Digital Hologram Compression Using Correlation of Reconstructed Object Images

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Abstract. An efficient digital hologram compression algorithm is proposed using the correlation in the complex valued object image. While the pure values are almost uncorrelated, the magnitude values exhibit a strong correlation between the real and imaginary part object images. Therefore, we adaptively employ the encoding result of one image to encode another image. Both images are first wavelet transformed and the wavelet coefficients are encoded using the SPIHT method. We used the significance encoding result of the real part image as the contexts of arithmetic coder for encoding the imaginary part image. Experimental results demonstrate that the proposed algorithm yields a better compression performance than the conventional method.

Keywords: Digital hologram, digital hologram compression, context-adaptive arithmetic coding.

1 Introduction

Three-dimensional (3D) images facilitate more realistic and immersive visual experiences, and therefore have drawn much attention in recent years. Holography is considered as one of the most promising techniques for 3D image representation, since it is free from eye fatigue and viewpoint constraints. Fig. 1 shows a typical principle of holography [7]. A light wave is split into two parts such that one directly travels to the recording medium as a reference light, and the other illuminates a 3D object. The interference pattern between the reference light and the reflected light on the 3D object is recorded, which is called hologram. When the hologram is illuminated with the reference light, the virtual object image is reconstructed.

With the aid of high resolution image sensors and high performance computers, holograms can be stored and processed in digital form [8,13,6,12]. To be specific, an interference pattern is captured by a CCD (charged coupled device) sensor and stored as a digital image which is called digital hologram. In addition, the object image can be numerically calculated with computers based on the mathematical modeling of the reconstruction process [12]. The numerically reconstructed image yields the artifacts such as the bright region of undiffracted
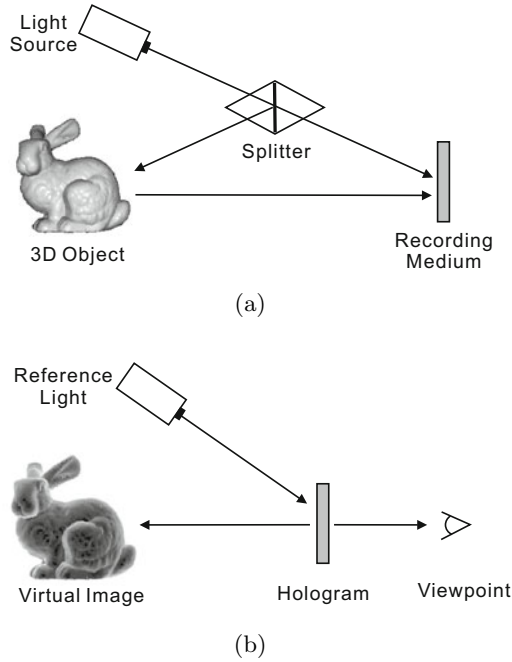
(a)

(b)

Fig. 1. Principle of holography. (a) Hologram recording and (b) image reconstruction

light wave and the conjugate object image. To remove such drawbacks, phase-shifting digital holography was developed which employs multiple interference patterns to directly extract the complex amplitude of the object wave [17].

In order to sufficiently sample the detailed shape of interference pattern, digital hologram is generally stored as a high resolution image which requires a huge amount of storage space. Therefore, a lot of research has been performed to efficiently compress the digital holograms [10,15,16,4,5,3]. Various entropy coding methods were used to losslessly compress the digital holograms [10]. Shortt *et al.* transformed the phase-shifting digital holograms using wavelets and encoded the quantized wavelet coefficients [15]. A nonuniform quantization method was also proposed based on the histogram of digital hologram [16]. While the most algorithms have directly compressed the digital hologram data, Darakis and Soraghan developed the compression algorithms for reconstructed images [4,5,3]. The wavelet-like Fresnelet transform is used and the transformed coefficients of the reconstructed image are encoded [4]. The compression performance of the digital hologram was also compared to that of the reconstructed image [5,3]. Note that, progressive transmission can be facilitated in [5] and [3], since the embedded bitstreams are derived by using the SPIHT (set partitioning in hierarchical trees) method [11].

