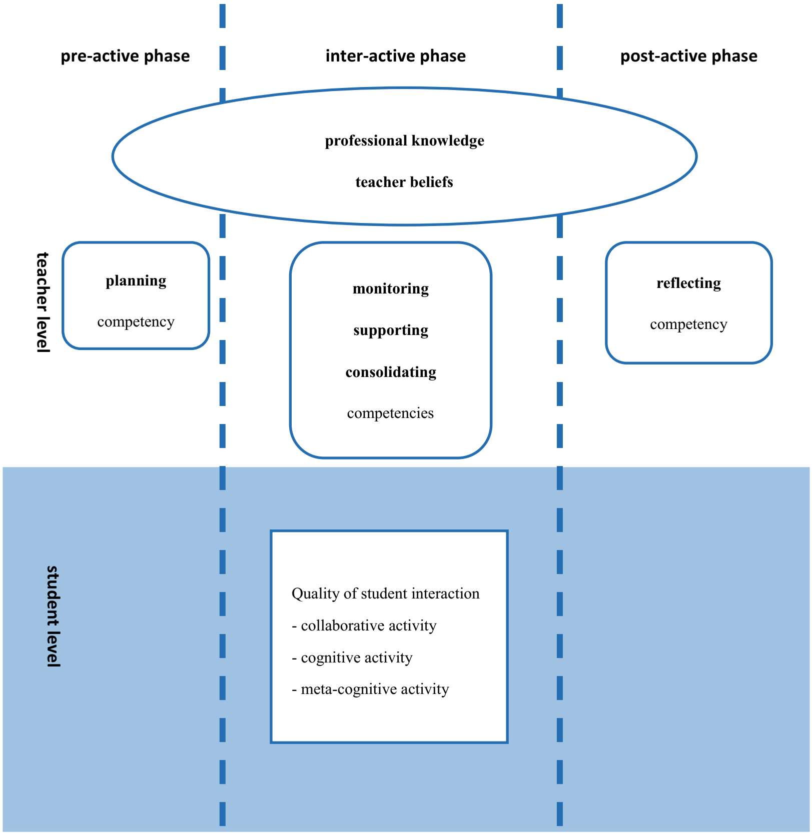

Fig. 10.1 The ICLC framework by Kaendler et al. (2015)

Our study operationally defines teacher intervention in collaborative problem solving as a verbal intervention at the individual or pair (group) level initiated by the students or teacher, excluding non-task-related instructions, during pair and group collaboration. The interventions defined here only focus on the interactive phase of the collaboration, excluding the pre-active and post-active phases. We also leave out discourse, such as reading out tasks to the whole class, explaining task requirements, or using directive language.

10.2.2 Review and Analysis of Empirical Research on Teacher–Group Interaction in China's Mathematics Classrooms

The concept of collaborative learning originated in the Western world but has been applied in China for many years. In the 1980s, scholars in China began to translate Western experimental research on collaborative learning; by the late 1980s and early 1990s, research and experiments on collaborative learning had emerged in China (Zeng & Tian, 2014).

In 2001, the State Council of the People's Republic of China promulgated the *Decision of the State Council on the Reform and Development of Basic Education*, which specified that collaborative learning should be encouraged to promote students' mutual communication and development (State Council of the People's Republic of China, 2001) Since then, collaborative learning has been fully promoted in mathematics classrooms and has been the subject of more studies. Collaborative learning has become a top topic in China's mathematics curriculum reform, with China's Compulsory Education Mathematics Curriculum Standards (2011 edition) intentionally infiltrating collaborative problem solving skills (Kong & Zhao, 2017). In recent years, the spread of new educational philosophies and technologies has informed new classroom teaching models—such as problem-based learning, problem solving teaching, and flipped classrooms—that emphasise group collaboration and require collaborative learning and problem solving activities.

Although group work is in full swing in Chinese mathematics classrooms, research into teacher–group interactions started late, and there are generally fewer teacher–group interactions than in other countries. The earliest study in China on teacher–group interaction in mathematics classrooms (Cao & He, 2009) coded videotaped classroom interactions into four categories (teacher–individual, teacher–group, teacher–class, and cross-interaction) and found that teacher–group interaction was relatively lacking. Subsequent research relying on that coding scheme studied teacher–student interactions in mathematics classrooms, yielding the findings in Table 10.1.

As seen from Table 10.1, the percentage of teacher–group interaction for the whole class was lower in earlier years. Researchers identified a lack of teacher–group interaction at that time, noting that collaborative learning in groups was less developed in mathematics classrooms and teachers had fewer interactions with students in group work. While the students worked together, teachers prepared content for subsequent lessons or patrolled the classroom without any interactions. Teachers only interacted with students when they found something wrong (Cao & He, 2009). In some classrooms, teachers would mainly 'observe,' 'nod,' and 'point' during their rounds, spending very little time with each student group (Wang & Wu, 2012).

The table also shows that as the curriculum began to call for more group work, teachers gradually took on more group guiding work, resulting in a significantly higher percentage of teacher-group interactions.

Table 10.1 Related research on teacher–group interaction in China

Year of classes	Teacher-group interaction percentage (average) (%)	Number of classes	Learning stage	Researchers (year of publishment)
1998–2001	9.0	10	Primary school	Wang and Wu (2012)
2005	7.6	10	Primary school	Wang and Wu (2012)
2005, 2007, 2009	0	3	Senior high	Wang (2010) (Master's dissertation)
2008	1.05	8	Junior high	Cao and He (2009)
2011	2.6	10	Primary school	Wang and Wu (2012)
About 2013	0	3	Senior high	Lu (2014) (Master's dissertation)
About 2014	0	1	Primary school	Xie (2014)
About 2014	2.59 (Before improvement) 9.81 (After improvement)	2 (Before improvement) 2 (After improvement)	Junior high	Li (2015) (Master's dissertation)
About 2016	22.45	1	Junior high	Cui and Dong (2017)
About 2019	35.51 (Experimental group) 21.57 (Control group)	5 (Experimental group) 2 (Control group)	Junior high	Li (2020) (Master's dissertation)

10.2.3 Teacher Intervention During Collaboration

Earlier studies have researched the effect of teacher intervention during students' collaboration, but their results are inconsistent. Most found a positive correlation between teacher presence in intervention and high levels of student knowledge processing, while others reported neutral findings. For example, Hogan et al. (1999) found that while groups performed moderately well overall in scientific reasoning when the teacher intervened, some presented higher reasoning when there was no intervention, and others presented lower levels. However, a few studies have also found a negative correlation between teacher presence in intervention and high levels of student knowledge processing. The timing and type of teacher support are possible explanations for the negative correlation results; Ros (1993) found that longer teacher–student contact correlated with lower levels of student knowledge processing, making brief acts of teacher–student interaction appear more desirable. Further, some studies have found that teacher presence and absence in the intervention are unrelated to student task engagement, while others have found that both are positively related to student task engagement. For example, Van de Pol et al. (2015) found that students could sustain task discussions well without the teacher's presence if the teacher had adequately and appropriately designed the instructional activity.

The equality of teacher intervention has been considered. Van Leeuwen et al. (2013) found that teachers get more involved with high-activity groups, while Pietarinen et al. (2021) found no correlation between prior competence and teachers' guidance, with teachers preferring to guide groups they perceived as motivated and willing to collaborate.

While the development and application of ICT (Information and Communications Technology) have resulted in many proven effective teacher intervention tools for computer-supported collaborative learning (CSCL) (e.g. Schwarz et al., 2021), in China, there is still a long way to go.

10.3 Current Study

10.3.1 Research Questions

This chapter studies teacher intervention in students' collaborative mathematics problem solving. Although similar studies have been conducted in several countries and regions, there is a lack of current research on teacher intervention at the elementary education level in China. While computer-supported collaborative learning (CSCL) has received more attention recently, this research studies teacher intervention in face-to-face collaborative problem solving, with open-ended tasks chosen to achieve better collaborative efficiency. While most studies have found that teacher intervention greatly impacts collaborative problem solving, a few have found that students can do as well or even better when there is no intervention; thus, intervention effectiveness should and has yet to be verified.

Previous research on collaborative learning and collaborative problem solving has mainly studied interventions at the behavioural level rather than investigating how teachers' interventions occur.

Based on the preceding discussion, the research questions for this paper are as follows:

RQ1: What is the effectiveness of teacher interventions on the outcomes of students' pair and group collaborative mathematics problem solving?

RQ2: What are the similarities and differences in the characteristics of teacher interventions with different effects on students' pair and group collaborative mathematics problem solving?

10.3.2 Research Design

This study adopted a mixed research approach with a Sequential Explanatory Design that combined quantitative and qualitative research methods. Quantitative tools were used to discover phenomena (RQ1), and qualitative research methods were used to explain and further explore those phenomena (RQ2).

A quantitative study was conducted to investigate the effectiveness of teacher interventions in collaborative problem solving by comparing the pair and group work scores of students in the intervention and control groups and analysing which aspects of the interventions were effective based on the response scores. Based on the pre-test scores, the pairs or groups were divided into different structures (high-score homogeneous group, low-score homogeneous group, and heterogeneous group). The effectiveness of teacher intervention was determined by comparing the performance of the pairs and groups within the different structures.

Based on the quantitative results, a typical teacher was selected to conduct a case study of classroom teaching in the intervention group, using an Interpretational Analysis data analysis model. Based on Van Leeuwen et al.'s (2013) framework, the teacher interventions were coded to explain how they occurred and affected students.

The case study needed to ensure the integrity of the context. This chapter relies on the original video recordings to analyse the data. It focuses on tacit knowledge, describing non-verbal clues as well as possible to recreate the real classroom context. The coding section draws on previous research on teacher intervention focus categories, intervention means, intervention initiators, and intervention targets to compare intervention situations and effects across teachers. In addition to analysing teacher–student interaction behaviours in the classroom, the analysis and discussion section also analyses parts of the teacher interviews, using different data sources for crystallisation.

10.3.3 Participants

A purposive sampling method is applied in the current research, all participating teachers are interested and have experience in collaborative learning, which grants the investigation of teachers' intervention is aligned with their daily classroom. The participants were 292 Grade 7 students selected from eight classes taught by four teachers in two secondary schools in District T of City B. The differences in mathematics performance between the two classes taught by the same teacher were not statistically significant, as shown in the following Tables 10.2 and 10.3. Each teacher designated one of their two classes as the control class and the other as the intervention class. They then divided each class into small groups based on mathematics performance to ensure that the average scores of each pair and group were not significantly different; adjustments were made based on discipline. Students first worked in pairs, and then two (or three) pairs worked in one group. Due to space limitations

Table 10.2 Basic information for each class

Teacher	Comparing classes			Intervention classes		
	N	Pairs	Groups	N	Pairs	Groups
A	34	17	8	34	17	8
B	32	16	8	33	16	8
C	40	20	9	39	19	9
D	39	19	9	41	20	9

Table 10.3 Mathematics performance of each class

Teacher	Comparing classes		Intervention classes			
	M_1	SD	M_2	SD	$M_1 - M_2$	p
A	83.55	16.54	84.06	13.08	−0.51	0.888
B	79.75	16.75	81.56	13.43	−1.81	0.635
C	82.95	14.32	81.84	13.51	1.11	0.727
D	79.14	16.68	77.65	19.28	1.49	0.720

and the need to involve all students, several pairs of three students and groups of five to six students emerged in each class.

Mathematics performance was based on the current semester's mid-term exam results; however, some students had missing grades, which were set as missing values. A *t*-test found no statistically significant differences in Table 10.5, despite some differences in the mean grades of each teacher's control and intervention groups.

