Chapter 20
Study on Efficient Complex Network Model

Cheng Wang, Qing Zhang and Jianping Gan

Abstract This paper summarizes the relevant research of the complex network systematically based on Statistical Property, Structural Model, and Dynamical Behavior. Moreover, it emphatically introduces the application of the complex network in the economic system.

Keywords Complex network · Small-world · Scale-free network

20.1 Introduction

One of the basic viewpoints in the systems science is that structure decides function. If we consider every element as panel point, the relationship of the elements as the connection, then the system will be a network [1], for example the nervous system can be regarded as network interconnected by lots of nerve cells through nerve fibers. The computer network can be regarded as a network connected by computer through communication medium such as optical cable, twisted-pair, and coaxial-cable. Besides the power grid, social networking and

C. Wang (✉) · Q. Zhang
College of Mathematics and Computer Science,
Huanggang Normal University, Huanggang, China
e-mail: wangc80@163.com

Q. Zhang
e-mail: zhangqing@hgnu.edu.cn

J. Gan
Institute of Demographic Ecology and Resources Management,
Huanggang Normal University, Huanggang, China
e-mail: jpgan@hgnu.edu.cn

Y. Yang and M. Ma (eds.), *Proceedings of the 2nd International Conference on Green Communications and Networks 2012 (GCN 2012): Volume 5*,
Lecture Notes in Electrical Engineering 227, DOI: 10.1007/978-3-642-35398-7_20,

transportation network, and so on are of the same kind [2]. Emphasis on the structure of the system and the system analysis from structure are the research thinking of the complex network. The difference is that the property of the topological structure of the abstracted real networks is different from the network discussed before, and has numerous nodes, as a result we call it complex network [3]. In recent years, a large number of articles are published in world leading publication such as Science, Nature, PRL, and PNAS, which reflects indirectly that complex network has been a new research hot spot.

The research in complex network can be simply summarized as contents of three aspects each of which has close and further relationships: Rely on the statistical property of the positivist network measurement; understanding the reason why the statistical property has the property it has through building the corresponding network model; forecasting the behavior of the network system based on the structure and the formation rule of the network.

20.2 The Statistical Property of the Complex Network

The description of the world in the view of the network started in 1736 when German mathematician Eular solved the problem of Johannesburg's seven bridges. The difference of complex network researching is that you should view the massive nodes and the properties they have in the network from the point of the statistics firstly. The difference of the properties means the different internal structures of the network; moreover the different internal structures of the network bring about the difference of the systemic function. Therefore, the first step of our research on complex network is the description and understanding of the statistical properties, sketched as follows:

1. Average path length

In the research of the network, generally speaking we define the distance between two nodes as the number of the shortest path edge of the two connectors; the diameter of the net as the maximum range between any two points; the average length of the net is the average value of the distance among all the nodes, it represents the degree of separation of the nodes in the net, namely the size of the net. An important discover in the complex network researching is that the average path length of the most of the large-scale real networks is much less than our imagine, which we call "Small-world Effect". This viewpoint comes from the famous experiment of "Milgram Small-world", the experiment required the participators to send a letter to one of their acquaintances making sure the letter reach the recipient of the letter, in order to figure out the distribution of the path length in the network, the result shows that the number of the average passing person is just six, in addition the experiment is also the origin of the popular theory "6° of separation".

2. Convergence factor

The aggregation extent of the nodes in the network is represented by Convergence factor C, that is how close of the network. For example in the social networks, your friend's friend may be your friend or both of your two friends are friends. The computational method is that: assuming node i connect other k_i nodes through k_i, if the k_i connected each other, there should be $k_i(k_i - 1)/2$ sides among them, however if the k_i nodes have E_i sides, then the ratio of E_i to $k_i(k_i - 1)/2$ is the convergence factor of node i. The convergence factor of the network is the average value of all the nodes' convergence factor in the network. Obviously only is in fully connected network the convergence factor equals 1, in most other networks convergence factor less than 1. However, it proves to be that nodes in most large-scale realworlds network tent to be flock together, although the convergence factor C is far less than 1, it is far more than N^{-1}.

