Particle Swarm Optimization Approach for Fuzzy Cognitive Maps Applied to Autism Classification

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Abstract. The task of classification using intelligent methods and learning algorithms is a difficult task leading the research community on finding new classifications techniques to solve it. In this work, a new approach based on particle swarm optimization (PSO) clustering is proposed to perform the fuzzy cognitive map learning for classification performance. Fuzzy cognitive map (FCM) is a simple, but also powerful computational intelligent technique which is used for the adoption of the human knowledge and/or historical data, into a simple mathematical model for system modeling and analysis. The aim of this study is to investigate a new classification algorithm for the autism disorder problem by integrating the Particle Swarm Optimization method (PSO) in FCM learning, thus producing a higher performance classification tool regarding the accuracy of the classification, and overcoming the limitations of FCMs in the pattern analysis area.

1 Introduction

Classification is a data processing technique in which each data set is assigned to a predetermined set of categories. Generally classification goal is the creation of a model which will be used later for the prediction-classification of future unknown data. Classification problems have been aroused the interest of researchers of different domains in the last decade like biology, medical, robotic and so on. Such classification paradigms can be the prediction of cancer cell by characterizing them as benign or malignant, the categorization of bank customers according to their reliability, the determination whether a child suffers from the autism disorder problem and so on [1],[2],[3]. Various learning approaches have been proposed for the classification of input instances and for the comprehension of complex systems function, like Artificial Neural Networks, Clustering methods and Genetic Algorithms.

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Fuzzy Cognitive Map [4] is a soft computing technique which is used for modeling and analysis of complex systems. FCM may be considered as a simple mathematical model in which the relations between the elements can be used to compute the "strength of impact" of these elements. FCM can also be considered as an integration of multifold subjects, including neural network, fuzzy logic, semantic network, learning algorithms. It is a dynamic tool involving feedback mechanisms [4], and this dynamicity leads the research community to work on it. Due to its advantageous features, such as simplicity, adaptability to system characteristics, support of inconsistent knowledge, analysis of complex systems, learning from historical data and previous knowledge, FCM has found large applicability in many different scientific fields for modeling, control, management and decision making [5].

The FCM learning, as a main capability of FCM, is a crucial issue in modeling and system analysis. It concerns the adaptation of the connection matrix (known as weight matrix) using diverse adaptive and evolutionary type learning methods, such as unsupervised learning based on the Hebbian method [6,7], supervised ones with the use of evolutionary computation [8-11] and/ or gradient-based methods [12,13].

Up to date to the literature, there is no any previous study on proposing a particle swarm optimization approach for FCM to perform classification. Previous studies related with the FCM application in classification tasks are described. The first work was presented by Papakostas et al. (2008) who implemented FCMs for pattern recognition tasks [14]. In their study, a new hybrid classifier was proposed as an alternative classification structure, which exploited both neural networks and FCMs to ensure improved classification capabilities. A simple GA was used to find a common weight set which, for different initial state of the input concepts, the hybrid classifier equilibrate to different points [14]. Next, Arthi et al. analyzed the performance of FCM using Non-linear hebbian algorithm for the prediction and the classification of autism disorder problem. The classification approach was based on human knowledge and experience, as well as on historical data (patterns). The proposed algorithm presented high classification accuracy of 80% [3]. In order to enhance the learning capabilities of this hebbian-based type of FCM learning, a new learning approach based on the ensemble learning, such as bagging and boosting, was integrated. FCM ensemble learning is an approach where the model is trained using non linear Hebbian learning (NHL) algorithm and further its performance is enhanced using ensemble techniques. This new approach of FCM ensembles, showed results with higher classification accuracy instead of the NHL alone learning technique [15]. Recently, Papakostas et al. (2012) presented some Hebbian-based approaches for pattern recognition, showing the advantages and the limitations of each one [16]. Another study of Zhang et al. [17] proposes a novel FCM, which is automatically generated from data, using Hebbian learning techniques and Least Square methods.

This research work is focused on the application of a new classification technique concerning the autism disorder. The FCM model constructed by physicians to assess three levels of autism (no autism, probable autism and autism) was trained using a new particle swarm optimization (PSO) clustering algorithm for forty real children cases. In other words, the main objective of this study is to present the PSO algorithmm for FCM learning applied to a classification case study.