10.3.4 Data Analysis

The data selected for this chapter consist of classroom videos, task sheets, and video interviews with teachers. Data on pair and group collaborative problem solving were selected (Task 1 and Task 2, respectively, see Appendix for details). The data analysis consisted of scoring students' collaborative problem solving outcomes and coding teacher interventions, including their intervention focus, means, initiators, and targets. In the problem solving process, students in the same pairs and groups reported their collaborative problem solving outcomes on the same task sheet, which served as the basis for scoring their collaborative problem solving results. The scoring framework was piloted and polished to ensure it was clear and covered the performance of all students, with items categorised to facilitate exploring specific student performances. The data for analysing teacher intervention characteristics were primarily derived from the follow-up teacher video, using the groups' videos for further confirmation. The frameworks for teacher intervention of focus, means, initiators, and targets were drawn from previous research.

10.3.4.1 Scoring of Task Sheet

The scoring schemes were based on pair (group) collaboration tasks; the score for each item represents whether the student understood the relevant task content reasonably well and represented it clearly. The task sheet scoring scheme is shown in Table 10.4.

The marking scheme for the paired task was based on Task 2, where the 'Comprehension of task' items corresponded to questions about 'one of the five people is a Year 7 student' and 'five people living in a house.' The 'Mathematical performance' items tested whether the ages of the other four people were calculated correctly. The 'Character roles' and 'Character relationships' items tested whether students reasonably represented how the five people in the house were related, with 'Character relationships' concerning the generational age difference. 'Expression' items corresponded with the instruction, 'Write a paragraph explaining your answer,' with each item's scoring depicting how the pairs performed at each level. The scores for each item are added together to give an overall score for pair collaboration problem solving, ranging from 0 to 5, discretely (Table 10.5).

The group task sheet scoring scheme was designed based on Task 1, where the item 'Number of rooms' was based on the task's requirement that an 'apartment has five rooms,' 'Marking of functions' and 'Geometric drawing' corresponded to 'Label each room and show the dimensions (length and width) of all rooms,' 'Geometric drawing' tested students' drawing performance, and 'Area' corresponded to the 'total area is 60 m^2' item. As with the pair tasks, the total scores for the group tasks were discrete (0–5 points).

Table 10.4 Pair task sheet scoring scheme

Items	Score	Description
Comprehension of task	0	Unreasonable age of students in Grade 7 or wrong number of people
	1	Reasonable age of students in Grade 7 and the right number of people
Mathematical performance	0	Mathematical mistakes relating to age
	1	Correct mathematical calculations relating to age
Character roles	0	No persons given or incorrect number of personas
	1	Everyone is given a role
Character relations	0	Character relationships do not make sense or are not age-appropriate
	1	Character relationships are reasonable and age-appropriate
Expression	0	Unclear or incomplete expressions
	1	A clear and complete expression

Table 10.5 Group task sheet scoring scheme

Items	Score	Description
Number of rooms	0	The number of rooms is wrong
	1	The number of rooms is correct
Marking of functions	0	The marking of functions is missing or incomplete
	1	The marking of functions is complete
Geometric drawing	0	The marking of room sizes is missing or incomplete
	1	The marking of room sizes is complete, but there are geometric mistakes in the drawing
	2	The marking of room sizes is complete without geometric mistakes in drawing
Area	0	The total area is incorrect
	1	The total area is correct

10.3.4.2 Analysis of Teacher–Student Dialogue

Units of Analysis

The dialogue units of analysis were closely related to the study's research questions. This chapter focuses on verbal teacher–student interactions, where the teacher moved between groups within the class, and who initiated the dialogue (i.e., uttered the first sentence in the conversation); thus, the units of analysis were divided based on each conversation. Each sentence spoken by the teacher may have a different intervention purpose, form, and target. The units of analysis were divided based on those in a typical two-person face-to-face discourse; teacher-student conversations always followed rounds, so the units of analysis were divided mainly by speaker changes. It is important to note that a turn sometimes contained more than one discourse and that an utterance was divided by 'perceptible pauses' (commas or full stops) in the transcribed text (van Boxtel et al., 2000).

Dialogue Initiators and Intervention Targets

The dialogue initiators classification was mainly based on Chiu's (2004) teacher intervention initiators classification, where the unit of analysis for a dialogue is the first person to speak in a single teacher–student dialogue (usually with multiple talking rounds). The categories were student-initiated and teacher-initiated.

In this study, teachers were suggested not to conduct class-oriented interventions or guidance. Therefore, the intervention targets were categorised depending on whom the teacher was talking to—individuals, pairs, or groups. Every turn was coded as a unique intervention target.

10.4 Results

10.4.1 Results of Pair Collaborative Problem Solving

10.4.1.1 Overall Results

First, based on the design, the results of pair tasks were compared between the intervention and control classes. Figure 10.2 shows that the intervention classes' scores were higher than the control classes,' indicating the intervention class students generally performed better in pair collaborative problem solving.

Further statistical tests compared the means for the intervention and control classes. Since neither data group followed a normal distribution, a non-parametric test (Mann–Whitney *U* test) was used to test the differences between the two groups.

Combining Cohen's *d* and *p*-values, Table 10.6 shows that the intervention classes outperformed the control classes in all items except 'Character role,' for which there was no difference. The most significant differences were found in 'Mathematical performance,' which showed a moderate difference based on effect size, and 'Expression,' which showed a moderate difference with borderline significance. The differences in the other items were not statistically significant and showed lower validity. Overall, the differences in the total scores of the two groups were marginally significant with moderate validity.

These results show that the teacher intervention had its most significant effect on 'Mathematical representation,' as evidenced by the fact that more pairs calculated correctly and expressed their responses clearly on the task sheet. Overall, teacher

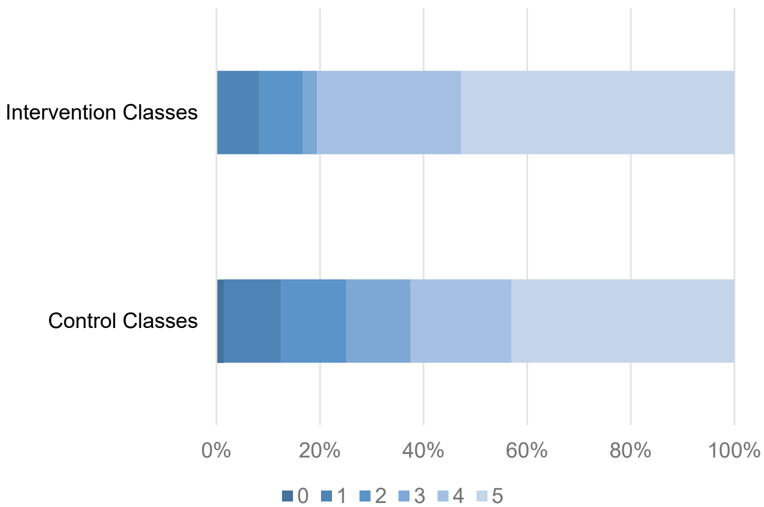

Fig. 10.2 Comparison of pair collaborative problem solving results

Table 10.6 Scoring result of pair collaborative problem solving

Items	Intervention classes (72)		Control classes (72)		*p*	Cohen's *d*
	M	*SD*	*M*	*SD*		
Comprehension of task	1.00	0.00	0.99	0.12	0.317	0.166
Mathematical performance	0.72	0.45	0.54	0.50	0.025	0.376
Character roles	0.85	0.36	0.85	0.36	1.000	0.000
Character relations	0.69	0.46	0.60	0.49	0.224	0.202
Expression	0.82	0.39	0.69	0.46	0.081	0.291
Total score	4.08	1.29	3.67	1.48	0.095	0.298

intervention improved student pairs' response outcomes in collaborative problem solving tasks.

10.4.1.2 Comparisons of Pairs

The pairs were divided into four pair types based on pre-test scores and pair structure: high-score homogeneous, middle-score homogeneous, low-score homogeneous, and heterogeneous (Table 10.7).

Based on Levene's test, the was a variance in homogeneity ($p = 0.196$); ANOVA could be performed to compare the differences in mean scores of different pair structures.

Table 10.8 shows a significant difference in mean values between the different pair structures. A further one-by-one comparison of data from the different groups was performed using the Least Significant Difference (LSD) method, the results of which are shown in Table 10.9.

Table 10.11 shows significant differences between the low-score homogeneous pair and the medium- or high-score homogeneous pairs and marginally significant differences between the heterogeneous pair scores and the low-score homogeneous and high-score homogeneous pair scores. All other differences were not significant.

Table 10.7 Basic information of groups of different pair structure

	High-score homogeneous pair		Middle-score homogeneous pair		Low-score homogeneous pair		Heterogeneous pair	
	N	*M*	*N*	*M*	*N*	*M*	*N*	*M*
Control classes	15	3.40	13	3.38	21	4.00	23	3.70
Intervention classes	11	3.09	31	4.38	12	4.92	28	3.89
Total	26	3.27	44	4.00	33	4.33	51	3.84

Table 10.8 Result of ANOVA

	Sum of squares	*df*	Mean square	*F*	*p*
Inter-group	17.262	3	5.754	3.069	0.030
Intra-group	262.488	140	1.875		
Total	279.750	143			

Table 10.9 One-to-one comparison of the different structures of pairs by LSD

	Middle-score homogeneous pair		High-score homogeneous pair		Heterogeneous pair	
	$M_1 - M_2$	*p*	$M_1 - M_2$	*p*	$M_1 - M_2$	*p*
Low-score homogeneous pair	−0.731	0.042	−1.064	0.004	−0.535	0.107
Middle-score homogeneous pair			−0.333	0.321	0.303	0.519
High-score homogeneous pair					0.306	0.086

A 4 (pair structure) × 2 (intervention or not) multifactorial between-group experimental design further explored the intervention's effects on pairs with different pair structures.

Correcting for the model term ($F = 2.576$, $p = 0.016$), the model was significant, where the experimental term ($F = 4.142$, $p = 0.044$) and the pair structure term ($F = 2.598$, $p = 0.055$) were statistically significant, while the crossover term was not. The above between-subjects effect test indicated that the model was reasonable and explained 11.7% of the variance ($R^2 = 0.117$) (Table 10.10).

Based on Fig. 10.3, the performance of the middle- and high-score homogeneous pairs was higher in the intervention classes than in the control classes. The performance of the heterogeneous pairs in the intervention classes was somewhat higher than in the control classes. Nevertheless, the performance of the low-scoring homogeneous groups in the intervention classes lagged behind their control classes peers. The intervention most improved the middle- and high-score homogeneous pairs' performance, particularly for the heterogeneous group, while the low-scoring homogeneous group's performance showed a decreasing trend.

