

ENCOUNTER: On Breaking the Nonce Barrier in Differential Fault Analysis with a Case-Study on PAEQ

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Abstract. This work exploits internal differentials within a cipher in the context of Differential Fault Analysis (DFA). This in turn overcomes the nonce barrier which acts as a natural counter-measure against DFA. We introduce the concept of *internal differential fault analysis* which requires only one faulty ciphertext. In particular, the analysis is applicable to parallelizable ciphers that use the counter-mode. As a proof of concept we develop an internal differential fault attack called **ENCOUNTER** on **PAEQ** which is an AES based parallelizable authenticated cipher presently in the second round of on-going CAESAR competition. The attack is able to uniquely retrieve the key of three versions of full-round **PAEQ** of key-sizes 64, 80 and 128 bits with complexities of about 2^{16} , 2^{16} and 2^{50} respectively. Finally, this work addresses in detail the instance of fault analysis with varying amounts of partial state information and also presents the first analysis of **PAEQ**.

Keywords: Fault analysis · Authenticated encryption · **PAEQ** · Internal differential · **AESQ** · Nonce · **AES**

1 Introduction

The popularity of cryptanalyzing a cipher by observing its behavior under the influence of faults is mainly attributed to the ease of such fault induction and overhead in incorporating a counter-measure. Among different types of fault based cryptanalysis, Differential Fault Analysis (DFA) [1–7] has garnered particular attention of the side-channel research community since it has been one of the most effective side-channel attacks on symmetric-key constructions. DFA puts in the hand of an attacker an interesting ability: *The possibility of performing a differential analysis starting from an intermediate state of the cipher.* This ability could be fatal in case of iterated symmetric-key designs since it is equivalent to cryptanalyzing a round-reduced version of the cipher. However, classical DFA has a specific requirement known as the *replaying criterion* which states that the attacker must be able to induce faults while **replaying** a previous fault-free run of the algorithm. In this scenario, the introduction of a *nonce* constraint comes in as a direct contradiction to the ability to replay.

The notion of nonce-based encryption was formalized by Rogaway in [8] where the security proofs relied on the pre-condition of the *uniqueness of the nonce* in every instantiation of the cipher. Thus, it can be easily inferred why nonces provide an in-built protection against DFA. Usage of nonces to counter fault attacks are already available in Public-Key literature. The famous Bellcore attack [9, 10] on RSA-CRT signatures can be prevented if the message is padded with a random nonce which is recoverable only when verifying a correct signature. It was shown by Coron et al. [11] that in some limited setting these nonces can be tackled. However, the techniques used rely on theoretical constructs which may not be applicable to their private-key counterparts. Though there have also been attempts to mount DFA attacks on symmetric-key designs in the presence of a nonce, the solutions are very specific to the underlying cipher. Such an instance can be found in [12], where the authors studied the impact of the nonce constraint on the fault-attack vulnerability of authenticated cipher APE [13] and demonstrated the idea of *faulty collisions* to overcome it.

This work tries to address the nonce barrier in a more general setting by totally bypassing the replaying criterion which amplifies the scope of the ideas presented here. This is made possible by a DFA strategy that no longer requires a fault-free run of the cipher. Instead the strategy needs only one multi-block plaintext and faulty ciphertext pair to mount an attack. The nonce constraint is no longer a threat to DFA if the analysis relies on a single faulty ciphertext. The idea of using single faulty ciphertext stems from the prospect of using internal differentials within the cipher. This type of analysis is well-studied and has been successfully used in cryptanalysis of symmetric-key designs [14, 15]. We explore the possibility of deploying this in the context of DFA. In particular, we look at parallelizable authenticated encryption (AE) schemes that use the counter mode of operation to separate the branches. The parallel branches of these schemes provide a good platform for injecting faults and studying the fault propagation in the internal difference of the branches. The main idea is to nullify very low hamming distance between the inputs of the parallel branches using a primary fault and subsequently employ internal differential fault analysis using a secondary fault. To undertake a case-study we select PAEQ which is among the 30 Round-2 candidates in the on-going CAESAR [16] competition on authenticated ciphers. PAEQ meets the basic criteria for analysis since it is parallelizable and uses the counter mode. Moreover, the underlying permutation follows AES [17] very closely which provides an edge in terms of fault analysis. In this work, we present the first analysis of PAEQ in form of an internal difference based fault attack called **ENCOUNTER** on full-round `paeq-64`, `paeq-80` and `paeq-128` using a 4-round distinguishing property. Two byte faults are required to be injected in one of the parallel branches during the encryption phase of PAEQ. **ENCOUNTER** uses only *one 255-block known plaintext and corresponding faulty ciphertext* to significantly reduce the average key-search space of the three versions of PAEQ. Finally, one might be tempted to believe that classical differential fault analysis results on AES would *suffice* to analyze an AE scheme like PAEQ which is inherently AES based. However, there is a fundamental difference given the fact that

the output here is truncated giving an attacker access to only partial information of the state. Thus the current work provides an instance of a fault based analysis of partially specified states.

Our Results

- Introduce the concept of internal differential fault analysis (IDFA) in the context of parallelizable ciphers in the counter mode
- Showcase that IDFA requires only one run of the algorithm thereby overcoming the *nonce barrier* of DFA.
- Present a 4-round internal differential distinguisher for PAEQ.
- Use the distinguisher to develop the **ENCOUNTER** attack on full-round PAEQ using only two faults in the same instance of PAEQ
- Reduces average key-space of primary PAEQ variants to practical limits viz., **paeq-64**: 2^{64} to 2^{16} , **paeq-80**: 2^{80} to 2^{16} , **paeq-128**: 2^{128} to 2^{50} .
- Present instances of fault analysis of an AES based design with various types of partially specified internal states.

The rest of the paper is organized as follows. Section 2 provides a brief description of PAEQ. The notations used in the work are given in Sect. 3. The concept of internal differential fault analysis is introduced in Sect. 4. A 4-round distinguisher of PAEQ is showcased in Sect. 5. Section 6 introduces the notion of fault quartets. The **ENCOUNTER** attack on PAEQ is devised in Sect. 7 and its complexity analysis is furnished in Subsect. 7.5. The experimental results are presented in Sect. 8 while the concluding remarks are given in Sect. 9.

2 The Design of PAEQ

PAEQ which stands for *Parallelizable Authenticated Encryption based on Quadrupled AES* was introduced by Biryukov and Khovratovich in ISC 2014 [18] along with a new generic mode of operation PPAE (Parallelizable Permutation-based Authenticated Encryption). It was also submitted to the on-going CAESAR competition for authenticated cipher and is presently one of the 30 round 2 candidates. The design of PAEQ was mainly driven by simplicity and to achieve a security level equal to the key-length. Hence the authors argued in favor of a permutation based design. It is fully parallelizable and on-line and offers a security level up to 128 bits and higher (up to $w/3$, $w \leftarrow$ width of internal permutation) and equal to the key length. An interesting aspect of the PPAE mode of operation (denoted PPAE_f) is that *the inputs to the internal permutation f are only linked by counters*. This property makes PAEQ a prime candidate to apply the concept of fault based internal differentials proposed in this work. Next we touch upon PPAE_f and the internal permutation of PAEQ called AESQ.

