A New Algorithmic Approach for Contrast Enhancement

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Abstract. A novel algorithmic approach for optimal contrast enhancement is proposed. A measure of expected contrast and a sister measure of tone subtlety are defined for gray level transform functions. These definitions allow us to depart from the current practice of histogram equalization and formulate contrast enhancement as a problem of maximizing the expected contrast measure subject to a limit on tone distortion and possibly other constraints that suppress artifacts. The resulting contrast-tone optimization problem can be solved efficiently by linear programming. The proposed constrained optimization framework for contrast enhancement is general, and the user can add and fine tune the constraints to achieve desired visual effects. Experimental results demonstrate clearly superior performance of the new technique over histogram equalization.

1 Introduction

The contrast of a raw image can be far less than ideal, due to various causes such as poor illumination conditions, low quality inexpensive imaging sensors, user operation errors, media deterioration (e.g., old faded prints and films), etc. For better and easier human interpretation of images and higher perceptual quality, contrast enhancement becomes necessary and it has been an active research topic since early days of computer vision and digital image processing.

Contrast enhancement techniques can be classified into two approaches: context-sensitive or point-wise enhancers and context-free or point enhancers. In context-sensitive approach the contrast is defined in terms of the rate of change in intensity between neighboring pixels. The contrast is increased by directly altering the local waveform on a pixel by pixel basis. For instance, edge enhancement and high-boost filtering belong to the context-sensitive approach. Although intuitively appealing, the context-sensitive techniques are prone to artifacts such as ringing and magnified noises, and they cannot preserve the rank consistency of the altered intensity levels. The context-free contrast enhancement approach, on the other hand, does not adjust the local waveform on a pixel by pixel basis. Instead, the class of context-free contrast enhancement techniques adopt a statistical approach. They manipulate the histogram of the input image to separate the gray levels of higher probability further apart from the neighboring gray levels. In other words, the context-free techniques aim to increase

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the average difference between any two altered input gray levels. Compared with its context-sensitive counterpart, the context-free approach does not suffer from the ringing artifacts and it preserves the relative ordering of altered gray levels. This paper is mainly concerned with a rigorous problem formulation for context-free contrast enhancement, and accordingly it develops a general optimization framework to solve the problem.

Despite more than half a century of research on contrast enhancement, most published techniques are largely ad hoc. Due to the lack of a rigorous analytical approach to contrast enhancement, histogram equalization seems to be a widely accepted synonym for contrast enhancement in the literature and in textbooks of computer vision and image processing. The justification of histogram equalization as a contrast enhancement technique is heuristic, catering to an intuition. Low contrast corresponds to a biased histogram and thus can be rectified by reallocating underused dynamic range of the output device to more probable pixel values. Although this intuition is backed up by empirical observations in many cases, the relationship between histogram and contrast has not been precisely quantified.

There is no mathematical basis for the uniformity or near uniformity of the processed histogram to be an objective of contrast enhancement in general sense. On the contrary, histogram equalization can be detrimental to image interpretation if carried out mechanically without care. In lack of proper constraints histogram equalization can over shoot the gradient amplitude in some narrow intensity range(s) and flatten subtle smooth shades in other ranges. It can bring unacceptable distortions to image statistics such as average intensity, energy, and covariances, generating unnatural and incoherent 2D waveforms. To alleviate these shortcomings, a number of different techniques were proposed to modify the histogram equalization algorithm $[1, 2, 3, 4, 5, 6]$. Very recently, Arici *et al.* proposed a histogram modification technique that first finds a histogram $\mathbf{h}$ in between the original input histogram $\mathbf{h}_i$ and the uniform histogram $\mathbf{u}$ and then performs histogram equalization of $\mathbf{h}$. The intermediate histogram h is determined by minimizing a weighted distance $\|\mathbf{h} - \mathbf{h}_i\| + \lambda\|\mathbf{h} - \mathbf{u}\|$. By choosing the Lagrangian multiplier λ the user can indirectly control undesirable side effects of histogram equalization. This latest paper also gave a good synopses of existing contrast enhancement techniques. We refer the reader to [7] for a survey of previous works, instead of reparaphrasing them here.