However, the previous compression techniques did not exploit the correlation in the complex valued reconstructed wave field from a digital hologram. In this paper, we investigate the correlation between the real and imaginary part data of

the complex valued object image, and propose an efficient compression algorithm for digital holograms. It is observed that the magnitude images of the real and imaginary parts are highly dependent. Therefore we use the encoding result of the real part image as contexts to adaptively encode the imaginary part image based on the context-adaptive arithmetic coding. Experimental results demonstrate that the proposed algorithm exhibits a better rate-distortion performance than the existing method to compress digital holograms.

This paper is organized as follows. Section 2 describes the digital hologra-phy. The correlation of digital hologram is investigated in Section 3. Section 4 explains the coding algorithm of digital hologram data. Section 5 presents the experimental results, and finally Section 6 concludes this paper.

2 Digital Holography

Fig. 2 illustrates a coordinate system in digital holography, where the digital hologram is captured in the hologram plane (or camera plane) and the recon-structed image is calculated in the plane of distance d from the hologram plane. Let $\mathcal{U}(x,y)$ be the complex amplitude of the reflected object wave in the holo-gram plane, which is represented by

$$\mathcal{U}(x,y) = a_u(x,y)\exp\{i\phi_u(x,y)\},$$

where $a_u(x,y)$ and $\phi_u(x,y)$ denote the real amplitude and the phase, respec-tively. Similarly, the reference light wave is given by

$$\mathcal{V}(x,y) = a_v(x,y)\exp\{i\phi_v(x,y)\}.$$

The intensity of the interference pattern of $\mathcal{U}(x,y)$ and $\mathcal{V}(x,y)$ is the scaled version of digital hologram, which is calculated as
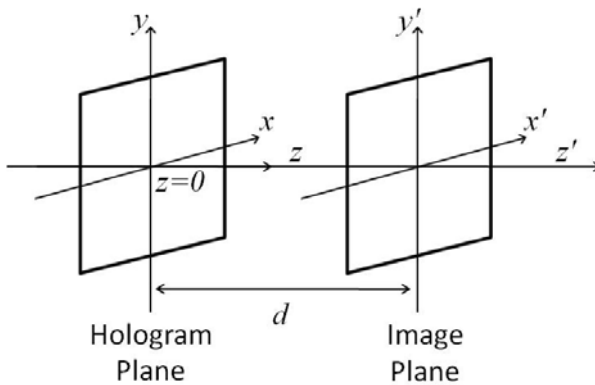
$$I(x,y) = |\mathcal{U}(x,y) + \mathcal{V}(x,y)|^2$$

Fig. 2. Coordinate system in digital holography

$$= (\mathcal{U}(x,y) + \mathcal{V}(x,y)) \left(\mathcal{U}(x,y) + \mathcal{V}(x,y)\right)^*$$
$$= a_u^2(x,y) + a_v^2(x,y) + \mathcal{U}(x,y)\mathcal{V}^*(x,y) + \mathcal{V}(x,y)\mathcal{U}^*(x,y), \qquad (1)$$

where $*$ means the complex conjugate.

When reconstructing an image from the digital hologram, $I(x,y)$ is multiplied by the reference wave as

$$\mathcal{V}(x,y)I(x,y) = \left(a_v^2(x,y) + a_u^2(x,y)\right)\mathcal{V}(x,y) + a_v^2(x,y)\mathcal{U}(x,y) + \mathcal{V}^2(x,y)\mathcal{U}^*(x,y).$$

The first term denotes the undiffracted component passing through the hologram. $a_v^2(x,y)\mathcal{U}(x,y)$ and $\mathcal{V}^2(x,y)\mathcal{U}^*(x,y)$ mean the virtual object image and the distorted real object image, respectively. In practice, as shown in Fig. 2, the complex amplitude $\mathcal{W}(x',y')$ in the image plane is reconstructed by propagating $\mathcal{V}(x,y)I(x,y)$ from the hologram plane to the image plane via the Fresnel-Kirchhoff integral in (2), which models the diffraction of light wave [14,12].

$$\mathcal{W}(x',y') = \frac{i}{\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{V}(x,y)I(x,y)\frac{\exp\left(-i\frac{2\pi}{\lambda}D\right)}{D}dxdy, \qquad (2)$$

where λ is the wavelength of the reference light and $D = \sqrt{(x-x')^2 + (y-y')^2 + d^2}$. Equation (2) is numerically approximated by

$$\mathcal{W}(x',y') = \frac{i}{\lambda d} \exp\left(-i\frac{2\pi d}{\lambda}\right) \exp\left\{-i\frac{\pi}{\lambda d}\left(x'^2 + y'^2\right)\right\}$$
$$\times \iint_{-\infty}^{\infty} \mathcal{V}(x,y)I(x,y) \exp\left\{\frac{-i\pi}{\lambda d}(x^2+y^2)\right\} \exp\left\{\frac{i2\pi}{\lambda d}(xx'+yy')\right\} dxdy, \quad (3)$$

which is called Fresnel transform. Moreover, the digitized version of Fresnel transform can be efficiently calculated by using the fast Fourier transform [14,12].

