Chapter 3
Frictionless Air Cannot Provide Lift: A Paradox

Wouldn't it be nice if there was no friction to retard motion we wish to embark on? No, it wouldn't be, for many reasons. We will, however, limit discussion of friction and its role in aerodynamics.

Before we tackle the vorticity distribution along airfoils, consider the flow field that would result if we apply the equations of motion *without friction*. Engineers know how to do this, but you don't have to! It is possible to write the governing equations for flow about an airfoil without the terms necessary to have friction acting between fluid elements. That is a mathematical trick one can use to see what happens. In analyzing flow of honey or motor oil about the airfoil, that approach would be pretty unrealistic. In air, however, it is much better because friction effects are largely confined to the region near boundaries and there, friction does become important enough to require inclusion in any realistic mathematical description … as we shall see, even as we avoid the mathematics. The 'model' air assumed to be free of viscous friction effects is said to be "inviscid."

The absence of friction means that no torques can be applied to any of the air and hence no vorticity is created. The airfoil will not carry circulation. The mathematical solution with uniform flow far upstream and far downstream being might look something like the streamlines shown in Fig. 3.1. Such a streamline pattern is easy for the engineer to calculate because exclusion of friction effects simplifies the descriptive (differential) equations enormously. In fact, the resulting streamline pattern is identical to one obtained for the constant voltage lines between the top and bottom of the field shown in the figure with the airfoil modeled as an electrical conductor. This similarity is the reason engineers call this *potential* flow.

While nature might seem kinder if the air were to be inviscid (no friction) and the flow field to look this way, the wing designer would not like it. An airfoil in inviscid air would provide no lift because the flow is not deflected downward as Newton's laws require (see Fig. 3.1).

R. Decher, *The Vortex and The Jet*,
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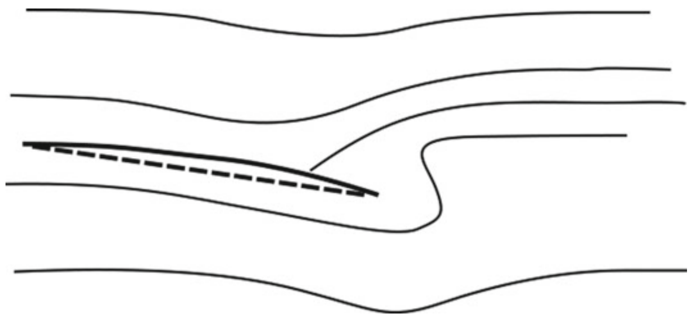

Fig. 3.1 Flow about a flow deflecting airfoil in the absence of flow rotation or vortices. Shown are 5 streamlines including one that impacts the airfoil at the leading edge (not shown) and departs from the top rear surface of the airfoil. The dotted line is the chord of the airfoil

3.1 The Trailing Edge

In a strange turn of events, friction comes into play to rescue the situation: The flow around the trailing edge is not possible because, in reality, the air that has slid along the two sides (particularly the lower surface) of the airfoil has been slowed by friction and thus robbed of the energy necessary to turn the corner at the trailing edge. If the air could travel to the (non-existent) stagnation point on the upper surface it would be going from a region of low static pressure at the trailing edge into a region of the higher pressure at the stagnation point. That is very difficult as we shall see. Instead, the air simply gives up by going in the direction given by the trailing edge angle. Thus, one could say that friction causes the flow to follow the angle dictated by the trailing edge. Two scientists came up with a rule as follows: The trailing edge assures that the wing circulation arises in just the amount required to have the flow leaving the trailing edge in its direction. The German mathematician Martin Kutta and the Russian scientist Nikolai Joukowski came up independently with that rule through observation of the flow physics in the early 20$^{\text{th}}$ Century. Another step of progress.

In our aerodynamic world, the friction experienced by the flow on its lower surface is largely responsible for the establishment of the vorticity that provides lift. We might wish we could live without the consequences of friction but, the reality is that we could not. Just picture yourself on a smooth ice sheet trying to go for a walk!

Figure 3.1 is an opportunity to introduce a little more nomenclature. The curved shape of the airfoil *mean line* is normally characterized by the maximum distance (called the *camber*) between the straight *chord* (dashed line) and the mean line, expressed as a few percent of the chord. The airfoil's *angle of attack* is the angle of the chord relative to the freestream direction, that is, the direction of the flow if the airfoil was not present.