2.1 PPAE Mode of Operation

PPAE_f (illustrated in Algorithm 1) can be instantiated with an n -bit permutation f . The inputs to the permutation are formatted as $(D_i || counter || N || K)$ for each plaintext block and $(D_i || counter || AD-block || K)$ for processing associated data (AD) where $D_i \leftarrow$ domain separator, $N \leftarrow$ nonce and $K \leftarrow$ key. The plaintext and AD are divided into blocks of size $n - k - 16$ and $n - 2k - 16$, respectively, where k is the key-size. Incomplete last blocks are padded using the byte-length of the block and domain separators are changed accordingly. Plaintext processing and authentication calls f twice while AD data is authenticated using a single call. Partial authentication data from all branches are passed to a final call to f , the output of which is optionally truncated to get the tag. The entire operation is depicted in Fig. 1. An interested reader can refer to [18, 19] for details.

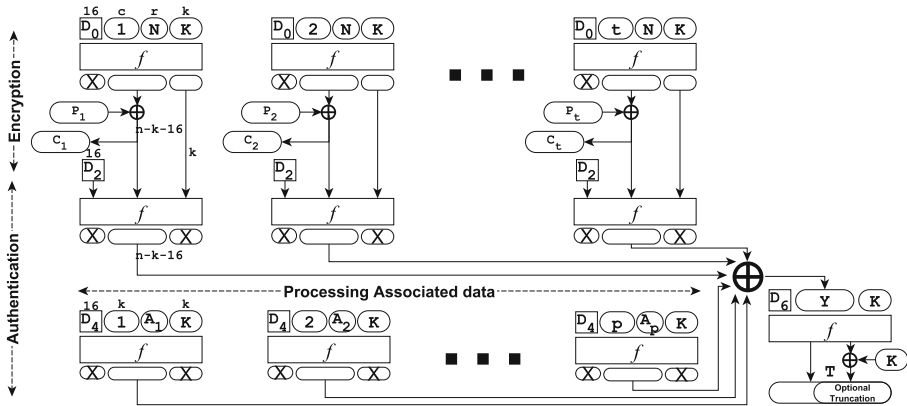


Fig. 1. Encryption and authentication with PPAEQ

2.2 The Internal Permutation: AESQ

AESQ operates on a 512-bit internal state which can be subdivided into 128-bit substates. Before going into details we introduce some definitions.

Definition 1 (Word). Let $\mathbb{T} = \mathbb{F}[x]/(x^8 + x^4 + x^3 + x + 1)$ be the field \mathbb{F}_{2^8} used in the AES MixColumns operation. Then a **word** is defined as an element of \mathbb{T} .

A **word** is just a **byte** redefined to account for the field arithmetic. In this work, we will come across *partially specified states/substates* where certain words might have unknown values. To capture this scenario, we use the symbol ‘X’ to represent unknown words. Thus, to be precise a **word** is an element of $\mathbb{T} \cup \{‘X’\}$.

Definition 2 (Substate, State). The internal **state** of the AESQ permutation is defined as a 4-tuple of substates where each substate is a (4×4) -word matrix.

Algorithm 1. $\text{PPAE}_f(P, N, K, A, n)$

Input: $\begin{cases} P \leftarrow \text{Plaintext}, & N \leftarrow \text{Nonce}, \quad |N| = r, \quad K \leftarrow \text{Key}, \quad |K| = k \\ A \leftarrow \text{Associated Data}, \quad f \leftarrow \text{Internal permutation}, \quad n \leftarrow \text{Internal state size} \end{cases}$

Output: $C, T \rightarrow \text{Ciphertext and Tag}$

1: $D_i = (k, (r + i) \bmod 256)$, $i = 1, 2, \dots, 6$ \triangleright Generating 2-byte domain separators
2: $\{P_1, P_2, \dots, P_t\} \leftarrow P$ $|P_i| = (n - k - 16)$ bits
3: $\{A_1, A_2, \dots, A_p\} \leftarrow A$ $|A_i| = (n - 2k - 16)$ bits
4: **if** $(|P_t| < n - k - 16)$ **then** $P_t \leftarrow P_t || a || a \dots || a$ $\triangleright a = |P_t|/8$ and $|a| = 1$ byte
5: **if** $(|A_p| < n - 2k - 16)$ **then** $A_p \leftarrow A_p || b || b \dots || b$ $\triangleright b = |A_p|/8$ and $|b| = 1$ byte
6: $Y = \mathbf{0}$ $\triangleright |Y| = n - k - 16$
7: **for** $i = 1$ to t **do**
8: $V_i \leftarrow D_0 || R_i || N || K$ $\triangleright \begin{cases} R_i \leftarrow \text{Branch Index}, R_i = i, |R_i| = n - k - r - 16 \\ D_0 \leftarrow D_1 \text{ for incomplete last block} \end{cases}$
9: $W_i \leftarrow f(V_i)$; $C_i \leftarrow W_i[17 \dots (n - k)] \oplus P_i$
10: $X_i \leftarrow D_2 || C_i || W_i[(n - k + 1) \dots n]$ $\triangleright D_2 \leftarrow D_3$ for incomplete last block
11: $Y_i \leftarrow (f(X_i))[17 \dots (n - k)]$; $Y \leftarrow Y \oplus Y_i$
12: **for** $i = 1$ to p **do** \triangleright Binding Associated Data
13: $X'_i \leftarrow D_4 || R_i || A_i || K$ $\triangleright \begin{cases} R_i = i, |R_i| = k \\ D_4 \leftarrow D_5 \text{ for incomplete last block} \end{cases}$
14: $Y'_i \leftarrow (f(X'_i))[17 \dots (n - k)]$
15: $Y \leftarrow Y \oplus Y'_i$
16: $T \leftarrow f(D_6 || Y || K) \oplus (0^{n-k} || K)$
17: $C = \{C_1, C_2, \dots, C_t\}$ \triangleright Truncate C_t for incomplete last plaintext block

A state is denoted by s , while each substate is represented by $s^m = [s^m_{i,j}]$ where $s^m_{i,j}$ are the elements of s^m and m denotes the **substate index**. We denote a column of $[s^m_{i,j}]$ as $s^m_{*,j}$ while a row is referred to as $s^m_{i,*}$.

$$s^m = [s^m_{i,j}], \text{ where } \begin{cases} s_{i,j} \in \mathbb{T} \cup \{ 'X' \} \\ 0 \leq i, j < 4; \quad m \in \{1, 2, 3, 4\} \end{cases} \quad s = (s^1, s^2, s^3, s^4)$$

AESQ is a composition of 20 round functions with a **Shuffle** operation (denoted by \mathcal{S} , Refer Table 1) after every 2 rounds. Each round-function is denoted by \mathcal{R}_r where the index r denotes the r^{th} round of AESQ. Every round applies on the internal state a composition of four bijective functions which are basically the standard AES round operations **SubBytes**, **ShiftRows**, **MixColumns**, **AddRoundConstants** applied individually on each substate. In the context, of a state we denote these functions as β_r, ρ_r, μ_r and α_r respectively. The reference to a substate is addressed by including the substate index in notation. For example, to refer to the **MixColumns** on the second substate in \mathcal{R}_{17} we use μ_{17}^2 . Similarly, when considering a substate in \mathcal{R}_r we refer to the round function applied individually to the substate as \mathcal{R}_r^m by including the substate index in the notation. This implies that for an internal state s the output of the r^{th} round of AESQ is $\mathcal{R}_r(s) = \mathcal{R}_r^1(s^1) || \mathcal{R}_r^2(s^2) || \mathcal{R}_r^3(s^3) || \mathcal{R}_r^4(s^4)$.

