THE PARTIAL SUM CRITERION FOR STEINER TREES IN GRAPHS AND SHORTEST PATHS

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Many optimization problems are defined in a weighted graph. Generally, the criterion is min sum, i.e., the total weight of all edges in the solution; here we consider a different criterion.

The partial sum criterion with parameter p sums the largest p edge weights in the solution, giving the criterion value of the considered solution.

For $p = 1$ the criterion is known as bottleneck or minmax criterion. For the Steiner Tree Problem in Graphs - see for this problem Duin and Voss (1994) - with minmax criterion we describe in Duin and Volgenant (1993) an $O(|E|)$ algorithm with E the set of edges in the problem. The algorithm can be extended easily to the min-max Steiner Forest Problem with a restriction on the number of components - see Duin and Volgenant (1987).

For special choices of K (the set of special nodes in this problem) the algorithm solves the bottleneck shortest path problem corresponding with $|K| = 2$, Punnen (1991), and the bottleneck spanning tree problem corresponding with $|K| = n$, Camerini (1978); thereby it unifies these two existing algorithms.

We show for larger values of p that the partial sum Steiner Trees in Graphs problem is NP-hard; this result holds both for the *connected* case, where the p chosen edges must form a subtree as well as for the case, where the set of these edges need not be connected.

For the shortest path problem we consider the criterion for arbitrary values of p, defining it for solutions with less than p edges as the total sum. The connected version of the problem appears to be NP-hard, while for the free case a polynomial algorithm is derived, that is related to the algorithm for the p-sum Linear Assignment Problem as given by Grygiel (1981).

Applications are present in communication networks. Indirect applications arise within other algorithms, e.g., the minmax path algorithm can be used as a subroutine within a minmax Linear Assignment Problem. The partial sum criterion can be useful for instance in problems in location theory or as a criterion in fitting a line to given data points, minimizing the 'distance' of p outlyers to the line through the data points.

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