# Temporary Independence

12/1. SEVERAL times we have used, without definition, the concept of one variable or system being ' independent ' of another. It was stated that a system, to be state-determined, must be ' properly isolated ' ; and some parameters in S. 6/2 were described as ' ineffective '. So far the simple method of S. 4/12 has been adequate, but as it is now intended to treat of systems that are neither wholly joined nor wholly separated, a more rigorous method is necessary.

The concept of the ' independence ' of two dynamic systems might at first seem simple: is not a lack of material connexion sufficient ? Examples soon show that this criterion is unreliable. Two electrical parts may be in firm mechanical union, yet if the bond is an insulator the two parts may be functionally independent. And two reflex mechanisms in the spinal cord may be inextricably interwoven, and yet be functionally independent.

On the other hand, one system may have no material connexion with another and yet be affected by it markedly: the radio receiver, for instance, in its relation to the transmitter. Even the widest separation we can conceive—the distance between our planet and the most distant nebulae—is no guarantee of functional separation; for the light emitted by those nebulae is yet capable of stirring the astronomers of this planet into controversy. The criterion of physical connexion or separation is thus useless.

12/2. Can we make the test for independence depend on whether one variable (or system) gives energy or matter to the other ? The suggestion is plausible, but experience with simple mechanisms is misleading. When my finger strikes the key of a typewriter, the movement of my finger determines the movement of the type; and the finger also supplies the energy necessary for the type's movement. The diagram
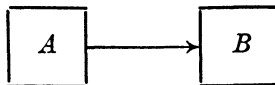
would state, in this case, both that energy, measurable in ergs, is transmitted from A to B, and also that the behaviour of B is determined by, or predictable from, that of A. If, however, power is freely available to B, the transmission of energy from A to B becomes irrelevant to the question of the control exerted. It is easy, in fact, to devise a mechanism in which the flow of both energy and matter is from B to A and yet the control is exerted by A over B. Thus, suppose B contains a compressor which pumps air at a constant rate into a cylinder, creating a pressure that is shown on a dial. From the cylinder a pipe goes to A, where there is a tap which allows air to escape and can thus control the pressure in the cylinder. Now suppose a stranger comes along; he knows nothing of the internal mechanism, but tests the relations between the two variables: A, the position of the tap, and B, the reading on the dial. By direct testing he soon finds that A controls B, but that B has no effect on A. The direction of control has thus no necessary relation to the direction of flow of either energy or matter when the system is such that all parts are supplied freely with energy.

Independence

12/3. The test for independence can, in fact, be built up from the results of primary operations (S. 2/10), without any reference to other concepts or to knowledge of the system borrowed from any other source.

The basic definition simply makes formal what was used intuitively in S. 4/12. To test whether a variable X has an effect on a variable Y, the observer sets the system at a state, allows one transition to occur, and notices the value of Y that follows. (The new value of X does not matter.) He then sets the system at a state that differs from the first only in the value of X (in particular, Y must be returned to its original initial state). Again he allows a transition to occur, and he notices again the value of Y that results. (He thus obtains two transitions of Y from two states that differ only in the value of X.) If these two values of Y are the same, then Y is defined to be **independent** of X so far as the particular initial states and other conditions are concerned.

By **dependent** we shall mean simply ' not independent '.

This operational test provides the 'atom' of independence. Two transitions are needed: the concept of 'independence' is meaningless with less.

12/4. In general, what happens when the test is applied to one pair of initial states does not restrict what may happen if it is applied to other pairs. The possibility cannot be excluded that the test may give results varying arbitrarily over the possible pairs. Often, however, it happens that, for some given value of all other variables or parameters, Z, W, . . . , Y is independent of X for all pairs of initial states that differ only in the value of X. In this case, for that particular field and for that particular value of the other variables and parameters, Y is **independent** of X in a more extended sense. Provided the field and the initial values of Y, Z, W, etc., do not change, Y's transition is unaffected by X's initial value. In this case, Y is independent of X over a region (in the phase space) represented by a line parallel to the X-axis, 'independent' in the sense that whenever the representative point, moving on a line of behaviour, leaves this region, Y will undergo the same transition.

Sometimes it may happen that Y is independent of X not only for all values of X but also for all values of the other variables and parameters—Z, W, etc. In the previous paragraph a change of Z's value might have changed the field or region so that Y was no longer independent of X. In the present case, Y's transition (from a uniform Y-value) is the same regardless of the initial values of X, Z, W, etc. Y is then **independent** of X unconditionally.