2 Fuzzy Cognitive Maps

An FCM is a soft computing technique which combines the main aspects of fuzzy logic and neural networks (NN) and avoids the mathematical complexity of system analysis. FCM was originated by Kosko [4] as an extension of cognitive maps in order to create an abstract modeling methodology to model and represent the behavior of a system and the human thinking. Concepts stand for states, variables, inputs, outputs and any other characteristics of the system. Each weight expresses the causal relationship between two interconnected concepts.

Generally there are two main approaches for the creation of a FCM, the expertbased in which the FCM is a manual created and the computational method in which the FCM is made by the processing of historical data. Several scientists have dealt with the computational creation of FCMs in the light of learning algorithms [18].

FCMs have an inference mechanism similar to those of Neural Networks (NN). Combining the Fuzzy Logic and NN, the inference process is accomplished using simple mathematical operations between weight matrices, minimizing in doing so the complexity of a system. The inference process implementation can be described by the following five steps.

Step 1. Read the input vector A.

Step 2. Read the weight matrix W.

Step 3. Calculate the value of each concept by the following equation.

$$A_{i}(t) = \left(A_{i}(t-1) + \sum_{j=1, j \neq i}^{n} W_{ji} * A_{j}(t-1)\right)$$
(1)

Step 4. Apply a threshold function, usually sigmoid, to the values which were calculated in Step 3.

Step 5. Until the Concept values reach an equilibrium state (steady state) we continue the process from Step 3.

Concepts and weight matrix values lie between the intervals [0~1] and [-1,+1], respectively. The main difference from NN is the initial determination of the weight matrix and its meaning after estimation. Despite the fact that the main characteristic of both techniques is the weight matrix adaptation, on the NN technique the weight matrix is initialized with random values for all possible connections among nodes and reach to the "global optima", whereas on FCM each weight value has a real meaning for the problem, representing a causal interconnection, so uncertain modification of initial values of weights may converge the system to a "local optima".

3 Learning Algorithms for FCMs

The learning approaches for FCMs are concentrated on learning the connection matrix, based either on expert intervention and/or on the available historical data (like the neural network learning process). In other words we target on finding weights that

better represent the relationships between the concepts. Learning approaches for FCMs can be divided into three categories [18]:

- The hebbian-based algorithms such as NHL, ddNHL, which produce weight matrices based on experts' knowledge that lead the FCM to converge into an acceptable region for the specific target problem.
- 2. The population-based algorithms such as evolutionary, immune, swarm-based, which compute weight matrices based on historical data that best fit the sequence of input state vectors or patterns.
- 3. The hybrid algorithms which are focused on computing weight matrices based on experts knowledge and historical data.

Although FCMs have not been widely used on classification tasks, the last decade some researchers have proved that the classification procedure is feasible with FCMs [18]. So far, the usage of FCMs on classification problems has been implemented mainly by hebbian learning approaches [9] and by exploiting both neural networks and FCMs to ensure improved classification capabilities [14]. First, Papageorgiou et al. presented a brain tumour characterization algorithm based on Active Hebbian Learning for FCMs [19]. Next, Papakostas et al. presented a pattern classification algorithm based on FCMs. To map the outputs of the classifier to a problem's classes, three different class mapping techniques were implemented. The first mapping refers to the Class per Output technique where a specific class is assigned to a single output. The second class mapping technique, the Threshold one, works by the extraction of specific output threshold for the output concept values. The last technique consists of the clustering of the values of the output concepts, and for each class the mapping is computed by the calculation of minimum distance of each cluster. Recently, Papakostas et al. used for the classification of the data an idea that stems from NN tactics, which modifies the structure of FCMs by adding Hidden Concept nodes [17]. An extension of FCMs which is also inspired by NN classification theory is also presented at [15] where ensemble learning approaches like bagging or boosting are implemented. One more novel FCM extension for classification of testing instanced has been presented in [17]. The inference process of the LS-FCM model is similar to other FCM approaches but it uses a Least Square methodology to overcome the most weakness of the existing FCM algorithm, namely the heavy calculation burden, convergence and iterative stopping criteria. Song and his coworkers [20] extended the application of the traditional FCMs into classification problems, while keeping the ability for prediction and approximation by translating the reasoning mechanism of traditional FCMs to a set of fuzzy IF-THEN rules. They focused to the contribution of the inputs to the activation of the fuzzy rules and quantified the causalities using mutual subsethood, which works in conjunction with volume defuzzification in a gradient descent-learning framework. In next section, we suggest a population-based algorithm using the Particle Swarm Optimization method in order to achieve higher classification accuracy for the autism disorder problem.

