Base Stock Inventory Systems with Compound Poisson Demand: Case of Partial Lost Sales

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Abstract. In this paper we extend earlier work that analyzes a single echelon single item base-stock inventory system where Demand is modeled as a compound Poisson process and the lead-time is stochastic. The extension consists in considering a cost oriented system where unfilled demands are lost. The case of partial lost sales is assumed. We first model the inventory system as a Makovian M/G/∞ queue then we propose a method to calculate numerically the optimal base-stock level. A preliminary numerical investigation is also conducted to show the performance of our solution.

Keywords: base-stock, compound Poisson, queuing system, service level, lost sales.

1 Introduction

Inventory control policies have been discussed extensively in the academic literature since the 1950s. A considerable amount of research has been conducted by considering the Normality assumption for modeling the demand ([6], [7], [10]). This is due to the fact that the Normal distribution is attractive from a theoretical perspective and it is also known to provide a good empirical fit to observed demand data. However, it should be noted that the normality assumption makes more sense in the context of fast moving items and in the case of intermittent demand items, such an assumption is judged to be far from appropriate [8].

Intermittent demand items are characterized by occasional demand arrivals interspersed by time intervals during which no demand occurs. As such, demand is built, for modeling purposes, from constituent elements (demand arrivals and demand sizes) that require the consideration of compound demand distributions. Compound Poisson demand processes have pretty much dominated the academic literature due to their comparative simplicity and their theoretical appeal since they may result in standard statistical distributions ([1], [4], [5], [9]). For a Poisson arrival process for example coupled with Logarithmic sizes the resulting distribution is Negative Binomial [9]. In addition, from a modeling perspective, the compound Poisson process may also

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model the demand in the context of fast moving items by considering very low demand time intervals or equivalently a high Poisson rate.

Recently, a research work has been conducted to analyze a single echelon single item base-stock system where Demand is modeled as a compound Poisson process and the lead-time is stochastic following a General distribution [2]. In order to determine the optimal base-stock level, the authors have considered cost oriented inventory systems where unfilled demands are backordered. The backordering assumption is realistic and often considered by researchers in order to determine the optimal parameters of inventory policies. However, other interesting cases could also be considered for that purpose, such as the lost sales case under which the system assumes that an excess demand is not carried over but is lost and the total cost include a lost sales cost. This case constitutes the objective of our research work.

It should be noted that the lost sales inventory models have received far less attention from researchers than the backorder models. This is because in one hand lost sales inventory models are much less analytically tractable and in the other hand, most of the optimality results in the base-stock systems have been derived under the backorder assumption. The lack of attention to the lost sales case, especially for base-stock inventory systems under compound Poisson demand has motivated this research work. In this paper, we extend the work of [2] by analyzing the same base-stock inventory system under the lost sales case. The optimal base-stock level is determined under a cost oriented system where unfilled demands are lost.

The remainder of the paper is organized as follows. Section 2 describes the inventory system considered in the paper and provides a method that can be used to calculate the optimal base-stock level. Section 3 presents some preliminary results of the numerical investigation. We end in Section 4 with conclusions and directions for further research.

2 System Analysis

2.1 System Description and Notation

We consider a single echelon single item inventory system where the demand and the lead time are stochastic. Demand is modeled as a compound Poisson process, i.e. the inter-demand arrivals are exponentially distributed and the demand size follows an arbitrary discrete probability distribution. The focus on the discrete case is motivated by the fact that it leads to solutions that are mathematically tractable. Note that if demand sizes are continuous, the same analysis holds but summations should be replaced by their analogous integral functions. The stock is controlled according to a base-stock policy where each replenishment order is associated with a stochastic lead-time. The inventory system will be analyzed in this paper by considering a cost oriented system where unfilled demands are lost.

