Incorporating Regularity of Required Workload to the *MMSP-W* with Serial Workstations and Free Interruption of the Operations

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Abstract. We propose a mathematical model to solve an extension to the mixed-model sequencing problem with work overload minimization *(MMSP-W*) for production lines with serial workstations and parallel homogeneous processors and regularizing the required workload. We performed a computational experience with a case study of the Nissan engine plant in Barcelona.

Keywords: Manufacturing, Sequencing, Work overload, Linear programming.

1 Introduction

Manufacturing lines with mixed products are very common in Just in Time (*JIT*) and Douki Seisan (*DS*) environments. These lines, composed of multiple workstations must be flexible enough to treat different product types.

These lines usually consist of a set (K) of workstations laid out in series. Each workstation ($k = 1,\ldots,|K|$) is characterized by the use of the human resources, tools and automated systems necessary to carry out the work assigned to the workstation. The set of tasks assigned to the workstation is called workload, and the average time required to process these tasks at normal activity rates is called workload time or processing time.

An important attribute of these production lines is flexibility. The products (such as engines or car bodies) that circulate through the lines are not completely identical. Although some of the products may be similar or of the same type, they may require different resources and components and therefore may require different processing times.

The desired flexibility of these mixed-product lines requires that the sequence in which the product types are manufactured follow two general principles: (1) to minimize the stock of components and semi-processed products and (2) to maximize the efficiency of the line, manufacturing the products in the least amount of time possible.

A classification of sequencing problems arising in this context was given in [1]:

1. *Mixed-model sequencing*. The aim in this problem is to obtain sequences that complete the maximum work required by the work schedule.

C. Emmanouilidis, M. Taisch, D. Kiritsis (Eds.): APMS 2012, Part I, IFIP AICT 397, pp. 405–412, 2013.

2. *Car sequencing*. These problems are designed to obtain sequences that meet a set of constraints related to the frequency in which the workstations are required to incorporate special options (e.g., sunroof, special seats or a larger engine) within the products.
3. *Level scheduling*. These problems focus on obtaining level sequences for the production and usage of components.

The *MMSP-W* [2, 3] consists of sequencing T products, grouped into a set of I product types, of which d_i are of type i ($i = 1,\ldots,|I|$). A unit of product type i ($i = 1,\ldots,|I|$), when is at workstation k ($k = 1,\ldots,|K|$), requires a processing time equal to $p_{i,k}$ for each homogeneous processor (e.g., operator, robot or human-machine system) at normal activity, whereas the standard time granted at each station to work on an output unit is the cycle time, c.

Sometimes a workstation, k, can work on any product a maximum time l_k, which is called time window, and is longer than the cycle time ($l_k > c$), which causes that the time available to process the next unit is reduced. When it is not possible to complete all of the work required, it is said that an overload is generated.

The objective of *MMSP-W* is to maximize the total work completed, which is equivalent to minimize the total work overload generated (see Theorem 1 in [4]), sequencing the units on the line, considering the interruption of the operations at any time between the time of completion of one cycle and the time of termination marked by the time window associated with that cycle [5]. In addition, in our proposal we will maintain constant the cumulative time of work required at the workstations in all positions of the product sequence.

2 Models for the *MMSP-W*

2.1 Reference Models

For the *MMSP-W* with serial workstations, free interruption of the operations and homogeneity of required workload, we begin with several models as reference (see table 1).

Table 1. Comparison of the major differences of models *M*1 to *M*4 and *M*4∪3

	*M*1	*M*2	*M*3	*M*4	*M*_4∪3
Objective	*Max V*	*Min W*	*Max V*	*Min W*	*Min W/ Max V*
Start instants	Absolute $s_{k,t}$	Relative $\hat{s}_{k,t}$	Absolute $s_{k,t}$	Relative $\hat{s}_{k,t}$	Relative $\hat{s}_{k,t}$
Variables	$v_{k,t}$	$w_{k,t}$	$v_{k,t}$	$w_{k,t}$	$w_{k,t}$, $v_{k,t}$
Window, t=T	$l_k \ \forall k$	$c \ \forall k$	$l_k \ \forall k$	$l_k \ \forall k$	$l_k \ \forall k$
Rank for b_k	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$
Links between stations	No	No	Yes	Yes	Yes

The models from the literature, *M*1 [2] and *M*2 [3], do not consider links between workstations. *M*1 is focused on maximize the total work performed, using an absolute time scale at each station and considering more than one homogeneous processor at each workstation. *M*2 is focused on minimize the total work overload with relative time scale at each station corresponding to each processed product unit and only considers one processor at each workstation.