In our view, directly processing histograms to achieve contrast enhancement is an ill-rooted approach. The histogram is an awkward, obscure proxy for contrast. The popularity of histogram equalization as a context-free contrast enhancement technique is apparently because no mathematical definition of context-free contrast has ever been given in the literature. This paper fills the aforementioned long-standing void by defining a measure of expected context-free contrast of a transfer function, with this contrast measure being one if the input image is left unchanged. Furthermore, to account for the distortion of subtle tones caused by contrast enhancement, which is inevitable in most cases, a counter measure of tone subtlety is also introduced. The notions of expected contrast and tone subtlety give rise to a new perceptual image quality measure called contrast-tone ratio. The new measure sets an ideal objective for the enhancement of perceptual image quality, which seeks to achieve high contrast and subtle tone reproduction at the same time. But using the contrast-tone ratio as an objective function for maximization is computationally difficult because the function is highly non-linear. Instead, we formulate contrast enhancement as a problem of maximizing the expected contrast subject to limits on tone distortion. Such a contrast-tone optimization problem can be converted to one of linear programming, and hence it can be solved efficiently in practice.

In addition, our linear programming technique offers a greater and more precise control of visual effects than existing techniques of contrast enhancement. Common side effects of contrast enhancement, such as contours, shift of average intensity, over exaggerated gradient, etc., can be effectively suppressed by imposing appropriate constraints in the linear programming framework. In the new framework, Gamma correction can be unified with contrast-tone optimization. The new technique can map L input gray levels to an arbitrary number Ł of output gray levels, allowing Ł to be equal, less or greater than L. It is therefore suited to output conventional images on high dynamic range displays or high dynamic range images on conventional displays with perceptual quality optimized for device characteristics and image contents.

Analogously to global and local histogram equalization, the new contrast enhancement framework allows the use of either global or local statistics when optimizing the contrast. However, in order to make our technical developments in what follows concrete and focused, we will only discuss the problem of contrast enhancement over an entire image instead of adapting to local statistics of different subimages. All the results and observations can be readily extended to locally adaptive contrast enhancement.

The remainder of the paper is organized as follows. In the next section we introduce some new definitions related to the intuitive notions of contrast and tone, and they lead to a new image quality measure called contrast-tone ratio. In section 3, we pose the maximization of the contrast-tone ratio as a problem of constrained optimization and develop a linear programming approach to solve it. In section 4 we discuss how to fine tune output images according to application requirements or users' preferences within the proposed contrast-tone optimization framework. Experimental results are reported in section 5, and they demonstrate the versatility and superior visual quality of the new contrast enhancement technique.

2 Contrast, Tone, and a New Perceptual Quality Measure

Contrast enhancement involves a remapping of input gray levels to output gray levels. In fact, such a remapping is required when displaying a digital image of L gray levels on a monitor of Ł gray levels, $L \neq \text{Ł}$. This remapping is carried out by an integer-to-integer transfer function

$$T : \{0, 1, \cdots, L-1\} \rightarrow \{0, 1, \cdots, \text{Ł}-1\} \tag{1}$$

The nature of the physical problem stipulates that the transfer function T be monotonically non-decreasing, because T should never reverse the order of intensities.[1] In other words, we must have $T(j) \geq T(i)$ if $j > i$. Therefore, any transfer function satisfying the monotonicity has the form

$$\begin{aligned} T(i) = \sum_{0 \leq j \leq i} s_j, \ 0 \leq i < L \\ s_j \in \{0, 1, \cdots, \text{Ł} - 1\} \\ \sum_{0 \leq j < L} s_j < \text{Ł}. \end{aligned} \tag{2}$$

The last inequality ensures the output dynamic range not exceeded by $T(i)$.