3. Degree distribution

The degree k_i of the node i in the graph theory is the total amount of the sides connected by node i, the average of the degree k_i of the node i is called average degree of the network, defined as $< k >$. The degree of the node in the network is represented by distribution function $p(k)$, the meaning of which is that the probability that any nodes with k sides, it also equals the number of nodes with k degree divide the number of all the nodes in the network.

The statistical property described above is the foundation of the complex networks researching; with the further researching we generally discover the realworld network has other important statistical property, such as the relativity among network resilience, betweenness, and degree and convergence factor.

20.3 Complex Network Model

The most simple network model is the regular net region; the same number around every node is its characteristic, such as 1 d chain-like, 2 d lattice, complete graph and so on. Paul Erdös and Alfred Rényi discovered a complete random network model in the late 50s twentieth century, it is made of any two nodes which connected with probability p in the graph made of N nodes, its average degree is $<k> \ = p(N-1) \approx PN$; the average path length $l : \ln N / \ln(<k>)$; the convergence factor $C = P$; when the value of N is very large, the distribution of the node degree approximately equals poisson distribution. The foundation of the random network model is a significant achievement in the network researching, but it can hardly describe the actual property of the realworld, lots of new models are raised by other people.

1. Small-world networks

As the experiment puts, most of the realworld networks has small-world (lesser shortest path) and aggregation (larger convergence factor). However, the regular network has aggregation, but its average shortest path length is larger, random graph has the opposite property, having small-world and less convergence factor. So the regular networks and random networks can not reflect the property of the realworld, it shows that the realworld is not well-defined neither is complete random. Watts and Strogatz found a network which contains both small-world and high-aggregation in 1988, which is a great break in the complex network researching. They connected every side to a new node with probability p,through which they build a network between regular network and random network (calling WS net for short), it has less average path length and larger convergence factor, while the regular network and random network are special case when p is 0 and 1 in the WS net.

After the WS model being put forward, many scholars made a further change based on WS model, the NW small-world model raised by Newman and Watts has the most extensive use. The difference between NW model and WS model is that NW model connects a coupe of nodes, instead of cutting off the original edge in the regular network. The advantage of NW model is that the model simplifies the theory analysis, since the WS model may have orphan nodes which NW would not do. In fact, when p is few while N is large, the results of the theory analysis of the two models will be the same; we call them small-world model now.

2. The scale-free network

Although the scale-free network can describe the small-world and high-aggregation of the realworld well, the theory analysis of the small-world model reals that the distribution of the node is still the index distribution form. As the empirical results put it is more accurate to describe the most of the large-scale realworld model in the form of the power-law namely $p(k) : k^{-\gamma}$.

Compared with index distribution power-law has no peak, most nodes has few connection, while few nodes have lots of connection, there is no characteristic scale as the random network do, so Barabási and some other people call this network distribution having power rate characteristics Scale-free network. In order to explain the foundation of the Scale-free network, Barabási and Albert found the famous BA model, they thought the networks raised before did not consider the two important property of the realworld—growth property and connection optimization, the former means the new nodes are constantly coming into the network, the latter means after their arriving the new nodes prefer to connect the nodes with large degree. Not only do they make the simulation analysis of the generating algorithm of the BA model, but also it has given the analytic solution to the model using the way of the mean field in statistical physics, as the result put: after enough time of evolution, the distribution of BA network don't change with time, degree distribution is power-law with its index number 3 steadily.

Foundation of the BA model is another great breakout in the complex network research, demonstrating our further understanding of the objective network world. After that, many scholars made many improvements in the model, such as non-linearity priority connection, faster growth, and local events of rewind side, being aging, and adaptability competition and so on. Note that: most instead of all of the realworld is Scale-free network, for some realworld network's degree distribution is the Truncation form of the power-law.

Scholars also found some other network model such as local area world evolution model, Weight evolution network model and certainty network model to describe the network structure of the realworld besides small-world model and scale-free network.

