Chapter 4
Measuring Objective Information

Abstract In the context of the sextuple model, this chapter presents a metrics system with 11 indicators to measure objective information. Under the measurement, potential, and features of the set, each indicator is mathematical explored from the methodological foundations, specific definitions, and related propositions for quantitative information analysis and information systems applications.

Keywords Objective information theory (OIT) · Information measurement · Metric system on on information · Metric system on on information system

4.1 Motivations to Measure Information

The philosophy of objective information and the related sextuple model provide a powerful and flexible tool to derive various information metrics, which is inspired by both theoretical research and practical experience in the light of purpose and principle to measure information.

4.1.1 Prerequisite to Well Manage Information

Peter Drucker said “If you can’t measure it, you can't manage it.” A set of reasonable and feasible metrics to measure information is the prerequisite for managing and using information well [1]. Measurement is a quantitative description to describe the relationship between objects. Information measurement is one of the basic tasks of information comprehensive effectiveness evaluation and comprehensive integration. It has a positive significance for the effective use of information resources and guiding the scientific development of information system construction.

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4.1.2 Need to Comprehensively Master Information

There is a lack of a comprehensive of system for information metrics. To the best of our knowledge, Shannon information entropy is the only widely adopted information metric [2]. However, because information systems have evolved into different functionalities with complex structures far beyond those of communications, metrics based on entropy cannot comprehensively measure the complex dynamics of various information systems. Although other studies on the measurement of information exist, they lack for strict mathematical definitions and being systemic [3], making it difficult to establish an essential, basic reference framework for studying information mechanisms.

4.1.3 Requirement to Thoroughly Analyze Information Systems

There has been a lack of a thorough analysis of the efficacies of information systems. A dynamic mechanism describes how the target objects affect each other in a specific field. For example, in Newtonian mechanics, the efficiencies of speed, energy, and power were measured and analyzed to establish the theoretical system of mechanical dynamics. This methodology has also been applied in other fields. However, although information is pervasive in everyday life, there is a lack of a systematic approach to measuring and analyzing different efficacies of information systems. It is clear that only by establishing a complete analytical system of information efficacy can we accurately analyze the dynamic patterns in complex large-scale information systems.

4.1.4 Criteria to Measure Information

The information sextuple model depicts information with noumenon, state occurrence time, state set, carrier, reflection time, and reflection set, i.e. $I = \langle o, T_h, f, c, T_m, g \rangle$. To guide and regulate the derivation of information metrics, we should have come up with the following criteria.

- Traceability. The metrics are mathematically defined and derived from the sextuple model. It determines the specific definitions and mathematical expressions of various metrics according to the definition model of information.
- Completeness. The metrics are systematically defined and closely related to the value of the objective information. It forms a complete metric system which is closely related to its value according to the practical connotation of information.

- Generality. The metrics should be applicable to different information systems, such as information acquisition, transmission, processing, action, and their combinations, rather than being limited to a specific system. It forms metric definitions based on various categories of information which is universally applicable to any information systems.
- Practicability. The metrics should be able to guide the design, implementation, and analysis of practical information systems. It forms a practical and workable metric system that could guide the analysis and research of information systems according to the requirements.
- Openness. Owing to the characteristics of information, the metrics should continually evolve to match the needs of theoretical research and engineering applications. It forms a developing metric system that can supervise more and more complex information systems with newer and newer requirements.

To address these fundamental challenges, OIT takes information as the reflection of things and their movement states in the objective world and subjective world. It simplifies the concept of information from a general arbitrary idea to an objective concept in the real world, defining information as a mathematical mapping from objects in the real world to objects in information space. Based on the above principles, we will define 11 indicators for the metric system of information under the sextuple model. And the relevant basic propositions are also drawn from the set property such as metric, potential, and distance.

4.2 Metric System on Information

The metric system of objective information mainly includes 11 indicators: volume, delay, scope, granularity, variety, duration, sampling rate, aggregation, coverage, distortion, and mismatch (see Table 4.1). In Table 4.1, all of these indicators are defined and inferred with basic mathematical tools, such as set and measure, which allows them to accommodate classical information theories. For example, Shannon information entropy is a special case of the volume to measure the capacity of a communication system that transmits messages. In fact, for each metric indicator, a corresponding example can be found in classical or common information theories.

4.2.1 Volume

The value of information is closely related to its inherent containability, which is reflected by the required volume for the carrier. When other conditions are the same, the smaller the required volume is, the higher the information value is; and conversely, the lower the value is. So information may be measured by the volume.