3   Correlation in Digital Hologram

The complex valued image $\mathcal{W}(x',y')$ of 3D object is reconstructed from a digital hologram using the diffraction of light waves via (2) or (3). Therefore, $\mathcal{W}(x',y')$ is composed of the real part image $Re(\mathcal{W}(x',y'))$ and the imaginary part image $Im(\mathcal{W}(x',y'))$,

$$\mathcal{W}(x',y') = Re(\mathcal{W}(x',y')) + iIm(\mathcal{W}(x',y')).$$

Fig. 3 (a) represents the 'Brahms' hologram. Fig. 3 (b) and (c) show the magnitude images of the corresponding $Re(\mathcal{W}(x',y'))$ and $Im(\mathcal{W}(x',y'))$, respectively. While the digital hologram yields a noise-like pattern and thus does not directly represent the shape of captured object, the reconstructed image exhibits a relatively higher spatial correlation by showing the shape of object. Therefore, a more improved compression performance can be achieved on the reconstructed images than the digital holograms [5,3]. Moreover, as observed in Fig. 3 (b) and
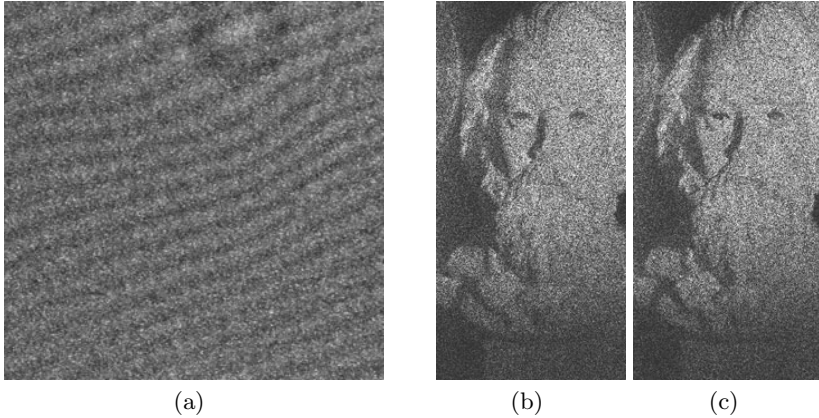
(a)   (b)   (c)

Fig. 3. Digital hologram and the reconstructed object images. (a) The 'Brahms' digital hologram. (b) Magnitude of the real part object image. (c) Magnitude of the imaginary part object image.

(c), the real and imaginary part images also yield the similar magnitude values each other, at the corresponding pixels.

In order to quantitatively investigate the dependence between $Re(\mathcal{W}(x', y'))$ and $Im(\mathcal{W}(x', y'))$, we measure the Pearson correlation coefficient

$$\rho(R, I) = \frac{E\left[R - E[R]\right] E\left[I - E[I]\right]}{\sqrt{E\left[(R - E[R])^2\right] E\left[(I - E[I])^2\right]}} \qquad (4)$$

where R and I are the random variables of $Re(\mathcal{W}(x', y'))$ and $Im(\mathcal{W}(x', y'))$, respectively. In general, $\rho(R, I) \approx 0$. It means that the pure values of $Re(\mathcal{W}(x', y'))$ and $Im(\mathcal{W}(x', y'))$ are almost uncorrelated each other, yielding different patterns of speckles. However, it is empirically observed that the magnitude values of $|Re(\mathcal{W}(x', y'))|$ and $|Im(\mathcal{W}(x', y'))|$ have a relatively strong correlation, for example 0.6 in the 'Brahms' hologram.

The similar characteristics are also observed in the wavelet transformed images. Fig. 4 (a) and (b) represent the magnitude images of the wavelet coefficients of $Re(\mathcal{W}(x', y'))$ and $Im(\mathcal{W}(x', y'))$, respectively, which are reconstructed from the 'Copper screw' hologram. The correlation coefficient for the wavelet coefficients is 0.53.