It will be seen that two variables may be 'independent' to varying degrees: at two points, over a line, over a region, over the whole phase-space, over a *set* of fields. The word is thus capable of many degrees of application. The definitions given above are not intended to answer the question (of doubtful validity) 'what *is* independence *really* ?' but simply to show how this word must be used if a speaker is to convey an unambiguous message to his audience. Clearly, the word often needs supplementary specification (e.g. does 'Y independent of X' mean 'over this field' or 'over all fields' ?); the supplementary specification must then be given, either by the context or explicitly.

The word 'independent' is thus similar to the word 'stable': both words are often useful in that they can convey information

about a system quickly and easily *when the system has a suitable simplicity* and when it is known that the listener will interpret them suitably. But always the speaker must be prepared, if the system is not simple, to add supplementary details or even to go back to a description of the transitions themselves; here there is always security, for here the information is complete.

12/5. Because there are various degrees of independence, so that Y may be independent of X over a small region of the field but not independent if the same region is extended, it follows that one system can give a variety of diagrams of immediate effects— as many as there are ranges and conditions of independence considered. This implication is unpleasant for us; but we cannot evade the fact. (Fortunately it commonly happens that the independencies in which we are interested give much the same diagram, so often one diagram will represent all the significant aspects of independence.)

12/6. So far we have discussed Y's independence of X. Whatever this is, it in no way restricts, in general, whether X is or is not independent of Y. If X is independent of Y, but Y is not independent of X, then X **dominates** Y.

12/7. The definition given so far refers to independence between two variables. It may happen that every variable in a system A is independent of every variable in a system B, all possible pairs being considered. We then say that system A is **independent** of system B.

Again, such independence does not, in general, restrict the possibilities whether B is or is not independent of A; A may **dominate** B.

12/8. To illustrate the definition's use, and to show that its answers accord with common experience, here are some examples.

If a bacteriologist wishes to test whether the growth of a microorganism is affected by a chemical substance, he prepares two tubes of nutrient medium containing the chemical in different concentrations (X) but with all other constituents equal; he seeds them with equal numbers of organisms; and he observes how the numbers (Y) change as time goes on. Then he compares the two

later numbers of organisms after two initial states that differed only in the concentrations of chemical.

To test whether a state-determined system is dependent on a parameter, i.e. to test whether the parameter is 'effective', the observer records the system's behaviour on two occasions when the parameter has different values. Thus, to test whether a thermostat is really affected by its regulator he sets the regulator at some value, checks that the temperature is at its usual value, and records the subsequent behaviour of the temperature; then he returns the temperature to its previous value, changes the position of the regulator, and observes again. A change of behaviour implies an effective regulator.

Finally, an example from animal behaviour. Parker tested the sea-anemone to see whether the behaviour of a tentacle was independent of its connexion with the body.

> 'When small fragments of meat are placed on the tentacles of a sea-anemone, these organs wind around the bits of food and, by bending in the appropriate direction, deliver them to the mouth.'

(He has established that the behaviour is regular, and that the system of tentacle-position and food-position is approximately state-determined. He has described the line of behaviour following the initial state: tentacle extended, food on tentacle.)

> 'If, now, a distending tentacle on a quiet and expanded sea-anemone is suddenly seized at its base by forceps, cut off and held in position so that its original relations to the animal as a whole can be kept clearly in mind, the tentacle will still be found to respond to food brought in contact with it and will eventually turn toward that side which was originally toward the mouth.'

(He has now described the line of behaviour that follows an initial state identical with the first except that the parameter 'connexion with the body' has a different value. He observed that the two behaviours of the variable 'tentacle-position' are identical.) He draws the deduction that the tentacle-system is, in this aspect, independent of the body-system:

> 'Thus the tentacle has within itself a complete neuro-muscular mechanism for its own responses.'

The definition, then, agrees with what is usually accepted. Though clumsy in simple cases, it has the advantage in complex

cases of providing a clear and precise foundation. By its use the independencies within a system can be proved by primary operations only.

12/9. The definition makes 'independence' depend on how the system behaves over a single unit of time (over a single step if changing in steps, or over an infinitesimal time if changing continuously). The dependencies so defined between all pairs of variables give, as defined in S. 4/12, the diagram of *immediate* effects.