4 Particle Swarm Optimization Algorithm for FCM Classification

Particle Swarm Optimization (PSO) is a computation method based on the social behavior of birds being in a flock. PSO algorithm [21] optimizes a problem by having a population of candidate solutions. The solutions called particles and their existence is at the problem hyperspace. The motion of each particle into the problem hyperspace over time according to a simple mathematical equation defines the Particle position and velocity. Each particle's position is influenced by its local best known position and is also guided toward the best known positions in the search-space, which are updated as better positions found by other particles.

To implement the PSO algorithm for FCM classification two steps are necessary. In the first step, a number of prototypes are positioned, in an unsupervised way, on regions of the input space with some density of the input data. For this, the Particle Swarm Clustering (PSC) [21,22] algorithm is used. In the second step the algorithm must decide about the decision boundaries that partition the underlying output vector from step one into three sets, one for each class. For this purpose, one-dimensional decision boundaries were determined by two methods. The first one is the bayesian statistical decision method [22] and the second one is the minimum Euclidean distance method [23]. The classifier accuracy is estimated by the leave-one-out cross-validation (LOOCV) method [10].

To implement the Particle Swarm Clustering algorithm for FCM we assume that we have a swarm consisting of k particles $\{P_1, P_2, P_3... P_k\}$. For our approach every particle position is a candidate FCM, meaning a weight matrix. This matrix can be initialized either by random values on the non-zero weights, thus keeping the main problem's signs constrains or by experts' suggestions. There is in general a plethora of weight matrices that lead the concepts to different values according to any input data. Let's consider a data set with T real cases, where each case is represented by a vector. For each estimated vector (which is calculated implementing the eq. (1) for a given weight matrix and an input vector), there is a particle of greater similarity to the input vector, obtained by the Euclidean distance between the particle and the input data. This is the winner particle, and its velocity is updated by eq (2).

$$v_i(t+1) = w * v_i(t) + \phi_1 * (p_i^j(t) - x_i(t)) + \phi_2 * (g^j(t) - x_i(t))$$
(2)

In eq (2), the parameter w, called inertia moment, is responsible for controlling the convergence of the algorithm and it is decreased at each step. The cognitive term $p_i^{\ j}(t) - x_i(t)$, associated with the experience of the particle winner, represents the best particle's winner position, in relation to the j_{th} input data so far. The social term $g^{\ j}(t) - x_i(t)$ is associated with the particle closest to the input data, that is, the particle that had the smallest distance in relation to the j_{th} input object so far. The parameters w, $\varphi 1$, $\varphi 2$, and $\varphi 3$ (used in eq. (4)) are selected by the practitioner and control the behavior and efficacy of the PSO method. They take values within the

range [0,1], to avoid the chaotic behavior of position and velocity vectors, in FCM equilibrium state. The winner particles position is updated by eq (3)

$$x_i(t+1) = x_i(t) + v_i(t+1)$$
 (3)

The procedure of the Particle Swarm Clustering Algorithm for FCM is shown in Pseudocode 1. Step 9 of Pseudocode 1 updates all those particles that did not move at iteration t. Thus, after all data sets were presented to the swarm, the algorithm verifies whether some particle did not win in that iteration. These particles are updated using eq (1) in relation to the particle that was elected the winner more often at iteration t. In the last step, the algorithm assigns a label to each estimated data. This task is feasible because we know a priori the correct labels for each data. This knowledge stems from the experts.

$$v_i(t+1) = w * v_i(t) + \phi_3 * (x_{winner} - x_i(t))$$
 (4)

There are two possible termination conditions for the algorithm, which are (a) a maximum number of iterations which is determined empirically and (b) the minimization of a cost function concerning the global optimization methods. In this study the following cost function is found to be appropriate:

$$\frac{1}{N} \sum_{i}^{N} \left(A_{i} \left(Out \right) - Y_{i} \left(Out \right) \right)^{2} \tag{5}$$

Where j is the winner particle, $A_i(Out)$ is the candidate FCM response of the output concept for the i_{th} data set and $Y_i(Out)$ is the given response for the i_{th} data set. N is the number of concepts.