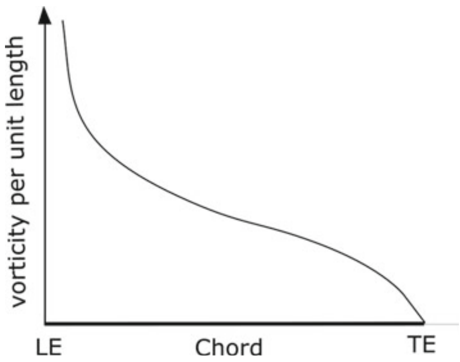

Fig. 3.2 Typical vorticity (per unit length) distribution along the chord for an airfoil at an angle of attack. The integrated vorticity per unit chord length is the strength of the equivalent bound vortex. LE and TE refer to the leading and trailing edges of the airfoil

Our understanding is at the point where we (a good mathematician) can determine the vortex strength distribution along any airfoil shape, provided it is reasonable. The calculated vorticity distribution along the chord allows the determination of the velocity distribution everywhere in the field and along the airfoil's two surfaces. At positive angles of attack, velocities on the upper surface will be larger than the incoming flow (the freestream) and those below are lower. The total of the distributed vorticity (i.e., the circulation) is directly related to the lift exerted by the airfoil at the angle of attack chosen (Fig. 3.2).

In summary, the trailing edge installs vorticity on the airfoil and distributes it along the mean line. The distributed vorticity, together with the freestream flow, determine the local flow velocity everywhere in the field, including on both sides of the mean line. A wing section with finite thickness would naturally impose small changes in velocities associated with the thickness, but because the wing is typically thin, the local velocity adjustments for thickness are quite small and we will ignore them here.

How good is this model? For substantiation of what we might measure in a wind tunnel or on an airplane, we must be able to relate the local velocity to the local pressure. Pressure is easier to measure than velocity itself, especially near the surfaces of the airfoil section. The relation between velocity and pressure turns out not to be simple and it will be addressed in due time. Assuming, for the time being, that such capability is in hand, the lift can be calculated for an airfoil at modest angles of attack using our vorticity model and compared to a wind-tunnel measurement. The variation of lift with angle of attack is given very closely as measured in the laboratory, even when airfoil thickness is considered. Another aspect of the model validity is that it correctly provides the location on the airfoil where the force is effectively applied. The engineer calls this the *aerodynamic center* which for most airfoil designs is, in theory and in practice, at the quarter chord location on the airfoil. There, the pitching moment of the wing is independent of angle of attack and can be easily calculated using the methodology used to determine lift. The pitching moment is a torque about a point on the mean line of an airfoil or, in the case of a wing, about a line parallel to the span that must be countered by a torque from the horizontal tail surface.

So far so good, but the model is not perfect. It ceases to be adequate at large angles of attack because the flow fails to follow the prescription imposed by its shape. In ordinary aerodynamic terms, we encounter *stall* of the wing at high angles of attack. The model also says nothing about the resistive force called drag.

The trailing edge function seems pretty straightforward in light of the arguments made above: it must be sharp and point downward. It may be noted that the local vorticity at the trailing edge must be zero, according to the Kutta-Joukowski rule so that the air velocities along the upper and lower sides of the airfoil are identical, in speed and direction.

At this point in the evolution of our understanding of aerodynamics we have the capability to analyze shapes (the airfoil section) we might consider to be the basis for constructing a three-dimensional wing. We are, however, a long way from having the ability to specify the lift that is desired and, with a mathematical model, determine the right airfoil or wing shape. We have not yet even considered that resistive forces, otherwise known as drag forces, will be in play. The road to a practical and well performing wing was not going to be easy! More breakthroughs in the necessary understanding will be made and we will tackle them.

3.2 The Leading Edge

As long as we are playing with the design of an airfoil to generate lift, we might say a word or two about its leading edge. The leading edge plays a different role from that of the trailing edge. First, we recognize that near it, skin friction has little role to play on flow behavior because this edge encounters clean freestream flow. In an airplane application, the angle of attack and the associated lift must be allowed to vary with the needs of a maneuvering airplane. Consider, for example the simple flat plate airfoil at a modest angle of attack. Figure 3.3 shows the oncoming stagnation streamline attaching to the airfoil at the lower surface. When the angle of attack is reduced, this attachment point wanders forward and closer to the real leading edge.