Table 1. Column mapping under Shuffle (\mathcal{S})

	s^1				s^2				s^3				s^4			
From	$s^1_{*,0}$	$s^1_{*,1}$	$s^1_{*,2}$	$s^1_{*,3}$	$s^2_{*,0}$	$s^2_{*,1}$	$s^2_{*,2}$	$s^2_{*,3}$	$s^3_{*,0}$	$s^3_{*,1}$	$s^3_{*,2}$	$s^3_{*,3}$	$s^4_{*,0}$	$s^4_{*,1}$	$s^4_{*,2}$	$s^4_{*,3}$
To	$s^1_{*,3}$	$s^4_{*,3}$	$s^3_{*,2}$	$s^2_{*,2}$	$s^1_{*,1}$	$s^4_{*,1}$	$s^3_{*,0}$	$s^2_{*,0}$	$s^1_{*,2}$	$s^4_{*,2}$	$s^3_{*,3}$	$s^2_{*,3}$	$s^1_{*,0}$	$s^4_{*,0}$	$s^3_{*,1}$	$s^2_{*,1}$

$$\text{AESQ} = \mathcal{S} \circ \mathcal{R}_{20} \circ \mathcal{R}_{19} \circ \dots \circ \mathcal{S} \circ \mathcal{R}_2 \circ \mathcal{R}_1$$

$$\mathcal{R}_r = \alpha_r \circ \mu_r \circ \rho_r \circ \beta_r; \quad \mathcal{R}_r^m = \alpha_r^m \circ \mu_r^m \circ \rho_r^m \circ \beta_r^m$$

Round-reduced AESQ permutation is denoted by AESQ^n where $n = 2k, 1 \leq k \leq 9$. Thus $\text{AESQ}^n = \mathcal{S} \circ \mathcal{R}_n \circ \mathcal{R}_{n-1} \circ \dots \circ \mathcal{S} \circ \mathcal{R}_2 \circ \mathcal{R}_1$. Since n is even, it implies that we consider reductions in steps of two-rounds and AESQ^n always ends in the \mathcal{S} operation. Finally, the round constant for substate m in round r of AESQ is given by: $rc_r^m = ((r - 1) * 4 + m)$. In α_r^m , rc_r^m is added to all words of row $s^1_{*,*}$.

2.3 Handling Partially Specified States/Substates

As mentioned earlier in this work we have to handle states or substates that may have multiple unknown values. We now define how the operations $\beta_r^m, \rho_r^m, \mu_r^m$ and α_r^m behave in case of a partially specified substate $s^m = [s^m_{i,j}]$. Here SBOX denotes the AES Substitution box and M_μ denotes the MixColumns matrix. ρ_r^m does not rely on values of s^m and just shifts the positions of unknown values.

$$s^m_{i,j} \xrightarrow{\beta_r^m} \begin{cases} \text{X} & \text{if } s^m_{i,j} = \text{X} \\ \text{SBOX}(s^m_{i,j}) & \text{Otherwise} \end{cases} \quad \left| \quad s^m_{*,j} \xrightarrow{\mu_r^m} \begin{cases} M_\mu \times s^m_{*,j} & \text{if } \forall i, s^m_{i,j} \neq \text{X} \\ \{\text{X}, \text{X}, \text{X}, \text{X}\}^T & \text{Otherwise} \end{cases} \quad \left| \quad s^m_{i,j} \xrightarrow{\alpha_r^m} \begin{cases} s^m_{i,j} & \text{if } i \neq 1 \\ s^m_{i,j} \oplus rc_r^m & \text{if } s^m_{i,j} \neq \text{X} \\ \text{X} & \text{Otherwise} \end{cases}$$

3 Notations

Definition 3 (Diagonal). A *diagonal* of a substate ($s^m = [s^m_{i,j}]$) is the set of words which map to the same column under the Shift-Row operation.

$$d_k^m = \{s^m_{i,j} : \rho^m(s^m_{i,j}) \in s^m_{*,k}\}, \text{ where } k = (j - \sigma(i)) \bmod 4; \sigma = \{0, 1, 2, 3\} \quad (1)$$

Definition 4 (Differential State). A *differential state* is defined as the element-wise XOR between an internal state $s = (s^1, s^2, s^3, s^4)$ and the corresponding state $s' = (s'^1, s'^2, s'^3, s'^4)$ belonging to a different branch of PAEQ.

$$\delta = (\delta^1, \delta^2, \delta^3, \delta^4) = ((s^1 \oplus s'^1), (s^2 \oplus s'^2), (s^3 \oplus s'^3), (s^4 \oplus s'^4))$$

Definition 5 (Column Vector). A *Column Vector*, is a set of columns where each element is a candidate for one particular column of a substate.

Definition 6 (State/Substate Vector). A *Substate Vector*, identified by $[s^m]^v$, is a set of substates where each element is a prospective candidate for substate s^m . A *State Vector* of state s is the cross-product of its substate vectors.

4 Internal Differential Fault Analysis

In this work, for the first time, we explore the idea of using internal differentials to mount fault based attacks. Though the concept of internal differentials is well-known in cryptanalysis [14, 15], the idea has never been applied in fault based Side Channel Analysis. Our research reveals that the best crypto-primitives to mount such a type of attack are the ones that employ a parallel mode of operation. In principle, internal differentials exploit self-symmetric structures present within a construction. In the parallel mode of operation all parallel branches are structurally similar exhibiting nice instances of such self-symmetry. We look at inputs of such branches. Parallelizable ciphers using the counter mode are generally characterized by the common property that the inputs to the parallel branches only differ in the counter value. Thus, the hamming distance of the inputs are quite low. More interestingly, we can find branches in which *the bit positions where the inputs differ are very localized*.¹ This localization can be fatal from the perspective of fault analysis since a fault injection in the counter could possibly lead to a collision in the counter values. So the faulty counter could become equal to a fault-free counter and since the inputs differ only in the counter value this would give us two branches with the same input. This forms a pre-condition for deploying internal differentials analysis since under this scenario, we could inject a second fault in the internal state of any of these two identical branches and study how the fault diffuses in internal difference of the corresponding states. The problem now reduces to classical DFA and hence we may apply standard techniques pertaining to differential fault analysis. Figure 2 depicts the concept visually in a very generic sense.