Table 4.1 Classical theories of information science corresponding to different metrics

Metrics	Classical/common theories	Basic inference
Volume	Shannon information entropy	The minimum reducible volume of random event information is its information entropy
Delay	Whole and partial delay principle	The overall delay of serial information transmission is equal to the sum of the delays of each link
Scope	Radar equation [4]	The extent of radar detection information is directly proportional to the square root of transmitting power, antenna aperture, and antenna gain, and inversely proportional to the square root of detection sensitivity
Granularity	Rayleigh criterion for optical imaging [5]	The granularity of optical imaging information is proportional to the wavelength of light and inversely proportional to the width of the sampling pore
Variety	Invariance principle of reducible information type [6]	Reducible information can keep the type of information unchanged
Duration	Average duration of continuous information monitoring	The average time of information collection of the continuous monitoring system is equal to the mean time between failures of the system
Sampling rate	Nyquist's sampling theorem [7]	The lowest reducible sampling rate of the periodic function information is equal to half of its frequency
Aggregation	Invariance principle of aggregation degree of reducible information	Reducible information can keep the aggregation degree of the information unchanged
Coverage	Metcalfe's law [8]	The value of a network system is equal to the product of the maximum scope and the maximum coverage of all contained information
Distortion	Kalman's filtering principle [9]	A minimum distortion estimation method for linear systems with known metric variances
Mismatch	Average search length principle [10]	The shortest search path for minimum mismatch information in a finite set of information

Let $g(O \times T)$ be the set of state sets, which contains $g(c, T_{\mathrm{m}})$ on the objective world and temporal domain, and $(g(O \times T), 2^{g(O \times T)}, \sigma)$ be a measure space, where σ is a measure for the space. Then, the volume of information I relative to measure σ (viz. $\mathrm{volume}_\sigma(I)$) is the measure $\sigma(g(c, T_{\mathrm{m}}))$ of $g(c, T_{\mathrm{m}})$, which is expressed as follows:

$$\mathrm{volume}_\sigma(I) = \sigma(g(c, T_{\mathrm{m}})) \tag{4.1}$$

In information systems, the volume of information is usually measured in bits, which is the most understandable information metric. In practice, the measure σ of $(g(O \times T), 2^{g(O \times T)}, \sigma)$ for a specific task is determined by the universe of discourse. Therefore, the volume metric defined here is in a general form, but can be defined differently according to the practical needs. The metrics defined in the reminder of this study follow the same principle when applied in practice.

Volume is one of the most familiar and applied information metrics. Almost all information resources in the form of files in information systems use volume as a typical metric. Many information technologies have been developed in response to the need to keep the substantive content unchanged and minimize the information volume of the files. Data compression is one of them. Many classical information principles are related to the volume metric, and it is not difficult to prove that the minimum restorable volume of information of random events is the Shannon information entropy.

4.2.2 Delay

The value of information I is related to its timeliness, embodied in the delay between reflection time and occurrence time. Generally, when other conditions are the same, the shorter the delay is, the higher the information value is; and conversely, the lower the value is. So I may be measured by the delay.

Delay reflects the speed of the carrier response to the state of noumena. Therefore, the delay of information I (viz. delay(I)) is the difference between the supremum of its reflection time ($\sup T_{\mathrm{m}}$) and the supremum of its occurrence time ($\sup T_{\mathrm{h}}$), which is expressed as follows:

$$\mathrm{delay}(I) = \sup T_{\mathrm{m}} - \sup T_{\mathrm{h}} \tag{4.2}$$

It is should be noted that this definition of delay allows for both positive and negative values. In particular, when $\sup T_{\mathrm{m}} < \sup T_{\mathrm{h}}$, $\mathrm{delay}(I) < 0$. This represents the prediction of the future states by the carrier prior to the occurrence time T_{h} of the state for noumena. For example, the motion of targeted objects and the occurrence of events of interest can be predicted in information systems.

Actually, delay is also one of the most concerned information metrics. In fact, it will be insignificant for most information if its delay is too large. For example, if the delay of a weather forecasting information is positive, indicating that the carrier's reflection time is later than the actual weather occurrence time in the relevant region, the information of weather forecasting loses the significance of forecast. Therefore, people develop high-speed communication and high-performance computing technologies often with the purpose of reducing the delay metric of information. It is not difficult to prove that the total delay of serial information transmission is equal to the sum of information delays of each link in an information system.

4.2.3 Scope

Information is the objective reflection of noumenon, and its value is related to the noumenon scope. When other conditions are the same, the wider the noumenon scope is, the higher the information value is; and conversely, the lower the value is. So information can be measured by scope (coverage, extensity).

Let $(O \cup S, 2^{O \cup S}, \sigma)$ be the measure space over $O \cup S$. Let σ be some measure on $(O \cup S)$. According to the definitions of O and S, noumena o are elements of O and S, that is, $o \in 2^{O \cup S}$. Then, the scope of information I relative to measure σ (viz. $\text{scope}_\sigma(I)$) is the measure $\sigma(o)$ of o, which is defined as follows:

$$\text{scope}_\sigma(I) = \sigma(o) \tag{4.3}$$

The scope indicator reflects the extent of information noumena or information content, and is also an information metric that is highly valued by people. The Chinese idiom of “Climbing higher and looking farther” refers to the fact that a person can get a greater scope of visual field information when he/she climbs higher. Telescopes, radar and other detection tools are also designed to enable users to obtain a greater scope of information. It is not difficult to prove that the radar equation reveals the positive and negative proportional relationship between the scope of detection information and radar transmission power, antenna aperture, antenna gain and detection sensitivity.