Pseudocode 1. Particle Swarm Clustering Algorithm for FCMs

Step 1. At t=0 Initialize the swarm $P(0) = \{P_1, P_2, ..., P_k\}$ with random weight matrices for the Position Vector X and the Velocity Vector V keeping only the non-zero weights and/or the weights signs based on the experts knowledge.

D: Data set with T real cases

Y: T-1 training cases of D

C-labels: The correct labels were determined by experts.

While stopping criterion is not met

For each input data row j

For each Particle i

Step 2. Compute the new concept values A^{j}_{new} by eq(1)

Step 3. Compute the distance between A_{new}^{j} and Y_{new}^{j}

End for

Step 4. Find the Particle with the minimum distance and declare it as the Winner Particle P^{j}_{\min}

Step 5. Compare the distance of Winners Particle position to its best position thus far.

d1: distance between P_{\min}^{j} and Y_{\min}^{j}

d2: distance between the winner's particle best position $pbest_P^j_{\,min}$ and Y^j

if d1<d2 then ${\tt pbest_P^j_{min}} \ {\tt P^j_{min}}$

Step 6. Compare the distance of Winner's Particle position to its global best position thus far.

 $\mbox{\bf d3:}$ distance between the winner's particle global best position gbest_P $^j_{\text{min}}$ and Y j

if d1<d3 then

gbest_P^j min= P^j min

Step 7. Change the Velocity of winner's particle using eq 2.

Step 8. Change the Position of winner's particle using eq 3.

End for

Step 9. Change the Velocity and the Position for the particles who did not win by eq4 and eq 3.

Step 10. Test the stopping criterion

End while

Step 11. Assign a label to each data set according to C-labels **Return:** The Predicted labels from Step 11 and the new data set which is estimated on step 6.

5 Experimental Analysis and Results

The autism disorder problem was selected as a very complex process and due to its previous use in classification tasks [10,15]. Forty real children cases from an Indian hospital were studied and diagnosed by the experts (doctors). Those forty datasets were collected for classification of three different categories, like twenty three as "Definite Autism" (DA), thirteen as "Probably Autism" (PA) and four as "No-Autism" (NA) children and gathered in [3]. There is previous experience from experts as well as historical data, and the classification objective is to classify these cases into three classes: DA, PA and NA in order to achieve higher classification accuracy. Experts decided about the concepts and their initial interconnections among them and defined that there are twenty three main symptoms for the autism disorder problem, such as climbing on things, bringing objects to parents, etc [3]. The decision concept concerns the autism class.

Table 1. Scenario (I): Classification accuracies of Particle Swarm Classification for FCM. Only the non-zeros weights are initialized by random values.

Boundaries	MED	BSM	MED	BSM	MED	BSM	MED	BSM
Decision								
Particles	K= 20	K= 20	K= 20	K= 20	K= 50	K= 50	K= 50	K= 50
Iterations	R =100	R = 100	R= 500	R =500	R =100	R =100	R= 500	R =500
True Positive	32,12%	33,27%	32%	34.18%	32,12%	34,1%	32,41%	33,18%
(All %)								
Model Accuracy	82,35%	85,3%	82.05%	87,64%	82,35%	87.43%	83.1%	87.64%
Correct Classes	89	89	426	432	89	91	417	432
(All)								
FCM Classifica-	89%	89%	85,2%	84,6%	89%	91%	83.4%	86,4%
tion Accuracy								

Table 2. Scenario (II): Classification accuracies of Particle Swarm Classification for FCM. The
non-zeros weights are initialized by random values to ± 0.2 of initial values, keeping the
problem constrains for weights.