Table 10.10 Between-subjects effect test

	F	*p*
Calibration model	2.576	0.016
Experimentation	3.628	0.059
Pair structure	3.810	0.012
Experimentation × pair structure	1.564	0.201

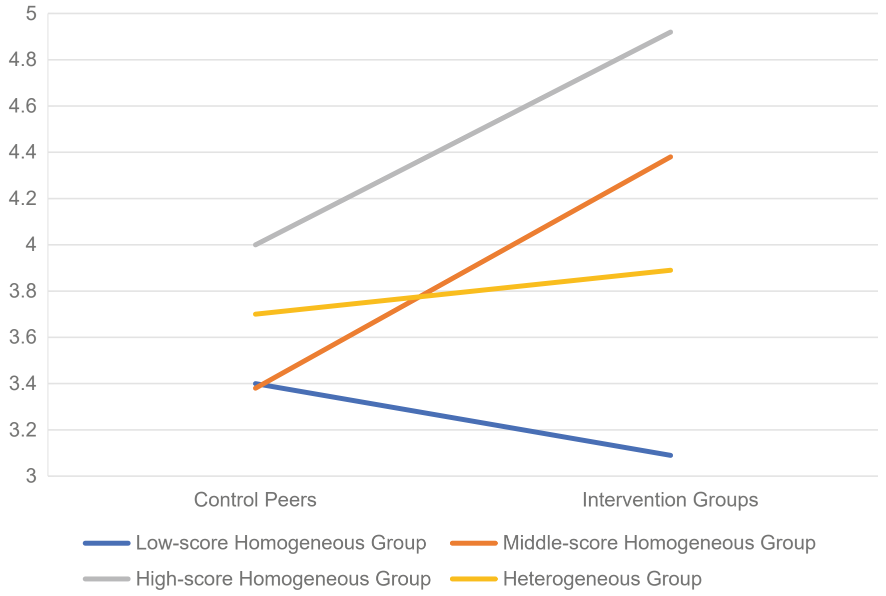

Fig. 10.3 Score of different pair structures

Table 10.11 Mean and standard deviation performance of pairs of different levels

Teacher	A	B	C	D
Control class	3.71 (1.36)	4.00 (1.21)	3.95 (1.40)	3.05 (1.78)
Intervention class	4.41 (1.00)	3.31 (1.78)	4.32 (1.16)	4.20 (0.95)

10.4.1.3 Comparisons of Differences in Interventions Across Teachers

Comparing the pair collaborative problem solving results for the intervention and control classes revealed that students in the intervention class outperformed those in the control class. Regarding each teacher's intervention effects, Teachers A, C, and D's intervention classes outperformed their control classes, while Teacher B's classes did the opposite (Table 10.11).

A comparison of means revealed that Teachers A, C, and D's intervention group class pairs scored higher on the collaborative task and had less in-class score differentiation, with Teacher D's class showing the most significant improvement. In contrast, Teacher B's intervention class pairs scored lower on the collaborative task and had a more pronounced divergence.

10.4.2 Results of Group Collaborative Problem Solving

10.4.2.1 Overall Results

The overall evaluation of the collaborative problem solving results in groups is shown in Fig. 10.4, based on the evaluation framework mentioned earlier.

The figure shows that the intervention classes outperformed the control classes in group collaborative problem solving. As neither group of classes' performance was normally distributed, a non-parametric test was used to test the difference between the two data groups.

Based on Table 10.12, the intervention class outperformed the control class on all items except 'Marking of functions.' Combining Cohen's *d* and *p*-values, the most significant differences were found for the 'Number of rooms' item, which showed a high level of validity, and the 'Geometric drawing' item, which was borderline significant and showed a medium level of validity. The differences in the other items were not statistically significant and showed lower validity. Generally, comparing the two groups' total scores revealed significant differences with moderate validity.

The above results show that, with teacher intervention, more students correctly marked the number of rooms, indicating they understood the concept of 'room' in the task through their experience and could solve questions such as, 'Is a balcony an apartment room?' This will be discussed in more depth later.

In addition, some groups with more than four members appeared to lag behind the four-member groups' overall performance but outperformed the intervention classroom group with more than four members and the control class, as shown in Table 10.13.

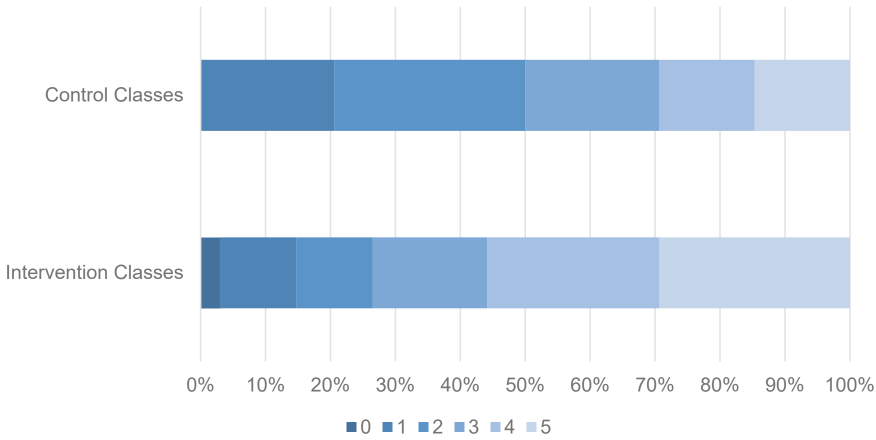

Fig. 10.4 Comparison of group collaborative problem solving results

Table 10.12 Scoring result of group collaborative problem solving

Items	Intervention classes (34)		Control classes (34)		p	Cohen's d
	M	SD	M	SD		
Number of rooms	0.85	0.36	0.62	0.49	0.029	0.539
Marking of functions	0.82	0.39	0.85	0.36	0.744	−0.078
Geometric drawing	1.15	0.82	0.79	0.84	0.084	0.419
Area	0.59	0.50	0.47	0.51	0.335	0.231
Total score	3.41	1.48	2.74	1.36	0.045	0.472

Table 10.13 Scoring result of groups of different sizes

Groups	4-Student		5-Student		6-Student		Total	
	N	$M(SD)$	N	M (SD)	N	M (SD)	N	M (SD)
Control class	29	2.90 (1.37)	1	1.00 (–)	4	2.00 (0.82)	34	2.73 (1.35)
Intervention class	27	3.52 (1.47)	3	3.00 (2.00)	4	3.00 (1.41)	34	3.41 (1.48)
Total	56	3.19 (1.44)	4	2.50 (1.91)	8	2.50 (1.91)	68	3.07 (1.45)

10.4.2.2 Comparisons of Groups

Based on the pre-test scores, the student groups were divided into three group structures: high-score homogeneous, low-score homogeneous, and heterogeneous. A 4 (group structure) × 2 (intervention or not) multifactorial between-group experimental design was used to explore the intervention's effects on pairs with different pair structures further (Table 10.14).

Adjusting the model term ($F = 2.069$, $p = 0.081$) showed the model was borderline significant, where the experimental group term ($F = 4.442$, $p = 0.040$) was statistically significant, while the group structure term and the crossover term were not. The above between-subjects effect test indicated that the model was reasonable and explained 14.3% of the variance ($R^2 = 0.143$) (Table 10.15).

Table 10.14 Basic information of groups with different group structures

	Low-score homogeneous group		How-score homogeneous group		Heterogeneous group	
	N	M	N	M	N	M
Control classes	6	2.33	13	2.69	15	2.93
Intervention classes	10	3.10	12	4.17	12	2.91
Total	16	2.81	25	3.40	27	2.93

Table 10.15 Between-subjects effect test

	F	*p*
Calibration model	2.069	0.081
Experimentation	4.412	0.040
Group structure	1.455	0.241
Experimentation × group structure	1.846	0.166

Fig. 10.5 Score of different group structures

Figure 10.5 shows that the intervention classes' low-score and high-score homogeneous groups outperformed the corresponding two control class groups, with no significant difference in the groups' performance. In other words, the intervention improved the performance of the low- and high-scoring homogeneous peer groups most significantly, while there was nearly no change in the performance of the heterogeneous group.

10.4.2.3 Comparisons of Differences in Interventions Across Teachers

As can be seen from the above results, Teachers A, B, and D's intervention classes performed significantly better than their control classes, while Teacher C's intervention and control classes had no significant performance difference. Group performances in intervention classes were less discrete than those in the control classes in all teachers' classes, except for the two classes taught by Teacher B (Table 10.16).

Table 10.16 Mean and standard deviation performance of pairs of different levels

Teacher	A	B	C	D
Control class	2.75 (1.58)	2.63 (1.41)	3.00 (1.41)	2.56 (1.23)
Intervention class	3.88 (1.55)	3.38 (1.85)	3.00 (1.41)	3.44 (1.23)

10.4.3 Results of the Case Study

As there were no statistically significant differences in academic performance between classes taught by the same teacher, the effect of teacher intervention on CPS outcomes can be determined by comparing the performance of the intervention and control classes. The previous statistical analysis revealed that teacher interventions generally contributed to students' collaborative pair and collaborative group problem solving outcomes. However, there were exceptions. The intervention class taught by Teacher B had lower overall pair CPS performance than the control class, while there was little difference in Teacher C's two classes' collaborative group problem solving.

The above discussions were oriented toward the peer or group collaboration results, and the intervention's impact and effectiveness were judged solely by the results. Cases had to be selected and analysed in depth to explore teacher interventions' impacts further. Accordingly, two teachers, Teachers B and D, were selected. Based on student response results alone, Teacher B's intervention was the least effective, with the intervention class performing poorer than the control class, while Teacher D's intervention was the most effective. The basic information about the two teachers' effective verbal interventions is shown in Table 10.17.

In general, the teachers initiated most interventions in both classes (Fig. 10.6). However, the percentage of intervention initiators in the two teachers' classes differed significantly at different stages. In Teacher B's class, the students initiated most of the pair collaboration stage conversations, while the teachers initiated the vast majority of group collaboration stage conversations. In Teacher D's class, the students initiated nearly two-thirds of the pair collaboration stage conversations, while the teachers initiated half of the conversations in the group collaboration stage. In Teacher D's class, nearly two-thirds of the pair collaboration phase conversations were teacher-initiated, while teachers and students each initiated half of the group collaboration phase conversations.

Table 10.17 Number of teacher intervention

	Pair collaboration phase	Group collaboration phase	Total
Teacher B	38	27	65
Teacher D	44	43	87
Total	82	70	152

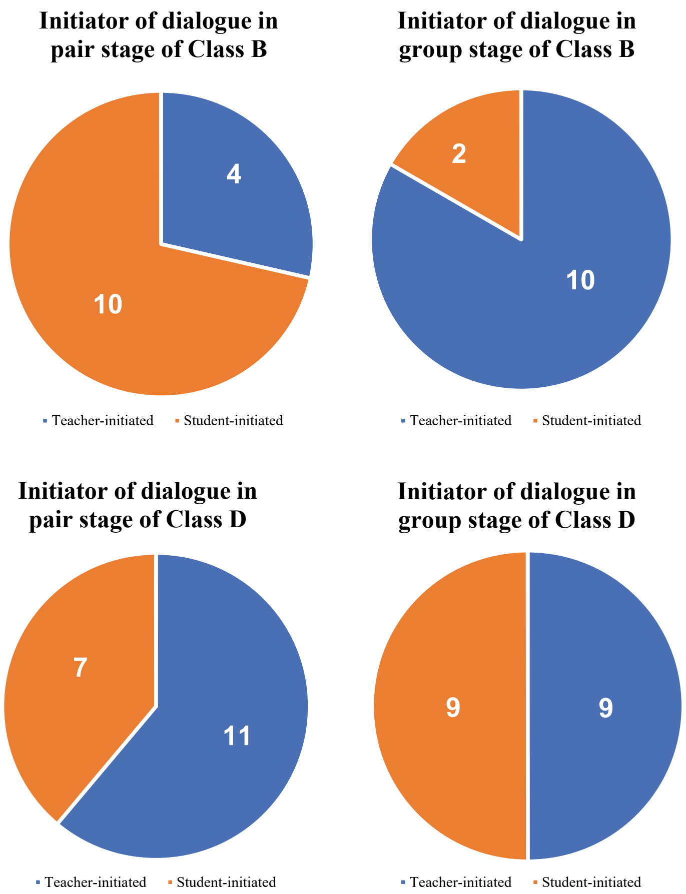

Fig. 10.6 Initiator of dialogue

In the two collaborative problem solving phases, the teachers' intervention foci were slightly different, with the interventions Teacher B initiated focusing on cognitive activities (17/20 in the pair phase and 12/23 in the group phase) and those Teacher D initiated focusing on social activities (19/34 in the pair phase and 17/26 in the group phase).