In (2), which is a general definition of the transfer function T, s_j is the increment in output intensity versus a unit step up in input level j. Therefore, s_j can be interpreted as context-free contrast at level j, which is the rate of change in output intensity without considering the pixel context. Note that a transfer function is completely determined by the vector $\mathbf{s} = (s_0, s_1, \cdots, s_{L-1})$, namely the set of contrasts at all L input gray levels.

Having associated the transfer function T with context-free contrasts s_j's at different levels, we induce from (2) a natural definition of expected (context-free) contrast of T for an image I:

$$C(\mathbf{s}) = \sum_{0 \leq j < L} p_j s_j \tag{3}$$

where p_j is the probability that a pixel in I has input gray level j.

The above defined expected contrast quantifies the colloquial meaning of contrast. To verify this let us examine some special cases.

Proposition 1. *The maximum expected contract $C(\mathbf{s})$ is achieved by $s_k = \text{Ł} - 1$ such that $p_k = \max\{p_i | 0 \leq i < L\}$, and $s_j = 0$, $j \neq k$.*

Proof: Assume for a contradiction that $s_j = n > 0$, $j \neq k$, would achieve higher expected contrast. Due to the constraint $\sum_{0 \leq j < L} s_j < \text{Ł}$, s_k equals at most $\text{Ł} - 1 - n$. But $p_j n + p_k(\text{Ł} - 1 - n) \leq p_k(\text{Ł} - 1)$, refuting the previous assumption. ■

Proposition 1 agrees with our perception that the highest contrast is achieved when the transfer function is a single step (thresholding) function that converts the input image from gray scale to binary. The binary threshold is set at level k such that $p_k = \max\{p_i | 0 \leq i < L\}$ for maximum expected contrast.

The lowest (zero) expected contrast is trivially achieved by a constant transfer function $T(i)$, namely $s_i = 0$ for all $0 \leq i < L$. Again, this agrees with our intuition of zero contrast.

In many applications it makes sense to preserve the average intensity while maximizing the expected contrast. In such cases, the average-preserving maximum expected contrast is achieved by $s_k = \text{Ł} - 1$, $s_j = 0$, $j \neq k$, such that

[1] This restriction may be relaxed in locally adaptive contrast enhancement. But in each locality the monotonicity should still be imposed.

$\sum_{0\le j<k} p_j \approx \sum_{k\le j<L} p_j$. Namely, $T(i)$ is the binary thresholding function at the average gray level.

If $L = Ł$ (i.e., when the input and output dynamic ranges are the same), the identity transfer function $T(i) = i$, namely, $s_i = 1$, $0 \le i < L$, achieves expected contrast $C(\mathbf{1}) = 1$ regardless the gray level distribution of the input image. Therefore, the unit expected contrast means a neutral expected (context-free) contrast level without any enhancement. The notion of neutral contrast can be generalized to the cases when $L \ne Ł$. We call $\tau = Ł/L$ the tone scale. In general, the transfer function

$$T(i) = \left\lfloor \frac{Ł-1}{L-1} i + 0.5 \right\rfloor, \ 0 \le i < L \tag{4}$$

or equivalently $s_i = \tau$, $0 \le i < L$, achieves the neutral contrast $C(\tau\mathbf{1}) = \tau$. We note the following simple and yet important property of context-free contrast.

Proposition 2. *The* $\max\min\{s_0, s_1, \cdots, s_{L-1}\}$ *is achieved if and only if* $C(\tau\mathbf{1}) = \tau$*, or* $s_i = \tau$*,* $0 \le i < L$*.*

Proposition 2 states that the simple linear transfer function, i.e., doing nothing in the traditional sense of contrast enhancement, actually maximizes the minimum of context-free contrasts s_i of different levels $0 \le i < L$, and the neutral contrast $C(\tau\mathbf{1}) = \tau$ is largest possible when satisfying this maxmin criterion.

