Chaos Driven Particle Swarm Optimization with Basic Particle Performance Evaluation – An Initial Study

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Abstract. In this paper, the novel concept of particle performance evaluation is introduced into the chaos driven particle swarm optimization algorithm (PSO). The discrete chaotic dissipative standard map is used here as a chaotic pseudo-random number generator (CPRNG). In the novel proposed particle performance evaluation method the contribution of each particle to the process of obtaining the global best solution is investigated periodically. As a reaction to the possible poor performance of a particular particle, its velocity calculation is thereafter altered. Through utilization of this approach the convergence speed and overall performance of PSO algorithm driven by CPRNG based on Dissipative map is improved. The proposed method is tested on the CEC13 benchmark set with two different dimension settings.

1   Introduction

Since its introduction in 1995 [1], the PSO algorithm [1], [2] has become one of the leading representatives of evolutionary computational techniques (ECTs) and also of the Swarm intelligence [2]. Various modifications have been proposed over the time [3–7] and the inner dynamic of the PSO has been subject of many detailed studies e.g. [8, 9]. One of the more recent trends with very satisfactory results is represented through the implementation of chaotic sequences into the PSO dynamic [10-17]. Within the previous research [13-17], the strong evidences have been collected to support the claim that utilization of CPRNGs based on different chaotic systems may lead to significantly different behavior of the swarm and overall performance of the algorithm. The dissipative standard map [18] in particular seems to be very promising chaotic sequence for the usage as the CPRNG inside the PSO algorithm. The performance of such algorithm has been investigated deeply in previous studies e.g. [13, 15, 17].

In this research a simple method called the "Particle Performance Evaluation" is designed to improve the performance of PSO algorithm driven by dissipative chaotic map and to address some of the main issues that were discovered during previous research.

The paper is structured as follows: In the next section the original PSO algorithm is described. Section three contains brief summary of the results and main points of previous research of PSO witch Dissipative map based CPRNG. The "Particle Performance Evaluation" is described in section four. Following sections contain the experiments design and results. The results discussion and conclusion follows afterwards.

2   Particle Swarm Optimization Algorithm

The original (canonical) PSO algorithm was introduced in 1995 by Eberhart and Kennedy [1, 2]. Each particle in the swarm is defined by its "position" - the combination of cost function (CF) parameters, and "velocity". The new position of the particle in the following iteration is then obtained as a sum of actual position and newly calculated velocity. The velocity calculation follows two natural tendencies of the particle: To move to the best solution found so far by the particular particle (known in the literature as personal best: *pBest* or local best: *lBest*). And to move to the overall best solution found in the swarm or defined sub-swarm (known as global best: *gBest*).

According to the method of selection of the swarm or subswarm for *gBest* information spreading, the PSO algorithms are noted as global PSO (GPSO) [7] or local PSO (LPSO) [8]. Within this research the PSO algorithm with global topology (GPSO) [1,2, 7] was utilized.

In the original GPSO the new velocity is calculated according to (1):

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c_1 \cdot Rand \cdot (pBest_{ij} - x_{ij}^{t}) + c_2 \cdot Rand \cdot (gBest_j - x_{ij}^{t}) \qquad (1)$$

Where:

v_i^{t+1} - New velocity of the *i*th particle in iteration *t*+1.

w – Inertia weight value.

v_i^{t} - Current velocity of the *i*th particle in iteration *t*.

c_1, c_2 - Priority factors (set to the typical value = 2).

$pBest_i$ – Local (personal) best solution found by the *i*th particle.

$gBest$ - Best solution found in a population.

x_{ij}^{t} - Current position of the *i*th particle (component *j* of the dimension *D*) in iteration *t*.

$Rand$ – Pseudo random number, interval (0, 1). The CPRNG is used here.