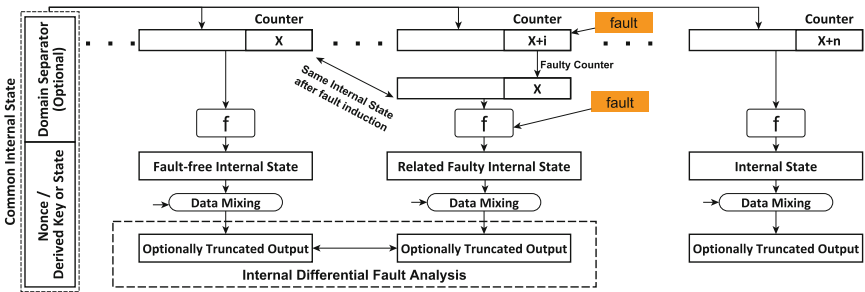


Fig. 2. Generic model for using internal differentials in fault analysis of parallelizable ciphers in the counter mode.

An interesting aspect of this strategy is that it requires *only one multi-block ciphertext with a single faulty block* thereby making the analysis independent of the effect of a nonce which prohibits usage of standard DFA. One could argue regarding the viability of the first fault in terms of achieving the counter collision.

¹ For instance, the differing bits could be localized within a byte.

However, in this work we introduce the concept of *Fault Quartets* (Refer Sect. 6) which can use a round-reduced distinguisher of the underlying cipher to locate the fault-free branch corresponding to a faulty branch due to the first fault under reasonable assumptions. Building upon these ideas a practical IDFA attack is mounted on PAEQ. Though the specifics rely on the underlying construction, the overall notion of IDFA can be adapted to other ciphers which meet the properties discussed earlier. In the next section we develop a four-round distinguisher of PAEQ based on internal differentials arising from counter values.

5 An Internal Differential Distinguisher for 4-Round PAEQ

In this section we showcase a 4-round internal differential distinguisher which surfaces due to low Hamming-distance between inputs to various parallel branches of the first call to the AESQ permutation during the encryption phase. Here we deal with AESQ reduced to 4-rounds. We start devising the distinguisher by making the following observation which is due to the PPAE mode of operation.

Observation 1. *Two parallel branches of PAEQ with the same domain separator differ only in the counter value.*

Based on Observation 1, we first choose any branch of PAEQ in the encryption phase with counter value i . We next find a branch with counter value j such that their corresponding c -bit² binary representations differ in exactly one³ byte. The main idea is to restrict the internal-difference to a byte and later study its diffusion from \mathcal{R}_1 to \mathcal{R}_4 . The differential propagation in an AES round is quite well-known. However, the presence of quadruple AES instantiations in AESQ and the inclusion of \mathcal{S} operation that mixes the substates make the analysis interesting. Here, we are particularly interested in studying how the bytes belonging to the same column become interrelated as the difference diffuses.

Four Round Differential Propagation in AESQ:

- \mathcal{R}_1 : Difference spreads to entire column, and the bytes are related by factors governed by the `MixColumns` matrix.
- \mathcal{R}_2 : Entire substate affected; columns of the substate are related by factors.
- \mathcal{S} : One column of each substate affected; factor relations unaffected.
- \mathcal{R}_3 : All substates affected. All columns exhibit byte inter-relations due μ_3 .
- \mathcal{R}_4 : All substates still affected with all relations destroyed due to β_4
- \mathcal{S} : Columns permute.

This process is illustrated in Fig. 3a. After every round the corresponding byte inter-relations are given. Recall that here we are dealing with a differential state (Definition 4). Here, the numbers in a particular column indicate the factor by

² Recall, the counter is of size $c = n - k - 16$ bits.

³ For instance, $i = 5$ and $j = 8$ differ only in the least significant byte.

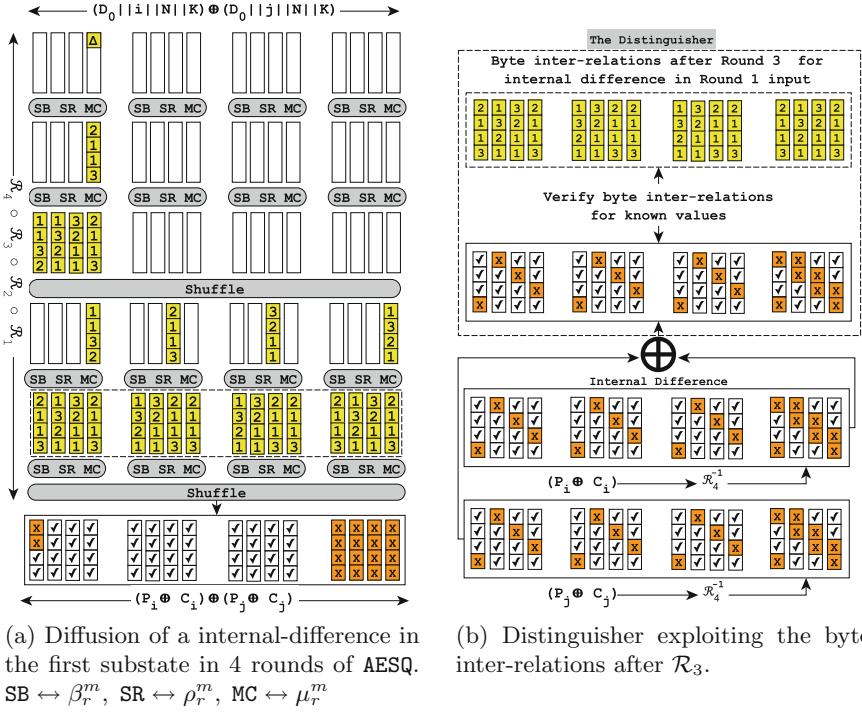


Fig. 3. A demonstration of the 4-round distinguishing strategy using paeq-128. A \checkmark denotes a known value while a X denotes an unknown value.

which the byte-wise differences in that column are related. In the first substate $\{2, 1, 1, 3\}$ imply that differences are of the form $\{2 \times f, 1 \times f, 1 \times f, 3 \times f\}$ where $f \in \mathbb{T} \setminus \{0\}$. The byte inter-relations after \mathcal{R}_3 translates into the following observation:

Observation 2. *The byte inter-relations after \mathcal{R}_r due to a difference in \mathcal{R}_{r-2} input is invariant based on which substate the initial difference was located.*⁴

The implication of Observation 2 is that there is a bijective relation between byte inter-relations after \mathcal{R}_3 and substate where we had the initial internal difference in \mathcal{R}_1 input. As there are four substates so we have four unique byte inter-relations (Fig. 6) after \mathcal{R}_3 . We can now present the proposed distinguisher in Algorithm 2 which passes with probability 1 for PAEQ if $n = 4$.

Remark 1. $\text{DISTINGUISHER}(P, C, i, j, n)$ will fail for any $n > 4$ because Observation 2 only holds for three rounds while the last round is handled by inversion to get a meet-in-the-middle scenario to verify the byte inter-relations (Fig. 3). Moreover, it might also fail if i and j differ in more than one byte since in that

⁴ It is understood that $r > 2$ and $2|(r - 1)$.