In terms of visual effects, smooth tone reproduction demands the transfer function to meet the maxmin criterion of proposition 1. This is because tone continuity requires small increment between adjacent gray levels to avoid contours or banding effects. Given a transfer function $T(i)$, define the tone subtlety of $T(i)$ as

$$\begin{aligned} \Phi(\mathbf{s}) &= \max_{1\le i\le Ł} \left\{T^{-1}(i) - T^{-1}(i-1)\right\} \\ T^{-1}(i) &= \min\{j : T(j) = i\} \end{aligned} \tag{5}$$

In the definition we account for the fact that the transfer function $T(i)$ is not a one-to-one mapping in general. The smaller the value of $\Phi(\mathbf{s})$ the smoother the tone reproduced by $T(i)$. It is immediate from the definition that the best achievable tone subtlety is $\tau = \min_{\mathbf{s}} \Phi(\mathbf{s})$. But since the dynamic range Ł of the output device is finite, the two visual quality criteria of high contrast and tone continuity are in mutual conflict. Therefore, the mitigation of such an inherent conflict is a critical issue in designing contrast enhancement algorithms, which is seemingly overlooked in the existing literature on the subject.

Following the discussions above, a new perceptual image quality measure presents itself, which we call the contrast-tone ratio (CTR)

$$CTR = \frac{C(\mathbf{s})}{\Phi(\mathbf{s})} \tag{6}$$

For the linear transfer function (4), $CTR = 1$ regardless of the intensity histogram of input image I. Also, if the input histogram is uniform then the highest possible CTR is 1, meaning that no further enhancement is possible. For a general input histogram, we are interested in finding the transfer function $T(i)$ that maximizes CTR, or achieves sharpness of high frequency details and tone subtlety of smooth shades at the same time.

3 Contrast-Tone Optimization by Linear Programming

In the proceeding section we formally defined the expected contrast $C(\mathbf{s})$ of a transfer function $T(i)$ on an image I. It also shown that the expected contrast is a good, meaningful measure of the overall contrast of an image. With the expected contrast $C(\mathbf{s})$ as a measurement of overall contrast one would attempt to perform contrast enhancement by finding the "optimal" transfer function $T(i)$, among all permissible ones, that maximizes C. But this single-minded approach would likely produce over-exaggerated, unnatural visual effects, as revealed by Proposition 1. The resulting $T(i)$ degenerates a continuous-tone image to a binary image. This maximizes the contrast of a particular gray level but completely ignores accurate tone reproduction.

In order to find a correct approach of improving visual quality it is helpful to model contrast enhancement as a problem of optimal resource allocation in competition with tone subtlety. The achievable expected contrast $C(\mathbf{s})$ and tone subtlety $\Phi(\mathbf{s})$ are physically confined by the output dynamic range Ł of the display. In (3) the optimization variables $s_0, s_1, \cdots, s_{L-1}$ represent an allocation of Ł available output intensity levels, each competing for a larger piece of dynamic range. While contrast enhancement necessarily invokes a competition for dynamic range (an insufficient resource), a highly skewed allocation of Ł output levels to L input levels can deprive some input gray levels of necessary representations. This causes unwanted side effects, such as flattened subtle shades, unnatural contour bands, shifted average intensity, and etc. Such artifacts were noticed by other researchers as drawbacks of the original histogram equalization algorithm, and they proposed a number of ad hoc. techniques to alleviate these artifacts while sticking to the baseline of histogram equalization.