In general, this diagram is not restricted: all geometrically drawable forms may occur in a wide enough variety of machines. This freedom, however, is not always possible if we consider the relation between two variables over an extended period of time. Thus, suppose Z is dependent on Y, and Y dependent on X, so that the diagram of immediate effects contains arrows:
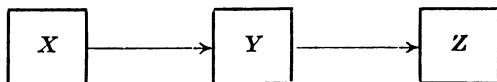

X may have no *immediate* effect on Z, but over two steps the relation is not free; for two different initial values of X will lead, one step later, to two different values of Y; and these two different values of Y will lead (as Z is dependent on Y) to two different values of Z. Thus after *two* steps, whether X has an immediate effect on Z or not, changes at X will give changes at Z; and thus X does have an effect on Z, though delayed.

Another sort of independence is thus possible: whether changes at X are followed at *any* time by changes at Z. These relations can be represented by a **diagram of ultimate effects.** It must be carefully distinguished from the diagram of immediate effects. It is related to the latter in that it can be formed by taking the diagram of immediate effects and adding further arrows by the rule that if any two arrows are joined head to tail,
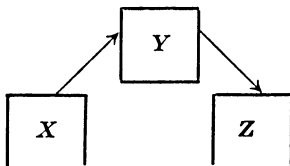
a third arrow is added from tail to head, thus
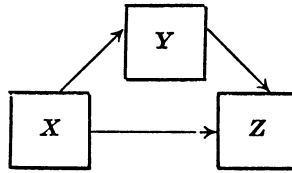
The rule is applied repeatedly till no further addition of arrows
is possible. Thus the diagram of immediate effects I in Figure
12/9/1 would yield the diagram of ultimate effects II.

FIGURE 12/9/1.

The diagram of ultimate effects shows at once the dependencies
in the case when we allow time for the effects to work round the
system. Thus from II of the Figure we see that variable 1 is
permanently independent of 2, 3, and 4, and that the latter three
are all ultimately dependent on each other.

The effects of constancy

12/10. Suppose eight variables have been joined, by the method
of *S.* 6/6, to give the diagram of immediate effects shown in
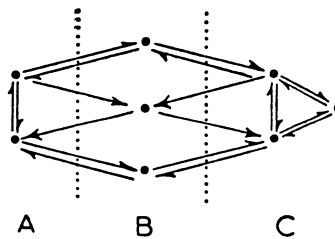Figure 12/10/1. We now ask: what *behaviour* at the three

A   B   C

FIGURE 12/10/1.

variables in *B* will make *A* and *C* independent, in the ultimate
sense, and also leave both *A* and *C* state-determined ? That is,
what behaviour at *B* will sever the whole into independent parts,
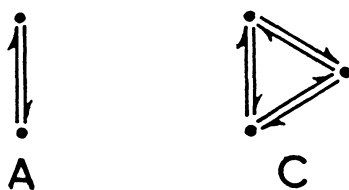giving the diagram of immediate effects of Figure 12/10/2:

FIGURE 12/10/2.

The question has not only theoretical but practical importance. Many experiments require that one system be shielded from effects coming from others. Thus, a system using magnets may have to be shielded from the effects of the earth's magnetism; or a thermal system may have to be shielded from the effects of changes in the atmospheric temperature; or the pressure which drives blood through the kidneys may have to be kept independent of changes in the pulse-rate.

A first suggestion might be that the three variables B should be removed. But this conceptual removal corresponds to no physical reality: the earth's magnetic field, the atmospheric temperature, the pulse-rate cannot be ' removed '. In fact the answer is capable of proof (S. 22/14): *that A and C should be independent and state-determined it is necessary and sufficient that the variables B should be null-functions.* In other words, A and C must be separated by a *wall of constancies.*

It also follows that if the variables B can be sometimes fluctuating and sometimes constant (i.e. if they behave as part-functions), then A and C can be sometimes functionally joined and sometimes independent, according to B's behaviour.

12/11. Here are some illustrations to show that the theorem accords with common experience.

(*a*) If A (of Figure 12/10/1) is a system in which heat-changes are being studied, B the temperatures of the parts of the container, and C the temperatures of the surroundings, then for A to be isolated from C and state-determined, it is necessary and sufficient for the B's to be kept constant. (*b*) Two electrical systems joined by an insulator are independent, if varying slowly, because electrically the insulator is unvarying. (*c*) The centres in the spinal cord are often made independent of the activities in the brain by a transection of the cord; but a break in physical continuity is not necessary: a segment may be poisoned, or

anaesthetised, or frozen; what is necessary is that the segment should be unvarying.