For the remainder of the paper, we denote by:

λ: mean demand arrival rate

X: demand size (random variable)

μ_X : mean of demand size

σ_X : standard deviation of demand size

f : probability density function of the random variable X

F : cumulative probability distribution of the random variable X

Y : lead-time (random variable)

L: mean lead-time

$I(t)$: inventory position at time t

$N(t)$: number of outstanding ordered units in the system at time t

S: order-up-to-level

h: inventory holding cost per unit per unit of time

B: unit lost sale cost per unit

X_k : the sum of k i.i.d. random variables X $(k \geq 1)$

f_k : probability density function of the random variable X_k

F_k : cumulative probability distribution of the random variable X_k $(k \geq 1)$

$(X)^+ = \max(X, 0)$

$P(x)$: probability of an event x

2.2 System Modeling and Analysis

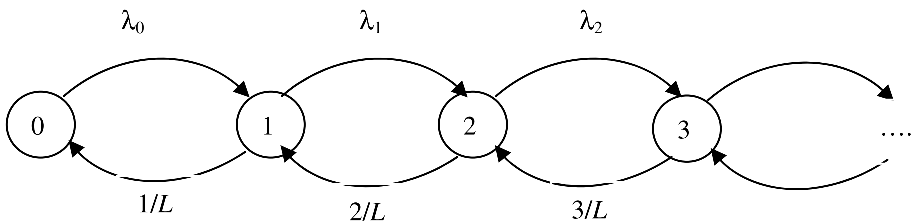
In this paper, we assume that partial lost sales are considered, i.e. a demand can be partially satisfied from the stock on hand and the excess demand is lost. The expected total inventory cost is composed of the expected holding cost and the expected lost sales cost. It is given as follows:

$$E[C(S)] = hSp_0^{(S)} + \sum_{i=1}^{\infty} \left[h(S - X_i)^+ + B\lambda(X_i - S)^+ \right] p_i^{(S)} \qquad (1)$$

which is equivalent to

$$E[C(S)] = hSp_0^{(S)} + \sum_{i=1}^{\infty} \left[h\sum_{n=0}^{S}(S - n) f_i(n) + B\lambda \sum_{n=S+1}^{\infty}(n - S) f_i(n) \right] p_i^{(S)} \qquad (2)$$

where $p_i^{(S)}$ is the probability that there are $I(t) = S - X_i$ items in stock at the stationary regime. Note that analyzing the inventory level $I(t)$ is equivalent to analyzing the number of customers $N(t)$ in a Makovian M/G/∞ queue, with demand arrival rate λ_i and departure rate $\mu_i = i\mu = i\ (1/L)$, as shown in the following Markovian chain, where the state i corresponds to the number of outstanding ordered units in the system. For more details about the modeling of inventory systems by using the queuing theory, the reader is referred to [3].



$$\lambda_i = \lambda\, P(\text{the quantity of the order (i - 1) is strictly less than } S) = \lambda\, F_i(S-1)$$

Thus, it is easy to show that in the stationary regime,

$$p_i^{(S)} = \frac{(\lambda L)^i}{i!}\, p_0^{(S)} \prod_{j=1}^{i-1} F_j(S-1) \ \text{ for all } i \geq 2 \tag{3}$$

where $p_0^{(S)} = 1 - \sum_{i=1}^{\infty} p_i^{(S)}$ and $p_1^{(S)} = (\lambda L)\, p_0^{(S)}$

The optimal base-stock level S^* should be calculated numerically such that:

$$S^* = \min\{S : E[C(S+1)] - E[C(S)] \geq 0\}$$

Where $E[C(S)]$ is given by (2) and $p_i^{(S)}$ is given by (3).

It is clear that in order to find numerically the optimal base-stock level S^*, the function $E[C(S+1)] - E[C(S)]$ should be calculated for different values of S (by using a dichotomy for example), which requires the calculation of the stationary probabilities $p_i^{(S)}$ for different values of S.

In the next section, we calculate numerically the optimal base-stock level and cost and we investigate the impact of some parameters' variation on the optimal base-stock level.

3 Numerical Investigation

For the purpose of the numerical investigation, we assume that the Poisson rate of the demand arrivals λ is varied from 0.1 to 10, which allows us to deal with demands that characterize both fast and slow moving items as it has been done by [2]. Regarding the distribution of the demand size X, in order to reduce the complexity of the of the numerical investigation, we should consider a demand size distribution that is discrete and regenerative, since in that case the distribution of the random variable X_i (the sum of i discrete demand size distributions) is of the same type, so that by knowing the type of the distribution f_i and the first two moments, the analysis is straightforward. For the purpose of this numerical investigation, we consider that demand sizes follow a Poisson distribution with $\mu_X = 10$. The results are presented by considering the case: $h = 1$, $B = 10$.

