Chaotic Pattern Recognition: The Spectrum of Properties of the Adachi Neural Network

Ke Qin[1,⋆] and B. John Oommen[2,⋆⋆]

[1] School of Computer Science and Engineering
University of Electronic Science & Technology of China
Chengdu, China, 610054
thinker_qk@hotmail.com
[2] School of Computer Science, Carleton University,
1125 Colonel By Dr., Ottawa, ON, K1S 5B6, Canada
oommen@scs.carleton.ca

Abstract. Chaotic Pattern Recognition (PR) is a relatively new subfield of PR in which a system, which demonstrates chaotic behavior under normal conditions, resonates when it is presented with a pattern that it is trained with. The Adachi Neural Network (AdNN) is a classic neural structure which has been proven to demonstrate the phenomenon of Associative Memory (AM). In their pioneering paper [1,2], Adachi and his co-authors showed that the AdNN also emanates periodic outputs on being exposed to trained patterns. This was later utilized by Calitoiu *et al* [4,5] to design systems which possibly possessed PR capabilities. In this paper, we show that the previously reported properties of the AdNN do not adequately describe the dynamics of the system. Rather, although it possesses far more powerful PR and AM properties than was earlier known, it goes through a spectrum of characteristics as one of its crucial parameters, α, changes. As α increases, the AdNN which is first an AM become *quasi*-chaotic[1]. The system is then distinguished by two phases which really do not have clear boundaries of demarcation. In the former of these phases it is *quasi*-chaotic for some patterns and periodic for others. In the latter of these, it exhibits properties that have been unknown (or rather, unreported) till now, namely, a PR capability (which even recognizes masked or occluded patterns) in which the network resonates sympathetically for trained patterns while it is *quasi*-chaotic for untrained patterns. Finally, the system becomes completely periodic. All these results are, to the best of our knowledge, novel.

Keywords: Chaotic NNs, Chaotic Pattern Recognition, Adachi NN.

⋆ The work of this author was done while he was in Canada as a Visiting Scholar at Carleton University in 2008.

⋆⋆ *Chancellor's Professor* ; *Fellow : IEEE* and *Fellow : IAPR*. This author is also an *Adjunct Professor* with the University of Agder in Grimstad, Norway. The work of this author was partially supported by NSERC, the Natural Sciences and Engineering Research Council of Canada. The authors would like to thank Dr. Dragos Calitoiu for his time and his insight during our mutual discussions.

[1] A formal explanation of this expression is given in the body of the paper.

N. da Vitora Lobo et al. (Eds.): SSPR&SPR 2008, LNCS 5342, pp. 540–550, 2008.

1 Introduction

Although the field of PR has been extensively studied, the entire area of designing and implementing Neural Networks (NNs) which are inherently chaotic, and which simultaneously possess PR capabilities is relatively new[2]. As far as we know, the pioneering papers in this regard are those due to Calitoiu and his co-authors [4,5]. The aim of this paper is to demonstrate the fundamental properties of the NN used in [4,5], the AdNN, (which was earlier proposed by Adachi and his co-authors [1,2]). Quite simply, we shall show that the AdNN is an extremely powerful and efficacious system, and that its properties that have been reported in the literature are but a shadow of its true underlying capabilities. Indeed, we shall demonstrate that the same NN can demonstrate the phenomena of Associate Memory (AM), *quasi*-chaos, pattern recognition and periodicity for different values of a certain parameter[3].

An AM permits its user to specify part of a pattern, and to thereafter retrieve the values associated with that pattern. One of the limitations of most ANN models of AM is the dependency on an external input. Once an output pattern has been identified, the ANN remains in that state until the arrival of a new external input. This is in contrast to real biological neural networks which exhibit sequential memory characteristics. To be more specific, once a pattern is recalled from a memory location, the brain is not "stuck" to it, it is also capable of recalling other associated memory patterns without being prompted by any additional external inputs. This ability to "jump" from one memory state to another *in the absence of a stimulus* is one of the hallmarks of the brain, and *this is one phenomenon that a chaotic PR system has to emulate.*

