Reliability Analysis on Horizontal Bearing of Pile Foundation in Sloping Ground Based on Active Learning Kriging Model

Hao Liang, Chang Liu, and Xiuqing Yan

Abstract The uncertainty of pile and soil and slope effect are two of the major factors affecting the horizontal bearing capacity of piles of transmission tower in sloping ground. In order to analyze the influence of the two factors on the reliability of pile, this paper proposes a reliability analysis method for horizontal bearing of pile foundation in sloping ground based on proxy model. Firstly, the analytical model of horizontal bearing of the pile foundation in sloping ground was derived, and corresponding performance functions were constructed. Secondly, by combining Kriging model method with the performance functions, the reliability analysis method of pile foundations in sloping ground is established. Finally, taking a typical transmission line project in mountainous area as an example, the horizontal bearing reliability of pile foundation was analyzed. The results show that the proposed analysis method can quickly converge to the horizontal bearing limit state of pile. Slope effect has more significant influence on horizontal deformation than that of material yield. Among the uncertainty parameters, the bearing capacity of pile foundation is sensitive to the dispersion degree of horizontal force, pile diameter and the elastic modulus of foundation pile.

Keywords Pile foundation in sloping ground · Kriging model · Reliability analysis theory · Horizontal bearing characteristics · Transmission line

1 Introduction

During the construction of transmission lines in mountainous areas, a large number of transmission towers are located in steep terrain and complex geological conditions. Unilateral soil of the pile foundation located on the slope is missing, and the internal

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force and deformation of the pile foundation under horizontal load are complicated [1]. Therefore, more attention should be paid to the safety of horizontal bearing capacity of pile foundation in sloping ground under special terrain conditions such as mountain area.

Numerous studies have been conducted on the horizontal bearing characteristics of pile foundation in sloping ground. Cheng et al. [2] analyzed the influence of distance between the foundation edge and the slope and slope ratio on the horizontal bearing deformation performance of pile foundations through model tests and numerical simulation. Sivapriya et al. [3] studied the key factors affecting the horizontal bearing capacity of pile foundations according to model test. Yin et al. [4] revealed the influence mechanism of slope spatial effect on displacement and internal force of pile foundation under horizontal load. The above research mainly focuses on the influence mechanism of slope effect through deterministic analysis method. However, in fact, the horizontal bearing performance of pile presents obvious uncertainties due to the strong random characteristics of pile and soil [5].

The uncertainty is one of the safety hazards which affect the horizontal bearing capacity of pile. In that case, reliability analysis method has been used to evaluate the horizontal bearing performance of pile. Yin et al. [6] derived the limit state formula of pile-column bridge pile and analyzed the reliability of its horizontal deformation by checking point method. Based on response surface and Monte Carlo theory, Chan et al. [7] proposed a new reliability analysis method and conducted reliability evaluation for horizontal bearing of pile foundations. However, current studies are mainly based on traditional reliability theories such as check point method, response surface and Monte Carlo. It is difficult to ensure the solution efficiency and convergence when the limit state equation of pile foundation is high-dimensional and complex. In addition, the research object is mostly pile foundation in level ground, while the horizontal bearing reliability of pile in sloping ground is less considered.

This paper establishes a theoretical model of horizontal bearing capacity of transmission tower pile foundation in sloping ground. An efficient analysis method of reliability is proposed by introducing Kriging model, which can greatly improve the calculation efficiency on the premise of accuracy. On this basis, the reliability of the transmission tower pile foundation in sloping ground in an actual project is evaluated, and the influence of different factors on the reliability is analyzed.

2 Theoretical Model of Horizontal Bearing of Pile Foundation in Sloping Ground

2.1 Analysis of Pile Foundation Deformation Mechanism Considering Slope Effect

As shown in Fig. 1, pile foundation in sloping ground affected by load includes the slope affected section (l_1) and the embedded section (l_2). Due to the free face in front of the foundation pile, the soil on both sides of the pile foundation is asymmetrical, which weakens the horizontal resistance of soil in slope affected section, namely 'slope effect'. The length of slope affected section is:

$$l_1=\lambda d \tan\theta \tag{1}$$

where λ is reduction coefficient of slope effect(λ ∈ [3, 5] [3, 5]); d is the pile diameter; θ is grade of slope.