Algorithm 2. DISTINGUISHER (P, C, i, j, n)

Input: $\begin{cases} P, C \rightarrow 1 \text{ known plaintext-ciphertext } (P = P_1 || \dots || P_t, C = C_1 || \dots || C_t) \\ (i \neq j) < t \rightarrow \text{Branch indexes which differ in one byte}^a \\ n \rightarrow \text{Total number of AESQ rounds considered} \end{cases}$

Output: 0/1

- 1: $s \leftarrow \mathcal{S}^{-1}(P_i \oplus C_i) \quad s' \leftarrow \mathcal{S}^{-1}(P_j \oplus C_j)$
 - 2: $s \leftarrow \mathcal{R}_n^{-1}(s) \quad s' \leftarrow \mathcal{R}_n^{-1}(s')$
 - 3: $\delta = s \oplus s'$ ▷ The internal difference
 - 4: **Result** $\leftarrow \overset{\text{Verify Byte Inter-relations (Fig. 6)}}{\delta}$
 - 5: **if** **Result** == **TRUE** **then return** 1
 - 6: **else return** 0
-

^a $i, j \neq t$ since the last block might have a different domain separator in which case Observation 1 will not hold.

case the internal difference in \mathcal{R}_1 input might span multiple substates. Finally, **DISTINGUISHER** constitutes a general 3-round differential characteristic which will hold for any 3 consecutive rounds of AESQ as long as the starting states have an internal difference of one byte. This is what is exploited to develop an internal differential fault attack on PAEQ.

From the above remark we get the impression that if *somehow* we could get a one-byte internal difference in \mathcal{R}_{17} then the 4-round distinguishing property could be verified from \mathcal{R}_{20} i.e. the full AESQ permutation. In order to achieve this scenario we introduce the concept of **Fault Quartets** in the next section.

6 The Fault Quartet

Definition 7. A fault quartet ($\mathcal{Q}_{i,j}$) is a configuration of four internal states of the AESQ belonging to two different branches i and j of an instance of PAEQ in the encryption phase. It is uniquely identified by the ordered pair of the branch indexes (i,j) . $\mathcal{Q}_{i,j}$ is of the following form and has the following constraints:

$$\mathcal{Q}_{i,j} = \{s, s^\#, t, t^\#\} \tag{2}$$

where $\begin{cases} s, t \rightarrow \text{branch input states, } s^\# = \text{AESQ}^{16}(s), t^\# = \text{AESQ}^{16}(t) \\ \text{Constraint 1: } s \oplus t = \mathbf{0} \\ \text{Constraint 2: } s^\# \text{ and } t^\# \text{ have an internal difference of one byte.} \end{cases}$

To generate a fault quartet, we take a plaintext of 255 blocks⁵ and induce two random byte faults during the encryption phase of PAEQ. The first fault, called the **equalizer** is injected in the last byte of the counter of any branch i . The second fault called the **differentiator** is injected anywhere in the input of \mathcal{R}_{17} of AESQ in the same branch. The **equalizer** achieves the first constraint

⁵ Last block is a complete block (i.e., block-size = $n - k - 16$) due to Observation 1.

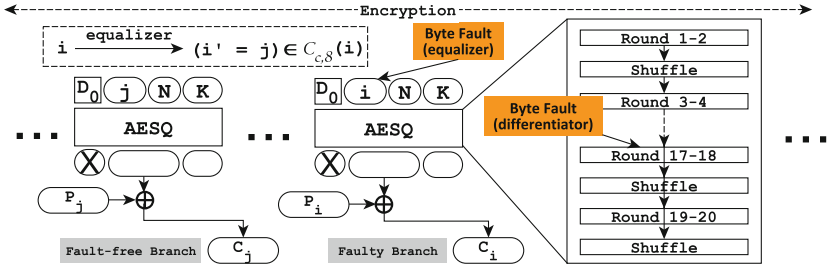


Fig. 4. Fault injection in PAEQ

of $\mathcal{Q}_{i,j}$ which states that the input states must have no internal difference while the differentiator induces a one-byte internal difference between the outputs of \mathcal{R}_{16} which constitutes the second constraint. Figure 4 demonstrates the fault injection. The following observation accounts for the choice of a 255-block message above.

Observation 3. *The number of the plaintext blocks required to guarantee⁶ the existence of a fault quartet with the equalizer fault injected in the last byte of the counter of any branch is 255 with a complete last block.*

The complete block at the end ensures that all inputs to AESQ have the same domain separator. The choice of 255 implies that all of these differ only in the last byte of the counter. Due to equalizer fault the counter value of branch i changes to j which is equal to the counter value of any⁷ one of the remaining 254 fault-free branches. Thus, this outlines the condition to guarantee that a fault quartet is generated. But how would one find such a quartet since neither the input states nor the output of of AESQ¹⁶ is visible to an adversary. This is addressed next where we give an algorithm to find such a quartet.

Finding $\mathcal{Q}_{i,j}$: Finding a fault-quartet translates into finding the branch-index ordered pair (i, j) where i corresponds to the branch of PAEQ where the faults have been introduced and the j corresponds to the fault-free branch. This is done by Algorithm 3 using the distinguisher developed earlier as a sub-routine. One can recall that due to the differentiator there will be a one-byte internal difference between branch i and j in the input of \mathcal{R}_{17} . Thus distinguishing property can be verified from \mathcal{R}_{20} i.e., the full AESQ permutation.

$\mathcal{Q}_{i,j}$ gives us the opportunity to exploit the distinguishing property of PAEQ in the last four rounds of AESQ. With these concepts we are in a position to finally introduce the ENCOUNTER attack which exploits the property further to recover the entire internal state of AESQ thereby revealing the key.

⁶ With a probability of $\frac{255}{256}$.

⁷ Except for the case when $j = 0$ when it matches none of the remaining branches.

Algorithm 3. FINDQ (P, C, i)**Input:** $P, C \rightarrow$ One 255-block plaintext-ciphertext; $i \rightarrow$ Index of faulty-branch**Output:** $(i, j) \rightarrow$ Branch-index ordered pair identifying the fault quartet $Q_{i,j}$

```

1: for all  $j \in \{1, 2, \dots, 255\} \setminus \{i\}$  do
2:   if (DISTINGUISHER( $P, C, i, j, 20$ ) == 1) then
3:     return  $(i, j)$  ▷ This line is reached exactly once

```

7 ENCOUNTER: Fault Analysis of PAEQ using Internal Differentials

The ENCOUNTER attack proceeds in two phases: INBOUND and OUTBOUND. The INBOUND phase is common to all PAEQ variants while the OUTBOUND phase varies. In this work we focus on paeq-64, paeq-80 and paeq-128 which constitute the primary set of PAEQ family as specified by the designers. We first provide a high-level description of the attack and then delve into the details.

High-level Description of ENCOUNTER

- **Fault-Injection:** Run PAEQ on a plaintext with 255 complete blocks. Inject the equalizer and differentiator faults in any branch i .
- **Find Fault-Quartet:** Use FINDQ to get index of the fault-free branch.
- **INBOUND Phase:** Invert states from output of AESQ for both branches and reconstruct internal differential state after \mathcal{R}_{19} . Guess diagonal of differentiator fault to get a set of four column vectors for the state after β_{19} .
- **OUTBOUND Phase:** Recovers candidates for all substates at the end of \mathcal{R}_{20} for the fault-free branch using the column vectors from INBOUND phase. Return them in the form of substate vectors.
- **Complete Attack:** Repeat INBOUND phase for every guess of the diagonal and consequently OUTBOUND too. Accumulate substate vectors from every guess. Their cross-product gives the reduced state-space for the state after \mathcal{R}_{20} . Inverting every candidate state and verifying the known part of the input reveals the correct key.