An extension of these models, considering links between consecutive stations, are models *M*3 (*M*1 extended) and *M*4 (*M*2 extended) proposed by [4]. Moreover, considering the equivalence of the objective functions of *M*3 and *M*4, we can combine them and obtain the *M*_4∪3 [6] model that considers the relative times scales used in *M*4.

2.2 Regularity of Required Workload

The overload concentrations at certain times during the workday may be undesirable. One way to avoid this occurrence is to obtain product sequences that regulate the cumulative time of required work at the workstations in all positions of the product sequence.

To do this, first we consider the average time required at the k^{th} workstation to process a product unit, which is the processing time for an ideal unit at workstation k. If $\dot{p}_k$ is the average time, then the ideal work rate for station k $\left(k=1,\ldots,|K|\right)$ is determined as follows:

$$\dot{p}_k = \frac{b_k}{T}\sum_{i=1}^{|I|} p_{i,k}\cdot d_i \qquad k=1,\ldots,|K| \tag{1}$$

Consequently, the ideal total work needed to complete t output units at workstation k is:

$$P_{k,t}^{*} = t\cdot \dot{p}_k \qquad k=1,\ldots,|K|\ ;\ t=1,\ldots,T \tag{2}$$

Moreover, if we consider the actual total work required at the k^{th} workstation to process a total of t product units, of which $X_{i,t}=\sum_{\tau=1}^{t} x_{i,\tau}$ are of type i $\left(i=1,\ldots,|I|\right)$, then we have:

$$P_{k,t} = b_k\sum_{i=1}^{|I|} p_{i,k}\cdot X_{i,t} = b_k\sum_{i=1}^{|I|} p_{i,k}\left(\sum_{\tau=1}^{t} x_{i,\tau}\right) \qquad k=1,\ldots,|K|\ ;\ t=1,\ldots,T \tag{3}$$

Where $x_{i,t}$ ($i=1,\ldots,|I|$; $t=1,\ldots,T$) is a binary variable that is equal to 1 if a product unit i is assigned to the position t^{th} of the sequence, and 0 otherwise.

One way to measure the irregularity of the required workload at a set of workstations over the workday is to cumulate the difference between the actual and ideal work required to each output unit at each workstation:

$$\Delta_Q(P) = \sum_{t=1}^{T}\sum_{k=1}^{|K|} \delta_{k,t}^2(P), \quad \text{where} \quad \delta_{k,t}(P) = P_{k,t} - P_{k,t}^* \tag{4}$$

If we consider the properties derived from maintaining a production mix when manufacturing product units over time, we can define the number of units of product type i, of a total of t units, which should ideally be manufactured to maintain the production mix as:

$$X_{i,t}^* = \frac{d_i}{T} \cdot t \qquad i = 1,\ldots,|I| \;;\; t = 1,\ldots,T \tag{5}$$

Therefore, the ideal point $\vec{X}^* = \left(X_{1,1}^*,\ldots,X_{|I|,T}^*\right)$ presents the property of leveling the required workload, because at that point, the non-regularity of the required work is optimal, $P_{k,t} - P_{k,t}^* = \delta_{k,t}(P) = 0$ and then $\Delta_Q(P) = 0$, as shown in (6) (see theorem 1 in [6]):

$$P_{k,t} = b_k \sum_{i=1}^{|I|} p_{i,k} \cdot X_{i,t}^* \Leftrightarrow P_{k,t} = b_k \sum_{i=1}^{|I|} \frac{p_{i,k} \cdot d_i \cdot t}{T} = t \cdot \left(\frac{b_k}{T} \sum_{i=1}^{|I|} p_{i,k} \cdot d_i \right) = t \cdot \dot{p}_k = P_{k,t}^* \tag{6}$$