The maximum velocity of particles in the GPSO is typically limited to 0.2 of the range of the optimization problem and this pattern was followed in this study. The new position of a particle is then given by (2), where x_i^{t+1} is the new position of the particle:

$$x_i^{t+1} = x_i^{t} + v_i^{t+1} \qquad (2)$$

Finally the linear decreasing inertia weight [5, 7] is used in the GPSO here. Its purpose is to slow the particles over time thus to improve the local search capability in the later phase of the optimization. The inertia weight has two control parameters w_{start} and w_{end}. A new *w* for each iteration is given by (3), where *t* stands for current iteration number and *n* stands for the total number of iterations. The values used for the GPSO in this study were $w_{start} = 0.9$ and $w_{end} = 0.4$.

$$w = w_{start} - \frac{((w_{start} - w_{end}) \cdot t)}{n} \qquad (3)$$

3   PSO with CPRNG Based on Dissipative Standard Map

Within the previous research [13-15, 17] strong evidences have been gathered hinting that through using of CPRNG based on Dissipative standard map inside the velocity calculation formula (1) of PSO algorithm the performance may be improved (in some cases). The detailed investigations [13 - 15] showed that the PSO driven by Dissipative map based CPRNG seems to achieve better results than canonical PSO in the cases of complex highly multimodal or high dimensional problems [13, 15, 17].

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. This one clause is of deep importance to evolutionary algorithms. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map as the pseudo random number generator.

It seems that the performance of PSO is improved by altering the swarm behavior in such manner that the convergence speed is significantly slower, which may result in the lower chance of falling into local extremes. However the better achieved results are usually compensated by the significantly higher computational time demands given the higher number of iterations required for the algorithm to converge (for further details please see [13 - 15]).

The aforementioned behavior lead to conclusion: when strict time restrictions are applied, the PSO driven by chaotic Dissipative map based CPRNG is not able to achieve good results. As a reaction to these issues, the novel "Particle performance evaluation" approach is investigated and implemented into the GPSO driven by Dissipative standard map in this presented research. The goal is to enhance the overall performance of the algorithm through the improvement of the convergence speed together with maintaining the advantages of the original PSO driven by Dissipative map based CPRNG.

The Dissipative standard map is a two-dimensional chaotic map [18]. The parameters used in this work are $b = 0.6$ and $k = 8.8$ based on previous experiments [17, 15] and suggestions in literature [18].The map equations are given in (4).

$$X_{n+1} = X_n + Y_{n+1} (\mathrm{mod}\, 2\pi)$$
$$Y_{n+1} = bY_n + k \sin X_n (\mathrm{mod}\, 2\pi)$$

$$(4)$$

4   PSO with Particle Performance Evaluation (PSO with PPE)

The novel approach proposed in this study is based on the simple premise that all particles should take part in the process of finding and improving of the final solution. The only way of communication among the particles in the GPSO design [1] represents the shared knowledge of the position of the best globally found solution (*gBest*). In other words: To be beneficial for the swarm, the particle has to update the *gBest*. Therefore the first step in the particle performance evaluation (PPE) is the exact monitoring of the *gBest* updaters. A counter is allocated to each particle. On the start of each iteration the counter is incremented by 1 and is set to 0 when the particle

triggers a *gBest* update. In this way, it is possible to measure the number of iterations since the last *gBest* was found by particular particle. The second step in the PPE approach is to alter the performance of the particular particle when it has not triggered the *gBest* update for a given maximum number of iterations. In this initial research the simple constant c_1 (1) is modified to vector (see (5)).

$$v_{ij}^{t+1} = w \cdot v_{ij}^t + c_{1i} \cdot Rand \cdot (pBest_{ij} - x_{ij}^t) + c_2 \cdot Rand \cdot (gBest_j - x_{ij}^t) \qquad (5)$$

Where:

c_{1i} – Priority factor 1 for the *i*th particle.

Subsequently when the particular particle does not trigger a *gBest* update for 1/10 of the total number of iterations, the c_1 value for that particle is set to 1. The value of c_1 is set back to 2 when the particle reaches the *gBest* update.

Through the following of this very simple pattern it is possible to reduce the number of particles with no *gBest* updates (triggers), further to reduce the number of iteration between *gBest* updates for the each particle and to improve the overall performance of the PSO algorithm driven by Dissipative map based CPRNG in some cases as it is presented in the following sections.

5   Experiment Setup

Within all performance testing three different GPSO versions were used. The first one was the original canonical GPSO with linear decreasing inertia weight (as described in the section 2), noted GPSO. The second version was the GPSO with Dissipative map based CPRNG in the velocity calculation (Noted GPSO Disi) and finally the third version represents the GPSO with Dissipative map based CPRNG and PPE as described in the section 4 (noted PSO Disi PPE).

For the performance tests the CEC 13 benchmark suite [19] was used. For each version, totally 20 separate runs were performed and statistically analyzed.