The differential equation of deformation deflection of pile foundation considering slope effect, can be written as follows:

$$EI\frac{d^4x}{dy^4}+C(y)Bx=0 \tag{2}$$

where EI is the bending stiffness of pile; x is the horizontal displacement of pile; y is the distance from the calculation point of deformation to the pile top; $C(y)$ is the resistance coefficient of pile side; B is the calculative width of pile; when $d \le 1$ m, $B = 0.9(1.5d + 0.5)$; otherwise, $B = 0.9(d + 1)$.

The m method is used to estimate resistance coefficient of pile side foundation:

$$C(y)=\begin{cases} m\frac{y^2}{l_1}, & 0<y\le l_1 \\ m(y-l_1), & l_1\le y\le H \end{cases} \tag{3}$$

where m is the proportional coefficient of foundation resistance.

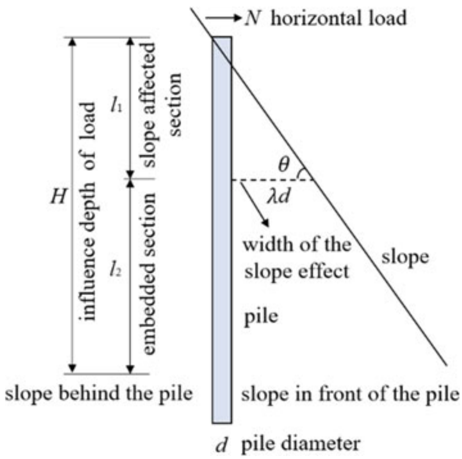


Fig. 1 Force analysis model of pile foundation in slope section

2.2 Theoretical Analysis Model

The central difference theory is used to solve the deflection differential equation of pile in sloping ground. Firstly, the influence depth of load is equally divided into ‘n’ parts, and two additional virtual nodes are set at both tips of pile. Then the boundary conditions need to be determined. For the boundary conditions of transmission tower pile, free pile top and embedded pile end can be considered. The difference schemes of boundary conditions are as follows:

$$\text{pile top}\begin{cases} x_2 - 2x_1 + 2x_{-1} - x_{-2} = \frac{2Ls}{EI} \\ x_1 - 2x_0 + x_{-1} = \frac{Ms^2}{EI} \end{cases} \tag{4}$$

$$\text{pile end}\begin{cases} x_{N-1} - x_{N+1} = 0 \\ x_N = 0 \end{cases} \tag{5}$$

where x_i is the horizontal displacement at different nodes; L and M are transverse load and bending moment respectively; s is the distance between nodes.

The controlling difference equation at different nodes of transmission tower pile in sloping ground is:

$$\text{slope affected section } x_{i-2} - 4x_{i-1} + \left(6 + \frac{Bmi^2s^6}{EIl_1}\right)x_i - 4x_{i+1} + x_{i+2} = 0 \tag{6}$$

$$\text{embedded section } x_{i-2} - 4x_{i-1} + \left(6 + \frac{Bm(is - l_1)s^4}{EI}\right)x_i - 4x_{i+1} + x_{i+2} = 0 \tag{7}$$

By combining the control difference equation of each node of pile foundation with the boundary condition equation, the horizontal displacement matrix equation of pile foundation in sloping ground can be obtained as follows:

$$\begin{bmatrix} a_{0,-2} & a_{0,-1} & \cdots & a_{0,N+1} & a_{0,N+2} \\ a_{1,-2} & a_{0,-1} & \cdots & a_{1,N+1} & a_{1,N+2} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ a_{N,-2} & a_{N,-2} & \cdots & a_{N,N+1} & a_{N,N+2} \\ b_{1,-2} & b_{1,-1} & \cdots & b_{1,N+1} & b_{1,N+2} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ b_{4,-2} & b_{4,-1} & \cdots & b_{4,N+1} & b_{4,N+2} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x_1 \\ \\ \cdots \\ \\ x_{N-1} \\ x_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \\ \cdots \\ 0 \\ 2Ls/EI \\ Ms^2/EI \end{bmatrix} \tag{8}$$

where $a_{i,j}$ is coefficient of each node in the difference equation of foundation pile, and i and j represent the calculation node from pile top to pile end and the serial number of each node in the recurrence relationship, respectively. $b_{k,h}$ is the coefficient of each node in the boundary condition, $k = 1, 2, 3, 4$, representing different boundary equations, and h is the serial number of each node in the corresponding boundary.