Physical separation, already noticed to give no certain independence, is sometimes effective because it sometimes creates an intervening region of constancy.

12/12. The example of Figure 12/10/1 showed one way in which the behaviour of a set of variables, by sometimes fluctuating and sometimes being constant, could affect the independencies within a system. The range of ways is, however, much greater.

To demonstrate the variety we need a rule by which we can make the appropriate modifications in the diagram of ultimate effects when one or more of the variables is held constant. The rule is:—Take the diagram of immediate effects. If a variable V is constant, remove all arrows whose heads are at V; then, treating this modified diagram as one of immediate effects, complete the diagram of ultimate effects, using the rule of S. 12/9. The resulting diagram will be that of the ultimate effects when
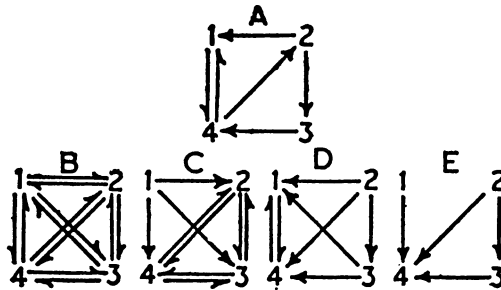

FIGURE 12/12/1 : If a four-variable system has the diagram of immediate effects *A*, and if 1 and 2 are part-functions, then its diagram of ultimate effects will be *B*, *C*, *D* or *E* as none, 1, 2, or both 1 and 2 become inactive, respectively.

V is constant. (It will be noticed that the effect of making V constant cannot be deduced from the original diagram of ultimate effects alone.) Thus, if the system of Figure 12/12/1 has the diagram of immediate effects A, then the diagram of ultimate effects will be B, C, D or E according as none, 1, 2, or both 1 and 2 are constant, respectively.

It can be seen that with only four variables, and with only two of the four possibly becoming constant, the patterns of inde-

pendence show a remarkable variety. Thus, in C, 1 dominates 3; but in D, 3 dominates 1. As the variables become more numerous so does the variety increase rapidly.

12/13. The multiplicity of inter-connexions possible in a tele-phone exchange is due primarily to the widespread use of temporary constancies. The example serves to remind us that 'switching' is merely one of the changes producible by a re-distribution of constancies. For suppose a system has the
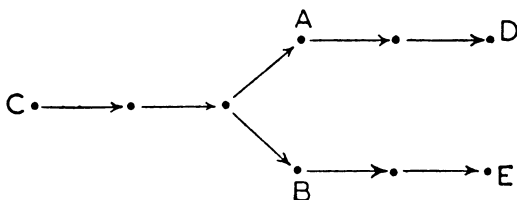

FIGURE 12/13/1.

diagram of immediate effects shown in Figure 12/13/1. If an effect coming from C goes down the branch AD only, then, for the branch BE to be independent, B must be constant. How the constancy is obtained is here irrelevant. When the effect from C is to be 'switched' to the BE branch, B must be freed and A must become constant. Any system with a 'switching' process must use, therefore, an alterable distribution of constancies. Conversely, *a system whose variables can be sometimes fluctuating and sometimes constant is adequately equipped for switching.*

The effects of local stabilities

12/14. The last few sections have shown how important, in any system that is to have temporary independencies, are variables that temporarily go constant. As such variables play a funda-mental part in what follows, let us examine them more closely.

Any subsystem (including the case of the single variable) that stays constant is, by definition, at a state of equilibrium. If the subsystem's surrounding conditions (parameters) are constant, the subsystem evidently has a state of equilibrium in the corresponding field; if it stays constant while its parameters are changing, then that state is evidently one of equilibrium in all the fields occurring. Thus, constancy in a subsystem's state implies that the state is

one of equilibrium; and constancy in the presence of small impulsive disturbances implies stability.

The converse is also true. If a subsystem is at a state of equilibrium, then it will stay at that state, i.e. hold a constant value (so long as its parameters do not change value).

Constancy, equilibrium, and stability are thus closely related.

12/15. Are such variables (or subsystems) common? Later (S. 15/2) it will be suggested that they are extremely common, and examples will be given. Here we can notice two types that are specially worth notice.