2.3 *MMSP-W* Model for Workload Regularity

Considering the properties described above and the reference model *M*_4∪3 [6], we limit the values of the cumulative production variables, $X_{i,t}$ ($i = 1,\ldots,|I|; t = 1,\ldots,T$), to the integers closest to the ideal values of production, $X_{i,t}^* = d_i \cdot t / T$, and then we obtain a new model, the *M*_4∪3_*pmr*. The parameters and variables are presented below:

Parameters	
K	Set of workstations ($k = 1,\ldots,|K|$)
b_k	Number of homogeneous processors at workstation *k*
I	Set of product types ($i = 1,\ldots,|I|$)
d_i	Programmed demand of product type *i*
$p_{i,k}$	Processing time required by a unit of type *i* at workstation *k* for each homogeneous processor (at normal activity)
T	Total demand; obviously, $\sum_{i=1}^{|I|} d_i = T$
t	Position index in the sequence ($t = 1,\ldots,T$)
c	Cycle time, the standard time assigned to workstations to process any product unit
l_k	Time window, the maximum time that each processor at workstation *k* is allowed to work on any product unit, where $l_k - c > 0$ is the maximum time that the work in progress *(WIP)* is held at workstation *k*

Variables	
$x_{i,t}$	Binary variable equal to 1 if a product unit i ($i=1,\ldots,\lvert I\rvert$) is assigned to the position t ($t=1,\ldots,T$) of the sequence, and to 0 otherwise
$s_{k,t}$	Start instant for the t^{th} unit of the sequence of products at station k ($k=1,\ldots,\lvert K\rvert$)
$\hat{s}_{k,t}$	Positive difference between the start instant and the minimum start instant of the t^{th} operation at station k. $\hat{s}_{k,t}=\left[s_{k,t}-(t+k-2)c\right]^{+}$ (with $[x]^{+}=\max\{0,x\}$).
$v_{k,t}$	Processing time applied to the t^{th} unit of the product sequence at station k for each homogeneous processor (at normal activity)
$w_{k,t}$	Overload generated for the t^{th} unit of the product sequence at station k for each homogeneous processor (at normal activity); measured in time.

Model *M_4∪3_pmr*:

$$Min \quad W=\sum_{k=1}^{|K|}\left(b_k\sum_{t=1}^{T}w_{k,t}\right)\Leftrightarrow Max \quad V=\sum_{k=1}^{|K|}\left(b_k\sum_{t=1}^{T}v_{k,t}\right) \tag{7}$$

Subject to:

$$\sum_{t=1}^{T}x_{i,t}=d_i \qquad i=1,\ldots,|I| \tag{8}$$

$$\sum_{i=1}^{|I|}x_{i,t}=1 \qquad t=1,\ldots,T \tag{9}$$

$$v_{k,t}+w_{k,t}=\sum_{i=1}^{|I|}p_{i,k}x_{i,t} \qquad k=1,\ldots,|K|;\ t=1,\ldots,T \tag{10}$$

$$\hat{s}_{k,t}\geq\hat{s}_{k,t-1}+v_{k,t-1}-c \qquad k=1,\ldots,|K|;\ t=2,\ldots,T \tag{11}$$

$$\hat{s}_{k,t}\geq\hat{s}_{k-1,t}+v_{k-1,t}-c \qquad k=2,\ldots,|K|;\ t=1,\ldots,T \tag{12}$$

$$\hat{s}_{k,t}+v_{k,t}\leq l_k \qquad k=1,\ldots,|K|;\ t=1,\ldots,T \tag{13}$$

$$\hat{s}_{k,t}\geq 0 \qquad k=1,\ldots,|K|;\ t=1,\ldots,T \tag{14}$$

$$v_{k,t}\geq 0 \qquad k=1,\ldots,|K|;\ t=1,\ldots,T \tag{15}$$

$$w_{k,t}\geq 0 \qquad k=1,\ldots,|K|;\ t=1,\ldots,T \tag{16}$$

$$x_{i,t}\in\{0,1\} \qquad i=1,\ldots,|I|;\ t=1,\ldots,T \tag{17}$$

$$\hat{s}_{1,1}=0 \tag{18}$$

$$\sum_{\tau=1}^{t}x_{i,\tau}\geq\left\lfloor t\cdot\frac{d_i}{T}\right\rfloor \qquad i=1,\ldots,|I|;\ t=1,\ldots,T \tag{19}$$

$$\sum_{\tau=1}^{t}x_{i,\tau}\leq\left\lceil t\cdot\frac{d_i}{T}\right\rceil \qquad i=1,\ldots,|I|;\ t=1,\ldots,T \tag{20}$$