7.1 The Fault Model

Firstly, as ENCOUNTER is an IDFA attack it needs only a single run of PAEQ. Secondly, it is based on a **random byte** fault model requiring 2 faults: **equalizer** and **differentiator**, induced in any branch of AESQ while encrypting the plaintext. **equalizer** targets the *last byte* of the counter while **differentiator** targets *any* byte at the input of \mathcal{R}_{17} (Fig. 4). While classical DFA generally deals with single block messages, IDFA uses a single multi-block message as it targets parallelizable ciphers. In the context of ENCOUNTER the plaintext itself is a 255-block message. After fault injection the attacker uses faulty and fault-free blocks

of the same ciphertext and corresponding plaintext blocks to mount the attack. Assuming the faulty branch to be i , faulty ciphertext block C_i^* is identified below.

$$\text{ENCOUNTER Input} \begin{cases} P = P_1 || P_2 || \dots || P_i || \dots || P_j || \dots || P_{255} \\ C = C_1 || C_2 || \dots || C_i^* || \dots || C_j || \dots || C_{255} || \text{Tag}^* \end{cases}$$

7.2 The INBOUND Phase

As the name suggests the **INBOUND** phase tries to invert the the faulty and fault-free branches using the observable part of the output. A pictorial abstraction of the process is provided in Fig. 5 for early reference. After identifying the fault-free branch using **FINDQ**, the attacker separately inverts the partially known output⁸ of **AESQ** for both branches up to the input of \mathcal{R}_{20} . He then computes the internal difference of the states which gives him a partially specified differential state. This state, by virtue of fault diffusion, has a special property that the differences in individual columns are related. These relations are given in Fig. 6 and help to recover the complete differential state. Further, by **Observation 2** the verified byte inter-relations will also *reveal the substate* where the **differentiator** fault got injected.

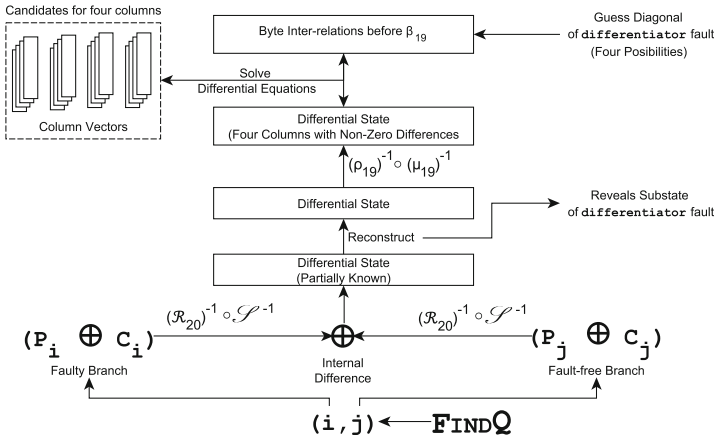


Fig. 5. The **INBOUND** phase. Returns candidates for columns after β_{19} .

The attacker inverts the reconstructed state up to input of ρ_{19} . Again, due to fault diffusion every substate of this state has exactly one column with non-zero related differences (Refer Fig. 3a). However, the relations differ based on the location of **differentiator** fault. By virtue of the *Diagonal Principle*⁹ [7] which

⁸ Computed using the **XOR** of plaintext and ciphertext blocks.

⁹ Faults injected in the same diagonal of an **AES** state in round r input lead to the same byte interrelations at the end of round $(r + 1)$.

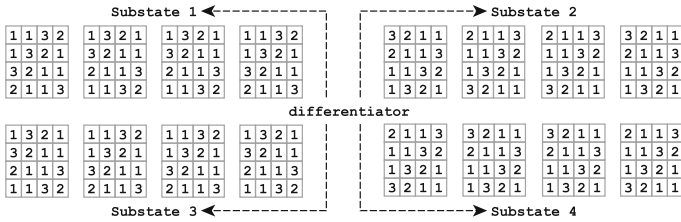


Fig. 6. Byte inter-relations after $\mathcal{R}_{19} \leftrightarrow$ Location of differentiator fault.

is a well-known result of DFA on AES, we know that for a particular substate there can be four kinds of relations based on the diagonal where the **differentiator** was injected. Figure 7 shows all these possible byte inter-relations based on the source of the byte-fault at the input of \mathcal{R}_{17} .

The attacker already knows which quadrant to look at in Fig. 7 since he knows the substate location of **differentiator**. However, he has no idea about the source diagonal and has to resort to *guessing*. So like classical DFA using the input and output difference of β_{19} , the attacker solves differential equations to generate candidates for the four columns which are stored in column vectors. At the end of **INBOUND** phase the attacker has a set of four column vectors for the particular guess of the fault diagonal.

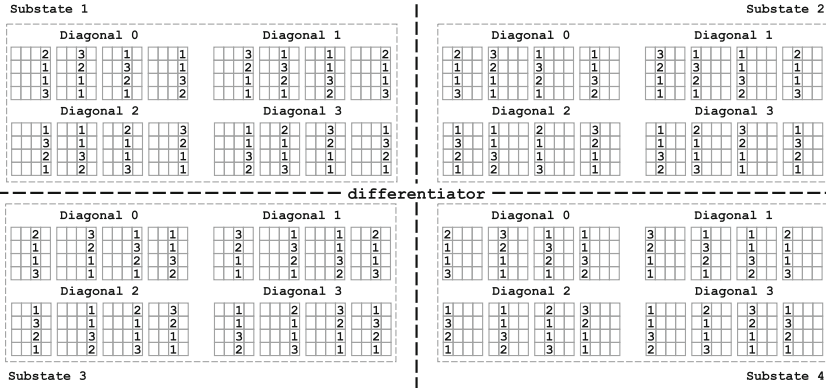


Fig. 7. Byte inter-relations at \mathcal{R}_{19} input vs location of **differentiator** fault. Each quadrant corresponds to a source substate. For each substate, the relations correspond to a source diagonal.

7.3 The OUTBOUND Phase

Unlike the **INBOUND** phase, the **OUTBOUND** phase works only on the fault-free branch of AESQ specifically on the partial state at the end of \mathcal{R}_{20} . Additionally, the column vectors generated above are also used. Let us look at the nature of

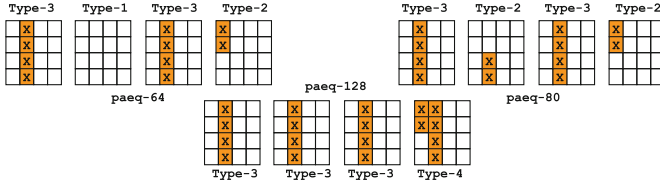


Fig. 8. The classification of substates observed in the internal state after $\mathcal{S}^{-1}(P_j \oplus C_j)$ based on number of unknown bytes.

the substate in the state determined by $\mathcal{S}^{-1}(P_j \oplus C_j)$. This is captured by Fig. 8 for PAEQ variants analyzed in this work.

