Chapter 18
Conclusions and Outlooks

1 Conclusions

We are now in the position to draw conclusions for this work. Motivated by the physical laws in higher dimensions leading to insight concerning those in lower dimensions, we have introduced wave equations in higher dimensions and put the mathematical and physical concepts and techniques like the wave equations and group theory related to the higher dimensions at the reader's disposal. In some sense we have provided a comprehensive description of the wave equations including the non-relativistic Schrödinger equation, relativistic Dirac and Klein-Gordon equations in arbitrary dimensions and their wide applications in quantum mechanics.

We have introduced the fundamental theory about the SO(N) group which has been used in the successive Chaps. 3–5 including the non-relativistic Schrödinger equation, relativistic Dirac and Klein-Gordon equations. As important applications in non-relativistic quantum mechanics, we have applied our theories proposed in Part II to study some quantum systems such as the harmonic oscillator, Coulomb potential, wavefunction ansatz method, Levinson theorem, generalized hypervirial theorem, exact and proper quantization rules and Langer modification, position-dependent mass Schrödinger equation. As for as those important generalized applications to relativistic quantum mechanics in higher dimensions, we have studied the Levinson theorem and generalized hypervirial theorem for the Dirac equation, the Klein-Gordon equation and Kaluza-Klein theory. A number of previous results are summarized and some new materials are presented.

2 Outlooks

Considering the interest in higher dimensional quantum physics, some new appearing fields could be studied. For example, the superstring theory, supergravity shall become interesting and challenging. On the other hand, the searching of the extra dimensions both in theory and in experiment also becomes exciting.

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