Chapter 2
Rock Mechanics in Hydraulic Fracturing Operations

Rock, fracture and fluid mechanics are crucial elements in understanding and engineering design of hydraulic fracture treatments. The combination of rock, fracture and fluid mechanics creates the study of fracture propagation, interaction and sensitivity caused by different treatment variables. The formation to be fractured and the resulting hydraulic fracture morphology is of paramount significance for hydrocarbon migration and extraction.

2.1 Stress

Mechanical stress is usually quantified as second-order tensor invariants. This tensor (N/m^2 or Pa) represents the force acting on a unit area of a surface or a unit volume of the material, which can be expressed by

$$\boldsymbol{\sigma} = \lim_{\Delta A \to 0}\left(\frac{\Delta \boldsymbol{F}}{\Delta A}\right) \tag{2.1}$$

where $\boldsymbol{\sigma}$ is the stress vector, $\boldsymbol{F}$ is the force (traction) vector and A is the contact area of $\boldsymbol{F}$.

The stress has both magnitude and direction. Since the area A of the contact surface is assumed close to zero, the stress reflects a point property. Note that there are some practical limitations in reducing the contact area of the force to zero. For easy calculation, the stress magnitude in experiments and fields is directly determined by dividing $|\boldsymbol{F}|$ by A. Stresses normal to the contact surface can be tensile or compression, while those parallel to the surface are called shear. In the Cartesian coordinate system, there are 9 stress components (σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{yx}, σ_{xz}, σ_{zy}, σ_{yz} and σ_{zx}) in terms of the stress in different directions, of which only 6 (σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{yz} and σ_{zx}) are independent for $\tau_{xy} = \tau_{yx},\ \tau_{xz} = \tau_{zx},\ \tau_{yz} = \tau_{zy}$. If there

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is no shear stress applied on the surface, the normal stresses become the principal stress, and the stress vector can be written as

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} \tag{2.2}$$

The three stress components are perpendicular to each other. In geologic applications, one of the principal stresses is often assumed in the vertical direction, and the other two are horizontally specified by default.

2.2 Stain

Strain represents the relative deformation between material points. If the original distance between the two points is l, after a period of action by force $\boldsymbol{F}$, the distance becomes $l + \Delta l$. The engineering stain is defined by

$$\varepsilon = \frac{\Delta l}{l} \tag{2.3}$$

The strains caused by tensile force correspond to extension whereas those under compressive force correspond to contraction. A shear strain is associated with surfaces sliding over each other. In the Cartesian coordinate system, each direction should have a corresponding strain component consistent with the stress. So, the strain can be expressed by

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix} \tag{2.4}$$

Similar to stress, six independent components (ε_{xx}, ε_{yy}, ε_{zz}, ε_{xy}, ε_{yz} and ε_{zx}) should also be specified to give the state of strain at a given point.

2.3 Linear Elastic Material and Its Failure

For a linear elastic material, the stress varies linear with the strain, which can be described by Hoek's law under uniaxial stress, i.e.,

$$\sigma_{xx} = E_{xx}\varepsilon_{xx} \tag{2.5}$$

where E_{xx} is the elastic modulus in the x-axis direction. In fact, the deformations in the normal direction (e.g., ε_{xx}, ε_{yy} and ε_{zz}) can affect each other. For instance, the relation between ε_{xx} and ε_{yy} in the x–y plane can be given by

$$\varepsilon_{yy} = -v\frac{\sigma_{xx}}{E} \tag{2.6}$$

where v is the Poisson's ratio ($0 < v < 0.5$).

In the Cartesian coordinate system, the complete relationship between stress and strain is reflected by the so-called elastic constitutive equation

$$\begin{aligned}
\varepsilon_{xx} &= \frac{1}{E}\left(\sigma_{xx} - v\left(\sigma_{yy} + \sigma_{zz}\right)\right) \\
\varepsilon_{yy} &= \frac{1}{E}\left(\sigma_{yy} - v(\sigma_{xx} + \sigma_{zz})\right) \\
\varepsilon_{zz} &= \frac{1}{E}\left(\sigma_{zz} - v\left(\sigma_{xx} + \sigma_{yy}\right)\right) \\
\gamma_{xy} &= \frac{1}{G}\tau_{xy},\ \gamma_{yz} = \frac{1}{G}\tau_{yz}, \gamma_{xz} = \frac{1}{G}\tau_{xz}
\end{aligned} \tag{2.7}$$

where G is the shear modulus, a function of elastic modulus and Poisson's ratio, i.e.,

$$G = \frac{E}{2(1 + 2v)} \tag{2.8}$$

When stresses exceed rock strength, the rock fractures and fails. A fracture criterion specifies the critical conditions for which failure occurs in a material. According to different failure mode and scales, the fracture criterion can be constructed by phenomenological theories (Mohr–Coulomb or Hoek–Brown) and mechanistic theories (Griffith, fracture mechanics models) [1].

For shear failure, Mohr–Coulomb criterion is often used, given by

$$\tau = \mu\sigma + C \tag{2.9}$$

where μ is the friction coefficient, C is the cohesion strength. This criterion is applicable for closely compacted rock without appreciable open cracks.