20.4 The Application of the Complex Network Model

Study of the network structure is important, but the ultimate purpose is that we can understand and explain the system's modus operand based on these networks, and then we can forecast and control the behavior of network system. This systemic dynamical property based on network is generally called dynamical behavior, it involves so many things such as systemic transfusion, synchronization, phase change, web search and network navigator. The researched above has strong theoretical, a kind of research of network behavior which has strong applied has increasingly aroused our interests, for example the spread of computer virus on computer net, the spread of the communicable disease among multitude and the spread of rumours in society and so on, all of them are actually some propagation behavior obeying certain rules and spreading on certain net. The traditional network propagation models are always found based on regular networks, we have to review the issue with the further research of the complex networks. We emphatically introduce the research of the application.

One of the uppermost and foremost purposes of network propagation behavior research is that we can know the mechanism transmission of the disease well. Substitute node for the unit infected, if one unit can associate with another in infection or the other way round through some way, then we regard that the two units have connection, in this way can we get the topological structure of network propagation, the relevant propagation model can be found to study the propagation behavior in turn. Obviously, the key to network propagation model studying is the formulation of the propagation rule and the choice of the network topological structure.

However, it does not conform to the actual fact simply regarding the disease contact network as regular uniform connect network. Moore studied the disease propagation behavior in small-world, discovering that the propagation threshold value of disease in small-world is much less than it does in regular network, in the same propagation degree, experience the same time, the propagation scope of disease in the small-world is significantly greater than the propagation scope in the regular network, that is to say: compared to regular network, disease in the small-world inflects easily; Paster Satornas and others studied the propagation behavior in the scale-free world, the result turns out to be amazing: there is always positive propagation degree threshold value in both of regular world and small-world, while the propagation degree threshold value approves to be 0. We can get the similar results when analyzing the scale-free world.

As lots of experiments put realworld network has both small-world and scale-free, the conclusion described above is quite frustrated. Fortunately, no matter virus or computer virus they all has little infectious ($\lambda = 1$), doing little harm. However, once the intensity of disease or virus reaches some degree, we have to pay enough attention to it, the measurement to control it can not totally rely on the improvement of medical conditions, we have to take measures to quarantine the nodes and turn off the relevant connections in order to cut off avenue of infection in which we can we change the topological structure of the propagation network. In fact, just in this way can we defeat the war of fighting SARS in 2003 summer in our country.

The study of the disease's mechanism transmission is not all of the questions our ultimate goal is that we can master how to control disease propagation efficiently. While in practical applications, it is hard to stat the number of nodes namely the number of units which have possibilities connect with other nodes in infection period. For example in the research of STD spread, researchers get the information about psychopath and high risk group only through questionnaire survey and oral questioning, while their reply has little reliability, for that reason, quite a lot of immunization strategy have been put forward by some scholars based on above-mentioned opinion, such as "who is familiar with the immune", "natural exposure", "vaccination".

Analyzing disease spread phenomenon is not just the purpose of researching network propagation behavior; what is more a large amount of things can be analyzed through it. For example we can apply it to propagation behavior's research in social network, the basic ideas showed as follows: first we should abstract the topological structure of the social network out from complex network theory, then analyze the mechanism transmission according to some propagation rules, analyze how to affect the propagation through some ways at last. Actually, this kind of work has already started, such as the spread of knowledge, the spread of new product network and bank financial risk; they have both relation and difference, the purpose of the research of the former is to contribute to its spread; the latter is to avoid its spread.

Acknowledgments This work is supported by the Natural Science Foundation of Hubei Province, China (No. 2011CDC028), and the Key Project of Hubei Provincial Department of Education (No. D20122701 and No. D20102902), the Ph.D. Fund of Huanggang Normal University (No. 2011CD229 and to Cheng Wang).

References

1. Xu GZ (2000) Systems science. Shanghai Scientific and Technological Educational Publishing House Pearson Education, Shanghai
2. Albert R, Barabási AL (2002) Statistical mechanics of complex network. Rev Mod Phys 74:47–97
3. Newman MEJ (2003) The structure and function of complex networks. SIAM Rev 45:56–167