4.2.4 Granularity

The value of information is directly related to the granularity of the noumenon it reflects. The granularity indicates the degree of coarseness of the particles into which the noumenon can be decomposed. When other conditions are the same, the finer the particles are, the higher the information value is; and conversely, the lower the value is. So information could be measured by granularity (detailedness).

For a pair of information I and I', if I' is proper sub-information of I and there is no other proper sub-information I'' of I such that $I'' \subset I'$, then I' is called the atomic information of I. Here, let $(O \cup S, 2^{O \cup S}, \sigma)$ be a measure space, and let σ be some measure on the set $(O \cup S)$. The set of all atomic information in information I is denoted as $A = \{I_\lambda = \langle o_\lambda, T_{\text{h}\lambda}, f_\lambda, c_\lambda, T_{\text{m}\lambda}, g_\lambda \rangle\}_{\lambda \in \Lambda}$, where Λ is an index set and λ is an index. In this case, let μ be the measure of the index set Λ and $\mu(\Lambda) \neq 0$. Then, the granularity of information I relative to measure σ (viz. $\text{granularity}_\sigma(I)$) is the ratio of the integral of all atomic noumenon information measures in A to the measure μ of index set Λ, which is expressed as follows:

$$\text{granularity}_\sigma(I) = \frac{\int_\Lambda \sigma(o_\lambda)\mathrm{d}\mu}{\mu(\Lambda)} \tag{4.4}$$

where it is most appropriate to take μ as the counting measure.

The granularity indicator reflects the degree of detail of information noumena or information content, and is also an indicator that people attach great importance to in their daily lives. With the development of display technology, the resolution of color TV is getting higher and higher, meaning that the TV image information is getting increasingly smaller granularity, which can provide viewers with an immersive watching experience. It is not difficult to prove that the Rayleigh criterion reflects the positive and negative proportional relationship between the granularity of optical imaging information and the light wavelength and sampling pore width, respectively.

4.2.5 Variety

For information I, let R be an equivalence relation on the set of states $f(o, T_\mathrm{h})$, and the set of equivalence classes of the elements in $f(o, T_\mathrm{h})$ relative to R is $[f(o, T_h)]_R$. Then, the variety of information I relative to R (viz. $\text{variety}_R(I)$) is the cardinality of set $[f(o, T_\mathrm{h})]_R$, which is expressed as follows:

$$\text{variety}_R(I) = \overline{\overline{[f(o, T_\mathrm{h})]_R}} \tag{4.5}$$

For reducible information, the equivalence relation within state set can be transferred to reflection set. Therefore, the reflection set of the carrier can fully reflect the variety metric of information.

A rich variety of information provides a comprehensive understanding of what is of interest. Therefore, variety is an important indicator for measuring information. For example, when people have access to text, picture, audio and video information of the same object of interest, the variety of the whole information is increased, which thus enables them to know the object more comprehensively and deeply. Restorability is an extremely important property of information. It is not difficult to prove that the restorable information can maintain the invariance of the variety indicator.

4.2.6 Duration

The value of information is related to the duration of the noumenon it reflects. The duration indicates the density and span of occurrence time. When other conditions are the same, the higher the density is and the longer the span is, the higher the information value is. Conversely, the lower the value is. So information could be measured by the duration (sustainability).

The duration of information I (viz. duration(I)) is the difference between the supremum and infimum of T_h, which is expressed as follows:

$$\text{duration}(I) = \sup T_h - \inf T_h \tag{4.6}$$

where infT_h is the infimum of the occurrence time T_h.

The duration indicator reflects the time span of information and is certainly an important indicator for measuring information. Manufacturers of monitoring equipment are always looking for ways to extend the stable working time of their products in order to meet the requirements of users to constantly increasing the information duration indicator. It is not difficult to prove that the average duration of information collected by a continuous monitoring system is equal to its mean time between failures (MTBF).