In the model, the equivalent objective functions (7) are represented by the total work performed (*V*) and the total work overload (*W*). Constraint (8) requires that the programmed demand be satisfied. Constraint (9) indicates that only one product unit can be assigned to each position of the sequence. Constraint (10) establishes the relation between the processing times applied to each unit at each workstation and the overload generated in each unit at each workstation. Constraints (11)-(14) constitute the set of possible solutions for the start instants of the operations at the workstations and the processing times applied to the products in the sequence for each processor. Constraints (15) and (16) indicate that the processing times applied to the products and the generated overloads, respectively, are not negative. Constraint (17) requires the assigned variables to be binary. Constraint (18) establishes the earliest instant in which the assembly line can start its operations. Finally, the constraints (19) and (20) are those that incorporate, indirectly, the regularity of required workload to the *MMSP-W*.

3 Computational Experience

To study the behavior of the incorporation of the regularity restrictions of work required into the $M\_4\cup 3$, we performed a case study of the Nissan powertrain plant in Barcelona. This plant has an assembly line with twenty-one workstations ($m_1,\ldots,m_{21}$) assembling nine types of engines ($p_1,\ldots,p_9$) that are grouped into three families (4x4, vans and trucks) whose processing times at stations ranging between 89 and 185 *s*.

For the experiment, we considered a set E of 23 ($\varepsilon = 1,\ldots,23$) instances associated to a demand plan of 270 engines, an effective cycle time c = 175 *s* and an identical time window for all workstations $l_k = 195$ *s* ($k = 1,\ldots,21$) (see tables 5 and 6 in [4]).

To implement the models, the Gurobi v4.5.0 solver was used on Apple Macintosh iMac computer with an Intel Core i7 2.93 GHz processor and 8 GB of RAM using MAC OS X 10.6.7. The solutions from this solver were obtained by allowing a maximum CPU time of 7200 *s* for each model and for each of the 23 demand plans in the NISSAN-9ENG set.

To estimate the quality of the experimental results, we use the following indicators:

$$RPD(f,\varepsilon) = \frac{f\left(S^*_{4\cup 3}(\varepsilon)\right) - f\left(S^*_{4\cup 3\_pmr}(\varepsilon)\right)}{f\left(S^*_{4\cup 3}(\varepsilon)\right)} \cdot 100 \qquad \left(f \in \Im = \{W, \Delta_Q(P)\};\ \varepsilon \in \mathrm{E}\right) \quad (21)$$

$$\overline{RPD}(f) = \frac{\sum_{\varepsilon=1}^{|\mathrm{E}|} RPD(f,\varepsilon)}{|\mathrm{E}|} \qquad \left(f \in \Im = \{W, \Delta_Q(P)\}\right) \quad (22)$$

Table 2 and figure 1 show the results obtained.

Table 2. Values of RPD for the functions W, $\Delta_Q(P)$ and average values ($\overline{RPD}(W)$, $\overline{RPD}(\Delta_Q(P))$) for the 23 instances of the NISSAN-9ENG set.

ε	W	$\Delta_Q(P)$	ε	W	$\Delta_Q(P)$	ε	W	$\Delta_Q(P)$	ε	W	$\Delta_Q(P)$
1	0.53	96.40	*7*	1.48	91.12	*13*	-17.48	95.59	*19*	0.00	94.96
2	-12.32	89.70	*8*	-15.11	94.25	*14*	-0.71	94.35	20	-7.91	96.31
3	0.94	89.24	*9*	-2.60	94.61	*15*	-2.08	94.58	21	-0.18	86.39
4	0.97	92.35	*10*	0.00	87.04	*16*	-10.57	90.42	22	0.30	90.16
5	-4.42	97.67	*11*	-56.41	95.25	*17*	-2.09	88.91	23	13.57	86.55
6	-15.74	94.26	*12*	-1.06	96.26	*18*	-2.31	92.08	$\overline{RPD}$	**-5.79**	**92.54**