As argued in the end of the proceeding section, a more principled solution of the problem is to maximize the contrast-tone ratio. Unfortunately, $C(\mathbf{s})/\Phi(\mathbf{s})$ is highly non-linear in $\mathbf{s}$. Instead of having $C(\mathbf{s})/\Phi(\mathbf{s})$ directly as the objective function, we develop a linear programming algorithm that maximizes $C(\mathbf{s})$ with linear constraints induced by $\Phi(\mathbf{s})$. Specifically, let us pose and examine the following constrained optimization problem:

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0\le j<L} p_j s_j \\ \text{subject to } &(a) \sum_{0\le j<L} s_j < \text{Ł}; \\ &(b)\ s_j \ge 0,\ 0\le j<L; \\ &(c) \sum_{j\le i<j+\phi} s_i \ge 1,\ 0\le j< L-\phi. \end{aligned} \tag{7}$$

In (7), constraint (a) is to confine the output intensity level to the available dynamic range; Constraints (b) ensure that the transfer function $T(i)$ be monotonically non-decreasing; Constraints (c) specify the coarsest level of tone subtlety $\Phi(\mathbf{s})$ allowed, where ϕ is an upper bound $\Phi(\mathbf{s}) \leq \phi$. The objective function and all the constraints are linear in $\mathbf{s}$.

Computationally, the original optimization problem of (7) is one of integer programming. This is because the transfer function $T(i)$ is an integer-to-integer mapping, i.e., all components of $\mathbf{s}$ are integers. But integer programming is NP-hard. To make the problem tractable we relax the integer constraints on $\mathbf{s}$ and convert (7) to a linear programming problem. By the relaxation any solver of linear programming can be used to solve the real version of (7). The resulting real-valued solution $\mathbf{s} = (s_0, s_1, \cdots, s_{L-1})$ can be easily converted to an integer-valued transfer function:

$$T(i) = \left\lfloor \sum_{0 \leq j \leq i} s_j + 0.5 \right\rfloor, \ 0 \leq i < L \tag{8}$$

For all practical considerations the proposed relaxation solution does not materially compromise the optimality. As a beneficial side effect, the linear programming relaxation simplifies constraint (c) in (7), and allows the contrast-tone optimization problem to be stated as

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0 \leq j < L} p_j s_j \\ &\text{subject to} \sum_{0 \leq j < L} s_j < L; \\ &\qquad s_j \geq 1/\phi, \ 0 \leq j < L. \end{aligned} \tag{9}$$

4 Fine Tuning of Visual Effects

The proposed contrast-tone optimization framework is general and it can achieve desired visual effects by adding proper constraints to (9). We demonstrate the generality and flexibility of the proposed linear programming approach to image enhancement by some examples among many possible applications.

The first example is the integration of Gamma correction into contrast-tone optimization. The optimized transfer function $T(\mathbf{s})$ can be made close to the Gamma transfer function by adding to (9) the following constraint

$$\sum_{0 \leq i < L} \left| (L-1)^{-1} \sum_{0 \leq j \leq i} s_j - [i(L-1)^{-1}]^{\gamma} \right| \leq \Delta \tag{10}$$

where γ is the Gamma parameter and Δ is the degree of closeness between the resulting $T(\mathbf{s})$ and the Gamma mapping $[i(L-1)^{-1}]^{\gamma}$.

In applications when the enhancement process cannot change the average intensity of the input image by certain amount Δ_μ, the user can impose this restriction easily in (9) by adding another linear constraint

$$\left| \frac{L}{\text{Ł}} \sum_{0 \le i < L} p_i \sum_{0 \le j \le i} s_j - \sum_{0 \le i < L} p_i i \right| \le \Delta_\mu \tag{11}$$