Decision	MED	BSM	MED	BSM	MED	BSM	MED	BSM
Boundaries								
Particles	K= 20	K= 20	K= 20	K= 20	K= 50	K= 50	K= 50	K= 50
Iterations	R =100	R =100	R= 500	R= 500	R =100	R =100	R= 500	R =500
True Positive	32,56%	33,95%	32.37%	33,78%	32,4%	34,2%	32,42%	34,28%
(All %)								
Model Accuracy	83,14%	87,05%	83%	86,43%	83,07%	87.79%	83.14%	87.91%
Correct Classes	81	85	389	401	81	78	399	397
(All)								
FCM Classifica-	81%	85%	77,8%	80.2%	81%	78 %	79,8%	79,4%
tion Accuracy								

The proposed PSO clustering algorithm for FCM was implemented at the 40 records to predict the classification category of each one. Figure 1 illustrates the proposed approach in the case of autism classification problem. Two different scenarios were examined: (I) the first concerns that the initial non-zero weights are initialized by random values and (II) the last concerns that the initial non-zero weights are initialized by random values within a ± 0.2 range of their initial values (belong in the interval [Weight-0.2, Weight+0.2]), thus keeping the signs and weight constraints. The classification performance results were gathered in Tables 1 and 2, respectively for each scenario.

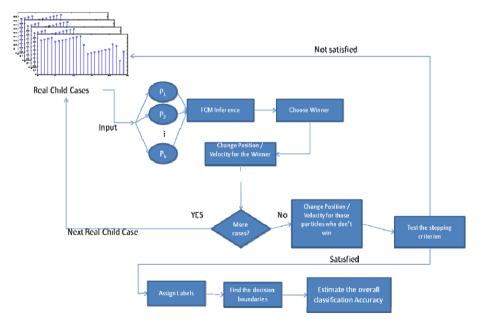
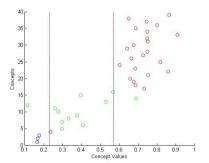


Fig. 1. Particle Swarm Algorithm for FCM Classification

In order to estimate the model accuracy (where all the 40 cases were considered) and the FCM system's accuracy (classification accuracy using the LOOVC method) two different decision boundaries methods were considered: the Minimum Euclidean Distance (MED) and the bayesian statistical decision boundary method (BSM). Additionally different numbers of Particles were considered, 20 and 50 and different numbers of iterations of the algorithm, 100 and 500.

Figures 2 and 3 illustrate the decision boundaries calculated for the decision concepts produced from one algorithm performance for K=20 and R=100.

For the algorithm performance, the row True Positive (TP) represents the average number of the correctly categorized cases according to the decision boundaries chosen. According to LOOCV method 39 random cases were used for the training procedure and the remaining one is used for testing. Thus, for the evaluation of the approach, 39 of the total 40cases were used for training, and only one for testing every time of cross validation. The total model accuracy was calculated by the division of the TP cases with 39. The "Correct Class" represents the total number of cases that have been classified correctly and the "Classification Accuracy" expresses the equivalent proportion.



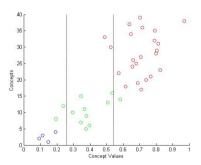


Fig. 2. BSM Classification Lines

Fig. 3. MED Classification Lines

The best accuracy in Scenario (I) is derived for the BSM decision method (92.31%), for K=50 and R=100, whereas in Scenario (II) the best accuracy is presented again for BSM method (87.17%), but for K=20 and R=100. Comparing our results with those previously presented using Hebbian-based learning algorithms (the result was 79.9%) [3] and ensemble-based learning algorithms (87.5%) [15], it is observed that the proosed method outperforms the previous one concerning the NHL approach for FCMs, in both scenarios considering random values for non-zero weights. However, the proposed PSO approach does not outperform the ensemble-based FCM learning approach in the cases considering random values in a ± 0.2 interval of the initial defined weights. Some modifications to the PSO clustering parameters will be investigated in order to increase further the performance of PSO algorithm for this task.

6 Conclusions

To sum-up, the PSO clustering approach for FCM learning is able to classify autism disorder with reasonably high overall accuracy, sufficient for this application area and therefore, it is established as an efficient learning approach for FCMs. This work presents our first investigation to explore the PSO system characteristics and capabilities in the FCM learning working on classification tasks and the results encourage us to further exploit it. Surely, more research work is needed to be done towards more investigation of the learning methodologies of FCMs and their implementation in pattern recognition.

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