4   Digital Hologram Compression

4.1   Progressive Coding

One of the state-of-the-art compression algorithms of digital holograms was developed on the reconstructed images using the wavelet-like transform and the SPIHT coding method [3]. We also apply the wavelet transform to the real part and the imaginary part of the complex valued reconstructed image individually,
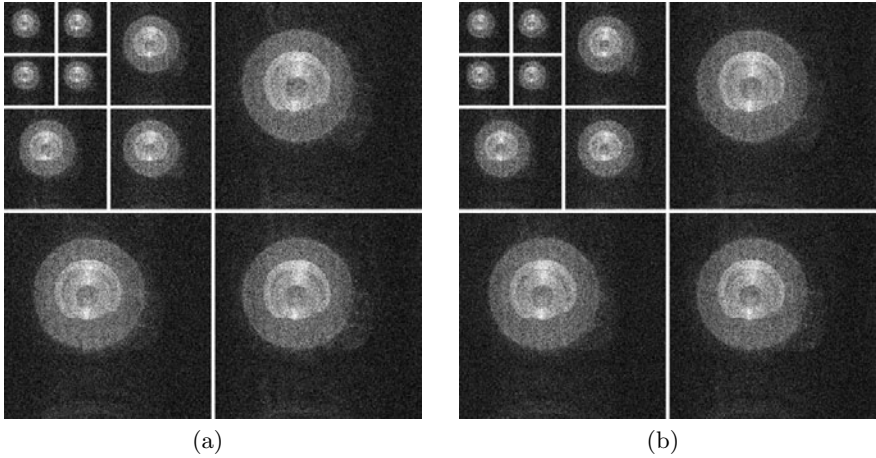
Fig. 4. Magnitude of wavelet coefficients in the object image reconstructed from the 'Copper screw' digital hologram. (a) Real part image. (b) Imaginary part image.

using the 9/7 filters [2]. The wavelet coefficients are uniformly quantized with the same step size, and then progressively encoded using the SPIHT method [11]. As shown in Fig. 4, the reconstructed image of a digital hologram is relatively sparse in the sense of significant pixels when compared with the general images, since it represents the sectional image of 3D objects focused on a specific depth. As a result, a coding gain can be expected by encoding a group of pixels together in the reconstructed image.

4.2   Context-Adaptive Coding

For each image of the real and imaginary parts, bitplanes of the quantized wavelet coefficients are progressively encoded one by one from the most significant bitplane (MSB) to the least one based on the SPIHT method. Let $c(x, y)$ be the quantized wavelet coefficient at the pixel coordinate (x, y). For a given bth MSB, the significance coding is performed. If $c(x, y)$ first becomes significant, i.e. $2^b \leq |c(x, y)| < 2^{b+1}$, then the significance bit $S_b(c(x, y)) = 1$ is encoded and the sign bit is encoded. If $c(x, y)$ is insignificant as $|c(x, y)| < 2^b$, then $S_b(c(x, y)) = 0$ is encoded. When $c(x, y)$ is already significant at the previous bitplane such that $|c(x, y)| \geq 2^{b+1}$, the magnitude refinement bit is encoded. The resulting bit sequence is entropy encoded using the arithmetic coder [9].

The significance of wavelet coefficient is directly related to the magnitude which yields a strong correlation between the real and imaginary part images. It means that if the coefficient $c_{real}(x, y)$ in the real part image is significant (or insignificant) at a given bitplane, the coefficient $c_{imag}(x, y)$ in the imaginary part image is highly probable to be significant (or insignificant) at the same bitplane. Therefore, the result of significance coding for one image can be used to predict the significance of another image. In practice, $S_b(c_{real}(x, y))$ is first initialized to be '0' for all b, and then updated while encoding $c_{real}(x, y)$. When encoding the

imaginary part image, if $S_b(c_{real}(x,y)) = 1$, then $S_b(c_{imag}(x,y))$ is arithmetic encoded with the context A_{pixel}, otherwise with the context B_{pixel}. Furthermore, we employ the different contexts according to the frequency subbands of wavelet decomposition, since the correlation coefficient between the real and imaginary part images exhibits different characteristics in each subband.

