

# Comparison between Mathematical Complexity and Human Feeling

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**Abstract.** Recently, we have often the opportunity to shop for something on the computer display connected to the Internet. However, it is often said that the product arrived at the home is something different from it presented on the computer display. We have studied into the differences between the shape evaluation in virtual space and in real space. And it is indicated that the shape evaluation in virtual space is slightly difference from it in real space, especially, in case of evaluation of product which has the complex contour[1]. In this research, we are focusing on the complexity of contour. The complexity is defined by mathematical methods, for example, Hurst exponent in fractal geometry. In this paper, we have proposed the algorithm which makes the curve lines based on the Hurst exponent. And using these curve lines made by proposed method, we have investigated that the mathematical complexity is equal to the complexity which people feel by sensory evaluation or not. The result shows that Hurst exponent almost can show the complexity which people feel, however, it is difficult in case of the cyclic curves.

**Keywords:** Curve complexity, Sensory evaluation, Bradley-Terry model.

## 1 Introduction

A complexity of curve is defined by mathematical method absolutely. Hurst exponent in fractal geometry can indicate the complexity as a number which is between 0 and 1. On the other hand, the complexity of curve by human feeling cannot be defined absolutely, but relatively. In this paper, we try to compare the human feeling complexity to mathematical complexity by sensory evaluation and statistical analysis. And we have tried to define the human feeling complexity as an absolute number.

First of all, we propose the curve generation method based on Hurst exponent. Then, some curves were made by the proposed method. Using these curves, the sensory evaluations by paired comparison were performed. To compare the mathematical complexity to human feeling complexity, we applied the Bradley-Terry model to the result of paired comparison. The model enables to determine the complexity based on

the paired comparison. Finally, we tried to define the human feeling complexity as an absolute number.

## 2 Curve Generating Algorithm Based on Hurst Exponent

### 2.1 Hurst Exponent

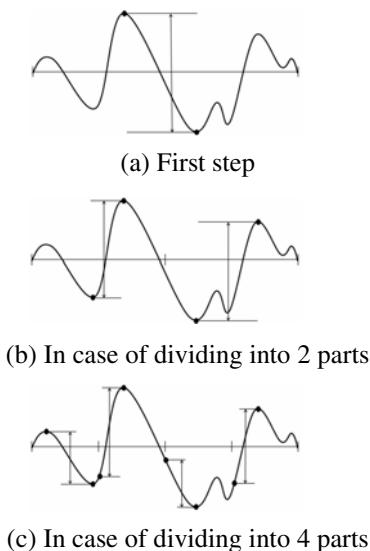
Complexity is able to be defined by Hurst exponent from mathematical viewpoint. Hurst exponent can be obtained by some methods. In this study, the scale transformation analysis has been used to define the complexity. Hurst exponent ( $He$ ) is shown by the expression as follows:

$$He = m - D \quad (1)$$

Here,  $m$  shows the Dimension [ $m = 2$  in this research],  $D$  shows the fractal dimension [ $1 < D < 2$ ]. Generally, it is said that an object which has the small Hurst exponent is complex.

The scale transformation analysis pays attention to the maximum fluctuation of the curved line. The curved line divided into  $2^{n-1}$  parts (See. Fig.1), and the maximum fluctuation of each part is measured from minimum value and maximum value of the part. The average of maximum fluctuation of  $2^{n-1}$  parts and  $n$  is plotted on the log-log graph, and the slope of the regression line is the Hurst exponent.

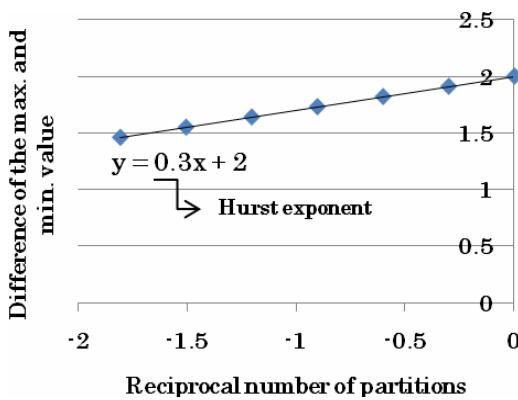
For example, to obtain the Hurst exponent of the curve shown in Fig.2, Table 1 shows the averages of fluctuations between the min. and max. values. Fig.3 shows the