The evidence that indicates the possible relevance of chaos to brain functions was first obtained by Freeman [7,13] through his clinical work on the large-scale collective behavior of neurons in the perception of olfactory stimuli. Based on his experiments, he conjectured that the quiescent state of the brain is chaos, while during perception, when attention is focused on any sensory stimulus, the brain activity becomes more periodic. The controlling of chaos gives rise to periodic behavior, culminating in the identification of the sensory stimulus that has been received. Thus, mimicing this identification on a neural network can lead to a new model of PR. encounters one of the memorized patterns as an input to the network, we want the network to *resonate* with that pattern, i.e., to generate *that* pattern with a certain periodicity. Between two consecutive appearances of the memorized pattern, the network can also be in an infinite number of states, but in none of the memorized ones. greater detail in [4,5], where we also show that in order to achieve recognition, one must decrease the level of chaos until a periodic behavior is obtained.

[2] The issue of how this sub-field of PR differs from the previously reported sub-fields and the sub-fields of NNs [3,8,9,11] (including those capable of associative memory and learning [6,12]), is described in [4,5,10].

[3] It is also very interesting to note that the parameter of interest in this paper, namely α, is one that has not been examined in the literature when it concerns any of these phenomena.

One of the primary differences between our results and the results reported earlier for the AdNN (and its variants) [1,2,4,5] is the demonstration that it is powerful enough to accomplish PR even if the pattern is masked or occluded. This phenomenon, which is almost "mystical" and inexplicable, has been both unknown and unreported. The second major difference between our results and the results reported earlier for the AdNN (and its variants) [1,2,4,5] is the concept of what we shall refer to as *quasi*-chaotic behavior, explained presently. The final difference of importance is that we show that all of the above-mentioned properties are a consequence of varying the parameter α and not the parameters k_r and k_f, as was previously anticipated.

1.1 Rationale and Contributions of the Paper

The entire field of Chaotic PR is in its infancy, and every step into the unknown leads to fascinating results. In particular, the papers of Adachi and his co-authors [1,2] first presented the AdNN with the conjecture that it demonstrated chaotic phenomena. Later, in [4,5], the authors showed by an analysis of the Lyapunov exponents, that the AdNN was *not* chaotic. But this was by virtue of only considering the coefficients, k_r and k_f. We emphasize here that no previous result delved into the importance of the parameter, α, the refractory scaling parameter[4].

The initial rationale for investigating the AdNN was to procure a clearer understanding of the role that α played. It turns out that unlike the coefficients k_r and k_f, α seems to have a more pronouncing effect on the chaotic, periodic and PR capabilities of the NN. Thus, by considering the range of values that α can assume, we encounter a fascinating spectrum of behavioral properties, which were previously unreported. Indeed, although the reason for the "peculiar" behavior is unknown, our experimental results (see Section 3.2) illustrate that the AdNN can truly be used to develop chaotic PR systems. Indeed, (a) We have shown that the chaotic behavior of the AdNN is not merely constrained by the coefficients k_r and k_f; (b) We have clearly demonstrated that unlike the parameters k_r and k_f, the refractory scaling parameter, α, plays a significant role in determining the phenomenon displayed by the NN. (c) As opposed to the previously claimed results, we show that the AdNN is not chaotic. Rather, it exhibits a behavior which is *quasi*-chaotic, implying that it is chaotic if the output is merely compared to the input, but if the output is examined as a stream in its own right, it is periodic all the same. Such a behavior was previously unknown; (d) We have shown that the same network, the AdNN can be made to demonstrate a wide spectrum of phenomena – Associative Memory, periodicity, pattern recognition and *quasi*-chaos. This fact was previously unknown too; and (e) Finally, and most importantly, we have shown that there is a "range" in the

[4] Adachi and his co-authors [1,2] set the value for α as 10 in their experiments, but this seemed rather arbitrary. No explanation was given as to how and why this value of α was used. The same comment can be made about the results of Calitoiu *et al.* [4,5].

value of the parameter α, in which the system is *quasi*-chaotic for untrained patterns, but periodic for trained patterns or their noisy incarnations.

We conclude this sub-section by observing that we are not aware of any comparable results in the field of NNs and CNNs.