There are four types of substates with varying number of unknown bytes. The primary aim of this phase is to reduce the search space for these substates. A **Type-1** substate is left unaltered since it has no unknown byte. **Type-2** and **Type-3** are equivalent in the sense that both have three completely known columns while **Type-4** can be converted to the same form by guessing 2 bytes of the first column. Thus for the rest of the analysis we assume that we are dealing with a substate with only one unknown column. We now describe how the attacker produces candidates for such a substate. Again Fig. 9 visually illustrates this process for easy reference. Let us consider the m^{th} substate.

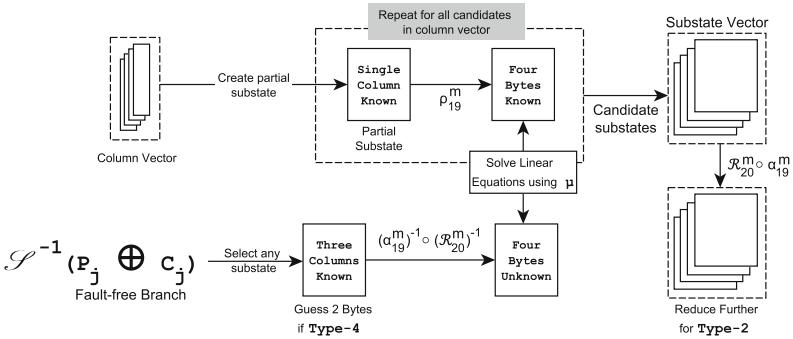


Fig. 9. The **OUTBOUND** phase. Returns candidates for substates after \mathcal{R}_{20} .

1. The substate is inverted up to **output** of μ_{19}^m . At this point the substate bears a property that every column has exactly one *unknown* byte.
2. Any column from the corresponding column vector¹⁰ is used to form a partial substate which is propagated forward up to the **input** of μ_{19}^m . At this point the substate has exactly one *known* byte in every column.
3. The attacker exploits linear relations between the partial input and partial output of μ_{19}^m to uniquely retrieve the substate. The recovered substate is propagated forward up to the end of \mathcal{R}_{20} .

¹⁰ Recall that the column vector corresponds to the state after β_{19} .

4. The process repeated for all columns in the column vector and all computed substates are stored in the corresponding substate vectors.

At the end of the **OUTBOUND** phase we have a set of substate vectors for all substates of the state after \mathcal{R}_{20} .

Remark 2. Unlike a **Type-3** substate, a **Type-2** substate has two extra known bytes in the fourth column which can be exploited. Thus, substate vector is reduced by comparing candidates with respect to these two bytes and eliminated if unmatched. *This should lead to large scale reduction of candidates.* As regards a **Type-4** substate 2 bytes need to be guessed to make it like a **Type-3**. Thus we have to repeat the process of candidate generation 2^{16} times.

7.4 The Complete Attack

At the outset **ENCOUNTER** repeats the **INBOUND** phase four times for different diagonals positions of **differentiator**. Every iteration returns four column vectors one for each substate of the state after β_{19} . The **OUTBOUND** phase then follows, iterating over different types of substates $\in \mathcal{S}^{-1}(P_j \oplus C_j)$ from the fault-free branch j returning corresponding substate candidates in substate vectors. The cross-product of the substate vectors forms the state vector which constitutes the reduced state-space for $\mathcal{S}^{-1}(P_j \oplus C_j)$. The last step is to invert every candidate state to get the input of branch j . Here, we can exploit the knowledge of the domain separator D_0 , the counter value j and the nonce N . This helps to eliminate all wrong candidates and reveals the master key \mathcal{K} . It must be noted that for a particular diagonal guess in **INBOUND** phase if the size of any substate vector returned by **OUTBOUND** is empty, all substate vectors generated for that guess are discarded. Algorithm 4 presents **ENCOUNTER** at an abstract level. In the next subsection, we perform a complexity analysis.

Remark 3. Retrieving diagonal for paeq-64: In case of **paeq-64** since second substate after \mathcal{R}_{20} (Refer Fig. 8) is completely known (**Type-1**), it can be inverted two rounds to reach input of \mathcal{R}_{19}^2 for both branches i, j . The byte inter-relations in the internal-difference of these inputs is verified against the ones given in Fig. 7. This reveals the fault diagonal of **differentiator**. Thus as a special case, for **paeq-64** the diagonal guess can be avoided.

7.5 Complexity Analysis

Here we are mainly interested in getting the size of set \mathcal{S} since from Algorithm 4 it is evident that AESQ^{-1} constitutes the most expensive operation of **ENCOUNTER** and the number of calls is bounded by $|\mathcal{S}|$. Equation 3 gives the upper bound on the size of \mathcal{S} assuming that $s \leftarrow \mathcal{S}^{-1}(P_j \oplus C_j)$.

$$|\mathcal{S}| \leq \sum_d |([s]^v)_d| = \sum_d \left(\prod_{x=1}^4 |[s^x]^v| \right)_d \tag{3}$$

Algorithm 4. ENCOUNTER(P, C, i)

Input: $\begin{cases} P, C \leftarrow \text{One known plaintext-ciphertext with 255 complete blocks} \\ i \leftarrow \text{Index of faulty-branch} \end{cases}$

Output: $\mathcal{K} \leftarrow \text{The Master Key}$

```

1:  $(i, j) \leftarrow \text{FINDQ}(P, C, i)$  ▷ Locate Fault-quartet  $\mathcal{Q}_{i,j}$ 
2:  $\mathcal{S} \leftarrow \emptyset$ 
3: for  $d \xleftarrow{\text{Guess}}$  Fault diagonal do ▷ Location of differentiator
4:   Four Column Vectors  $\xleftarrow{\text{INBOUND}} (P_i \oplus C_i, P_j \oplus C_j, d)$ 
5:   Four Substate Vectors  $\xleftarrow{\text{OUTBOUND}} (P_j \oplus C_j, \text{Column Vectors})$ 
6:   if (Any Substate Vector =  $\emptyset$ ) then Go to 3
7:   State Vector  $\xleftarrow{\text{Cross-Product}}$  Four Substate Vectors
8:    $\mathcal{S} \leftarrow (\mathcal{S} \cup \text{State Vector})$  ▷ Reduced state-space
9: for all  $e \in \mathcal{S}$  do  $(D_x || j_x || N_x || \mathcal{K}) \leftarrow \text{AESQ}^{-1}(\mathcal{S}(e))$ 
10:  if  $(D_x || j_x || N_x) == (D_0 || j || N)$  then return  $\mathcal{K}$ 

```

Table 2. Substate vector sizes in terms of Type-3 substate vector size

GetType(s^x)	Type-1	Type-2	Type-3	Type-4
$ s^x ^v$ (Refer Remark 2)	1	$p \ll q$	q	$r \leq 2^{16} \times q$

It implies that it suffices to study the sizes of substate vectors. It was seen in **OUTBOUND** phase that **Type-2** and **Type-4** substates are related to **Type-3**. Accordingly, the sizes of the corresponding substate vectors can be expressed in terms of a **Type-3** substate vector. This is furnished in Table 2 where q denotes the size of a **Type-3** substate vector while p and r denote sizes of **Type-2** and **Type-4** substate vectors respectively. Table 3 enumerates the theoretical upper bounds of the complexities individually identifying sizes of the substate vectors.