Besides the use of constraints in the linear programming framework, we can incorporate context-based or semantics-based fidelity criteria directly into the objective function of contrast-tone optimization. The expected contrast $C(\mathbf{s}) = \sum p_j s_j$ and the CTR depend only on the point statistics of the input image. We can complement $C(\mathbf{s})$ and CTR by weighing in the semantic or perceptual importance of increasing the contrast at different gray levels by w_j, $0 \le j < L$. In general, w_j can be set up to reflect specific requirements of different applications. In medical imaging, for example, the physician can read an image of L gray levels on an Ł-level monitor, $\text{Ł} < L$, with a certain range of gray levels $j \in [j_0, j_1] \subset [0, L)$ enhanced. Such a weighting function presents itself naturally if there is a preknowledge that the interested anatomy or lesion falls into the intensity range $[j_0, j_1]$ for given imaging modality. In combining point statistics and domain knowledge or/and user preference, we introduce a new objective function

$$\max_{\mathbf{s}} \left\{ \sum_{0 \le j < L} p_j s_j + \lambda \sum_{0 \le j < L} w_j s_j \right\} \tag{12}$$

where the Lagrangian multiplier λ regulates the relative importance of the expected contrast and a user-prioritized contrast.

In summarizing all discussions above we finally present the following general linear programming framework for visual quality enhancement.

$$\begin{aligned}
&\max_{\mathbf{s}} \sum_{0 \le j < L} (p_j + \lambda w_j) s_j \\
&\text{subject to} \quad \sum_{0 \le j < L} s_j < \text{Ł}; \\
&\qquad s_j \ge 1/\phi, \; 0 \le j < L; \\
&\qquad \sum_{0 \le i < L} \left| (\text{Ł} - 1)^{-1} \sum_{0 \le j \le i} s_j - [i(L-1)^{-1}]^\gamma \right| \le \Delta \\
&\qquad \left| \frac{L}{\text{Ł}} \sum_{0 \le i < L} p_i \sum_{0 \le j \le i} s_j - \sum_{0 \le i < L} p_i i \right| \le \Delta_\mu .
\end{aligned} \tag{13}$$

5 Empirical Results

Fig. 1 through Fig. 4 present some sample images that are enhanced by the proposed contrast-tone optimization technique in comparison with those produced by histogram equalization. In addition to visual inspection we compare the two methods by the new image quality measure CTR as well in Table 1. As expected, in all cases the proposed technique achieves significantly higher CTR than histogram equalization.

In image Beach (Fig. 1), the output of histogram equalization is too dark in overall appearance because the original histogram is skewed toward the bright range. But the proposed method enhances the original image without introducing unacceptable distortion in average intensity. This is because of the constraint that bounds the relative difference ($< 20\%$) between the average intensities of the input and output images. Fig. 2 shows an example when the user assigns higher weights w_j in (13) to gray levels j, $j \in (a, b)$, where $(a, b) = (100, 150)$ is a range of interest (brain matters in the head image). Fig. 3 compares the results of histogram equalization and the proposed method when they are applied to a typical portrait image. In this example histogram equalization overexposes the input image, causing an opposite side effect as in image Beach, whereas the proposed method obtains high contrast, tone continuity and small distortion in average intensity at the same time. In Fig. 4, the result of joint Gamma correction and contrast-tone optimization by the new technique is shown, and compared with those in difference stages of the separate Gamma correction and histogram equalization process. The image quality of the former is clearly superior to that of the latter.