The optimal base-stock level and the optimal cost when λ is varied in [0.1,1] are shown in Figure 1 which corresponds to the case of slow moving stock keeping units (SKUs). The optimal base-stock level and the optimal cost when λ is varied in [1,10] are shown in Figure 3 which corresponds to the case of fast moving SKUs.

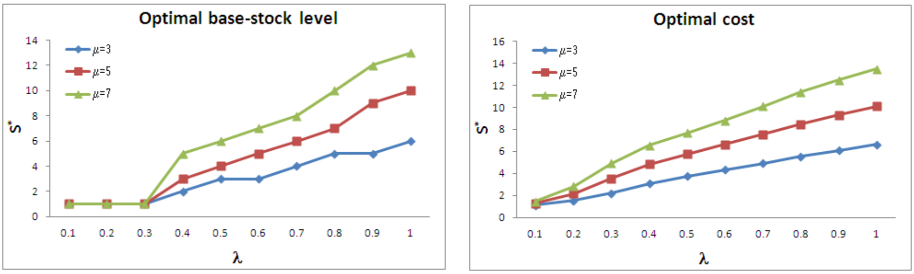


Fig. 1. Variation of the optimal base-stock level and cost in the case of slow moving SKUs

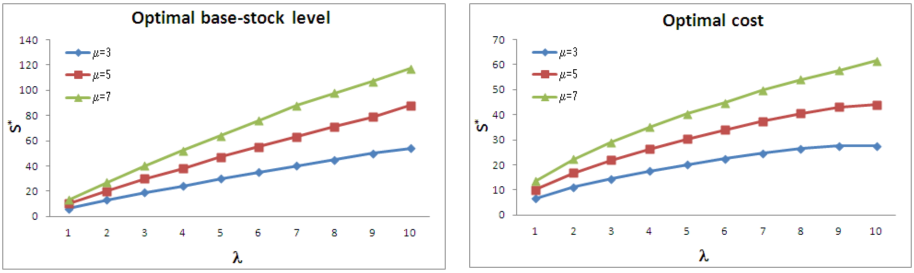


Fig. 2. Variation of the optimal base-stock level and cost in the case of fast moving SKUs

Figures 1-2 show that the optimal base-stock level is an increasing function of the demand arrival rate λ which is expected since the average demand per time unit increases which necessitates higher stock level to reduce the total cost. It should be noted that in the case of slow moving SKUs and for very low λ values, the optimal base-stock level is equal to 1 even when the average demand size increases which can be explained by the fact that the demand per time unit remains relatively low and does not necessitates more than one unit to be kept in stock. Obviously, when the demand size and the demand arrival rate increase considerably in the cases of both slow and fast moving SKUs, the optimal base-stock level and cost increase considerably.

4 Conclusions and Future Research

In this paper, we have analyzed a single echelon single item inventory system under a compound Poisson demand and stochastic lead-time. We assume that the system in controlled with a continuous review base-stock policy. Based on a Markovian M/G/∞ queue model, the optimal base-stock level is determined under a cost oriented system where unfilled demands are lost. Some preliminary numerical results, related to the optimal base-stock level and cost, have been given in the case of both slow and fast moving SKUs. The results show that in the case of slow moving SKUs and for very low λ values, the optimal base-stock level is very low. Moreover, in the cases of fast moving SKUs, when the demand size and the demand arrival rate increase considerably, the optimal base-stock level and cost increase considerably.

An interesting way to extend this research is to conduct an empirical investigation with real data to show the behavior of the optimal base-stock level and cost in such a context. Another important issue to consider is the hypothesized distribution when modeling the demand in the context of slow moving items (i.e. representation of demand sizes and demand intervals). This issue has been repeatedly addressed in the academic literature [8]. A compound Erlang process would be an interesting option to consider for the demand modeling.

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