By solving the above matrix equation, the horizontal displacements of transmission tower pile foundation under horizontal load can be obtained. In addition, the difference scheme of rotation angle, bending moment and shear of pile in sloping ground can be expressed as:

$$\begin{cases} \gamma_i = (x_{i+1} - x_{i-1})\big/ 2s \\ M_i = (x_{i-1} - 2x_i + x_{i+1})EI\big/ s^2 \\ L_i = (-x_{i-2} + 2x_{i-1} - 2x_{i+1} + x_{i+2})EI\big/ 2s^2 \end{cases} \tag{9}$$

where γ_i, M_i and L_i are the rotation angle, bending moment and shear at different nodes of the pile foundation respectively.

By substituting the calculated horizontal displacement into Eq. (9), the corresponding rotation angle, bending moment and shear can be obtained.

3 Reliability Analysis Method of Pile Foundation Based on Kriging Model

3.1 Horizontal Bearing Performance Function of Pile Foundation in Sloping Ground

The failure modes of pile under lateral load mainly include excessive horizontal displacement and material yield. When excessive horizontal displacement is considered, the performance function is:

$$g_d(\boldsymbol{s}) = d(\boldsymbol{s}) - D_l \tag{10}$$

where $g_d(\boldsymbol{s})$ is the performance function of the horizontal displacement; $\boldsymbol{s}$ is system random variables; $d(\boldsymbol{s})$ is the horizontal displacement value; D_l is the horizontal displacement limit.

When material yield is considered, the performance function is:

$$g_M(\boldsymbol{s}) = M(\boldsymbol{s}) - M_l \tag{11}$$

where $g_M(\boldsymbol{s})$ is performance function of material yield of pile; $M(\boldsymbol{s})$ is the bending moment value; M_l is flexural capacity.

3.2 Active Learning Kriging Mode

The performance function of pile foundation includes complex process of horizontal bearing analysis. Using traditional reliability theory may lead to imprecise solution and excessive calculation scale. Kriging model technology can replace complex analytical process by fitting physical process, making it possible to evaluate performance functions efficiently and accurately.

In Kriging model, complex system is regarded as Gaussian static random process. In that case, the optimal linear unbiased estimation and mean square deviation of the system response at the unknown input point can be deduced as:

$$\hat{y}(\boldsymbol{x}) = f^T(\boldsymbol{x})\hat{\boldsymbol{\beta}} + r^T(\boldsymbol{x})\boldsymbol{R}^{-1}\left(\boldsymbol{Y} - \boldsymbol{F}\hat{\boldsymbol{\beta}}\right) \tag{12}$$

$$\sigma^2 = \sigma_z^2\left(1 + \boldsymbol{u}^T\left(\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{F}\right)^{-1}\boldsymbol{u} - \boldsymbol{r}^T\boldsymbol{R}^{-1}\boldsymbol{r}\right) \tag{13}$$

where **r** is correlation function vector between the unknown input point and training sample; $\boldsymbol{u} = \boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{r} - \boldsymbol{f}$.

The accuracy of initial Kriging model is greatly affected by selected samples. Therefore, it is necessary to update the model through active learning function. In this paper, efficient global reliability analysis method (EGRA) which is widely used in the reliability field, is adopted for model optimization.