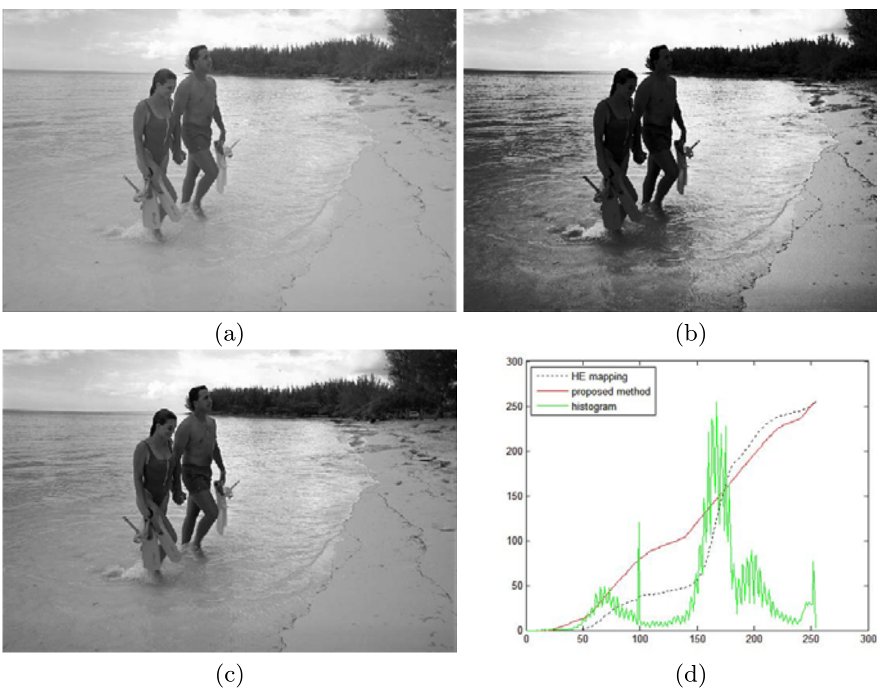


Fig. 1. (a) the original, (b) the output of histogram equalization, (c) the output of the proposed method, and (d) the transfer functions and the original histogram

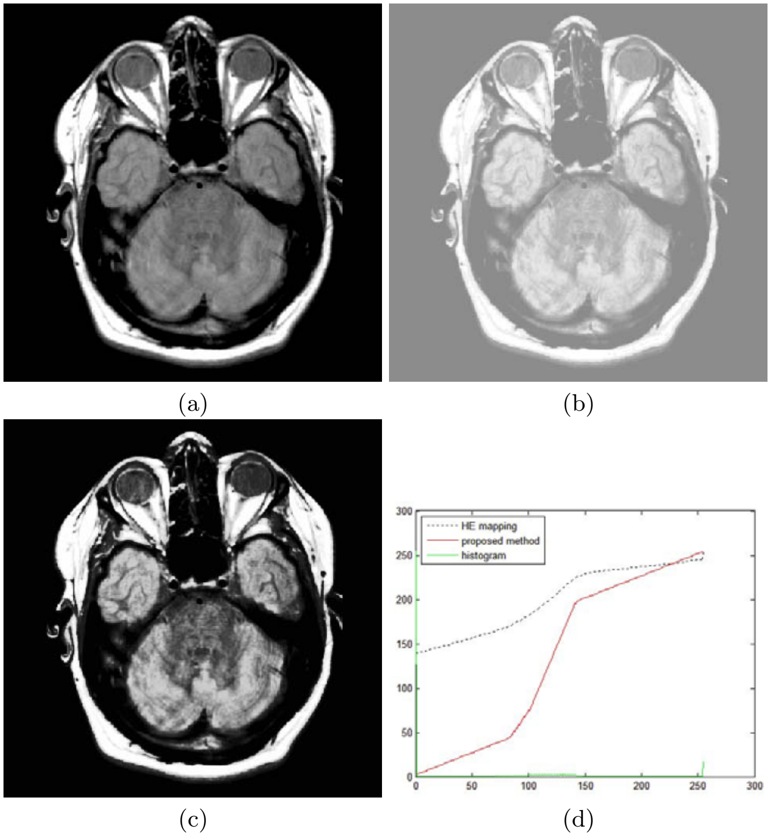


Fig. 2. (a) the original, (b) the output of histogram equalization, (c) the output of the proposed method, and (d) the transfer functions and the original histogram

Fig. 3. (a) the original, (b) the output of histogram equalization, (c) the output of the proposed method, and (d) the transfer functions and the original histogram

