Enhancing Biomedical Images Using the UFIR Filters with Recursive Responses

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Abstract. In this paper we present a novel computational scheme to determine the impulse response of the UFIR filters. A recursive form of impulse response is developed using the theory of the discrete orthogonal polynomials. An example of an enhanced of medical image is considered to compare its performance versus the matrix formulation of the impulse response UFIR filters. Finally, some quantitative and qualitative evaluations are carried out to verify its efficiency based on RMSE analysis.

Keywords: UFIR filters, biomedical images, discrete orthogonal polynomials, recurrent relation.

1 Introduction

Biomedical imaging is a powerful tool in many medical procedures, such as detection of abnormal mass, diagnosis of some diseases, proper treatment for illnesses, among others. Nowadays, exist a variety of medical imaging techniques, such as: X-ray, Magnetic Resonance Imagery (MRI), Positron Emission Tomography (PET), and Ultrasound images [1, 2]. These images are classified in two categories: images formed with electromagnetic waves and images formed from acoustic waves. The first, are formed with particles provided by a particles beam so that the penetration in tissue and the organs is used for forming images. On the other hand, the second category to group images formed with the scatter waves that return from both the tissue and organs, respectively. Particular features are identified in such images for example: attenuation, reflection, dispersion, and impedance [3, 4]. However, these features are the result of the change in density between different tissues. In addition, the noise is an important issue in image formation and the most important types of noise in biomedical imaging field are speckle and Gaussian noise.

The problem of saving sharp edges whilst enhancing the image corrupted by either speckle or Gaussian noise is typical in biomedical image processing [2]. An overall

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overview of nonlinear filtering has been given in [5] along with the important modifications for a large class of nonlinear filters employing order statistics. The algorithm problems for the FIR filter design have been discussed in [6]. In [7] the finite impulse response (FIR) median hybrid filters (MHF) strategy has been proposed with applications to image processing.

In the other hand, the family of Unbiased Finite Impulse Response (UFIR) filter is relatively new in signal processing applications [8]. This digital FIR filter was developed by Shmaliy in order to reduce the Time Interval Error (TIE) in Global Position System (GPS) signals. Later, in [9-11] a modification in the impulse response of the UFIR filters is introduced, this new approach makes use of the size of the step (p) of process which is very important, this development presented the next three processes: Filtering, when, $p = 0$, Prediction when, $p > 0$, and Smoothing when, $p < 0$. Some applications of UFIR [12, 13] and p-UFIR [14-16] filters in biomedical image processing were developed by Morales-Mendoza, for ultrasound images applications. Furthermore, in [17] a ramp UFIR filter was used in the new computational scheme for synthetic image processing also targeted the ultrasound image processing [18]. Finally, a recursive scheme for computing the UFIR filters impulse response was proposed in [19, 20].

In this paper, we present an application of UFIR filters with recursive response to preserve sharp edges with a simultaneous enhancing of biomedical images. The rest of the paper is organized as follows. In Sect. 2, we derive the polynomial image model. In Sect 3, we briefly discuss the derivation of the impulse responses of the filters UFIR using the matrix method approach. The orthogonality UFIR functions and the development of recursive impulse response of UFIR filters are discussed in detail in Sect. 4. An example of biomedical image processing is given in Sect. 5 and concluding remarks are drawn in Section 6.

2 Polynomial Image Model

A two-dimensional image is often represented as a $k_c \times k_r$ matrix, $\mathbf{M} = \{\mu_{i,j}\}$. In order to perform two dimensional filtering, this matrix can be written in the form of a row-ordered vector (known as lexicographic form) or a column-ordered vector, respectively

$$\mathbf{y}_r = \left[\mu_{1,1} \cdots \mu_{1,k_r} \cdots \mu_{k_c,1} \cdots \mu_{k_c,k_r}\right]^T , \quad (1)$$

$$\mathbf{y}_c = \left[\mu_{1,1} \cdots \mu_{k_c,1} \cdots \mu_{1,k_r} \cdots \mu_{k_c,k_r}\right]^T . \quad (2)$$

The filtering procedure is then often applied twice, first to (1) and then to (2), or vice versa.