3.3 Reliability Analysis Method of Pile Foundation in Sloping Ground

By combining Kriging model with the Monte Carlo simulation process (MCS), the failure probability of pile foundation can be calculated efficiently. According to the statistics of Kriging model response corresponding to numerous pile and soil random variables, failure probability is:

$$P_f \approx \hat{P}_f = \sum_{i=1}^{n_{mc}} I(g(\boldsymbol{s}_i) \le 0) \Big/ n_{mc} \tag{14}$$

where P_f is failure probability; $\hat{P}_f$ is simulated failure probability; n_{mc} is the number of samples; $I(\cdot)$ is the fault indication function, and its expression is:

$$I(g(\boldsymbol{s}_i)) = \begin{cases} 0\, g(\boldsymbol{s}_i) > 0 \\ 1\, g(\boldsymbol{s}_i) \le 0 \end{cases} \tag{15}$$

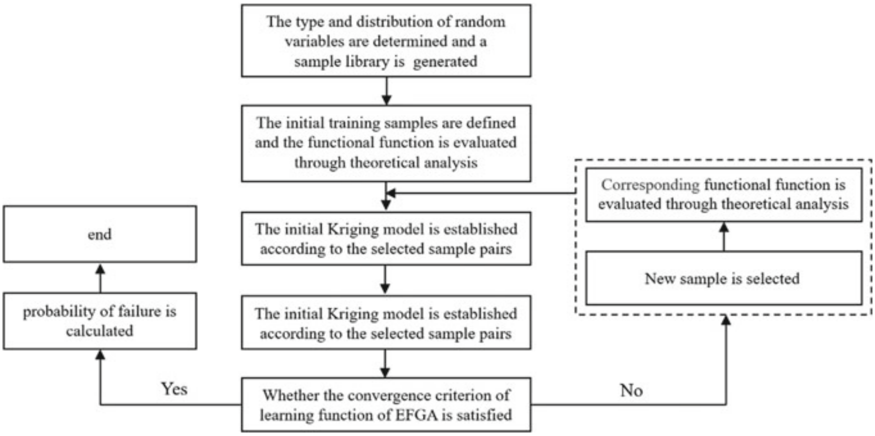


Fig. 2 Flow chart of reliability analysis of foundation pile in sloping ground

To sum up, the specific reliability analysis process of pile foundation in sloping ground based on Kriging model is shown in Fig. 2.

4 Reliability Analysis of Pile Foundation in Sloping Ground

4.1 Calculation Conditions and Parameters

A UHV DC transmission line passes through the mountainous area of Ganzi Prefecture. The slope of this area varies from 25º to 55º with an average gradient of 40º. The transmission towers in this area adapt pile-type foundation using C25 concrete. The pile length *H* is 15 m, the pile diameter *d* is 2.5 m, the elastic modulus *E* is 28 GPa, and the coefficient of variation of pile parameters are 0.025. The average proportional coefficient of soil resistance *m* is 60 MN/m^4. The horizontal load *L* and bending moment *M* acting on the pile top is 1000 kN, 1500 kN m respectively. The coefficient of variations of *L*, *M* and *m* are 0.25. The flexural bearing capacity of pile is 8295 kN m, and the displacement limit of pile top is 10 mm.

4.2 Analysis of Calculation Result

Failure Probability of Horizontal Bearing. Based on the above calculation conditions and parameters, failure probability of pile is calculated through the reliability analysis process shown in Fig. 2.

Table 1 Comparison of calculation results of different reliability analysis methods

Failure mode	Analysis method	Number of numerical analyses	Failure probability (‰)	Coefficient of variation	Relative error
Material yield	Kriging	234	3.81	0.0114	2.1%
	MCS	2×10^6	3.73	0.0116	–
Excessive horizontal displacement	Kriging	311	5.01	0.01	0.8%
	MCS	2×10^6	4.97	0.01	–