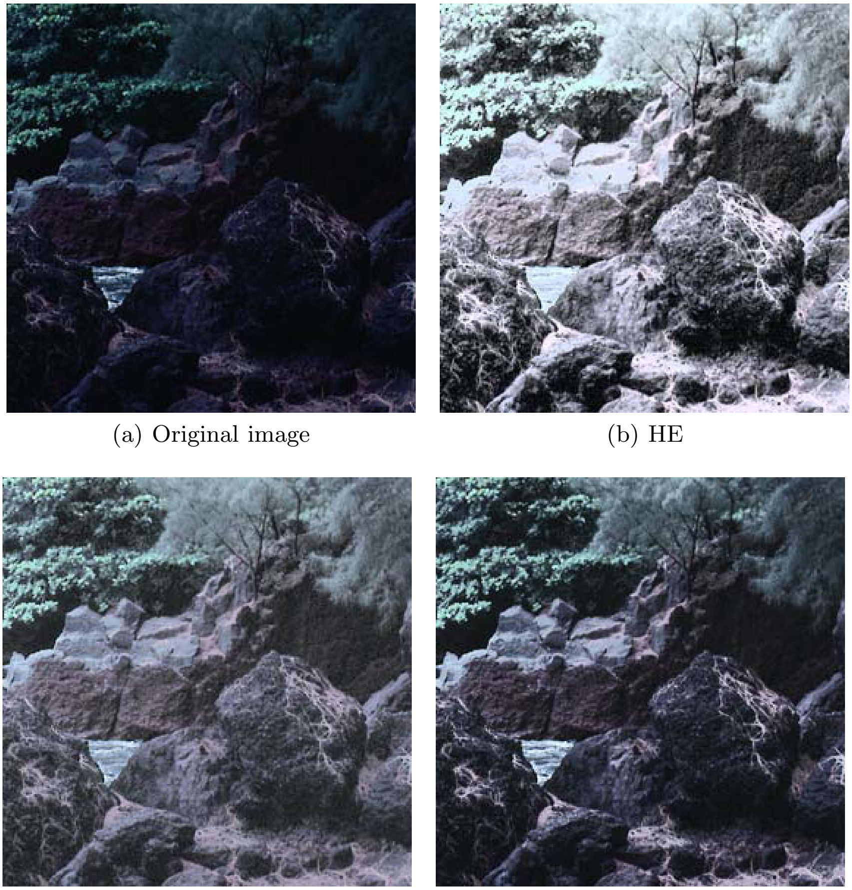
(a) (b) (c) (d)

Fig. 4. (a) the original image before Gamma correction, (b) after Gamma correction, (c) Gamma correction followed by histogram equalization, and (d) joint Gamma correction and contrast-tone optimization by the proposed method

Table 1. Comparison in CTR between histogram equalization and the proposed method

Image	Histogram equalization			Proposed method		
	Expected contrast	Tone Subtlety	CTR	Expected contrast	Tone subtlety	CTR
Beach	2.81	25	0.11	1.41	2	0.71
Head	0.58	8	0.07	0.73	2	0.36
Portrait	2.11	51	0.04	1.60	6	0.27

The proposed approach is also compared with the well-known contrast-limited adaptive histogram equalization (CLAHE) [8] in visual quality. CLAHE is considered to be one of the best contrast enhancement techniques, and it alleviates many of the problems of histogram equalization, such as over- or under-exposures, tone discontinuities, and etc. Fig. 5 is a side-by-side comparison of the proposed method, CLAHE and HE. CLAHE is clearly superior to HE in perceptual quality, as well recognized in the existing literature and among practitioners, but it is somewhat inferior to the proposed method in overall image quality, particularly in the balance of sharp details and subtle tones.



(a) Original image (b) HE

(c) CLAHE (d) The proposed

Fig. 5. Comparison of different methods on image Rocks

6 Conclusion

A new, general image enhancement technique of optimal contrast-tone mapping is proposed. The resulting problem can be solved efficiently by linear programming. The solution can increase image contrast while preserving tone continuity, two conflicting quality criteria that were not handled and balanced as well in the past.

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