8 Experimental Results

Computer simulations of **ENCOUNTER** were performed over 1000 randomly chosen nonces, keys. The results for **paeq-64/80** are shown in the form of bar diagrams in Figs. 10 and 11 respectively. The bars segregate the substate vectors in terms of their sizes (the value at the base) with the frequency of occurrence given at the top. The figures mainly show that in the average case q is concentrated around 2^8 . It was mentioned in Remark 2 that in the presence of additional information p could be further reduced such that $p \ll q$. This is confirmed by the results which show that $p = 1$ with a few exceptions when $p = 2$. Table 3 summaries the results while the details are given below:

- **paeq-64**: By Remark 3 we know that the diagonal for **differentiator** can be recovered thereby avoiding the guessing step in Algorithm 4 and reducing the complexity by a factor of four. So the final experimentally verified size of \mathcal{S} for **paeq-64** stands at $72292 \approx 2^{16.14}$.

- **paeq-80**: During simulation it was found that for **Type-2** substates **OUTBOUND** phase returned empty substate vectors for a wrong guess of faulty diagonal. This made Step 6 of Algorithm 4 to be TRUE reducing $|\mathcal{S}|$ by four times. So the final verified size $72578 \approx 2^{16.14}$ is very close to **paeq-64**.
- **paeq-128**: It has three **Type-3** substates contributing around 2^{24} while a **Type-4** substate is supposed to contribute over $2^{16} \times 2^8$. Finally, the complexity is increased four times due to diagonal guess. Thus the *estimated* value of $|\mathcal{S}|$ is around 2^{50} .

Table 3. ENCOUNTER Complexities

PAEQ	Substate vector size $s = \mathcal{S}^{-1}(P_j \oplus C_j)$				Theoretical complexity($ \mathcal{S} $)	Experimental result($\approx \mathcal{S} $)
	$\ s^1\ ^v$	$\ s^2\ ^v$	$\ s^3\ ^v$	$\ s^4\ ^v$		
paeq-64	q (Type-3)	1 (Type-1)	q (Type-3)	p (Type-2)	$q^2 p (\lll q^3)$	$2^{16.14}$
paeq-80	q (Type-3)	p (Type-2)	q (Type-3)	p (Type-2)	$4p^2 q^2 (\lll 4q^4)$	$2^{16.14}$
paeq-128	q (Type-3)	q (Type-3)	q (Type-3)	r (Type-4)	$4q^3 r (\leq 2^{18} q^4)$	2^{50} (estd.)

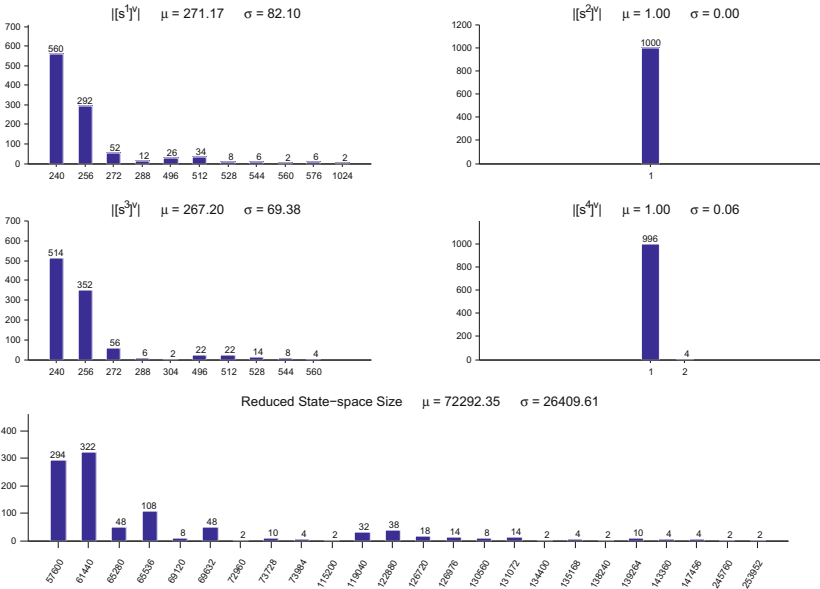


Fig. 10. Bar diagram for sizes of substate vectors and reduced state-space for 1000 experiments on **paeq-64** with mean (μ) and standard-deviation (σ) indicated.

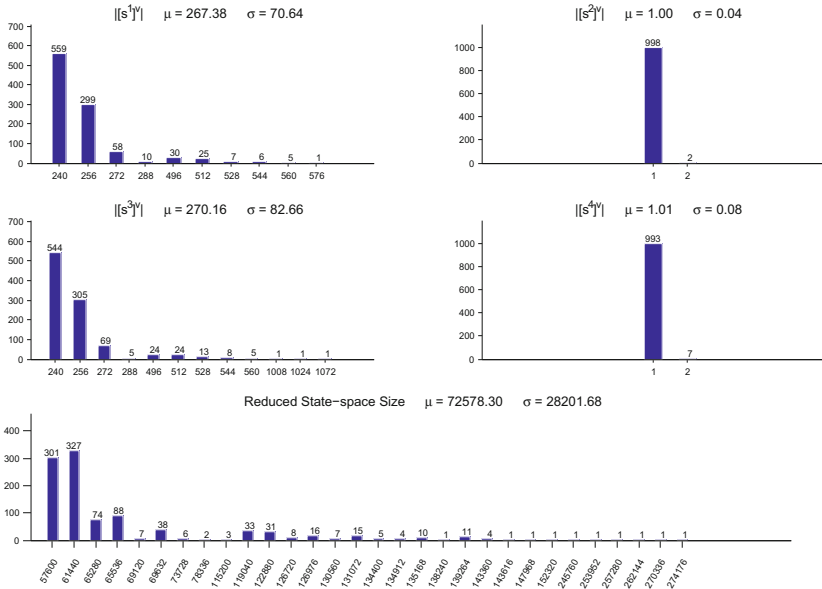


Fig. 11. Bar diagram for sizes of substate vectors and reduced state-space for 1000 experiments on `paeq-80` with mean (μ) and standard-deviation (σ) indicated.

9 Conclusion

This work introduces the notion of fault analysis using internal differentials. Parallelizable ciphers using the counter mode are found to be good targets for such kind of analysis though the real attack relies on the underlying construction. A 4-round distinguisher for authenticated cipher PAEQ is demonstrated. Using this the idea of fault quartets is proposed which can locate the fault-free branch corresponding to a faulty branch. Finally, an internal differential fault attack **ENCOUNTER** is devised against PAEQ using just two random byte faults with only a single faulty ciphertext and the corresponding plaintext. The attack reduces the key-space of `paeq-64`, `paeq-80` and `paeq-128` to around 2^{16} , 2^{16} and 2^{50} respectively. The ability to mount an attack using a single faulty run of the cipher makes IDFA independent of the effect of nonce thereby breaking the *nonce barrier* of DFA. Moreover, the fault analysis presented here is of particular interest since it deals with internal states that are partially specified which deviates it from classical DFA. Finally, this work constitutes the first analysis of CAESAR candidate PAEQ.

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