4.2.7 Sample Rate

For information I, if $\inf T_h \neq \sup T_h$, let $\{U_\lambda\}_{\lambda \in \Lambda}$ be a family of pairwise disjoint connected sets that satisfy the following: for any $\lambda \in \Lambda$, there are $U_\lambda \subseteq [\inf T_h, \quad \sup T_h]$, and $T_h \cap U_\lambda = \varnothing$. Then, the sampling rate of information I (viz. sampling _ rate(I)) is simply the ratio of the cardinality of Λ to the Lebesgue measure $|U|$ of $U = \bigcup_{\lambda \in \Lambda} U_\lambda$, which is expressed as follows:

$$\text{sampling\_rate}(I) = \frac{\overline{\Lambda}}{| U |} \tag{4.7}$$

Here, if $\inf T_h = \sup T_h$ or the Lebesgue measure of U is $|U| = 0$, then sampling _ rate (I) $= \infty$ is defined, which indicates that the state set of information I is completely continuous in time.

The sampling rate indicator reflects the sampling interval of information, which is closely related to the restorability of the information. It is not difficult to prove that Nyquist's sampling theorem reveals that the minimum restorable sampling rate of periodic function information is equal to half of its frequency, which is the essential requirement of information digitization.

4.2.8 Aggregation

The aggregation measures the similarity between different information, gathers similar information, separates dissimilar information, and divides the information into different clusters. The information similarity in the same cluster is the greatest, and the information dissimilarity between different clusters is the greatest. The aggregation starts from sample data, simplifies data through data modeling, and automatically clusters, which can be used as the basis for information classification.

For information I, if the cardinality of set $f(o, T_h)$ is $\overline{f(o,T_h)} \neq 0$, let $\mathfrak{R}$ be the set of relations between all elements on the state set $f(o, T_h)$. Then, the aggregation of I (viz. aggregation(I)) is the ratio of the cardinality of set $\mathfrak{R}$ to that of set $f(o, T_h)$, which is expressed as follows:

$$\text{aggregation}(I) = \frac{\overline{\mathfrak{R}}}{\overline{\overline{f(o,T_h)}}} \tag{4.8}$$

The aggregation metric characterizes the distance between the elements of the state set $f(o, T_h)$ in the information space. In general, the closer the distance is between the elements of the state set $f(o, T_h)$, the higher the degree of aggregation and the higher the value of the information.

The aggregation indicator reflects the degree of relevance of information content. Data fusion technology exactly uses the relevance of information content for in-depth processing to optimize other information metrics and meet the requirements of users in many aspects. It is not difficult to prove that the restorable information can maintain the invariance of the degree of aggregation.

4.2.9 Coverage

The value of information is often related to its distribution, which is reflected by the coverage of the carrier. For some information, the wider the coverage is (within a certain extent), the higher the Information value is. For other information, the narrower the coverage is, the higher the value is. So, in any case, information could be measured by the coverage (distribution).

For information I and I', if there are inverse mappings I^{-1} and I'^{-1}, such that $I^{-1}(g(c,T_m)) = I'^{-1}(g'(c',T'_m)) = f(o,T_h)$, then information I and I' are called to be copies of each other. Here, let $\{I_\lambda = \langle o_\lambda, T_{h\lambda}, f_\lambda, c_\lambda, T_{m\lambda}, g_\lambda \rangle\}_{\lambda \in \Lambda}$ be a set containing information I and all of its copies. Then, the coverage of information I relative to some measure σ on a measurable set of c (viz. coverage$_\sigma(I)$) is the integral of all measures c_λ, which is expressed as follows:

$$\text{coverage}_{\sigma}(I) = \int_{\Lambda} \sigma(c_{\lambda})\mathrm{d}\mu \tag{4.9}$$

The coverage indicator reflects the extent of distribution and knowledge of information content. Information encryption technology exactly adopts the approach to minimizing the coverage of information to the most extent in accordance with the right to know. Notwithstanding, the general online propaganda seeks to find ways to increase the coverage indicator of information. Therefore, the number of clicks and views has become a characteristic indicator to measure the value of a website in today's era, which is essentially the coverage of information. It is not difficult to prove that Metcalfe's law can also be expressed as the value of the network system is equal to the product of the maximum scope of information it can obtain and the maximum coverage of information it can distribute.

4.2.10 Distortion

As the mapping between noumenon state and carrier state, information should not be labeled true or false. According to the property of restorability, there always exists an inverse mapping of information in theory that could restore the actual state of information noumenon at the occurrence time. However, due to the complexity of the mapping process and the existence of subjective factors in the perception process, it is often impossible to acquire the exact inverse mapping of information. Therefore, only by deducing as far as possible can we get close to the actual state of noumenon at the occurrence time.

For information I and its reflection J, let the state set $f(o, T_{\mathrm{h}})$ and reflection state set $\tilde{f}(\tilde{o}, \tilde{T}_{\mathrm{h}})$ be elements in a distance space $\langle \mathcal{F}, d \rangle$, where $\mathcal{F}$ is the set of reflection sets and d is the distance on $\mathcal{F}$. Then, the distortion of reflection J of information I (viz. $\text{distortion}_J(I)$) is the distance between $\tilde{f}(\tilde{o}, \tilde{T}_{\mathrm{h}})$ and $f(o, T_{\mathrm{h}})$ in the distance space $\langle \mathcal{F}, d \rangle$, which is expressed as follows:

$$\text{distortion}_J(I) = d(f, \tilde{f}) \tag{4.10}$$

The distortion metric measures the degree of deviation between the reflection state and reduction state. The reflection state of information I is its reduction state if and only if the $\text{distortion}_J(I) = 0$.