In the different problem solving stages, all student-initiated questions concerned cognitive activities. Most of both teachers' subsequent interventions were also directed toward cognitive activities, especially for Teacher B, who directed almost all interventions at cognitive activities (Pair stage Teacher B: 17/18, Teacher D: 9/13; Group stage Teacher B: 4/4, Teacher D: 12/17) (Fig. 10.7).

Both teachers generally prefer to intervene in the whole pair/group. Teacher B's pair stage interventions focused more on individuals, while her group stage interventions focused more on the whole group (Table 10.18). Teacher D's interventions were

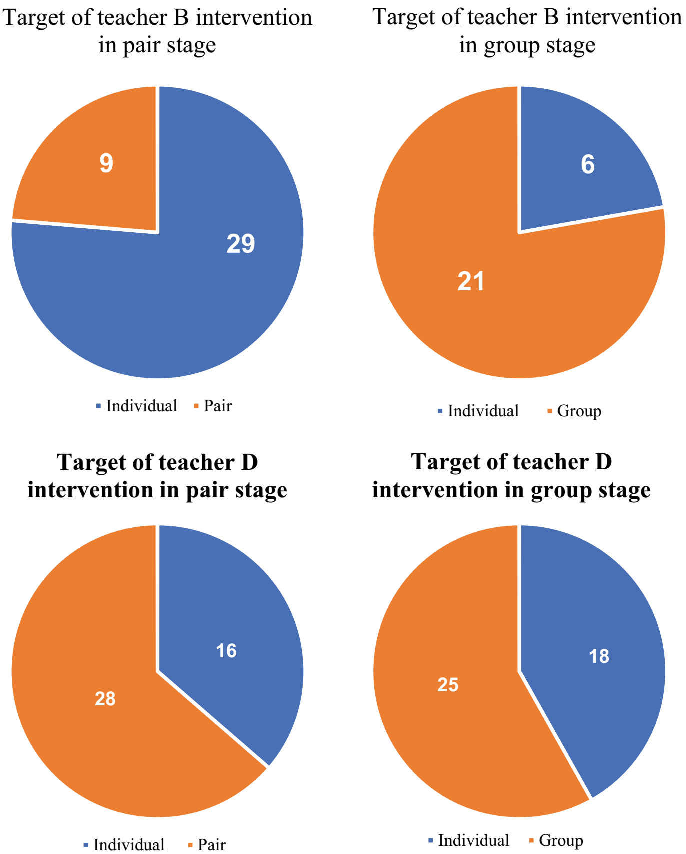

Fig. 10.7 Target of teacher intervention

almost the same at both stages, mostly focusing on pairs/groups while still paying sufficient attention to individual students (Fig. 10.8).

Figure 10.7 shows the disruption of intervention by Teachers B and D, indicating that both teachers initiate interventions among nearly equally groups. As the teachers claimed that they made students in different groups have similar performance in previous tests, the current study could not support information on the correlation between the previous test score and teachers' favour in intervention. Compared with students' CPS performance, there was possibly a negative correlation between distribution and performance. It could be that both teachers preferred to pay more attention to students in difficulty, which seemed ineffective for difficult pairs and groups.

Table 10.18 Performance of each pairs (average in groups) and groups

Teacher B's class	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	
Pair (Ave.)	3.5	5	3	5	1.5	2.5	1.5	4.5	
Group	2	3	5	5	1	5	1	5	
Teacher D's class	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	Group 9
Pair (Ave.)	3.5	4	4	3	5	5	4.5	4	4.67
Group	4	2	4	5	4	4	1	3	4

10.5 Discussion

10.5.1 Intervention Was Generally Effective, but Limited for Heterogeneous Groups

Regarding collaboration results, pairs' or groups' outcome scores were significantly higher in the intervention class than in the control class. Most intervention classes scored higher on the task, suggesting that the interventions in this study generally provided effective scaffolding for student problem solving and metacognition to enhance student response outcomes. The most significant improvements in the pair stage were in mathematical performance and expression, while the most significant improvement in the group stage was in 'Number of the room,' indicating problem comprehension. Analysis of the cases revealed that both teachers' interventions may have facilitated students' understanding of the problem's meaning. The model both teachers adopted to encourage reflection and discussion was more conducive to students' problem solving through discussion. The interventions that encouraged students to collaborate may have facilitated their mutual expression, leading students to make better explanations and representations on the task sheets. Among these, Teacher D's intervention for a more balanced cognitive and social activity was more effective.

This study also found that teacher interventions had a more limited effect on heterogeneous groups. For pairs or groups with different structures, the most effective were high homogeneous groups, followed by low homogeneous groups (homogeneous medium groups among pairs), while there was little difference in performance between the intervention and control classes for heterogeneous groups. The case study revealed that teachers' individual interventions for the latter students in pairs (or groups) had limited effect, especially for silent, reluctant students, who did not engage in collaboration after the intervention.

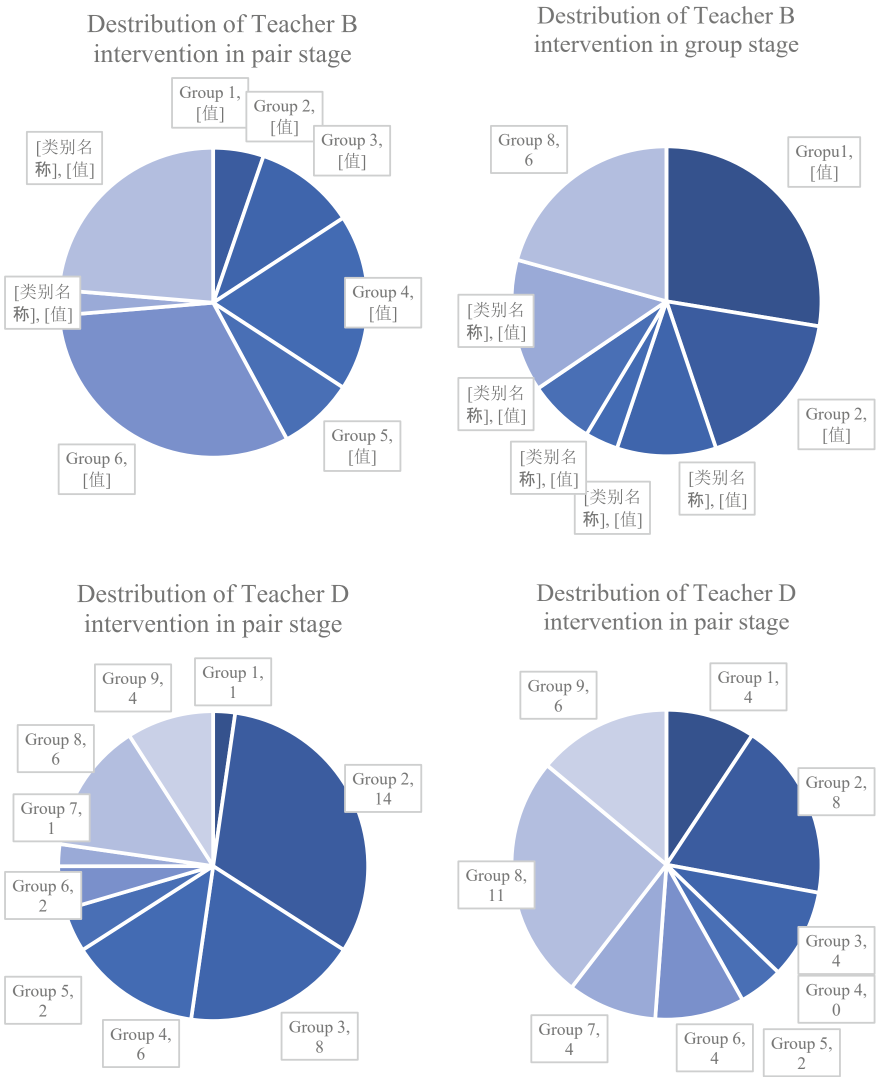

Fig. 10.8 Distribution of teacher intervention

10.5.2 Differences in the Level of Control of the Intervention

Both teachers more frequently implemented high-control-level interventions like straitly introduction and explanation to the students (Vermunt & Verloop, 1999) than low-control interventions like hinting or heuristics (Vermunt & Verloop, 1999). This result indicates that both teachers' intervention styles were biased toward high levels of control.

The two teachers presented different intervention styles when responding to students' questions, which could be divided into two categories: asking the teacher to explain the questions and asking the teacher to check the students' results. Both teachers responded less directly, allowing students to make decisions and encouraging them to give 'reasonable' and 'realistic' answers. The differences between the two teachers were that while Teacher B generally explained the nature of the open-ended tasks, encouraging students to think about them and reflect on them even after completion, Teacher D promoted student discussion, urging the pairs/groups to reach a consensus.

Throughout the student pair and group collaboration stages, Teacher D's intervention behaviour performance was relatively stable, whereas Teacher B's showed instability. Regarding intervention characteristics, the frequency, focus, and targets of Teacher B's interventions differed greatly between the collaboration stages. In the pair stage, Teacher B initiated interventions less often, focused more on cognition, and targeted individuals more while doing the opposite in the group stage.

The two teachers presented different intervention styles, with Teacher B's interventions being relatively mild and Teacher D's more strict. Teacher D more frequently initiated interventions than Teacher B. Video analysis showed that Teacher D had a relatively higher level of control in her interventions, was more prepared, systematically targeted some students with direct interventions, and corrected some students' off-task words, while Teacher B did not. Teacher B always observed students' performance with a smile on his face and had fewer physical interactions with them, whereas Teacher D would put her hand on students' shoulders when interacting with them individually to achieve one-to-one instruction, which might have exerted pressure on students.

In Dong et al.'s (2013) study, teacher-initiated instruction was relatively lower, accounting for only 9.9% of collaborative communication. In contrast, in the present study, excluding Teacher B's performance during the pair collaboration phase, the percentage of teacher-initiated instruction for collaborative communication was significantly higher for both teachers, and students performed better. Teacher D initiated a relatively higher proportion of interventions, and her students performed better, further illustrating the need for teacher instruction in collaborative communication.

The state of the two teachers' students' collaboration also differed. Excluding groups that did not fully discuss, Teacher B's students showed more agreement, while Teacher D's showed more argument. Previous CPS research has shown that a successful group always features arguments in which group members evaluate others' perspectives, promote further understanding of the problem, adjust their views, and seek a common solution (Cobb, 1995). Students in Teacher B's class may have argued less because Teacher B did not adequately encourage it; too little arguing leads group members to suppress or ignore disagreements and form a superficial consensus, resulting in no one challenging wrong ideas. It was observed that students in Teacher B's classroom were sometimes afraid to express disagreement.

10.5.3 Interventions Based on the Understanding of the Students and Emphasis the Equality

In one part of their interview, teachers were asked about the effectiveness of each group's learning. In their responses, both teachers paid more attention to students with outstanding performance and the most potential for improvement when evaluating group members' performance, based on how they understood the students in their daily classroom. Some teacher interventions referenced students' previous performance, comparing it to their current classroom performance. Van Leeuwen et al. (2015) found similar findings in online collaborative learning cases, where teachers based approximately 35% of their interventions on students' prior knowledge.