If a two-dimensional image is accurately represented using both (1) and (2), then each of the vectors may also considered as a deterministic one-dimensional signal, y. The unbiased FIR filter estimate is provided by the discrete convolution as follows

$$\hat{s}(t) = h_n(t, N) \otimes y(t) \quad (3)$$

where, $h_n(t, N)$ is the filter's impulse response with order approximation n, the constant N is the averaging boundary of the samples and $s(t)$ is the measurement signal (lexicographic image). It has shown in [8] it is possible to compute an unbiased estimation of $s(t)$ at time t from $y(t)$ by using the convolution defined in (3). The UFIR filter of n-degree, $n \in [0, K - 1]$, with impulse response function $h_n(t, N)$ and monic polynomial is defined as follows

$$h_n(t,N) = a_{0n}(N) + a_{1n}(N)t + a_{2n}(N)t^2 + \cdots + a_{nn}(N)t^n = \sum_{j=0}^{n} a_{jn}(N)t^j . \tag{4}$$

3 Matrix Model of the Impulse Response UFIR Filter

In this Section, we derive the general mathematical model for signals and images based on (4). First, we substitute t by discrete variable x so we have a polynomial that exist from 0 to $N - 1$ with the following fundamentals properties [8, 10],

- Unit area

$$\sum_{x=0}^{N-1} h_n(x,N) = 1 . \tag{5}$$

- Zero Moments

$$\sum_{x=0}^{N-1} x^u h_n(x,N) = 0 , \qquad 1 \le u \le n. \tag{6}$$

- Finite norm (energy)

$$\sum_{x=0}^{N-1} h_n^2(x,N) = a_{0n} < \infty . \tag{7}$$

In order to compute the coefficients of (4), lets first recall the properties described previously in (5), (6) and (7), respectively. Thus, the coefficients are defined as follows

$$a_{jn}(N) = (-1)^j \frac{M_{(j+1)1}(N)}{|\mathbf{D}(N)|} , \tag{8}$$

where, $|.|$ is the determinant of $\mathbf{D}(N)$ and $M_{(j+1)1}(N)$ is the minor of the Hankel matrix $\mathbf{H}_n(N) \in \Re^{(n+1)\times(n+1)}$,

$$\mathbf{D}(N) \equiv \mathbf{H}_n(N) = \begin{bmatrix} c_0(N) & c_1(N) & \cdots & c_n(N) \\ c_1(N) & c_2(N) & \cdots & c_{n+1}(N) \\ \vdots & \vdots & \ddots & \vdots \\ c_n(N) & c_{n+1}(N) & \cdots & c_{2n}(N) \end{bmatrix} , \tag{9}$$

each component in (9) can be found using the recursive relation of the Bernoulli polynomials $B_n(x)$ established in [8, 10]. Note that, so far, the following low-degree polynomials $h_n(x, N)$ were found and investigated so far

$$h_0(x,N)=\frac{1}{N} \tag{10}$$

$$h_1(x,N)=\frac{2(2N-1)-6x}{N(N+1)} \tag{11}$$

$$h_2(x,N)=\frac{3(3N^2-3N+2)-18(2N-1)x+30x^2}{N(N+1)(N+2)} \tag{12}$$

$$h_3(x,N)=\frac{8(2N^3-3N^2+7N-3)-20(6N^2-6N+5)x+120(2N-1)x^2-140x^3}{N(N+1)(N+2)(N+3)} \tag{13}$$

4 Recurrent Relation of the Impulse Response UFIR Filter

4.1 Orthogonality of the UFIR Functions

By definition, a set of orthogonal polynomials is a special system of polynomials $\{P_m(x)\}$, $m = 0, 1, 2\ldots$ that are orthogonal with respect to some weight function $\rho(x)$ on an interval $[a, b]$. Given the n-degree polynomials functions $h_n(x, n)$ defined by (4) and (8) from 0 to $N - 1$ with the properties (5)-(7), a class of functions $\{h_n(x, N)\}$ is orthogonal on $x \in [0, N - 1]$, satisfying

$$\sum_{x=0}^{N-1} h_k(x,N)h_n(x,N)\rho(x,N)=d_n^2(N)\delta_{kn}\,, \tag{14}$$

where, the subscripts k and $n \in [0, K - 1]$, δ_{kn} is Kronecker delta, $d_n^2(N)$ is defined as the square of the weighted norm of $h_n(x, N)$ and $\rho(x, N)$ is the ramp probability density function defined earlier, both functions are defined in [19, 21] as follows

$$\rho(x,N)=\frac{2x}{N(N-1)}\geq 0 \tag{15}$$

and $d_n^2(N)$ is defined as the weighted norm of $h_n(x, N)$.