Control parameters were set up based on the previous numerous experiments and literature sources [1, 14, 15, 19] as follows:

Population size: 30
Dimension: 10, 50
Iterations: 2500, 12500 (according to [19])
Runs: 20
$v_{max} = 0.2 \cdot Range$
$w_{start} = 0.9$
$w_{end} = 0.4$

6   Results

The mean results for all versions of PSO described in the previous section are given and compared in Tables 1 and 2. The bold numbers represents the best results. The mean results are presented alongside the total number of best results obtained.

Furthermore the performance of pairs of algorithms is compared, where 1 stands for "win" of the "algorithm 1" (the first from the pair - left); number 2 stand for "win" of algorithm 2 (the second from the pair - right) and 0 stands for draw. The final score is also given in Tables 1 and 2 as a sum of points for wins (1 point) and draws (0.5 point).

Table 1. Mean results comparison for dim = 10

$f(x)$	GPSO	GPSO Disi	GPSO Disi PPE	A1 vs. A2	A1 vs. A3	A2 vs. A3
f(1)	**-1400**	**-1400**	**-1400**	0	0	0
f(2)	**116137.6**	416672.8	238461.5	1	1	2
f(3)	2.73E+06	1.90E+06	**289907.1**	2	2	2
f(4)	-806.79	-471.1	**-957.38**	1	2	2
f(5)	**-1000**	**-1000**	**-1000**	0	0	0
f(6)	**-894.08**	-886	-890.78	1	1	2
f(7)	-795.82	**-796.74**	-793.61	2	1	1
f(8)	-679.67	**-679.69**	-679.67	2	0	1
f(9)	**-597.39**	-596.65	-596.82	1	1	2
f(10)	-499.51	-499.32	**-499.6**	1	2	2
f(11)	**-398.16**	-397.14	-397.61	1	1	2
f(12)	-287.02	-278.4	**-288.41**	1	2	2
f(13)	**-181.14**	-177.32	-180.41	1	1	2
f(14)	**73.36**	169.38	128.84	1	1	2
f(15)	835.16	949.55	**674.34**	1	2	2
f(16)	200.89	200.95	**200.79**	1	2	2
f(17)	314.23	329.04	**313.66**	1	2	2
f(18)	431.55	439.58	**422.38**	1	2	2
f(19)	500.68	501.48	**500.67**	1	2	2
f(20)	**602.48**	602.87	602.6	1	1	2
f(21)	1100.19	**1090.18**	1100.19	2	0	1
f(22)	**1031.51**	1045.12	1121.89	1	1	1
f(23)	**1623.05**	1726.28	1711.32	1	1	2
f(24)	1204.71	1209.16	**1204.64**	1	2	2
f(25)	1307.02	**1305.78**	1306.93	2	2	1
f(26)	1350.92	**1345.93**	1347.28	2	2	1
f(27)	**1640.26**	1656.97	1691.26	1	1	1
f(28)	1754.15	**1737.21**	1745.87	2	2	1
Best:	12	8	12 Score:	20 : 8	13 : 15	9.0 : 19.0

Table 2. Mean results comparison for dim = 50

f(x)	GPSO	GPSO Disi	GPSO Disi PPE	A1 vs. A2	A1 vs. A3	A2 vs. A3
f(1)	**-1400**	-1348.687	**-1400**	1	0	2
f(2)	**9.65E+06**	6.29E+07	2.18E+07	1	1	2
f(3)	3.01E+08	6.91E+09	**2.45E+08**	1	2	2
f(4)	1964.64	8114.369	**-742.66**	1	2	2
f(5)	**-1000**	-975.08	**-1000**	1	0	2
f(6)	-846.59	-832.96	**-853.21**	1	2	2
f(7)	**-749.31**	-732.56	-743.87	1	1	2
f(8)	-678.88	-678.879	**-678.9**	0	2	2
f(9)	**-558.06**	-550.86	-554.49	1	1	2
f(10)	-498.69	-377.11	**-499.88**	1	2	2
f(11)	-345.973	-202.754	**-346.22**	1	2	2
f(12)	-38.699	120.79	**-137.82**	1	2	2
f(13)	136.46	217.1	**88.27**	1	2	2
f(14)	**1582.63**	5858.16	1657.56	1	1	2
f(15)	12130.01	12897.36	**7891.71**	1	2	2
f(16)	203.04	**202.68**	202.89	2	2	1
f(17)	**432.469**	764.932	448.87	1	1	2
f(18)	868.08	947.15	**611.54**	1	2	2
f(19)	**507.03**	541.05	508.97	1	1	2
f(20)	621.37	621.67	**620.03**	1	2	2
f(21)	**1615.7**	1726.1	1661.8	1	1	2
f(22)	**2915.96**	7466.41	3205.45	1	1	2
f(23)	12395.47	14554.47	**10168.74**	1	2	2
f(24)	**1290.55**	1314.49	1306.99	1	1	2
f(25)	**1484.49**	1521.45	1510.91	1	1	2
f(26)	**1563.87**	1582.17	1586.17	1	1	1
f(27)	**2687.23**	2843.66	2863.93	1	1	1
f(28)	**1800**	1861.448	1960.31	1	1	1
Best:	13	1	12 Score:	26.5 : 1.5	14.0 : 14.0	4.0 : 24.0