Hoek–Brown criterion is an empirical law obtained from a variety range of triaxial tests on intact rock samples. It is fitted with three parameters (A, B and C) and its expression is

$$\tau = A(\sigma_N + B)^C \tag{2.10}$$

In 1921, Griffith proposed a criterion for tensile failure in brittle materials initiating at the tips of defects (flat elliptical cracks). It is suitable for quasi-static single tensile crack growth based on specific surface energy. For rock failure with a certain tensile strength T_0 later extended by [2], it can be written as.

$$\begin{array}{lll} (\sigma_1 - \sigma_3)^2 - 8T_0(\sigma_1 + \sigma_3) = 0 & \text{if} & \sigma_1 > -3\sigma_3 \\ \sigma_3 = -T_0 & & \sigma_1 < -3\sigma_3 \end{array} \tag{2.11}$$

Among the above rock failure criteria, Mohr–Coulomb and Griffith are more frequently used in hydraulic fracturing operations as the critical conditions for the initiation of hydraulic fracture. Plain strain is a reasonable approximation in a simplified description of hydraulic fracturing. On this basis, a KGD hydraulic fracture is introduced in the horizontal plane, and a PKN hydraulic fracture model is proposed in the vertical plane (normal to the fracture propagation. For a short fracture (a few meters of length) with considerable height (tens of meters) and small width (millimeters), one can assume the state of plain strain in every horizontal plane (KGD fracture). For a long fracture with the length of hundreds of meters, a limited height of tens of meters and small width in millimeters, one can assume the plane strain in every vertical plane orthogonal to the length direction (PKN fracture). In this book, only KGD model is used for theoretical analysis (see Chaps. 7 and 8).

2.4 Pressurized Crack

Linear elasticity deals with static equilibrium issues. If the fracture propagates stably or at a constant velocity, a “snapshot” of this fractured state can be considered quasi-static, and such a state of equilibrium will be introduced in the following part.

In an infinite plane, there is a hollow two-dimensional “crack” without any appreciable opening and is completely pressurized by internal fluid. The stress state around the fracture should be analyzed if its propagation state needs to be determined. For simplicity, the plane is assumed in the x–y axial plane, and the fracture is propagating in a direction aligned with the x-axis with its center as the origin (Fig. 2.1). The boundary condition of this problem is

$$\begin{array}{ll} \sigma_{yy}(x,0) = -P(x), & 0 \le x \le l \\ u_y(x,0) = 0, & x \ge l \\ \tau_{xy}(x,0) = 0, & x \ge 0 \end{array} \tag{2.12}$$

Muskhelishvili [3] has accomplished the pioneering work of the above mathematical model by solving integral equations or applying the integral transformation. This method starts with a function $g(\xi)$ constructed by

$$g(\xi) = \int_0^{\xi} \frac{P(x)dx}{(\xi^2 - x^2)^{1/2}},\ 0 < \xi < l \tag{2.13}$$

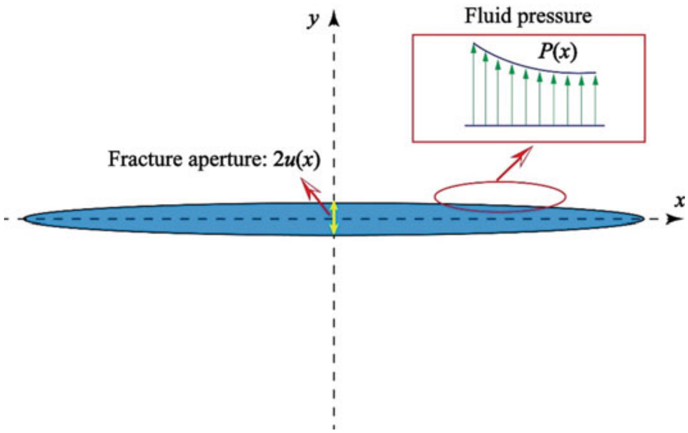

Fig. 2.1 Fracture pressurized by internal fluid

where $g(\xi)$ is modified fluid pressure summing up the fluid effect along the fracture length of l. With known $g(\xi)$, the fracture aperture can be calculated by twice of the normal displacement of any point on the upper side of the crack [4], given by

$$u_{\mathrm{y}}(x,0) = -\frac{4}{\pi E'}\int_{x}^{l}\frac{\xi g(\xi)d\xi}{(x^2-\xi^2)^{1/2}},\ x \le \xi \le l \tag{2.14}$$

where E' is the plain strain elastic modulus and can be expressed by $\frac{E}{1-\nu^2}$.

To solve this problem, $g(\xi)$ needs to be differentiable and the fluid pressure should be a function of the location inside the crack. For specific fluid pressure distribution along the crack, the above integrals can be solved in closed form (see Chap. 7).

References

1. Zang A, Stephansson O (2013) Stress field of the earth's crust
2. Griffith AA (1924) Proceedings of the 1st international congress on applied mechanics. Delft, The Netherlands, pp 55–63
3. Muskhelishvili NI (1953) Some basic problems of the mathematical theory of elasticity. Noordhoff, Groningen
4. Griffith AA (1921) Philosop Trans R Soc Lond (Ser A) 221(2):163–198