4.2 Recurrent Equation of UFIR Polynomials

It is known that any real orthogonal polynomials $P_n(x)$ such as (4) satisfy the Favard's theorem they are a sequence of polynomials satisfying a suitable three-term recurrence relation. The recursive relationship for determining the impulse response of UFIR filters [18-21] is defined as

$$h_n(x,N) = 2\frac{n^2(2N-1) - x(4n^2-1)}{n(2n-1)(N+n)} h_{n-1}(x,N) - \frac{(2n+1)(N-n)}{(2n-1)(N+n)} h_{n-2}(x,N) \tag{16}$$

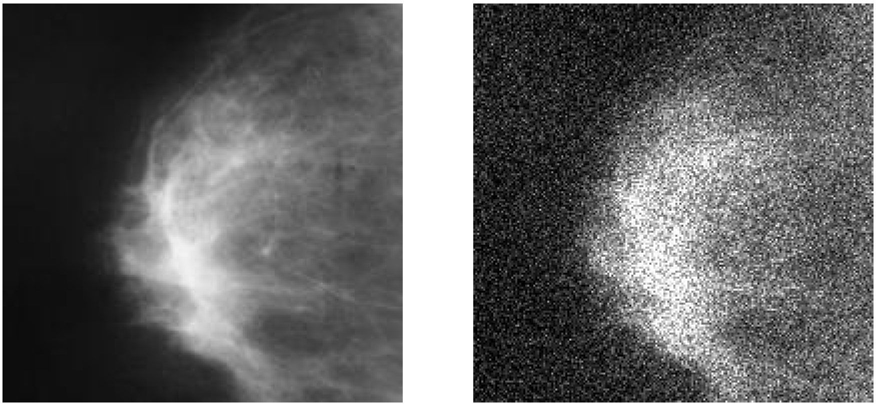
where, the above ratio, are constrained to the following initial conditions

$$h_{-1}(x, N) = 0 , \quad \text{and} \quad h_0(x, N) = 1/N \tag{17}$$

using the relation (16) with the initial conditions defined in (17) yields the impulse response defined in (11) to (13), respectively.

5 Simulations

For further investigation, we chose a biomedical image of 200 × 200 pixels to grayscale showed in Fig. 1a. The image was contaminated with both additive white Gaussian and speckle noise as shown in Fig. 1b. The simulation conditions were set as follows: The average of horizon is $N = 11$ samples and the noises added to image are the white and speckled noise with a variance of 0.1 each one.



a) b)

Fig. 1. a) Original biomedical Image, b) Noisy Image with $\sigma^2 = 0.1$

The computational evaluation is carried out using a biomedical imaging. Image improvement is obtained using low-order polynomial approximation of the impulse response of the filter. Fig. 2 shows the enhanced images using different impulse responses. The evaluation in terms of RMSE metric is shown in Table 1.

The computational evaluation is carried out in a personal computer that used an Intel® CORE 2 DUO processor with 2.4GHz of speed and 2GB of RAM. In Fig. 3, the time requested to pixel processing using the matrix and recursive forms of the impulse response of UFIR filter is shown.