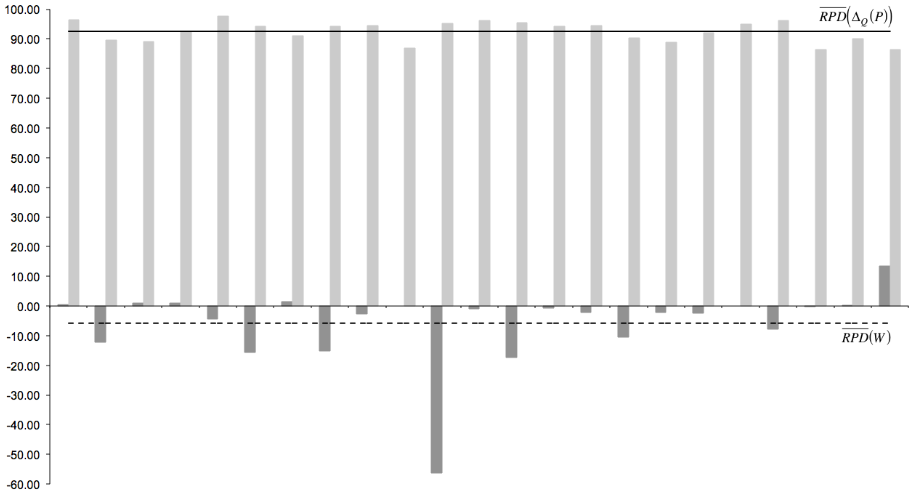

Fig. 1. Values of RPD for the functions W (dark grey), $\Delta_Q(P)$ (grey) and average values ($\overline{RPD}(W)$ (dotted line), $\overline{RPD}(\Delta_Q(P))$ (continuous line)) for the 23 instances of the NISSAN-9ENG set.

According to the results (see table 2 and figure 1) we can conclude the following:

- We can only guarantee the optimal solutions for instances 10 and 19, with the limitation of a run time of 7200 *s*,
- The reference model *M*_4∪3 achieves a better average overload than *M*_4∪3_*pmr* (a difference of 5.79% in $\overline{RPD}(W)$) on the set of 23 instances.
- The incorporation of constraints (8) and (9) into the reference model *M*_4∪3 produces a significant improvement in the regularity of the required work ($\overline{RPD}(\Delta_Q(P)) = 92,54\%$).

4 Conclusions

We have formulated a model for the *MMSP-W*, *M*_4∪3_*pmr*, that minimizes the total work overload or maximizes the total work completed, considering serial workstations, parallel processors, free interruption of the operations and with restrictions to regulate the required work.

A case study of the Nissan engine plant in Barcelona has been realized to compare the new model with the reference model *M*_4∪3.

The case study includes the overall production of 270 units of 9 different types of engines, for a workday divided into two shifts, and assuming that the particular demands of each type of engine may vary over time. This is reflected in 23 instances, each of them representing a different demand plan.

For the computational experience, the solver Gurobi 4.5.0 was used. The solutions have been found for the 23 instances, allowing a maximum CPU time of 7200 s for each instance. Using this CPU time, we can only guarantee the optimal solutions for the instances 10 and 19.

The results show that the incorporation of the restrictions to regulate the required work into the reference model $M\_4\cup 3$ produces an average gain of 92,54%, in terms of regularity of required work, while gets worse by an average of 5,79%, in terms of work overload.

We propose as future research lines: (1) to design and to implement heuristics and exact procedures to solve the problem under study; (2) to consider the minimization of the work overload and maximizing the regularity of the work required as simultaneous objectives of the problem; and (3) to incorporate to the proposed models, other desirable productive attributes such as maintenance of the production mix and the regular consumption of products parts, for example.

Acknowledgements. The authors greatly appreciate the collaboration of Nissan Spanish Industrial Operations (NSIO). This work was funded by project PROTHIUS-III, DPI2010-16759, including EDRF funding from the Spanish government.

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