Distortion is a negative indicator of the authenticity of information and is certainly one of the most concerned information metrics. It is hard to imagine that normal people need information that has lost its authenticity. Therefore, reducing errors by all means, that is, reducing information distortion, is one of the objectives for people to apply various technical means to process information since ancient times. It is not difficult to prove that Kalman filtering principle reveals the estimation method of the minimum information distortion for linear systems with known variance.

4.2.11 *Mismatch*

The value of information eventually depends on what degree it matches users' demand, embodied in the overall degree to which the various elements of information [11]. Generally speaking, the higher the overall degree is, the higher the information value is; conversely, the lower the value is. So information could be measured by mismatch (suitability).

Let information $I_0 = \langle o_0, T_{h0}, f_0, c_0, T_{m0}, g_0 \rangle$ be the target of information $I = \langle o, T_h, f, c, T_m, g \rangle$, let o_0 and o, T_{h0} and T_h, f_0 and f, c_0 and c, T_{m0} and T_m, and g_0 and g be elements in the sets $\mathcal{P}_o, \mathcal{P}_{T_h}, \mathcal{P}_f, \mathcal{P}_c, \mathcal{P}_{T_m}$, and $\mathcal{P}_g$, respectively, and let I_0 and I be elements in the distance space $\left\langle \left(\mathcal{P}_o, \mathcal{P}_{T_h}, \mathcal{P}_f, \mathcal{P}_c, \mathcal{P}_{T_m}, \mathcal{P}_g \right), d \right\rangle$. Then, the mismatch of information I to target information I_0 (viz. $\text{mismatch}_{I_0}(I)$) is the distance between I and I_0 in the distance space $\left\langle \left(\mathcal{P}_o, \mathcal{P}_{T_h}, \mathcal{P}_f, \mathcal{P}_c, \mathcal{P}_{T_m}, \mathcal{P}_g \right), d \right\rangle$, which can be expressed as follows:

$$\text{mismatch}_{I_0}(I) = d(I, I_0) \tag{4.11}$$

For the distortion and mismatch of information, we have not got the relationship between the measurements of the part and the whole, showing that neither the distortion nor mismatch would change along with the amount of information. This conforms to both common sense and the operation rules of information systems.

Mismatch is a negative indicator reflecting the degree to which information meets the requirements of users. It is the eternal goal of information systems to deliver the right content to the right object at the right time and place, which actually requires the use of various information technologies to continuously reduce the mismatch of all output information of information systems. It is not difficult to prove that the average search length principle reveals the shortest search path for information with minimum mismatch in a finite set of information.

4.3 Metric System on Information System

The metrics system on information can be further used to evaluate the effectiveness of complex information system, and the quality of informatization.

4.3.1 System of Systems

An system-of-systems (SoS) is a collection of task-oriented systems that offer more functionality and a greater performance than simply the sum of the constituent systems. With the ever-growing scales and complexities of information systems, it is becoming increasingly difficult for people to understand and grasp information systems, particularly SoS. Owing to the influence of various internal and external uncertainties, the dynamic behavior of these complex SoS may deviate from their original purpose, and unstable phenomena may appear. Moreover, in the construction and application of such large-scale SoS, an emphasis on order alone, while ignoring vitality, will lead to system rigidity, whereas an emphasis on vitality alone, while ignoring order, will lead to system chaos. Therefore, there is an urgent need for measuring the information during the process of the design, development, application, and evaluation of large-scale SoS.

4.3.2 Metric Effects and Efficacies of Information Systems

Any information system can be simplified as a basic process of information input, process, and output. The significance of information systems lies in their various efficacies. That is, the abilities of information systems to act on the input information and the effects expressed through the output information. In a large scale SoS, different efficacies are usually intertwined owing to complex information movements. Without a comprehensive analysis, reasonable deconstruction, and quantitative expression of these efficacies, it is difficult to deeply understand the inherent rules of the operation mechanism of information systems. Consequently, it is impossible to develop theoretical information systems dynamics (ISD) to guide the construction and development of large-scale SoS. Therefore, accurately and comprehensively measuring various efficacies of information systems is of decisive significance for an in-depth study on ISD.

Efficacy cannot be expressed quantitatively without certain metrics; thus, there must be an effective metric for a specific efficacy. The aforementioned 11 metrics can be used to measure various aspects of the effect on the input and output of information systems, which we refer to as the metric effect. It is therefore natural to apply these metrics to comprehensively and quantitatively describe and analyze the main efficacies of information systems. Specifically, there are 11 information system efficacies that can be established through 11 types of metric effects—namely, the volume, delay, scope, granularity, class, duration, sampling rate, aggregation, coverage, distortion, and mismatch efficacies.