2 The Adachi Neural Network Model and Its Variants

The AdNN[5] is a network of neurons with weights associated with the edges, a well-defined Present-State/Next-State function, and a well-defined State/Output function. It is composed of N neurons (Adachi set $N = 100$), topologically arranged as a completely connected graph, i.e, each neuron communicates with every other neuron, including itself. It is described by means of the following equations relating the two internal states $\eta_i(t)$ and $\xi_i(t)$, $i = 1...N$, and the output $x_i(t)$ as:

$$x_i(t+1) = f(\eta_i(t+1) + \xi_i(t+1)), \tag{1}$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{N} w_{ij} x_j(t), \tag{2}$$

$$\xi_i(t+1) = k_r \xi_i(t) - \alpha x_i(t) + a_i. \tag{3}$$

In the above, $x_i(t)$ is the output of the neuron i which has an analog value in [0,1] at the discrete time t. The internal states of the neuron i are $\eta_i(t)$ and $\xi_i(t)$, $f(\cdot)$ is the logistic function with the steepness parameter, ε, satisfying $f(y) = 1/(1+e^{-y/\varepsilon})$. k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. $\{w_{ij}\}$ are the synaptic weights from the i^{th} constituent neuron to the j^{th} constituent neuron, and a_i denotes the temporally constant external inputs to the i^{th} neuron. From the perspective of this paper, most importantly, α is the refractory scaling parameter, whose significance has, to date, not been investigated.

While the network dynamics are described by Equations (2) and (3), the outputs of the neurons are obtained by Equation (1). The feedback interconnections are determined according to the following symmetric auto-associative matrix of the p stored patterns by:

$$w_{ij} = \frac{1}{p} \sum_{s=1}^{p} (2x_i^s - 1)(2x_j^s - 1), \tag{4}$$

where x_i^s is the i^{th} component of the s^{th} stored pattern.

Calitoiu *et al* [5] proposed a model of CNNs which modifies the AdNN to enhance its PR capabilities. This NN, referred to as the Modified AdNN (M-AdNN), is actually also a Hopfield-like model, and manipulates the internal

[5] This review is necessarily very brief in the interest of space. However, we mention that the AdNN was initially modeled to serve as a dynamical Associative Memory. More details of the AdNN are found in [1,2,4,5,10].

structure and Present-State/Next-State equations of the original AdNN. Structurally, it is also composed of N neurons, topologically arranged as a completely connected graph. Again, each neuron i, $i = 1...N$, has internal states $\eta_i(t)$ and $\xi_i(t)$, and an output $x_i(t)$. Calitoiu *et al* [5] presented a brief rationale for each modification. The fundamental difference between the AdNN and the M-AdNN in terms of their Present-State/Next-State equations is that the latter has only a *single* global neuron (and its corresponding two global states) which is used for the state updating criterion for *all* the neurons.

Calitoiu *et al* [4] later enhanced the M-AdNN to yield an even more interesting NN, the Modified Blurring AdNN (Mb-AdNN). The latter was designed to solve the inverse PR problem, namely that of understanding how a neural system, which receives an exact pattern, can perceive it accurately in certain settings and yet see it as "blurred" in other settings. The authors of [4] accomplish this by forcing a set of neurons to have Present-State/Next-State functions which are updated using their current values, while the corresponding functions for the other neurons involve only a single global neuron as in the case of the M-AdNN. As we will not be investigating the Mb-AdNN in any detail here, the details of its design and analysis are omitted, but can be found in [4].

Since a detailed survey of the AdNN, the M-AdNN and the Mb-AdNN is not possible here, in the interest of completeness and continuity, we mention that its salient features are listed in [4,5,10]. However, we mention that the drawbacks of the current schemes are its excessive computational burden, the long transient phases, its imprecise PR capability, and the resonance for the untrained patterns. These have been tested for the data sets used by [1,2] and [4,5] given in Figure 1. Graphs demonstrating this are given in [10], and additional experimental results which clarify this phenomenon (which was earlier unreported) are listed in Table 1 for these data sets. We attempt to rectify these disadvantages here.

