Erratum to: The Proper Treatment of Linguistic Ambiguity in Ordinary Algebra

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Chapter "The Proper Treatment of Linguistic Ambiguity in Ordinary Algebra" in: A. Foret et al. (Eds.): *Formal Grammar, LNCS 9804*, https://doi.org/10.1007/978-3-662-53042-9_18

This is a correction note to the paper starting on p. 306. There is an error in the proof of Lemma 1, which states that the axiom

$(\|3)$ At least one of $a \leq a\|b$ or $b \leq a\|b$ holds

together with the other axioms entails the stronger statement that either $a = a\|b$ or $b = a\|b$. The proof of this lemma is incorrect, and the claim is wrong: we can construct an algebra with 4 elements $\{0, 1, 0\|1, 1\|0\}$ with the obvious Boolean algebra order, and $\|$ defined by the *margin property*: $a\|b\|c = a\|c$, with $a, b \in \{0, 1\}$, c an arbitrary term. It is not difficult to check that this is an ambiguous algebra in the sense of Section 4 of the paper, yet $0 \neq 0\|1 \neq 1$. As almost all later results are based upon Lemma 1, they are technically unproved. However, all problems can be remedied very easily by changing $(\|3)$ from the paper to:

$(\|3')$ At least one of $a = a\|b$ or $b = a\|b$ holds

Hence to make the paper correct, all we need is a slightly different axiom $(\|3')$, and Lemma 1 becomes basically part of the definition, so all problems are solved. So far our correction; there are two notes which might be interesting to the reader:

The original online version of this chapter can be found at https://doi.org/10.1007/978-3-662-53042-9_18

A. Foret et al. (Eds.): FG 2015/2016, LNCS 9804, pp. E1–E2, 2016.
https://doi.org/10.1007/978-3-662-53042-9_19

Note 1 From the point of view of the linguistic motivation of the axioms, $(\|3')$ is actually more natural than the original $(\|3)$, because it basically states that an ambiguous meaning is supposed to *intend* one of the meanings between which it is ambiguous. The weaker $(\|3)$ just states that it is supposed to *entail* one of these meanings, which is not what we would intuitively think. Actually, the authors of the paper preferred $(\|3)$ over $(\|3')$ not on a conceptual base, but rather because it is simply weaker and they believed the two to be equivalent anyway (this is what Lemma 1 of the mentioned paper states).

Note 2 In (Wurm, 2017), the authors have introduced the class of **universal distribution algebras (UDA)**, which is still weaker. If we take the class of ambiguous algebras as introduced in the 2016 paper and add an axiom for $\|$-associativity ($a\|(b\|c) = (a\|b)\|c$, which does not seem derivable so far), then it is not difficult to show that **UDA** subsumes this class. What is interesting is that **UDA** seems to have the same equational theory as ambiguous algebras in the strong sense (with $(\|3')$). Now since the class as defined in 2016, with associativity added, lies in between the two, it is neatly characterized by this (yet unpublished) result.

Reference

Wurm, C.: The logic of ambiguity: the propositional case. In: Foret, A., Muskens, R., Pogodalla, S. (eds.) Formal Grammar. 22th Conference, FG 2017, Toulouse, France, July 2017, Proceedings. Lecture Notes in Computer Science, vol. 10686. Springer (2017)