The two teachers had slightly different foci on proactive diagnosis and subsequent interventions. Teacher D focused more on the individual, calling students by name and asking for their thoughts directly in the classroom, whereas Teacher B did not. Teacher B tended to diagnose students' performance through observation, whereas Teacher D tended to rely more on words. When pairs were not collaborating, Teacher D diagnosed their status and encouraged discussion and communication, whereas Teacher B seemed to focus more on task-related outcomes and less often encouraged discussion. Though both teachers paid special attention to non-collaborating students, Teacher D tended towards direct verbal encouragement, while Teacher B was more observant; he commented on these students' performance during the interview, confirming his concern for them. Verbal diagnosis is important in facilitating student collaboration. Engeness and Edwards (2017) found that when teachers checked students' ideas, it helped students summarise and further explain their current progress.

Even though the limited effect of the intervention, the current study finds that both teachers emphasised equality for poorly-behaved pairs and groups. Unlike what has been found in previous studies (Pietarinen et al., 2021; Van Leeuwen et al., 2013), both teachers noticed students with poor academic performance, as confirmed in the interviews. However, in the group stage, teachers preferred interacting with poor discussion groups rather than individuals. Both teachers paid attention to groups that failed to collaborate during the group stage and encouraged them to work on collaborative tasks. However, this encouragement was given only when they observed that half of the group members were not collaborating or the group was still working in pairs. Neither teacher intervened if only one student failed to participate.

10.6 Suggestions

This study has analysed the basic situation and problems in teacher intervention in collaborative problem solving mathematics classrooms and summarised effective teacher intervention strategies. The following strategies provide ideas for effective teacher intervention in collaborative problem solving classrooms, combining the results of the above study with strategies for and discussions of timing interventions and selecting intervention content to overcome existing problems.

Forming Different Types of Collaborative Activities Based on Teaching Objectives

This study found that the two teachers' diagnoses shared common characteristics, suggesting selective pair (table) or group collaborative learning and collaborative problem solving, depending on the teaching situation. Although there were some differences in their intervention styles, both teachers focused on the individual, with specific interventions in the pair stage, and on the group as a whole in the group collaboration stage, easily ignoring silent group individuals. This is acceptable for a limited time. Given a similar situation in the daily classroom, we suggest that if the task is relatively simple and tends to test individual ability, pair (table) discussion could be considered to facilitate discovering silent individuals (because it is hard to have a discussion if one of the two is silent); if the task is relatively difficult, tends to be more exploratory, and requires more collective wisdom, then group discussion could be considered so the teacher can diagnose overall group performance more intuitively.

It is recommended that teachers effectively choose to conduct different forms of collaboration in daily classroom teaching—table (pair) discussion or group discussion. We suggest forms be chosen based on task difficulty and the collaboration object, in conjunction with the actual teaching situation, to enhance classroom efficiency. Previous studies have found that class and group size significantly impact teacher-student interaction; specifically, it is more likely that spectators or smaller collaborative groups will emerge within oversized groups (Kreijns et al., 2003). A similar phenomenon was found in this study, where groups with more than four students performed relatively worse; in the interview, Teacher D indicated that smaller, more silent collaborative groups existed in groups of six. It can be seen that when designing and conducting collaborative learning and collaborative problem solving, teachers should also arrange and design appropriate group sizes based on class size to maximise collaboration efficiency and achieve instructional goals.

Strategies for Sustained Interaction

Research has shown that most teachers only intervene when they find problems in students' collaborative processes. While this situation occurs more frequently in classes unfamiliar with collaborative learning, frequent active interventions often interfere with the normal process of student discussions and make students dependent on the teacher. It is recommended that teachers train students' communication skills through interventions. This can be done through a continuous interaction strategy in which the teacher constantly interacts with the group while participating in the group discussion. The relevant literature and the data in this study identify four strategies teachers commonly adopt when intervening with whole groups: (1) repeating ideas presented by students; (2) asking students to explain their proposed ideas; (3) prompting students to explain the source of their ideas; and (4) encouraging students to compare their respective ideas and reasoning processes.

Encourage students to explain their thoughts through questions like:

– Can you please explain why you think that way?
– Can you explain to the group what you mean?
– Can you please explain how you came to this conclusion?

Follow up with students on their evaluation of others' ideas through questions like:

– Could you ask other group members what they think?
– What do you think of the idea the student just presented? Could you comment on it?
– Could you please explain why you disagree with the other student's idea?
– What is the problem with the other solution?
– Could you please help him explain his idea?

Group agreement (without precluding outcome diversity) is key to a smooth discussion and successful task completion as a collaborative process. Teachers can use these interventions to guide groups to learn to communicate consistently, keep groups interacting, and get to the root of the problem of frequent teacher-initiated interventions. Long-term training helps students think about these issues spontaneously while engaging in discussion, improving their mathematical communication and collaborative problem solving skills.

Motivational Strategies

Focusing on enlightening intervention content can facilitate the problem solving process. The enlightenment strategy is closely related to how the teacher initiates the intervention and subsequently interacts with students. These are critical moments in the collaborative mathematics problem solving classroom, where the teacher must decide how to intervene and determine the intervention's focus and content. The data from this study showed that teachers could (1) encourage group members to speak, (2) only observe student discussions (without the teacher speaking), and (3) remind students to read the material carefully. This is a more effective motivational strategy that allows most group members to join the discussion with the teacher's encouragement; the task-solving process is not 'contracted' by a few students, and the teacher only observes and listens to the students' discussion after initially encouraging them. Even after the teacher leaves, students continue to share their opinions and ideas for a certain time. If students have questions about how to proceed with problem solving, the teacher can help reduce their teacher-dependence by reminding them to read the task materials carefully and review the existing conditions and problems to see where they are having difficulty. However, if students are experiencing difficulties that cannot be solved using pair resources alone, it is important that the teacher promptly provide additional information, which involves information supplementation strategies.

Rule-Making Strategies

In addition to the teacher-initiated strategies used in the intervention process described above, teachers can use rule-making strategies to address the problem of over-frequent teacher-initiated interventions and conduct whole-class interventions before initiating collaborative problem solving. Rule-making strategies involve the long-term development of students' collaborative problem solving skills. They can be divided into developing rules related to problem solving and developing rules related to collaborative discussions and usually take the form of whole-class teacher interventions. As the researcher did not make this a mandatory aspect of this study, the teachers only explained the task completion rules to their class and did not set rules for the collaborative group discussion process. However, this study found that the lack of discussion rules generally led to conflicts in group work, which led to teacher intervention. As group discussions can be made more efficient by establishing rules, the following suggestions are offered to help teachers develop task resolution rules and discussion rules:

Task resolution rules

- Clarify the task(s) to be completed
- Clarify the time needed to complete (each) task
- Clearly define the participants in the task (e.g., how many people are involved in the discussion) and divide the task among team leaders, reporters, recorders, etc. based on the needs of the activity
- Clarify the presentation of the task completion results, e.g., filling out task sheets, making posters, etc.

Discussion rules

- Share your ideas and listen to each other during the discussion
- When discussing, one person speaks at a time, one after the other
- Fully explain your ideas
- Ask 'why' if there is a difference of opinion
- Try to agree on the outcome of the discussion

Once the ground rules have been established in the classroom, they can be used as common-sense guidelines for teachers and students to follow in collaborative problem solving classrooms, facilitating teachers' efficient guidance and students' thinking together through verbal communication. After developing these ground rules, collective deliberation (through class meetings and other formats) can ensure they are appropriate and enforceable.

Complementary Information Strategies

Information supplementation strategies involve teacher interventions that offer additional mathematical or task-related information rather than directly providing solutions. Teachers can prompt students to consider the resources available to them. For example, in a house design problem, the teacher could prompt students to consider the size of the classroom bricks and provide the dimensions of the classroom bricks, from which students could estimate the approximate size of each room, enabling them to solve the problem by using available resources. Students may have a greater need for additional information to solve more complex problems. Teachers should be aware of and properly scaffold students' information needs during interventions.

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Chapter 11
Research on the Evaluation of Students' Collaborative Problem-Solving

Bingxuan Du

11.1 Introduction

The research on collaborative problem-solving (CPS) in China and abroad is gradually improving. In this process, experts in the education field have shown interest in the performance of Chinese students (Bai & Lin, 2016; Tan, Li, & Wan, 2018). CPS ability contains relatively complicated including cognitive, social, and metacognitive aspects (Krieger et al., 2022).

Discourse analysis, first and foremost as a linguistic study, has its own unique methodological system, with the main research methods containing critical discourse analysis (Oppong, 2017; Wang & Wu, 2017), *Foucauldian* discourse analysis methods (Yang & Yu, 2018), multimodal discourse analysis, etc. In the field of education, discourse analysis is particularly important in the field of classroom teaching research, and many valuable conclusions have been obtained through discourse analysis (Rogers, 2005). In recent years, the study of classroom discourse has received attention in mathematics education research, and many studies at home and abroad have provided insights into the hot topics of discourse-related research in mathematics classrooms. By studying two primary school mathematics classrooms, Gana E et al. raised the issue of mathematics classroom configuration, arguing that the semantic potential of classroom space design in mathematics teaching needs to be positively facilitated by appropriate management by teachers. Linking the management of space in the classroom (students' desk arrangement/teacher's position in the classroom space) to classroom discourse, a concrete dissection is made using authentic discourses from classroom videos, with the main object of study being the teacher's behavior and discursive performance. The semantics of the "perceptual space" created by the particular arrangement of the classroom desks clearly supports

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the methodological approach, and therefore the mindset of learning through participation in small groups. The significance of this study is that it validates the classroom effectiveness of cooperative group learning through the semantic methodology of discourse analysis, highlighting the feasibility of cooperative learning in the classroom (Gana et al., 2015). By introducing the concept of "Deliberative Dialogue", the study explores key features of mathematics classroom practice and identifies the importance of teacher discourse in classroom teaching and learning. The study also focuses on identifying the difference between "dialogue" and "communication" from the student's perspective, in terms of the teacher's willingness to facilitate the democratic participation of all students. That is, whether it is a teacher–student dialogue or a student–student dialogue, it is crucial to give meaning to the judgments made in the discussion dialogue. Students often use judgment as a neutral act of language, thinking aloud and expressing meaning when understanding and accepting or not accepting their dialogue partner. This identification provides the basis for the definition of discourse in this study (Ana et al., 2015).

In the current study, I selected four Beijing LH middle school classes and collected videos and recordings of CPS in Grade 7 pairs and groups. This research focuses on the embodiment of CPS in students' classroom discourse, analyzes the content of students' classroom discourse, and combines the 12 three-level sub-dimension skills in the PISA 2015 CPS evaluation framework, based on quantitative research discourse. The standard for assessing ability level in PISA 2015 is to evaluate and research students' CPS in math classroom videos. Based on the literature on CPS and discourse analysis and the status quo of collaborative group teaching in domestic mathematics classrooms, this chapter attempts to explore the following issues:

(1) How to evaluate students' ability to solve collaborative problems through classroom discourse?
(2) What sub-dimensions of CPS abilities are reflected in students' discourse, and what differences in CPS abilities exist in teams with different numbers of students?
(3) What is the students' CPS level in the experimental classroom?

High-level abilities such as CPS skills have gradually become the focus of the education field and key abilities required for future talent development. The research of this paper is based on previous research and evaluation and has the following research purposes and significance.

This section of the study expands on previous research and assessments to examine students' discourse performance in Chinese classrooms and improve students' CPS skills in a targeted manner, using international assessment framework items. It is hoped that identifying the strengths and weaknesses of students' CPS processes will provide a basis for subsequent targeted development of students' CPS skills. This study is aligned with international education and explores the elements of skills for developing international talents.