**Fig. 1.** How to obtain Hurst exponents

**Fig. 2.** Example of curve**Table 1.** Parameters of curve in Fig.2

Partitions	Reciprocal number of partitions(log10)	Average of difference between max. and min. value(log10)
1 part	0	2
2 parts	-0.301	1.909716
4 parts	-0.602	1.819412
8 parts	-0.903	1.729094
16 parts	-1.204	1.638795
32 parts	-1.505	1.548497
64 parts	-1.806	1.458196

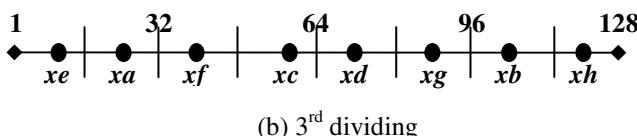
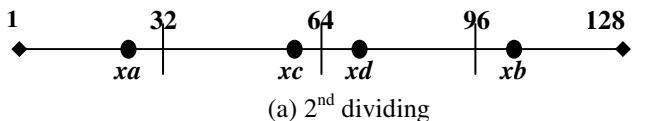
**Fig. 3.** How to obtain Hurst exponent

log-log graph in which the data from Table 1 are plotted on. Then, the slope of the regression line is the Hurst exponent.

## 2.2 Curve Generation Algorithm

We are proposing the method to generate a curve based on the Hurst exponent. The procedure for generating the curve  $f(x)$  which is consisting of 128 points is as follows.

( i ) Firstly, 2 points from 128 are chosen randomly. Then, one point  $xa$  is chosen from 1-64 and the other one  $xb$  is done from [65-128]. These will be the points that indicate global maximum and minimum values of the curve. Secondly, dividing the 128 points into two parts, one is consisting of 1-64 and the other is consisting of 65-128. If  $f(xa)$  is global maximum value and  $xa$  is smaller than 32, the point  $xc$  is chosen from [33-64] randomly and  $f(xc)$  is second minimum value and  $f(xc)$  is larger than  $f(xb)$ . When  $f(xa)$  is global minimum value,  $f(xc)$  is second maximum value at [33-64] and  $f(xc)$  is smaller than  $f(xa)$ . The point  $xd$  is also chosen from [65-128] which is shown in Fig.4(a) and Fig.4(b). In this way, 64 points are chosen from 128 points randomly after dividing into 32 parts. Then we can obtain the 64 points which take charge of maximum or minimum values.



**Fig. 4.** How to choose max. and min. points

( ii ) The global maximum and minimum value of creating curve is restricted to 100 and 0. The average of difference between the maximum and minimum values for each part,  $\text{average}(n-1)$  which is divided  $n-1$  times can be obtained from Hurst exponent. When  $f(xa)$  is maximum value (=100) and  $f(xb)$  is minimum value (=0). Then  $f(xc)$  and  $f(xd)$  are restricted as follows:

$$\begin{aligned} f(xa) - f(xb) &= \text{average}(0) = 100 \\ \{f(xa) - f(xc) + f(xd) - f(xb)\}/2 &= \text{average}(1) \\ f(xd) - f(xc) &= 2\text{average}(1) - \text{average}(0) \end{aligned} \quad (2)$$

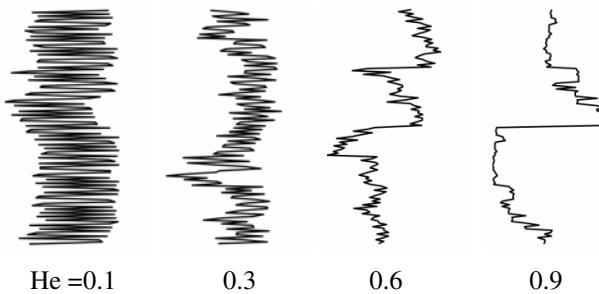
Here,  $f(xa) > f(xd) > f(xc) > f(xb)$

Secondly,  $f(xe)$  to  $f(xh)$  are restricted as follows:

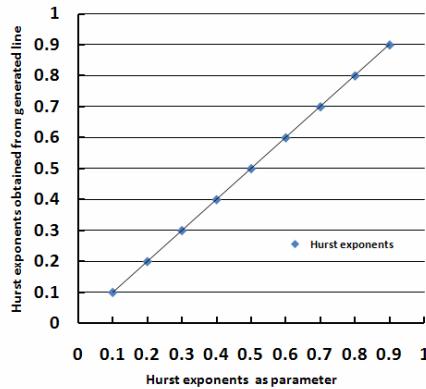
$$\begin{aligned}
 & \{ f(xa) - f(xe) + f(xf) - f(xc) + f(xd) - f(xg) + f(xh) - f(xb) \} / 4 \\
 & = \text{average}(2) - f(xe) + f(xf) - f(xg) + f(xh) \\
 & = 4\text{average}(2) - \text{average}(0) - 2\text{average}(1) + \text{average}(0) \\
 & = 4\text{average}(2) - 2\text{average}(1)
 \end{aligned} \tag{3}$$

Here,  $f(xa) > f(xf) > f(xe) > f(xc)$  and  $f(xd) > f(xh) > f(xg) > f(xb)$

Then, maximum and minimum values in each part are decided by the same way as above. Some generated curves by using the proposed system are shown in Fig.5. And, Fig.6 shows the Hurst exponents of generated curve based on the specific it. From this figure, the proposed method can supply curves based on the Hurst exponents.



**Fig. 5.** Examples of generated curves



**Fig. 6.** Relation between Hurst exponents of curves and parameters

### 3 Human Feeling Complexity

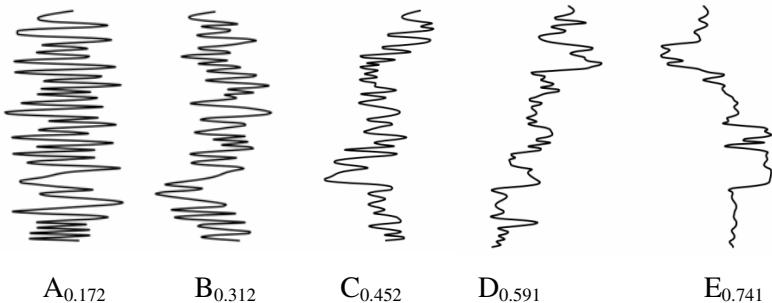
#### 3.1 Experimental Object

It is difficult to determine the human feeling complexity for a curve absolutely. Therefore, the sensory evaluation by paired comparison using some generated

curves made by the proposed method, and relative complexities are resolved by statistical analysis.

### 3.2 Experimental Method

The curves as shown in Fig.7 were used in the sensory evaluation by paired comparison. Two out of five curves were presented to a subject at the same time, and the subject watched the curves. Then the subject was urged to answer which curve he or she felt more complex. The subjects are 20 students in their twenties.



**Fig. 7.** Generated curves used in sensory evaluation

### 3.3 Experimental Method

Table 2 shows the results of paired comparison. The number in each table shows that of subjects who felt the line curve was more complex than the row curve.

**Table 2.** Result of paired comparison

	A <sub>0.172</sub>	B <sub>0.312</sub>	C <sub>0.452</sub>	D <sub>0.591</sub>	E <sub>0.741</sub>	Total
A <sub>0.172</sub>		12	13	11	11	47
B <sub>0.312</sub>	8		15	14	18	55
C <sub>0.452</sub>	7	5		12	18	42
D <sub>0.591</sub>	9	6	8		16	39
E <sub>0.741</sub>	9	2	2	4		17

The Bradley-Terry model is assumed to evaluate the complexities of curves quantitatively, defined as follows [2];

$$P_{ij} = \frac{\pi_i}{\pi_i + \pi_j} \quad (4)$$

$$\sum_i \pi_i = const.(=100) \quad (5)$$

Where  $\pi_i$  : intensity of  $i$ ,

$P_{ij}$  : probability of judgment that  $i$  is more complexity than  $j$ .