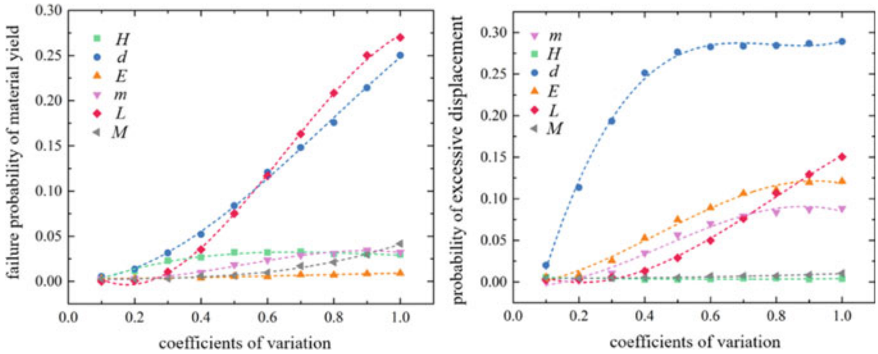
Assuming that the uncertainty parameters (s = [H, d, E, m, L, M]) obey normal distribution and are independent of each other, the sample library is firstly generated using the normal distribution theory, and the sample size is 105. Secondly, 50 training samples are selected by Latin hypercube sampling and substituted into the analytical model to obtain the corresponding displacement and bending moment of pile, so as to evaluate the performance functions. According to the sample pairs composed of input samples and performance function values, the initial Kriging model is constructed, and the model is optimized with EFGA method until it met the convergence criterion and the failure probability is finally determined. The comparison of results based on Kriging model method and Monte Carlo method is shown in Table 1.

The failure probability calculated by MCS can be regarded as exact result. As can be seen from Table 1, compared with material yield, the probability of excessive displacement is higher. The failure probabilities simulated by Kriging method and MCS method are basically the same, and the maximum relative error is only 2.1%. In addition, 2 × 106 numerical analysis processes are needed in MCS to get failure probability, while Kriging method needs 311 numerical analysis processes at most. The comparison indicates that the reliability analysis method based on Kriging model can significantly reduce the calculation cost and solve the complex limit state equation of pile foundation efficiently on the premise of accuracy.

Influence of Uncertainty Parameters on Reliability of Pile Foundation. In this section, the influence of the dispersion degree of random variables on the reliability of pile foundation are analyzed based on the proposed method.

Figure 3 shows the failure probability of transmission tower pile foundation in sloping ground under different coefficients of variation.

It can be seen from Fig. 3 that the reliability of pile is greatly affected by the variability, and the influence is also different under different failure modes. When the pile foundation fails due to material yield, the variability of the horizontal force and pile diameter has the greatest influence on the reliability, followed by bending moment, proportional coefficient of soil resistance and pile length. Compared with the above parameters, the influence of variability of elastic modulus on the pile is almost negligible. Among them, the failure probability keeps increasing approximately linearly with the increase of the variation coefficient of horizontal force, pile diameter and bending moment. When pile fails due to excessive displacement, the



(a) failure probability of material yield (b) failure probability of excessive displacement

Fig. 3 Influence of variability on pile foundation reliability under different failure modes

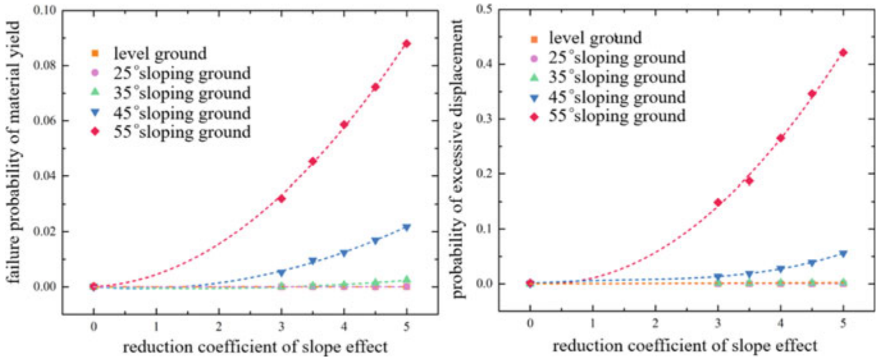
variability of pile diameter has the greatest influence on the reliability of pile foundation, followed by the horizontal force, elastic modulus and proportional coefficient of soil resistance, while the variability of bending moment and pile length have the least influence on the horizontal deformation.