Table 11.1 Study samples

Class	Number of pairs	Number of groups
1*a*	11	6
1*b*	12	6
3*a*	10	5
3*b*	8	4

11.2 Research Method

11.2.1 Subject and Methodology

11.2.1.1 Participants

Based on the specific research process, the author selected student pairs and groups from the project that meet the following screening criteria. First, the selected classes must use the same set of collaborative problems. At the same time, the teacher must rarely interfere with the students' collaborative process, limiting themselves to issuing and explaining tasks, patrolling, and solving students' problems not related to the collaborative problems in the classroom. Second, the names in the student pairs and groups must correspond with those of the students in the video, and the recordings of their conversations must be clear and legible. Third, the composition of the pairs and small groups must meet the project design requirements. Specifically, two pairs formed one group, the collected pair and group student task sheets had to be fully completed, and the pair group names and numbers had to be clear.

Based on the above screening criteria, videos and recordings of six classes were screened to select appropriate classes with pairs and groups. After screening, the selected classes for the study were *1a*, *1b*, *3a*, and *3b*, which included 41 pairs and 21 groups. The specific number of pairs and groups in each class is described in the subsequent data analysis section, as shown in Table 11.1.

11.2.1.2 Research Methodology

A combination of qualitative and quantitative research methods was used to study the CPS processes of students' pairs and groups in the experimental classroom videos.

1. Qualitative research methods

This study used the PISA 2015 CPS framework (Fig. 11.1) to briefly analyze the CPS of the students who participated in the experiment. We evaluated student performance in each dimension of the CPS process based on the levels of each ability dimension given in PISA 2015. This is also the main body of this study.

This was done by textual transcription of the video and recording samples. First, we analyzed specific questions from the pair and group collaborations in the video.

	(1) Establishing and maintaining shared understanding	(2) Taking appropriate action to solve the problem	(3) Establishing and maintaining team organisation
(A) Exploring and understanding	**(A1)** Discovering perspectives and abilities of team members	**(A2)** Discovering the type of collaborative interaction to solve the problem, along with goals	**(A3)** Understanding roles to solve the problem
(B) Representing and formulating	**(B1)** Building a shared representation and negotiating the meaning of the problem (common ground)	**(B2)** Identifying and describing tasks to be completed	**(B3)** Describing roles and team organisation (communication protocol/rules of engagement)
(C) Planning and executing	**(C1)** Communicating with team members about the actions to be/being performed	**(C2)** Enacting plans	**(C3)** Following rules of engagement, (e.g. prompting other team members to perform their tasks)
(D) Monitoring and reflecting	**(D1)** Monitoring and repairing the shared understanding	**(D2)** Monitoring results of actions and evaluating success in solving the problem	**(D3)** Monitoring, providing feedback and adapting the team organisation and roles

Fig. 11.1 Matrix of CPS skills for PISA 2015

We set specific discourse criteria for each level of the 12 three-level dimensions of competence in the PISA 2015 framework, classified students' discourse competence and level based on the criteria, and then conducted statistical analysis to conclude the study. The findings of the study are also summarized and analyzed.

2. Quantitative research methods

Based on the students' discourse in each group during the collaboration, the specific number of words and discourse rounds in the students' pairs and groups were counted to obtain relevant statistical results, which were combined with the results of the qualitative analysis to draw relevant research conclusions.

11.2.1.3 Data Collection Methods

Since this experiment focuses on students' discourse, the main data sources were recordings of students' actual collaborative processes. Using the project team's recording equipment, the video recorder and audio channel parameters were adjusted before recording to facilitate subsequent transcription of each student's discourse. At the same time, students' pair and group work produced paper results. The paper results from the pairs and groups needed to be accurately collected to evaluate the CPS level. To match the paper results with the experimental pairs and groups, the project team contacted the classroom instructor in advance and asked them to assist in pre-grouping the pairs and groups on the printed pair and group problem-solving task sheets. In this way, the groups were tailored to the actual classroom situation and met the experimental requirements.

Following experimental design and data collection principles, the project team experimented smoothly and collected several experimental groups with better effects, including 41 pairs and 21 groups. Among them, the experimental groups with better effects had the following characteristics: the names and genders of group members were identified, two pairs corresponded with one group, the recorded discourse was clear and legible, and the task sheets were collected completely.

11.2.1.4 Discourse Analysis and CPS Evaluation

The main body of this study is an analysis of student discourse in the CPS process, so student pair and group recordings were the original data used and analyzed. Based on the research questions, the analysis of student discourse data was conducted based on the following analysis steps.

First, recordings were transcribed to obtain the actual conversation of each student participating in the discussion. The transcription must match the words with the students' names to ensure the study's authenticity, and so must be done against the video.

After obtaining the transcripts, each student's discourse was analyzed in combination with CPS levels. The discourse that showed CPS was marked and the levels classified based on the PISA 2015 framework. After level delineation, the approximate level of CPS ability demonstrated by each student in the experimental situation could be obtained and the relevant research conclusions reached by analyzing and briefly comparing the differences in students' abilities in conjunction with the specific CPS process.

After getting each student's score, a comprehensive evaluation of students' CPS ability within the same group was conducted to obtain the comprehensive ability level of group cooperation. Then, the experimental groups' differences in CPS ability were analyzed to obtain relevant research conclusions.

Since each student was involved in two collaborations (pair and group), depending on the different collaborative problems, it was possible to obtain the same student's performance twice in two collaborative problem situations and analyze it based on the level of competence dimensions demonstrated in both collaborative processes, enabling the researcher to make relevant research conclusions.

11.2.2 Scientific Justification of Evaluation Criteria

The PISA 2015 discourse framework was the main evaluation criteria used in this study. The scientific validity of this evaluation criterion is demonstrated to ensure the rationality, rigor, and validity of the ensuing analysis and demonstrate its real value.

11.2.2.1 Rationality of the Theoretical Level

The evaluation criteria in this study used the PISA 2015 framework to assess CPS and were modified and improved based thereon to make the evaluation criteria more suitable for the complete assessment of students' CPS skills in real classrooms.

At the theoretical level, the PISA 2015 framework is relatively well developed. It has been administered to 9,800 15-year-old students in China, and the corresponding results have been obtained. Some educational researchers have evaluated the rationality of this framework, arguing that PISA 2105 focuses more on the collaborative dimension in defining skills for CPS. The four problem-solving skills dimensions intersect with the three collaborative skills dimensions, implying that the collaborative and problem-solving dimensions are interwoven in evaluating students' skills in each sub-dimension. The 12 skill sub-dimensions contain information on both the collaborative and problem-solving dimensions (OECD, 2017a). That is, this framework decomposes the CPS process into 12 sub-processes where collaboration and problem-solving intersect, and the skills embodied in each process are evaluated separately. In this framework, 12 sub-dimensional CPS skills form an organic whole and do not cross over and work together.

The PISA 2015 framework was selected for refinement in this study, taking into account students' cognitive characteristics and how CPS skills are reflected in students' discourse. The PISA 2015 test questions on CPS show that students' responses to items reflect their cognition in specific problems. Therefore, in this study, the essence of the evaluation was to concretize students' "responses" in the CPS process, analyze their discourse, and evaluate the concrete expression of their CPS from their discourse, in line with the Item Response Theory used in the PISA 2015 assessment of students.

11.2.2.2 Rigor of the Development Process

As the evaluation criteria are important for this study's authenticity and reliability, their development process is described and its rigor demonstrated.

After analyzing and sorting the PISA 2015 and ACT21S frameworks, we selected the PISA 2015 framework as it is more suitable for our students and has been used in their assessment. The researcher provided experts and teachers with evaluation examples, invited their comments and suggestions for improvement, and made modifications and improvements based on their opinions.

In the above process, the contact between the researcher and the experts and teachers was one-way. The experts and teachers did not interact. The researcher summarized their opinions and synthesized them for revision to ensure they were objective and rational. Furthermore, the experts and teachers focused on the evaluation criteria differently: the education research experts paid more attention to differences in ability levels and the match between student discourse and ability; the secondary school mathematics teachers focused on student performance and discourse content and evaluating students' discourse performance. Combining the two perspectives resulted in a more refined version.

11.2.2.3 Practice-Level Validity

To verify the PISA 2015 framework's validity in this application, three project group members were invited to evaluate the same student's discourse in a CPS process about the discourse evaluation criteria; the researcher then analyzed and compared the abilities and levels based on the three members' evaluations.

Analysis of the same discourse fragment, conducted after the project team members had become familiar with the evaluation criteria, revealed a concentration of sub-dimensional competence items and subtle differences in level delineation. After the discourse evaluation, project team members were invited to analyze this fragment briefly. The project team members centered their analysis on the skills reflected in the students' discourse, described the main skill characteristics of two students in the peer group, and analyzed why their abilities were at a certain level.

In summary, the discourse evaluation criteria for CPS skills in this study were valid and helped the researcher grasp students' CPS skills and corresponding levels efficiently.

11.2.2.4 Realistic Value

Evaluation studies of group collaboration have been the focus of many educational researchers and front-line teachers and are important for evaluating the learning effects of group collaboration and students' collaboration levels. In many studies, the evaluation of group collaboration focuses on students' collaborative outcomes, usually using paperwork, group reporting, or developing a series of process evaluation forms for students to self-evaluate (Cao & Bai, 2018). Such an evaluation approach is biased toward evaluating students' competence or skills in the problem-solving dimension and is weaker in evaluating the level of students' competence in the collaboration dimension.

The CPS sub-dimensional ability level classification and discourse evaluation criteria in this study were based on students' classroom discourse, which provides a specific and detailed evaluation of each student involved in the cooperative, fully taking into account students' equal participation in the cooperative, the organic integration of the collaborative and problem-solving dimensions, and avoiding situations where collaborative results or group reports are done by fixed members alone.

CPS is a student-led process, and the teacher cannot intervene in each student group's collaboration. However, in guiding their collaboration, teachers can capture students' discourse in the collaboration process and make timely evaluations in conjunction with this discourse evaluation standard to grasp students' performance of related abilities, which can help teachers deeply grasp students' learning, thinking, and ability development.

Moreover, due to the rapid development of information technology, more tools and techniques can be added to classroom teaching to assist teaching and learning and evaluate teaching effectiveness. It is believed that the popularization of related technology facilities will make collecting and transcribing students' classroom discourse simpler and enable teachers to obtain and evaluate students' discourse easily and quickly. The PISA 2015 framework will provide teachers with powerful guidelines for assessing students' CPS.

11.3 Study Results

11.3.1 Results of Quantitative Statistics and Qualitative Analysis of Students' Collaborative Discourse

11.3.1.1 Description of the Experimental Object Number

The data selected for this study were drawn from four classes and included 41 pairs and 21 groups. Two classes were experimented on each time, numbered *1a* and *1b*, and *3a* and *3b*.

After transcribing the pair and group recordings from the four selected CPS classrooms, 41 pair collaborative discourse texts and 21 group collaborative discourse texts were obtained. These transcribed discourses were first counted to obtain visual statistics on the number of words and conversation rounds.

Based on existing research, quantitative classroom discourse analysis provides strong evidence for researching and improving teacher–student relationships. In CPS research, the number of words and conversation rounds can complement qualitative research and provide valuable results and conclusions.

The pairs and groups were numbered as follows: "class - group number - pair number"; thus, 1*a*-3–2 means the second pair in the third group of study class 1*a*, with students in 1*a*-3–1 and 1*a*-3–2 forming the third group. Four data groups from each studied class were used for data comparison clarity.

11.3.1.2 Discourse Volume Statistics

The number of words between students' pairs and groups was counted, and the results for each studied class are as follows (Tables 11.2, 11.3, 11.4, 11.5).