- The volume is the most fundamental one that information systems act on information amongst various efficacies. In practice, each link of information collection, transmission, processing, data space and action can affect the volume efficacy by changing the capacity of the system. For example, the insufficient storage of information collection, data space and information action will lead to the reduction of the volume efficacy of information systems. The insufficient channel bandwidth of information transmission system will lead to discard part of the information, thus reducing the volume efficacy. Information processing also needs enough storage space to support, thus also affects the volume of information. Note that information processing systems can affect volume efficacy by the data compression processing, that is, the total information volume of whole system is increased by the decompression of some information.

- The delay is the ability of information systems to change the delay metric. In fact, all information flows and processing require a period of time, so each link of information collection, transmission, processing, data space and information action will certainly affect the delay metric. However, through the improvement of hardware or algorithms, each link can also achieve the lowest possible delay, so as to optimize the delay efficiency of the whole system. In particular, in the information processing link, the state set of the noumenon in the future can be predicted by the extrapolation algorithm in time dimension, which can actually reduce the delay metric of information and thus improve the delay efficiency of the information system.
- The scope characterizes the ability of information systems to change the scope metric of information noumena. In the information collection link, the scope metric of information acquisition is affected by the energy, distribution and other physical attributes of the acquisition equipment. For instance, the physical parameters such as antenna aperture, transmitter power, receiver sensitivity of radar determine the radar's detection range, viz. the scope of acquired information. The information processing link can also affect the scope effect of the output information due to the differences in the algorithms or the equipment. Note that although the information processing link does not directly involve the information noumena, it can also extend the scope of information noumena through the extrapolation algorithm in spatial dimension, thereby improving the scope effect of information. Data space, as the reflection of the real-world in information systems, certainly affects the scope effect of information by the integrity of the data model, database capacity and etc. Remarkably, the volume efficacy of information transmission step can also affect the scope effect of information, but in an indirect manner. Here, we can assume that the information transmission does not have the scope efficacy.
- The granularity is the ability of information systems to change the information granularity metric. Granularity metric characterizes the meticulousness of information noumena. In the information acquisition link, the granularity metric of information can be affected by the aperture area of the acquisition device, the number of sensors and other physical attributes. For instance, the number of photoelectric sensors integrated in the video information acquisition device determines the resolution or pixels of the video picture, which is the granularity efficacy of information acquisition. Information processing will also affect the granularity-effect of information due to the differences in the algorithms or the equipment. For instance, through interpolation algorithms in spatial dimension, it

is possible to optimize the granularity of information. Data space can also affect the granularity effect of information by factors such as the model integrity, granularity, and database capacity. Similar to the analysis of scope efficacy, the information transmission does not directly have the granularity efficacy on information.

- The variety is the ability of information systems to change the information variety metric, which characterizes the richness [12] of the state set types of information subjects. Each link in information space can affect the variety metric of information. The information acquisition and action can obtain and output different types of information owing to the differences in input and output methods. For example, microwave acquisition and audio acquisition devices can obtain different input information, and optical output and audio output devices can also produce different output information.
- The duration is the ability of information systems to change the information duration metric. Duration characterizes the time span of information continuity. Therefore, the duration of information collection directly determines the duration metric of information, and the duration of information processing also affects the duration metric of output information. Although in many cases, the duration of information transmission does not necessarily affect the duration metric of output information, there are some special cases that the duration of information transmission will directly affect the duration metric. For instance, the case of live broadcasting, usually occurs in radio and television. In general, information processing does not directly affect the duration metric of information, but by extrapolating, the state set of information can be expanded in time dimension, which actually affects the duration metric of information. Obviously, the storage capacity and structural design of the data space can directly affect the duration metric of information. Therefore, all links in information space have duration efficacy.
- The sampling rate is the ability of information systems to change the information sampling rate metric, which characterizes the occurrence density of the information state set in time. Obviously, the density of information collection directly determines the sampling rate metric of information. For instance, Nyquist sampling theorem shows that for periodic sine function curve, as long as the sampling rate is higher than half of its frequency, the original function curve can be restored by sampling information. Similarly, the frequency of information action can affect the intensity of the output information, viz. the sampling rate metric. In information transmission, if the bandwidth of the communication system is higher than the sampling rate of the input information, it will not affect the sampling rate of the output information, otherwise it will inevitably reduce the sampling rate. In general, information processing does not directly affect the sampling rate metric of information. However, through interpolation, the state set can be predicted in time. Thus, information processing can also affect the sampling-rate metric of information. The storage capacity and structure design of data space can directly affect the sampling rate metric of information. Therefore, each link in information space has sampling rate efficiency.