Influence of Slope Effect on Reliability of Pile Foundation. In order to reveal the influence of slope effect on the reliability of pile foundation, this section calculates the failure probability and corresponding average safety margin of pile in sloping ground under different reduction coefficient and slope gradient through the constructed Kriging model.

Transmission tower piles located in level ground, 25º, 35º and 45º slopes are selected respectively, and their failure probabilities are respectively calculated under the reduction coefficients of 3, 3.5, 4, 4.5, 5 and no reduction, as shown in Fig. 4.

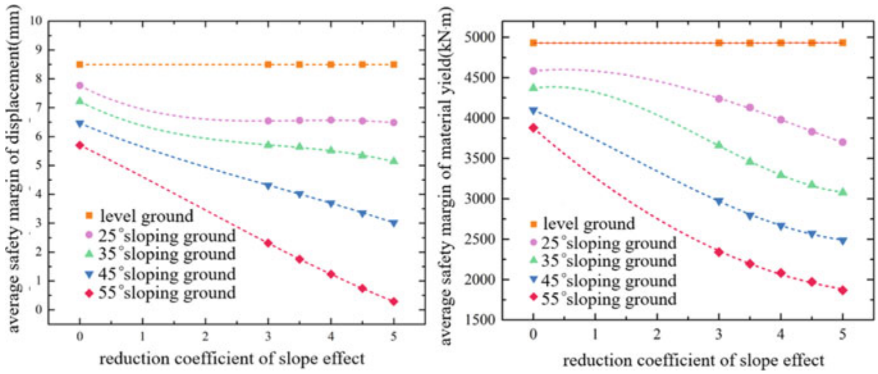
As shown in Fig. 4, when reduction coefficient or slope gradient is not considered, the failure probability of pile foundation is almost zero. Under the condition of constant reduction coefficient, the increase of slope leads to the decrease of reliability. When the slope or reduction coefficient is large, the failure probability increases significantly with the increase of slope. For the same slope gradient, failure probability increases with the reduction coefficient, and the increase amplitude is also affected by the gradient. The reliability decreases rapidly under the condition of large slope. In addition, the maximum failure probability of excessive displacement and material yield caused by the slope effect are 42.13% and 8.80%, respectively, indicating that the slope effect has a greater influence on the horizontal deformation.

The value of performance function is the safety margin. Figure 5 shows the average safety margin of pile foundation under different reduction coefficient and slope gradient.



(a)failure probability of material yield (b) failure probability of excessive displacement

Fig. 4 Influence of slope effect on foundation pile reliability under different failure modes



(a) average safety margin of displacement (b) average safety margin of material yield

Fig. 5 Average safety margin of each failure mode under slope effect

As can be seen from Fig. 5, the change of reduction coefficient does not affect the average safety margin for pile in level ground while the increase of the reduction coefficient reduces the safety margin of pile in sloping ground. The reduction amplitude of safety margin of displacement increases with the slope gradient, while the change amplitude of safety margin of bending moment is not affected by the gradient. The influence of slope gradient on the safety margin is similar to the reduction coefficient. The safety margin decreases with the slope gradient. The average safety margin of horizontal displacement and bending moment is minimized to 0.29 mm and 1867.91 kN m when the gradient and reduction factor are 55° and 5, respectively.

5 Conclusion

In this paper, an active learning reliability analysis method of pile foundation in sloping ground is proposed. Horizontal bearing reliability of transmission tower pile foundation under actual conditions is evaluated using proposed method, and the influence mechanism of random factors and slope effect on the reliability is further analyzed. The main conclusions are as follows:

(1) Compared with traditional method, the proposed Kriging method can rapidly significantly improve the computational efficiency.
(2) The displacement of pile in sloping ground is more likely to occur than that of the material yield when the horizontal load is carried.
(3) Horizontal bearing capacity of pile in sloping ground is greatly affected by the dispersion degree of elastic modulus of pile, horizontal force and pile diameter.
(4) Slope effect decreases horizontal bearing safety margin of transmission tower pile, which may greatly increase the failure probability; slope effect has more significant influence on horizontal deformation under two failure modes.

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