Table 11.2 Statistical table of the number of words of students in class 1a (unit: words)

		1a-1–1	1a-1–2	1a-3–1	1a-3–2	1a-6–1	1a-6–2	1a-7–1	1a-7–1
Pairs	Total words	1281	1552	1627	1298	1602	1859	742	1329
	Average number of words/min	144.26	152.91	136.95	109.81	145.37	186.83	73.32	129.41
Panel	Total words	4960		1956		5152		3607	
	Average number of words/min	291.25		106.77		278.04		208.74	

Table 11.3 Statistical table of the number of words of students in class 1b (unit: words)

		1b-1–1	1b-1–2	1b-3–1	1b-3–2	1b-6–1	1b-6–2	1b-7–1	1b-7–1
Pairs	Total words	1348	1141	1683	1298	857	262	1213	1332
	Average number of words/min	119.50	103.92	141.67	109.81	72.94	29.87	121.30	112.88
Panel	Total words	3149		2039		1205		2867	
	Average number of words/min	183.29		111.30		81.31		265.96	

Table 11.4 Statistical table of the number of words of students in class 3a (in words)

		3a-1–1	3a-1–2	3a-3–1	3a-3–2	3a-6–1	3a-6–2	3a-7–1	3a-7–1
Pairs	Total words	1723	653	884	489	505	789	698	817
	Average number of words/min	226.71	79.83	108.73	62.69	64.91	94.72	78.60	87.85
Panel	Total words	5606		4329		1956		3275	
	Average number of words/min	246.96		188.79		97.17		150.44	

11.3.1.3 Statistics of the Number of Conversation Rounds of Conversation

The number of conversation rounds in the words of the collaborative process between students' pairs and groups was counted for each studied class, as follows (Tables 11.6, 11.7, 11.8, 11.9).

As can be seen, the number of words and conversation rounds varied widely among pairs and groups. It is impossible to tell students' CPS process and related ability level from the data, and in-depth qualitative discourse analysis is needed.

Table 11.5 Statistical table of the number of words of students in class 3b (in words)

		3b-1–1	3b-1–2	3b-3–1	3b-3–2	3b-6–1	3b-6–2	3b-7–1	3b-7–1
Pairs	Total words	1834	665	1208	918	2265	1364	1476	942
	Average number of words/min	215.26	81.30	86.16	65.48	159.28	118.61	104.09	65.87
Panel	Total words	3246		1688		3068		2483	
	Average number of words/min	161.09		95.10		140.73		114.16	

Table 11.6 Statistics table of the number of conversation rounds of students in class 1a (unit: rounds)

	1a-1–1	1a-1–2	1a-3–1	1a-3–2	1a-6–1	1a-6–2	1a-7–1	1a-7–1
Total number of rounds in the pair	57	50	71	125	57	89	30	54
Total number of rounds in the group	374		139		359		199	

Table 11.7 Statistics table of the number of conversation rounds of students in class 1b (unit: rounds)

	1b-1–1	1b-1–2	1b-3–1	1b-3–2	1b-6–1	1b-6–2	1b-7–1	1b-7–1
Total number of rounds in the pair	74	62	68	123	61	18	95	121
Total number of rounds in the group	221		142		110		233	

Table 11.8 Statistics table of the number of conversation rounds of students in class 3a (unit: rounds)

	3a-1–1	3a-1–2	3a-3–1	3a-3–2	3a-6–1	3a-6–2	3a-7–1	3a-7–1
Total number of rounds in the pair	107	48	53	23	20	43	35	40
Total number of rounds in the group	417		289		163		211	

Table 11.9 Statistics table of the number of conversation rounds of students in class 3b (unit: rounds)

	3b-1–1	3b-1–2	3b-3–1	3b-3–2	3b-6–1	3b-6–2	3b-7–1	3b-7–1
Total number of rounds in the pair	116	44	46	62	136	54	65	52
Total number of rounds in the group	288		138		230		246	

11.3.2 Results of Discourse Analysis in Student Pair and Group CPS

After determining the sub-dimensional ability and level classification of the 41 pair and 21 group collaboration discourses, the comprehensive data were processed to obtain the analytic results.

The 12 three-level sub-dimensional skills in the PISA 2015 assessment framework were numbered to facilitate the subsequent description of the specific levels of collaboration problems. The four second-level dimensions of the problem-solving competency dimension corresponded to A, B, C, and D, and the three second-level dimensions of the collaboration competency dimension to 1, 2, and 3, respectively; as such, the 12 third-level dimensions corresponded to A1-A3, B1-B3, C1-C3, and D1- D3, as follows:

Discovering team members' perspectives and abilities (A1); Discovering the type of collaborative interactions and goals used to solve the problem (A2); Understanding students' roles in solving the problem (A3); Building a shared representation and negotiating the meaning of the problem (common ground) (B1); Identifying and describing tasks to be completed (B2); Describing roles and team organization (communication protocol/rules of engagement) (B3); Communicating with team members about the actions to be/being performed (C1); Enacting plans (C2); Following the rules of engagement (e.g., prompting other team members to perform their tasks) (C3); Monitoring and repairing shared understanding (D1); Monitoring results of actions and evaluating success in solving the problem (D2); Monitoring, providing feedback, and adapting the team's organization and roles (D3).

These 12 three-level dimensions are expressed in four, letter grade levels running from high to low: E (Excellent), G (Good), F (Fair), and P (Poor).

The frequencies of the demonstrated CPS three-level competence dimensions for all participating students are shown in Table 10, with separate statistics for different groups.

As can be seen from Table 11.10, the sub-dimensional skills appearing with high frequency (frequency $\geq$ 10%) in pair collaboration were B1 and D1; the sub-dimensional skills appearing with high frequency (frequency $\geq$ 10%) in group collaboration were A1, B1, B3, C1, C3, and D1. At the same time, it can be seen that the frequency of sub-dimensional skills' occurrence in pair and group collaboration differed significantly (|difference in frequency| $\geq$ 4%): A2, B3, and D1.

Based on the above ideas, the same can be derived for each pair and group level in Table 11.11.

More than half of the students in both pair and group collaboration (56.89% and 66.86%, respectively) were "Good," while 29.64% and 31.41%, respectively, were "Fair." More students (12.60%) were "Excellent" in pair collaborations than in group collaborations (0.43%), while slightly more were "Poor" in group collaborations (1.30%) than in pair collaborations (0.87%). Table 11.12 shows the level of each competency dimension demonstrated by the students, based on the above statistics.

Table 11.10 Frequency of occurrence of the student pair and group overall competence dimensions

Competence dimension	A1	A2	A3	B1	B2	B3
Pairs	9.01%	8.69%	1.85%	15.74%	2.28%	9.01%
Groups	11.67%	3.17%	1.15%	15.42%	3.46%	13.54%
Competence dimension	C1	C2	C3	D1	D2	D3
Pairs	8.90%	1.41%	8.79%	17.37%	9.55%	7.38%
Groups	11.67%	1.44%	11.82%	12.10%	9.22%	5.33%

Table 11.11 Statistics of the percentage of each level of student pair and group overall

	E (%)	G (%)	F (%)	P (%)
Pairs	12.60	56.89	29.64	0.87
Groups	0.43	66.86	31.41	1.30

Table 11.12 Statistics of the percentage of the level of each competence dimension

	Pair				Group			
	E (%)	G (%)	F (%)	P (%)	E (%)	G (%)	F (%)	P (%)
A1	32.53	44.58	22.89	0.00	0.00	64.20	35.80	0.00
A2	2.53	48.10	48.10	1.27	0.00	54.55	45.45	0.00
A3	11.76	76.47	11.76	0.00	0.00	87.50	12.50	0.00
B1	13.79	57.93	26.90	1.38	0.00	72.90	24.30	2.80
B2	19.05	52.38	28.57	0.00	0.00	66.67	33.33	0.00
B3	4.82	51.81	40.96	2.41	3.19	59.57	34.04	3.19
C1	24.39	48.78	26.83	0.00	0.00	64.20	34.57	1.23
C2	7.69	69.23	23.08	0.00	0.00	100.00	0.00	0.00
C3	12.35	60.49	27.16	0.00	0.00	63.41	35.37	1.22
D1	6.25	71.88	21.25	0.63	0.00	76.19	23.81	0.00
D2	4.55	63.64	30.68	1.14	0.00	57.81	42.19	0.00
D3	16.18	52.94	29.41	1.47	0.00	75.68	21.62	2.70

In pair collaboration, the highest percentage of students showing an "Excellent" level were in A1 (32.53%), followed by C1 (24.39%). B3 performed poorly in pair collaboration compared to other sub-dimensions, with a "Poor" percentage of 2.41%. There were almost no "Excellent" sub-dimension levels in group collaborations, except for B3, which had an "Excellent" level of 3.19%. This competency also showed more "Poor" levels (3.19%), while the remaining sub-dimensions had a higher percentage of "Good" levels.

11.3.3 Level of Students' CPS Skills

The weights assigned to each sub-dimension of CPS by the PISA 2015 framework are shown in Fig. 11.2. The weights of the sub-dimensional skills were calculated based on this criterion.

The weights for the"Exploring and understanding (A)" and "Representing and formulating (B)" secondary sub-dimensions equalled 40% when combined as the "AB dimension," reducing the 12 sub-dimensions to nine. The weights of dimensions 1, 2, 3, and dimensions AB, C, and D Were calculated as follows: the weight vector of secondary dimensions AB, C, and D was $\alpha = (40\% \ , 30\%, 30\%)^T$, and the weight vector of secondary dimensions 1, 2, and 3 was $\beta = (45\% \ , 25\%, 30\%)^T$, , based on which the following 3*3 weight matrix can be formed:

$$P = \left(\rho_{ij}\right)^{3*3} = \alpha\beta^T = \begin{bmatrix} 18\% & 10\% & 12\% \\ 13.5\% & 7.5\% & 9\% \\ 13.5\% & 7.5\% & 9\% \end{bmatrix}$$

The four levels were scored from 4 to 1 (highest to lowest) based on the above matrix. The level scores for each sub-dimensional competency were obtained by multiplying the percentage of each level in Table 11.12. The weight matrix *P* was then multiplied by each ability level score to obtain the overall pair and group CPS.

As can be seen from Table 11.13, only A1 in pair collaboration scored more than 3; A3 in pair collaboration and C2 in group collaboration scored 3.00. The level score of C2 in pair collaboration was 3.00. The remaining sub-dimensions scored below 3, with the lowest scores being 2.52 and 2.55 for pair and group collaboration in A2.

Based on the scores in Table 11.13 and the calculation method shown above, the overall competence scores for all subjects in pair and group problem-solving were calculated to be 2.85 and 2.69, respectively, which is at the "Fair" level.

	Establishing and maintaining shared understanding	Taking appropriate action to solve the problem	Establishing and maintaining team organisation	Total
Exploring and understanding				~40%
Representing and formulating				
Planning and executing				~30%
Monitoring and reflecting				~30%
Total	40%-50%	20-30%	30-35%	100%

Fig. 11.2 PISA 2015 weights for each sub-dimension of CPS skills

Table 11.13 Statistical competency level scores of the three dimensions (unit: points)

Pair	Dimension	A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3
	Score	3.10	2.52	3.00	2.84	2.90	2.59	2.98	2.85	2.85	2.84	2.72	2.84
Group	Dimension	A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3
	Score	2.64	2.55	2.88	2.70	2.67	2.63	2.63	3.00	2.62	2.76	2.58	2.73

11.3.4 Description of Discourse Analysis Results

11.3.5 Results of the Qualitative Study Analysis

Based on the PISA 2015 assessment framework and data from previous calculations, more detailed sub-dimensional level results were obtained, and the experimental groups reflected some more common results. The more prominent results are now described in preparation for the subsequent conclusion analysis.

1. Students' discourse performance was more prominent and occurred more frequently in certain sub-dimensions.

Table 11.10 shows that the sub-dimensions D1 and B1 were more prominent in both pair and group collaboration; the sub-dimensional skills that emerged were richer. The sub-dimensional skills that emerged in group work were richer, with more CPS sub-dimensions represented in student discourse than in pair collaboration.