- The aggregation is the ability of information systems to change the aggregation metric, which characterizes the closeness of the elements in the information state set. Normally, information collection and transmission does not directly affect the information aggregation. Through the analysis, association and fusion of information processing, the internal pattern of information state set can be revealed, which improves the information aggregation metric. The internal structure and model design of data space can directly determine the aggregation metric of information. The information action, which is based on information processing and data space, certainly has the aggregation metric of information. Therefore, the information processing, data space and information action have the aggregation efficacy.
- The coverage is the ability of information systems to change the information coverage metric, which reflects the pervasiveness of information carrier and its copies. In general, information collection does not involve the formation of information copies, so it has not the coverage efficacy. Information action ultimately produces output information, and the scale of action directly affects the coverage metric of information. In information transmission, the distribution of communication networks determines the coverage of information, so information transmission has the coverage efficacy. Although information processing does not directly interact with users, the targeting or distribution processing determines the targets of information action, so it also affects the coverage metrics. Both the distributed structure design and the replica distribution range of data space are directly related to the coverage metric. Therefore, there are information transmission, processing, data space and information action that have the coverage efficacy.
- The distortion is the ability of information systems to change the information distortion metric. Obviously, most of information collection are physical or human-in-loop processes, which often produce errors due to various reasons, thus increases the distortion of information. Similarly, most information actions are also physical or human-in-loop processes, which can also affect the distortion metric of information. Information transmission can increase the distortion of information due to the limitation of communication bandwidth, error code, packet loss or other reasons. Information processing can increase the distortion of information due to computation errors, whereas the filtering, smoothing or other processing algorithms can reduce the distortion metric of information. Information representation and storage in the data space can also affect the distortion metric of information. Therefore, all links in information space have the distortion efficacy.
- The mismatch is the ability of information systems to change the information mismatch metric. The information mismatch metric reflects the degree of deviation to the needs of users. Note that all users concern the distortion metric. As each link in information space has distortion efficiency, it can be simply inferred that each link also has mismatch efficiency.

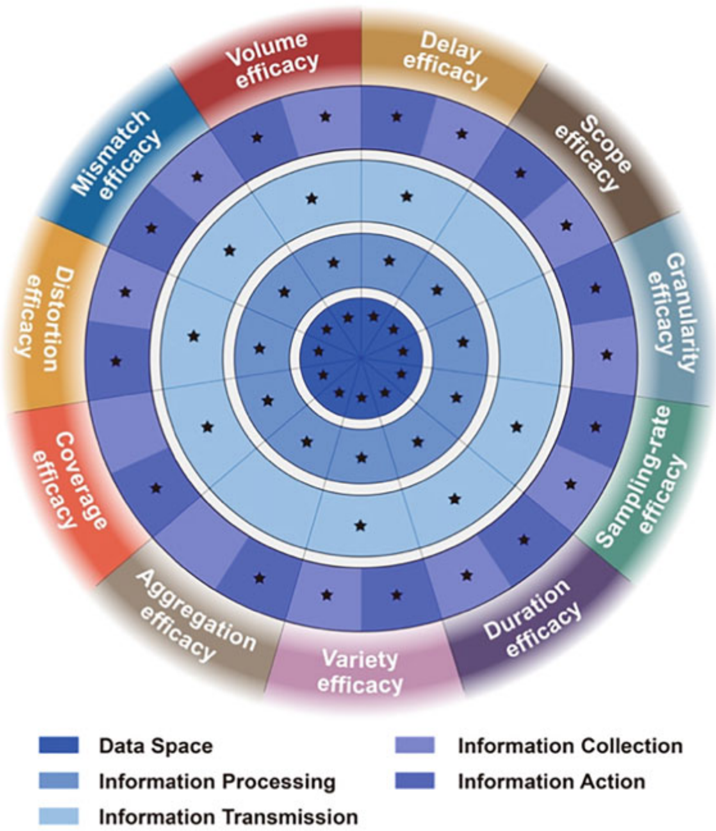

Fig. 4.1 Efficacy distribution for each link in information space [13]

Figure 4.1 illustrates the information efficiency distribution across the information space. In the figure, the star symbol indicates the existence of the corresponding efficiencies, represented by the sectors, of some link. The collection and action of information are positioned in the same ring at the periphery, and are distinguished by two different shades of blue: The dark blue represents the information collection link, and the light blue represents the information action link. Therefore, the functionality and performance of an SoS can be deconstructed through the efficacy distribution, which provides a sufficient and quantitative basis for the design, analysis, testing, and integration of an SoS.