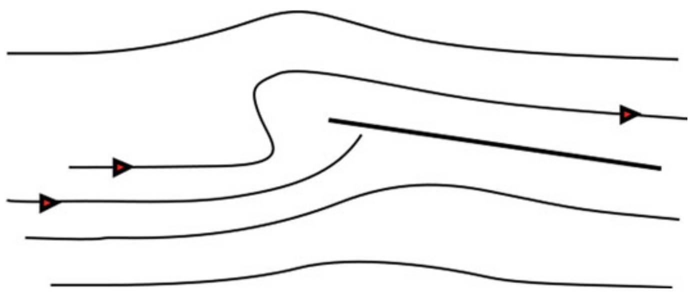

Fig. 3.3 Sketch of the flow about a lifting airfoil. This figure is not unlike Fig. 3.1 rotated 180 degrees

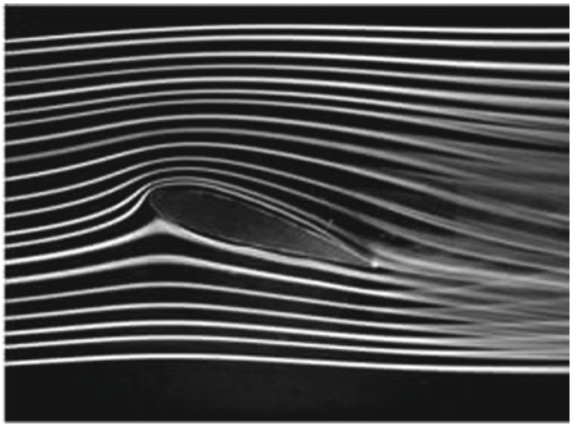

Fig. 3.4 Photo of wind tunnel test with smoke tracers. Flow is from left to right (Image: H. Babinsky, University of Cambridge)

In any case, the flow ahead of the stagnation point must be able to negotiate the flow around the leading edge. A nicely rounded leading edge allows that to take place. Figure 3.4 shows the flow in a wind tunnel with smoke tracing a number of streamlines.

In the later discussion of the jet engine, these ideas about the proper design of wing leading and trailing edges will, amazingly, also apply to the design of inlets and nozzles.

3.3 A Better Model

The representation of an airfoil as a line of vortices in an inviscid (not viscous) flow is a bit simplistic. The mathematicians tackling the problem of developing a decent model of the airfoil were fortunate in that tools were developed to analyze the characteristics of a more realistic airfoil. Among these was the ability to add flow elements to our combination of vortices and freestream to arrive at a good-looking airfoil with finite thickness. For example, a *source* can be added to obtain a flow that looks like the front end of a streamlined body (Fig. 3.5). Another negative source[1] (a sink) can be located behind the source to create a closed, elongated body with flow around it. More realistically, a number of sources followed by sinks behind the point of maximum thickness can be assembled to make a nice-looking airfoil. The beauty of these tools is that they are mathematically relatively simple (for those with that interest[2]) to describe the entire flow field around an airfoil. For simple examples,

[1] A source is described by a *radial* flow velocity issuing from a point and varying inversely with radius. See Fig. 7.8 for an example.

[2] A rich resource on the mathematics of the inviscid flow is available on the internet and classical texts under the heading “potential flow.”.

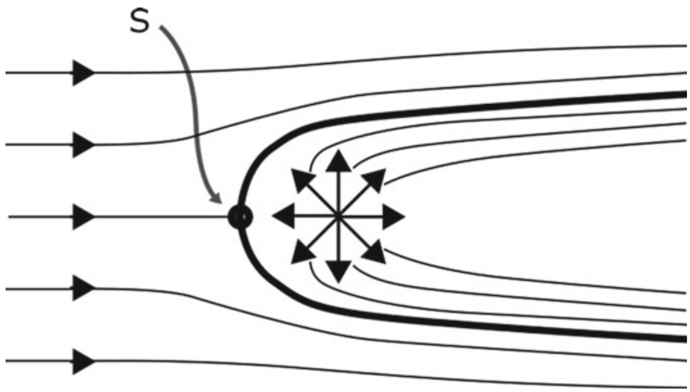

Fig. 3.5 Streamlines resulting from the superposition of a source and a uniform flow from the left. The dividing streamline defines a so-called "Rankine half-body." *S* points to the flow stagnation point

the results can be generated on a piece of paper, although for complex geometries, a computer will be required.