2. Students performed better in "establishing and maintaining shared understanding (1)" during the CPS process.

As can be seen in Table 11.11, students' performance in sub-dimensions A1 and C1 under the "Establishing and maintaining shared understanding(1)" secondary sub-dimension in the pair collaboration showed an "Excellent" level of discourse performance. C1 showed outstanding discourse performance at the "Excellent" level and the two sub-dimensions also performed better in group collaboration, indicating that students' inquiry in and understanding of the problem-solving perspective were better than in the other sub-dimensions.

3. Students' discourse in pair and group showed slight differences in sub-dimensions, but the differences were not significant.

In comparing the sub-dimensions of the same students' discourse in the two cooperative groups, the main differences were that some students showed more ability in team management and role adaptation with a larger number of collaborators. In contrast, some students showed less discourse in describing and analyzing problems, and more passive discourse in monitoring, refining, and giving feedback. As can be seen, in addition to their differences in discourse content, students also showed slight differences in dimensions. From the data, B3 was higher in group collaboration than in pair collaboration, while D1 was lower.

4. Students' CPS skills were at a "Fair" level.

Students' CPS level scores generally ranged from 2.50 to 3.00 (i.e., from middle- to upper-level "Fair" to "Good"), with the highest score being only "Good." The overall pair and group problem-solving ability scores were between "Fair" and "Good," but still belonged to the "Fair" level.

11.3.6 Quantitative Analysis Results

1. The number of words reflected students' general CPS level from the side.

The typical case study of CPS in pair and group from the previous chapter showed that the fewer words students spoke, the fewer their possible competency dimensions and the weaker their CPS level. Therefore, frequency statistics on student discourse can provide evidence of students' CPS level in pairs and groups: students with fewer words have fewer types and numbers of CPS competence sub-dimensions in pairs and groups and vice versa. However, there are still exceptional cases. For example, students who engaged in discussions about unrelated topics during the collaborative process showed a lot of discourse, but their CPS sub-dimensions were less diverse and lower.

2. The number of rounds students spoke also reflected their approximate level of CPS in mathematics.

In two different CPS processes, students with a significantly lower number of discourse rounds than other team members showed a more homogeneous dimension of competence in their discourse. Students with more discourse rounds communicated more with other team members, monitored and provided more feedback to other team members in their discourse, and showed more sub-dimensions of competence, with each competence at the "Good" level.

11.3.7 General Discussion

Some conclusions can be drawn by combining the two perspectives, as described below.

An overall analysis of the participating students' discourse revealed that among the 12 dimensions, students generally excelled in the secondary sub-dimension of the collaborative competency dimension ("Establishing and maintaining team organisation") and its four tertiary competency dimensions, which intersect with all secondary sub-dimensions under the problem-solving dimension and reflected the highest percentage of student discourse (i.e., A1, B1, C1, and D1 of the framework).

The experimental students' approximate CPS level can be obtained based on the CPS sub-dimensions reflected in their discourse.

Table 11.12 shows that students showed more of the highest levels for each ability dimension in pair collaboration but almost none at the highest levels in group collaboration; the frequency of the lowest levels of ability was also low, mainly in the "Good" and "Fair" levels, consistent with the PISA 2015 test results.

11.4 Conclusion and Implications

11.4.1 Discussion

The results and conclusions from the previous section showed the approximate level of CPS exhibited by some students in the experimental setting. This study provides an experimental basis for teachers to organize collaboration and develop students' higher-order abilities in authentic classrooms and offers implementation suggestions and strategies.

From the PISA 2015 CPS assessment results, it appears that the 9800 15-year-old students from four provinces and cities (Beijing, Shanghai, Guangdong, and Jiangsu) who participated in the test did not perform well (26th globally, the international average) and performed poorly compared to test rankings in mathematics and science subjects (OECD, 2017b).

PISA 2015 assessed students' CPS abilities through conversational agents (OECD, 2017a), while the current research is situated in a face-to-face context. Detailed analysis of the PISA 2015 test results indicates that the dimensions in which participating students' performance differed significantly from that of top-ranked Singaporean students were similar to the results of this experiment: the students participating in this study performed worse in the "Enacting plans" (C2) and "Identifying and describing tasks to be completed" (B2) CPS sub-dimensions, consistent with Chinese students' performance in PISA 2015.

It can be seen that Chinese students' CPS in mathematics is still somewhat different from that of the international leaders and that their performance in some important skills still needs to be improved and depends on teachers' guidance and development. Therefore, based on the PISA 2015 and this study's findings, some strategies can be offered for implementation in real classrooms.

11.4.2 Conclusion

1. Students' awareness and mastery of the sub-dimensional skills in CPS was inadequate.

Some dimensions were not reflected or rarely reflected in the student discourse, such as: "Discovering the type of collaborative interaction to solve the problem, along with goals (A2)," "Understanding roles in solving the problem (A3)," and "Enacting plans (C2)," all of which occurred less than 10% of the time, indicating that students' awareness and mastery of these competency dimensions was low.

2. Students' collaboration in the CPS process was inadequate, with low levels of relevant skills and disorganized discourse.

Of the two dimensions of discourse performance, the "problem-solving" dimension performed better, and students' inquiry and understanding of the problem was the most frequent part of the CPS process. The collaboration dimension's discourse performance was less frequent and at a lower level. Previous research found student attitude toward collaboration may have an influence on their CPS skill performance (Wang & Ning, 2019), while in current research through analysis of students' dialogue revealed that, with minimal teacher guidance and intervention, students' CPS process was chaotic and progressed in a slightly disorganized manner, and they analyzed problems in a continuous cycle, always with unclear goals. The skill and level evaluation revealed that when students could not reach an agreement within a group, they usually sought help from the teacher rather than solving the problem through collaborative inquiry, indicating they paid insufficient attention to CPS skills and were not conscious of using collaborative inquiry to solve problems.

3. Students had slightly higher levels of CPS skills in pair collaboration than in group collaboration, and team organization, management, and monitoring discourse were more likely to be reflected in group collaboration.

Student pairs' problem-solving skills levels were slightly higher than their group problem-solving levels. Students' discourse ability and content showed that pair collaborations were slightly more efficient in problem-solving than group collaborations, with students reaching agreement through dialogue more efficiently, thus completing the requirements of the problem. In contrast, group collaboration discourse performance revealed the weakness of students' ability to collaborate. Given more open-ended questions and more time for collaboration, group collaboration progressed significantly less than pair collaboration. Organization, management, and monitoring during solving problems are critical for successful teams to transfer discussion to an executable plan (Chang et al., 2017). While in the current research, the sub-dimensions of team organization, management, and monitoring, such as B3 and D3, were more likely to emerge in groups with larger numbers of collaborators, suggesting that the number of students involved in the collaboration affected CPS effectiveness.

4. The diversity of sub-dimensional skills reflected in individual student discourse was low, and the CPS skills level was "Fair."

Setting aside the dimensions that rarely appeared in the collaborative process, the remaining dimensions were unevenly represented in the student discourse. Students' discourses contained a fixed number of dimensions, indicating that the diversity of the skill dimensions reflected therein was low. Students' CPS skills scores were low, with both pair and group CPS skills being scored "Fair," below higher-level requirements and indicating a need for additional training.

11.4.3 Insights

1. Teacher-Targeted Development Recommendations and Strategies

Students already have some experience with and understanding of group work. They do not need instruction in the areas where they have a better grasp of the skills, but should learn more in those areas where they perform less well (Hao & He, 2019). Based on this study's findings, teachers should target the following aspects through developmental education.

This study's findings clearly show some sub-dimensional skills should be focused on when instructing students, beginning with the most basic. "Discovering the type of collaborative interaction to solve the problem and setting goals (A2)" and "Enacting plans (C2)" are skills that reflect students' ability to take appropriate actions to solve problems; however, most students do not demonstrate them, indicating that they are weak in this area. Teachers should make a conscious effort to develop students' skills in these two areas in their daily teaching.

Increasing students' awareness of types of interactions is one strategy that can be implemented. Since students are unaware of the interactions needed to solve problems, they may have never heard of a given type, similar to a machine facing an interacting object. Therefore, before engaging in CPS, teachers need to introduce students to interaction, a key element in the social networking domain, by simply and easily introducing students to one-to-one, one-to-many, many-to-one, and many-to-many types of interaction and their meanings (Cao & He, 2009). This will give students a basic understanding of how computers and networks work and provide some insight into how to conduct CPS, prompting them to discover and select the type of interaction needed to solve problems in a collaborative process, increasing their awareness of this competency dimension, and improving their level of competence.

Strengthening students' planning skills and developing their plan-making habits is another strategy. Planning ability rarely appeared in previous cooperative learning instruction and improvement strategies, because teachers' guidance often accompanies students' cooperative learning process. It is rare for students to carry out complete problem-solving on their own in domestic classrooms. To develop students' planning skills, teachers need to give them more freedom to think independently and design their problem-solving steps, thereby improving their planning skills.

2. Suggestions and Strategies for Systematic Teacher Development

CPS is a higher-order ability with high complexity, involving the compounding of many dimensional abilities (Yuan & Liu, 2016). Targeted cultivation is significant for improving students' sub-dimensional ability, with the ultimate goal of enhancing students' CPS ability. Therefore, once students' sub-dimensional abilities have been improved, teachers should systematically guide students' CPS to strengthen their cooperative awareness and collaboration ability.

Organizing CPS at the right time helps train students' CPS skills in real situations. Students' ability to improve must be trained in real-life situations, using appropriate methods to improve their overall quality, ability, and knowledge. Therefore, teachers should design problems that fit the teaching schedule and are suitable for cooperative learning, while completing their normal teaching. Let students experience the CPS process completely, then evaluate and self-evaluate to improve their CPS ability in a continuous improvement process.

Teachers should provide timely guidance in student collaboration to strengthen students' mastery of the CPS process. Their leadership is integral to students' learning process and their collaborative process guidance helps students collaborate more efficiently, gradually find the rhythm of and methods for collaborative learning, and improve their CPS skills.

This study has some limitations. The samples selected for this study were four classes from two schools in the same area. Through analyzing data in PISA 2015, it was found that CPS skills varied between different schools in China mainland (Tang, Liu & Wen, 2021). Due to the small sample size, more samples are needed to obtain more generalizable findings. Moreover, the classroom video used in the study captured an experimental scenario, which is somewhat different from a normal teaching schedule and a real classroom situation.

CPS skills are receiving attention in various countries due to international talent development needs. As a large-scale assessment linking countries, the PISA test is a powerful tool and model for research in this area. In future research, education field research in China could conduct a series of localized studies based on students' performance in the PISA test to assess and study their intellectual and cultural characteristics and provide more rigorous and accurate ways of cultivating talents with higher-order abilities.

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Appendix

Task 1 (Group task)

Xiao Ming's apartment has an area of 60 m^2. There are five rooms in Xiao Ming's apartment. Draw a possible plan of Fred's apartment. Label all rooms and show the dimensions (length and width) of each room.

Task 2 (Pair task)

The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer.

Task 3 (Group task)

A tower is made of 48 cube-shaped wooden blocks. The tower is 2 blocks wide, 3 blocks long, and 8 blocks high. The entire outside of the tower, including the bottom, is painted red. The tower is then taken apart.
How many of the 48 blocks have only one side painted red?
How many of the 48 blocks have two sides painted red?
How many of the 48 blocks have three sides painted red?
How many of the 48 blocks have more than three sides painted red?
Explain your answer to each question (use diagrams if you think this would help).

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