$\pi_i$  shows the intensity of complexity of the curve  $i$ . The model enables to determine the complexity based on the paired comparison. Here, the maximum likelihood method is used to solve the  $\pi$ . We obtain  $\hat{\pi}_i$  by the following formula where  $\hat{\pi}_i^0 (i=1,2,3)$  were used as initial values,

$$\hat{\pi}_i = \frac{T_i}{\sum_{j(\neq i)} \frac{N}{\hat{\pi}_i^0 + \hat{\pi}_j^0}} \quad (6)$$

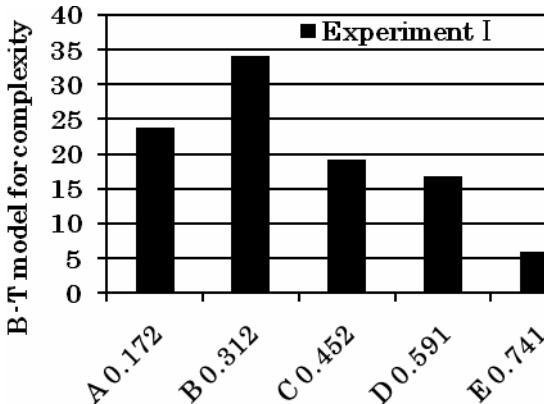
$N$  is the number of objects, and  $T_i$  is the total number of  $i$ 's win. Then  $\hat{\pi}_i$  is scaled up or down to satisfy the next formula.

$$\sum_i \hat{\pi}_i = K \quad (7)$$

where  $K$  is 100.

$$\hat{\pi}_i^1 = \frac{K \hat{\pi}_i}{\sum_i \hat{\pi}_i} \quad (8)$$

We iterated the series of calculation until  $\hat{\pi}_i$  was settled. The result of B-T models is shown in Fig. 8.

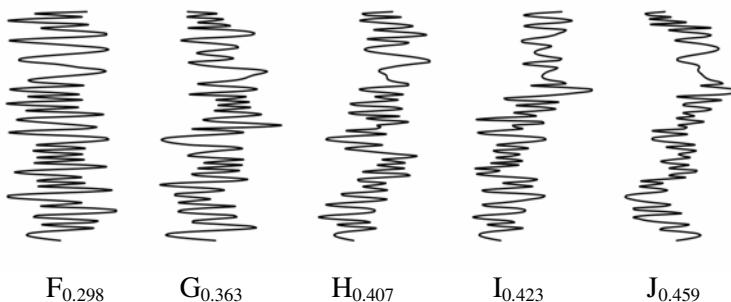


**Fig. 8.** Bradley-Terry model for Experience I

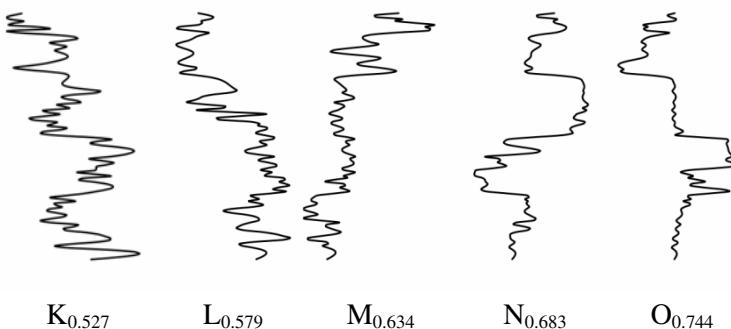
However, the curve A is more complex than B from the viewpoint of mathematics, the subjects tend to feel that B is more complex than A. This indicates that there is possibility of disagreement between the human feeling complexity and the mathematical complexity.

### 3.4 Experimental II

As a result of Experiment I, there is possibility that there is the disagreement between the human feeling and mathematical complexity when the Hurst exponents are small comparatively. Therefore, two experiments were performed in which Experiment II-1 used the curves which have small ( $<0.5$ ) Hurst exponents, and Experiment II-2 used the curves which have large ( $>0.5$ ) Hurst exponents comparatively. Fig.9 and 10 show the curves and their Hurst exponents used in Experiment II-1 and II-2.



**Fig. 9.** Generated curves used in Experiment II-1



**Fig. 10.** Generated curves used in Experiment II-2

The sensory evaluation by paired comparison was performed as same as Exp.I. Table 3 shows the results of paired comparison in Experiment II-1 and II-2. The Bradley-Terry models are also assumed to evaluate the complexities of curves quantitatively, and these results are shown in Fig. 11.