In addition, the SPIHT method improves the coding gain by encoding the significance of a group of coefficients together. Let $G(x,y)$ be the group of all pixel coefficients belonging to the hierarchical tree derived from the coordinate (x,y). If all the coefficients in $G(x,y)$ are insignificant at the bth MSB, then the significance bit $S_b(G(x,y)) = 0$ is encoded. Otherwise $S_b(G(x,y)) = 1$ is encoded, and the group $G(x,y)$ is divided into the smaller sub-groups. The significance coding is iteratively performed on each sub-group until the coordinates of significant coefficients are encoded. The group $G_{real}(x,y)$ in the real part image and the group $G_{imag}(x,y)$ in the imaginary part image are derived from the same spatial region, and thus yield the similar characteristics of significance. Therefore, we employ the other contexts A_{group} and B_{group} to adaptively encode $S_b(G_{imag}(x,y))$, according to the result of $S_b(G_{real}(x,y))$ which is updated during the encoding of the real part image.

5   Experimental Results

The performance of the proposed algorithm is evaluated using the three digital holograms, 'Brahms', 'Dice', and 'Copper screw', whose reconstructed images are shown in Fig. 5. Table 1 provides the capturing conditions for these digital holograms. The distortion of the reconstructed image is measured by the normalized root mean square (NRMS) error [3], and given by

$$\sqrt{\frac{\sum_x \sum_y \left(|\mathcal{W}(x,y)|^2 - |\tilde{\mathcal{W}}(x,y)|^2 \right)^2}{\sum_x \sum_y \left(|\mathcal{W}(x,y)|^2 \right)^2}} \tag{5}$$

where $\mathcal{W}(x,y)$ and $\tilde{\mathcal{W}}(x,y)$ denote the original image and the decoded one from the compressed bitstream, respectively.

Fig. 6, Fig. 7, and Fig. 8 show the rate and distortion compression performances for the imaginary part object images of the 'Brahms', 'Dice', and 'Copper screw' holograms, respectively. The dashed lines exhibit the results of the conventional coding method [3] and the solid lines represent the results of the proposed context-adaptive coding method. In order to compare the performances from only the coding schemes, we also used the 9/7 filters [2] to get the wavelet coefficients in the conventional method. It is observed that the proposed compression algorithm provides a better rate-distortion performance compared with the conventional method which separately encodes the real and imaginary part images. The bitrate reduction is over 20% especially at low bitrates. For example, while the conventional method consumes 11417 bytes to encode the imaginary part image to provide the distortion 7.65×10^{-2} for the 'Brahms' hologram, the proposed algorithm only requires 8863 bytes to achieve the same distortion.

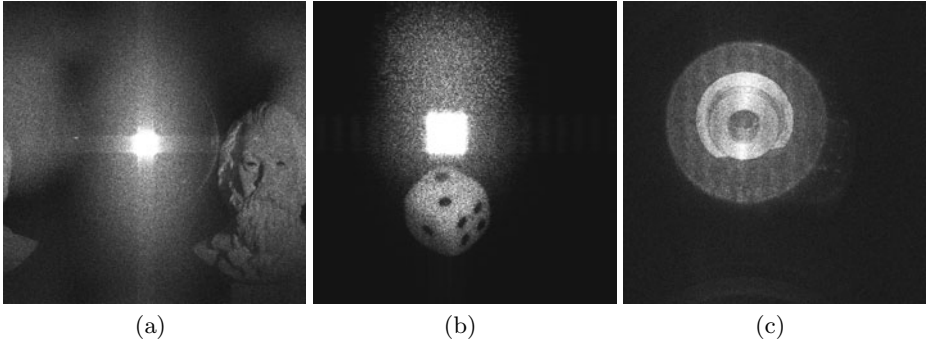(a)   (b)   (c)

Fig. 5. Reconstructed images of the test digital holograms. (a) Brahms (Courtesy of the HoloVision project [1]). (b) Dice (Courtesy of Dr. U. Schnars). (c) Copper screw (Courtesy of Dr. F. Zhang). The 'Brahms' and the 'Dice' are the classic digital holograms and the 'Copper screw' is the phase-shifting digital hologram.