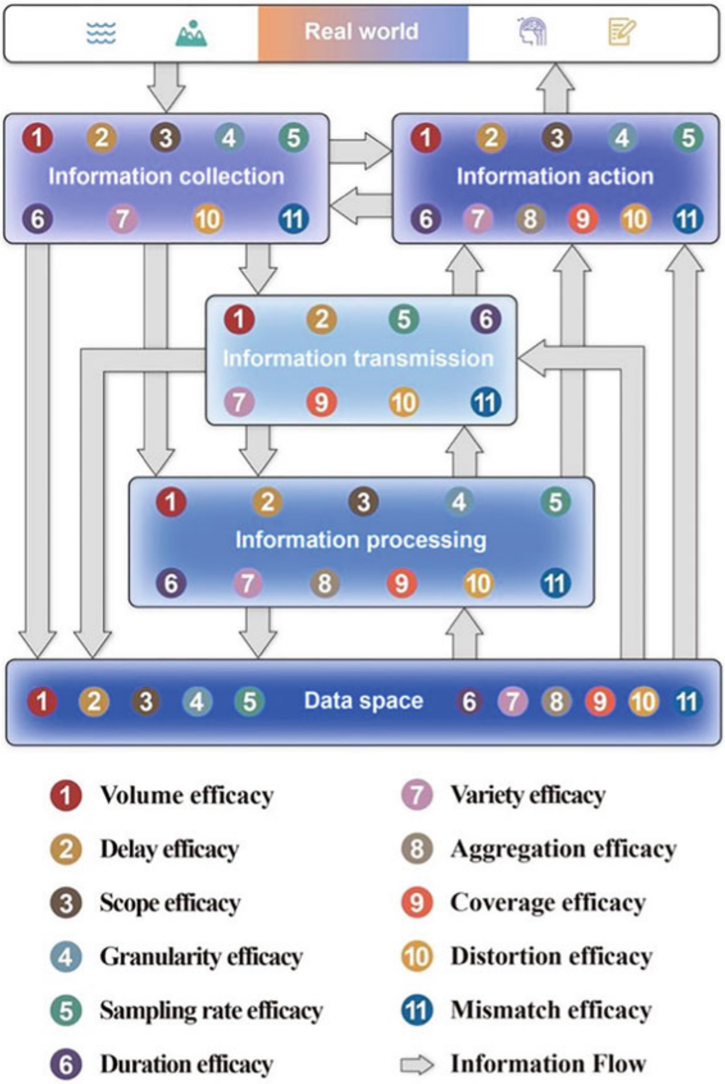

Fig. 4.2 Complete dynamic configuration of information systems [13]

4.3.3 Dynamic Configurations of Information Systems

In Fig. 4.2, the operating mechanism and efficacy distribution of various information links—that is, information collection, transmission, processing, storage, and action are presented, along with the possible information flows between the links; these are referred to as the integral dynamic configuration of an SoS. Information flow is the form and carrier of information movement in an SoS. In general, as long as information flow maintains its continuity, we can use the local metric effects at various links to analyze the global functionalities and performance of an entire SoS, which is the starting point to investigate ISD and the original intent of applying ISD to guide the planning, design, research and development, and integration of information systems.

In Fig. 4.2, each link in an SoS can affect the functionality and performance of an entire system. In general, the effects of the same class at each link can have mutual superposition or mutual restraint. For example, the delay effect at each link can be superimposed to form the delay effect of the entire SoS. In addition, the volume metric of the previous link forms the volume requirement for the subsequent link. If the requirement is not satisfied at the subsequent link, the volume efficiency of the entire SoS is affected.

It should be noted that there are mutual effects among the different metrics. For example, the volume effect impacts the distortion effect of the system. With insufficient volume, the elements in the reflection set will be abandoned, which will result in an increase in the distortion metric. The degree of mismatch reflects the degree to which the information output of the system deviates from the needs of specific users. The volume, delay, scope, granularity, duration, class, sampling rate, and aggregation metrics are closely related to the needs of specific users, meaning that any of these metrics can affect the mismatch metric of the entire SoS. For example, to control the range of information acquisition, the coverage metric must be adjusted according to the desires of the users. Therefore, the coverage metric is not related to the mismatch metric in a single direction. In addition, the distortion metric is not positively correlated with the mismatch metric. For example, in an encrypted information system, higher distortion results in lower mismatch for a specific user.

In engineering practices, users do not always need to apply the integral dynamic configuration of information systems. In many cases, some links of the SoS may not determine or affect the key efficacies of the entire SoS. In such cases, it is possible to limit the consideration of system designers to relatively minor links and form the simplified dynamic configurations for the information systems. Studying the mechanism of the efficacies with various configurations to reveal the inherent operating regularities of information systems provides a powerful means of guiding the planning and development of the large-scale SoS in engineering practices.

4.4 Chapter Summary

This chapter systematically establishes 11 efficiencies of measurement systems for information, defines the mathematical expression of each measurement from the basic model, and discusses the basic properties of each measurement, providing a theory for quantitative research and analysis of information operation and information system applications, and method basis.

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