Furthermore Fig. 1 depicts the illustrative comparison of the number of iterations since the last *gBest* update for each particle for GPSO Disi and GOPSO Disi PPE. It can be clearly observed the positive influence of PPE approach.

Finally two examples of mean *gBest* history plots are given in Fig. 2 and 3 to highlight the change in behavior of the chaos driven GPSO especially in terms of convergence speed. The results are further discussed in following section.
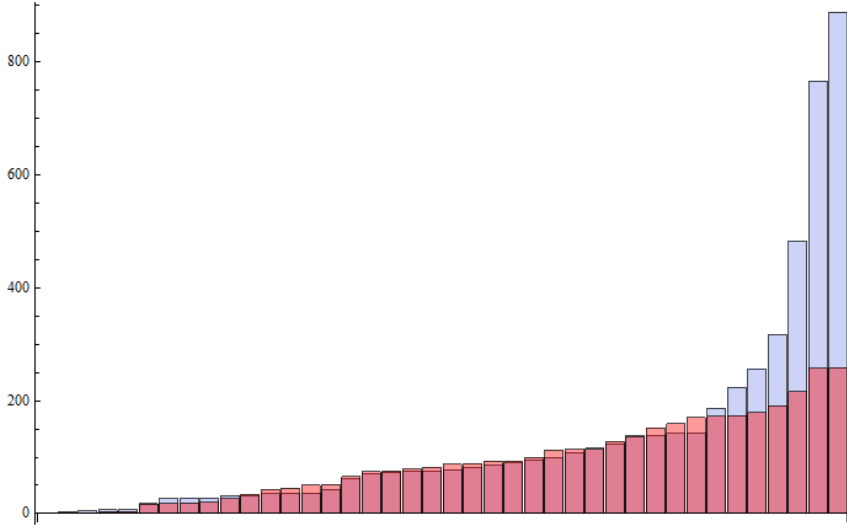

Fig. 1. Number of iterations since last *gBest* update for each particle (sorted). *f(1), dim =* 10; comparison of GPSO Disi– blue, GPSO Disi PPE – red
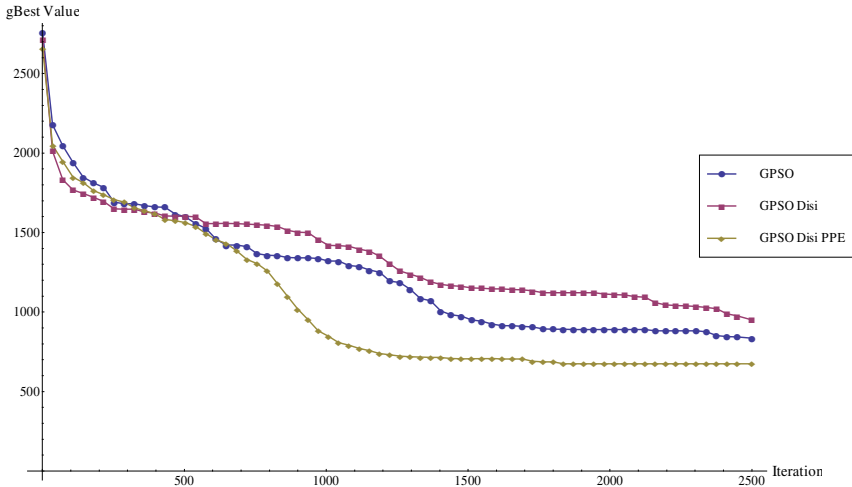