Table 1. Capturing conditions of digital holograms

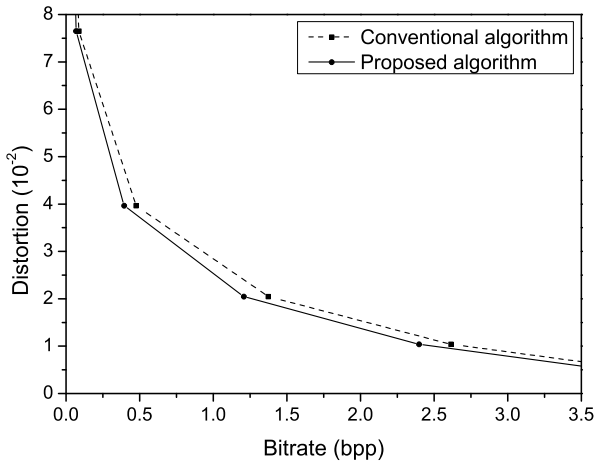|	Brahms	Dice	Copper screw
Type	classic	classic	phase shifting
Image resolution	1024 × 1024	1024 × 1024	1024 × 1024
Pixel size	6.8 μm	6.8 μm	6.45 μm
Wavelengh of reference light	632.8 nm	632.8 nm	830 nm
Captured distance	1290 mm	1054 mm	285 mm

Fig. 6. Rate and distortion curve of the proposed algorithm compared with the conventional method. The rate means the average bits per pixel to encode the imaginary part object image reconstructed from the 'Brahms' digital hologram.
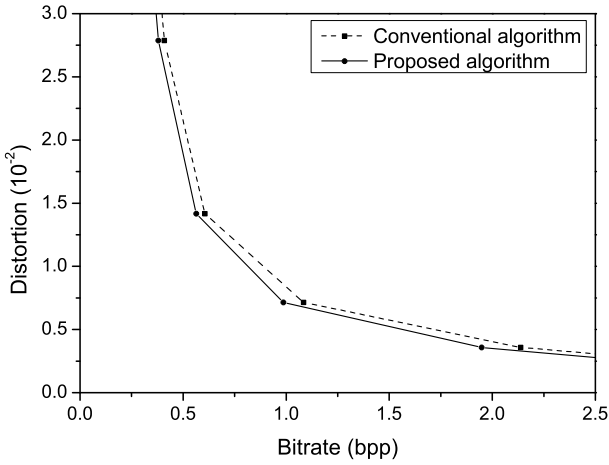
Fig. 7. Rate and distortion curve of the proposed algorithm compared with the conventional method. The rate means the average bits per pixel to encode the imaginary part object image reconstructed from the 'Dice' digital hologram.
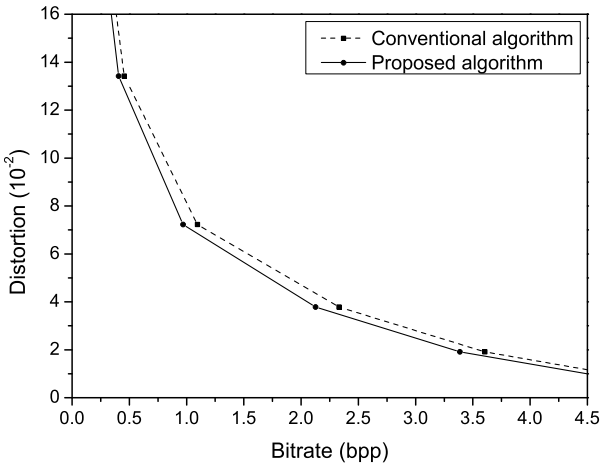

Fig. 8. Rate and distortion curve of the proposed algorithm compared with the conventional method. The rate means the average bits per pixel to encode the imaginary part object image reconstructed from the 'Copper screw' digital hologram.

6   Conclusion

In this paper, we proposed an adaptive compression algorithm for digital holograms using the correlation in the reconstructed images. The object image reconstructed from a digital hologram is inherently a complex wave field, and thus composed of the real and imaginary parts. The real and imaginary part images are wavelet trasnformed and the transform coefficients are progressively encoded using the SPIHT method. The two images yield a relatively strong correlation each other, which can be exploited to improve the compression performance for digital holograms. Specifically, the result of significance coding for the real part image is employed as contexts of arithmetic coder to encode the imaginary part image. Experimental results showed that the proposed context-adaptive compression algorithm for digital holograms improved the coding gain compared with the conventional state-of-the-art method.

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