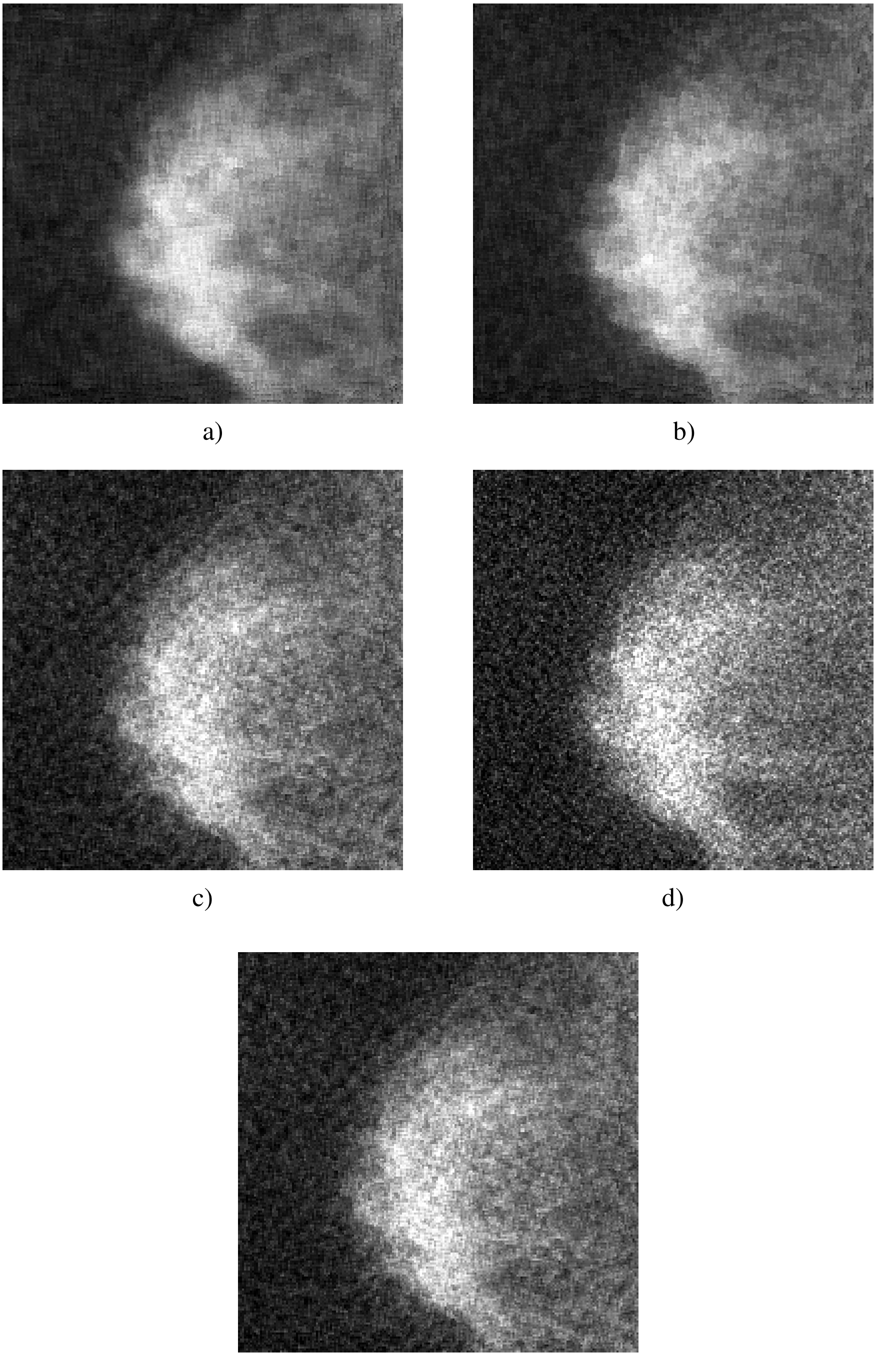


Fig. 2. Enhancing images with a) $h_1(N, x)$, b) $h_5(N, x)$, c) $h_{10}(N, x)$, $h_{15}(N, x)$ and e) $h_{20}(N, x)$

Table 1. I. Quantitative evaluation

RMSE	$h_1(N, x)$	$h_5(N, x)$	$h_{10}(N, x)$	$h_{15}(N, x)$	$h_{20}(N, x)$
Matrix	0.1238	0.2934	0.4594	0.6256	0.6874
Recursive	0.1283	0.2301	0.4376	0.6211	0.6834

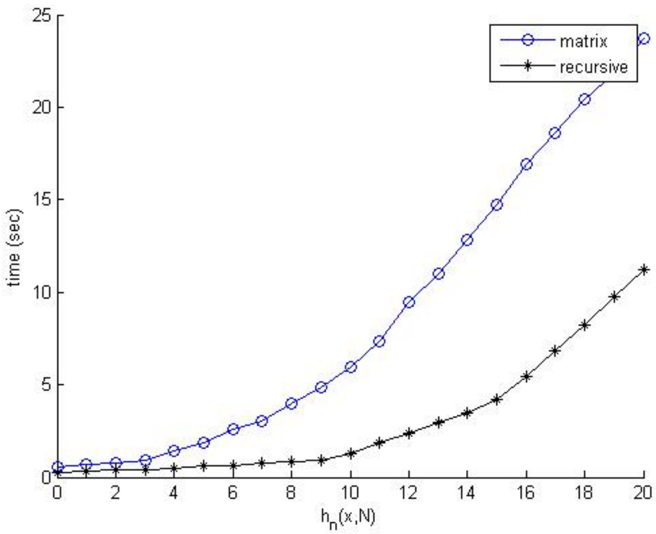


Fig. 3. Pixel processing using the Impulse Response of UFIR filter vs Time

6 Conclusions

In this paper we presented the development of two forms to derive the impulse response of the UFIR filter. The matrix and recursive form can be used to filter biomedical images interchangeably. The quantitative evaluation (Table I) of both forms is quite similar. The processing time of each form is the main factor to consider. In Fig. 3 shows that the higher order approximation, the greater the processing time required is, and we can conclude that recursive approach requires less processing time than the matrix form. Finally, this comparison would help to significantly reduce the computational complexity required to implement the algorithm in a signal processor.

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