As same as the result of Experiment I, there are some disagreements between the human feeling and mathematical complexity in both Experiment II-1 and II-2. However the difference between them in Experiment II-2 is slight, it between them in Experiment II-1 is not so small.

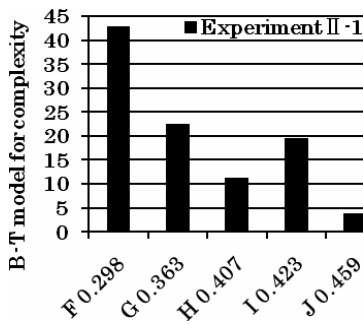
**Table 3.** Result of sensory evaluation in Experiment II

(a) paired comparison in Experiment II-1

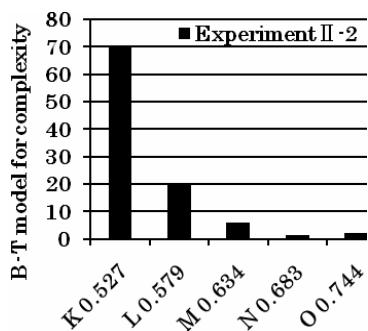
	F <sub>0.298</sub>	G <sub>0.363</sub>	H <sub>0.407</sub>	I <sub>0.423</sub>	J <sub>0.459</sub>	Total
F <sub>0.298</sub>		17	15	13	16	61
G <sub>0.363</sub>	3		15	13	17	48
H <sub>0.407</sub>	5	5		6	17	33
I <sub>0.423</sub>	7	7	14		17	45
J <sub>0.459</sub>	4	3	3	3		13

(b) paired comparison in Experiment II-2

	K <sub>0.527</sub>	L <sub>0.579</sub>	M <sub>0.634</sub>	N <sub>0.683</sub>	O <sub>0.744</sub>	Total
K <sub>0.527</sub>		17	18	19	19	73
L <sub>0.579</sub>	3		18	19	17	57
M <sub>0.634</sub>	2	2		18	15	37
N <sub>0.683</sub>	1	1	2		8	12
O <sub>0.744</sub>	1	3	5	12		21



(a) Bradley-Terry model for Exp. II-1

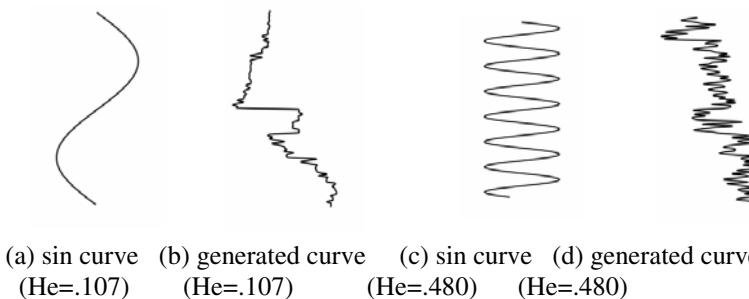


(b) Bradley-Terry model for Exp.II-2

**Fig. 11.** Bradley-Terry models for Experiment II

## 4 Discussions

Some subjects pointed out the curves which they felt the periodic round were not felt so complex. Fig. 12 shows some generated curves which have same Hurst exponents as the sin curve. It is obvious that generated curves are felt more complex than sin curves. It is indicated that the curve which has periodic round is not so complex as its Hurst exponents by human feeling.



**Fig. 12.** Sin curve and generated curve which have same Hurst exponent

## 5 Conclusions

A complexity of curve is defined by mathematical method absolutely. Hurst exponent in fractal geometry can indicate the complexity of a curve as a number which is between 0 and 1. On the other hand, the complexity of curve by human feeling cannot be defined absolutely, but relatively. In this paper, we have tried to compare the human feeling complexity to mathematical complexity by sensory evaluation and statistical analysis.

As a result of experiment, there is possibility that there is the disagreement between the human feeling and mathematical complexity when the Hurst exponents are small comparatively. Moreover, the periodicity of curve may effects on the human feeling complexity.

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