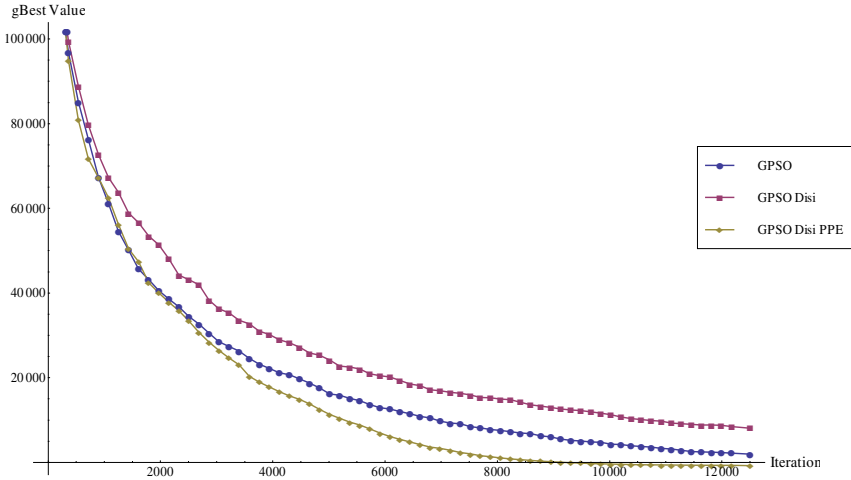**Fig. 2.** Mean *gBest* history comparison *f(15), dim =* 10

Fig. 3. Mean *gBest* history comparison *f(4), dim* = 50

7   Results Discussion

The data presented in previous section seem to indicate that when the PPE is applied, the performance of GPSO driven by Dissipative standard map was in some cases significantly improved. In the first experiment (Table 1) the original chaotic GPSO (GPSO Disi) managed to obtain better results than the canonical version (GPSO) only for 8 functions. However when PPE was applied the number of better results (for the restricted number of iterations) has increased to 15. In direct comparisons the chaos driven GPSO with PPE (GPSO Disi PPE) managed to find better results for 19 different benchmark functions than the simple chaos driven GPSO (GPSO Disi). As an illustrative example, the effect of PPE (in case of *f(15)*) can be further observed from Fig. 2, where the *gBest* history is depicted and the improvement of convergence speed is clearly visible. It also seems that the GPSO Disi would improve its performance further if the number of iterations was increased.

The second experiment (with higher dimension setting) has brought up surprising results (Table 2) that were unprecedented in any previous research [13-15]. It seems that the performance of GPSO Disi is not satisfactory within the given dimension setting, function complexity and number of iterations. However when PPE is applied once again the performance was significantly improved. The example of mean *gBest* history for *dim*=50 and *f(4)* is given in Fig. 3. It seems that on this type of problem [19] the chaos driven PSO with PPE manages to obtain significantly better results than both the canonical (GPSO) and original chaos driven version (GPSO Disi).

8   Conclusion

In this initial research the behavior of GPSO algorithm driven by Dissipative standard map based CPRNG was altered by applying the "Particle performance evaluation".

The performance of newly designed algorithm was tested on the CEC 13 benchmark suite and compared to the performance of original GPSO with both canonical and chaotic PRNG.

It is necessary to mention, that the main aim of this paper was not to test the chaos embedding approach for evolutionary/swarm based algorithms, as the utilization of different CPRNGS and the very positive influence of inner chaotic dynamics to the performance of embedded algorithms was proved in many previous studies. Since the PPE approach was already successfully tested with canonical versions of PSO and with default PRNGs (no chaos embedded), the contribution of this paper is to show/prove the influence of PPE approach also in the different case of the chaos driven PSO algorithm, which takes advantages from the different swarm environment given by the chaotic dynamics.

The initial results presented in this paper seem to indicate that this approach may lead to significant performance improvements for certain types of optimization problems. Also it seems that the number of redundant cost function evaluations may be significantly reduced by using the PPE approach. There is however still a need for significant number of experiment with higher variety of problem types and control parameters settings to prove these claims. The future research will focus mainly on further improvements of this method by enhancing the PPE with variety of different behavior changes (rules) for the particles.

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