One form, uncommon perhaps in the real world but of basic importance as a type-form in the strategy of S. 2/17, is that in which the subsystem has a definite probability p that any particular state, selected at random, is equilibrial. We shall be concerned with this form in S. 13/2. (In explanation, it should be mentioned that the sample space for the probabilities is that given by a *set* of subsystems, each a machine with input and therefore determinate in whether a given state, with given input-value, is or is not equilibrial.) The case would arise when the observer faced a subsystem that was known (or might reasonably be assumed) to be a determinate machine with input, but did not know which subsystem, out of a possible set, was before him; the sample space being provided by the set suitably weighted, the observer could legitimately speak of the probability that *this* system, at *this* state, and with *this* input, should be in equilibrium.

The other form, very much commoner, is that which shows ' threshold ', so that *all* states are equilibrial when some parametric function is less than a certain value, and few or none are equilibrial when it exceeds that value. Well-known examples are that a weight on the ground will not rise until the lifting force exceeds a certain value, and a nerve will not respond with an impulse until the electric intensity, in some form, rises above a certain value.

What is important for us here is to notice that threshold, by readily giving constancy, can readily give what is necessary for the connexions between variable and variable to be temporary. Thus the changes in the diagram of Figure 12/12/1 could readily be produced by parts showing the phenomenon of threshold.

12/16. These deductions can now be joined to those of S. 12/10. If three subsystems are joined so that their diagram of immediate effects is

$$A \longrightarrow B \longrightarrow C$$
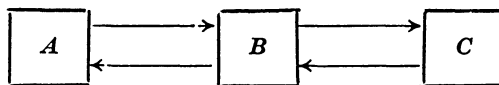
and if B is at a state that is equilibrial for all values coming from A and C, then A and C are (unconditionally) independent. Thus, B's being at a state of equilibrium *severs the functional connexion between A and C.*

Suppose now that B's states are equilibrial for some states of A and C, but not for others. As A and C, on some line of behaviour of the whole system, pass through various values, so will they (according to whether B's state at the moment is equilibrial or not) be sometimes dependent and sometimes independent.

Thus we have achieved the first aim of this chapter: to make rigorously clear, and demonstrable by primary operations, what is meant by ' temporary functional connexions ', when the control comes from factors within the system, and not imposed arbitrarily from outside.

12/17. The same ideas can be extended to cover any system as large and as richly connected as we please. Let the system consist of many parts, or subsystems, joined as in S. 6/6, and thus provided with basic connexions. If some of the variables or subsystems are constant for a time, then during that time the connexions through them are reduced functionally to zero, and the effect is as if the connexions had been severed in some material way during that time.

If a high proportion of the variables go constant, the severings may reach an intensity that cuts the whole system into subsystems that are (temporarily) quite independent of one another. Thus a whole, connected system may, if a sufficient proportion of its variables go constant, be temporarily equivalent to a set of unconnected subsystems. *Constancies*, in other words, *can cut a system to pieces.* (*I. to C.*, S. 4/20, gives an illustration of the fact.)

12/18. The field of a state-determined system whose variables often go constant has only the peculiarity that the lines of behaviour often run in a sub-space orthogonal to the axes. Thus,

over an interval in which all variables but one are constant, the corresponding line of behaviour must run as a straight line parallel to th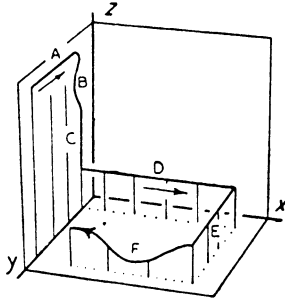e axis of the variable that is changing. If all but two are inactive (along some line of behaviour), that line in the phase-space may curve but it must remain in the two-dimensional plane parallel to the two corresponding axes; and so on. If all the variables are constant, the line naturally becomes a point—at the state of equilibrium. Thus a three-variable system might give the line of behaviour shown in Figure 12/18/1.

FIGURE 12/18/1. In the different stages the active variables are : A, y ; B, y and z ; C, z ; D x ; E y ; F x and z.

In the interval before they reach equilibrium, such variables will, of course, behave as part-functions. Through the remaining chapters they will show their importance. For convenience of description, a part-function (described in time by a variable) will be said to be **active** or **inactive** (at a given point on a line of behaviour) according to whether the variable is changing or remaining constant.