Table 1. Periodicities for trained and untrained patterns for the data set used by Adachi *et al* (the first two rows), and the digital data set used by Calitoiu *et al* (the last two rows). The entries of particular interest are in a **bold** font.

Pattern	1	2	3	4	With Noise	Random
Periodicity	20	20	39	39	39	**39**

Pattern	1	2	3	4	5	6	7	8	9	0	With Noise	Random
Periodicity	7,7,8	26	26	27	26	27	26	26	26	26	26	**25**

3 New Periodicity, *Quasi*-Chaotic and PR Properties of the AdNN

The goal of the field of Chaotic PR systems can be vocalized as follows: To be more specific, let us suppose that we want the chaotic PR system to recognize patterns P_i and P_j. To accomplish this, we train the system using both the patterns by a mere straightforward computation. This training phase assigns the weights between the neurons of the CNN, which effectively memorizes the

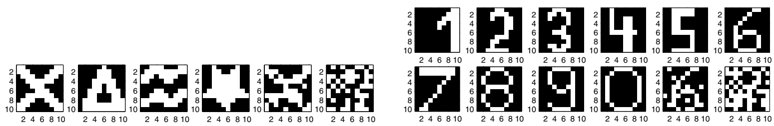


Fig. 1. (a) is the set of patterns used in Adachi *et al*'s and our experiments. The first four patterns constitute the set used by Adachi *et al.* Pattern5 is a noisy version of Pattern1 with 15% noise. Pattern6 is a randomly generated pattern. (b) is the set of patterns used in Calitoiu *et al*'s and our experiments. The first ten patterns constitute the set used by Calitoiu *et al.* Pattern11 is a noisy version of Pattern6 with 10% noise. Pattern12 is a randomly generated pattern, and is the reverse of Pattern6 in (a).

training patterns. Subsequently, on testing, if any pattern other than P_i or P_j is presented, the CNN must continue to be chaotic since it is not trained to recognize such a pattern. However, if P_i or P_j (or a pattern resembling either of them) is presented, the CNN must switch from being chaotic to being periodic. Note that as opposed to traditional PR systems, the output is not a single value. It is a sequence of values, which is chaotic (i.e., displays no periodicity) unless one of the trained patterns is presented.

We now present a sequence of results concerning the AdNN which were earlier unreported.

3.1 Periodicity Analysis Using Quasi-energy Functions

Although the AdNN was initially thought to be chaotic, Calitoiu *et al* [4] showed, by a Lyapunov analysis, that this was not the case. Rather, we present below an informal analysis for the observed periodicity.

As per the definition of Adachi *et al* [1], an alternate formulation for the dynamic equations of the AdNN is given by:

$$\begin{aligned} x_i(t+1) &= f(\sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_f^d x_j(t-d) - \alpha \sum_{d=0}^{t} k_r^d x_i(t-d) + a_i) \\ &= f(\sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_f^{t-d} x_j(d) - \alpha \sum_{d=0}^{t} k_r^{t-d} x_i(d) + a_i). \end{aligned} \quad (5)$$

Based on the above, we argue that the AdNN can never get "absorbed" into a fixed point. The details of the argument are rather lengthy and are omitted due to space limitations (they can be found in [10]) but can be summarized as:

1. If any neuron tends to converge towards the value '0' as a fixed point, it can be seen from the above equations that the dynamics of the system forces the output value of *that* neuron towards unity.

2. If any neuron tends to converge towards the value '1' as a forces the output value of *that* neuron towards '0'.

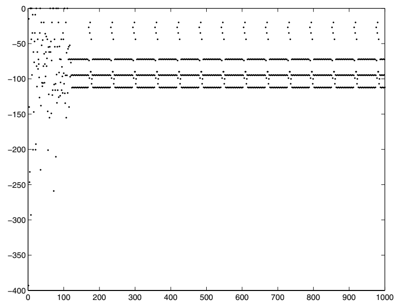


Fig. 2. A plot of the quasi-energy function with the parameters $\alpha = 17, k_f = 0.2, k_r = 0.9, \varepsilon = 0.00015$, and $a_i = 2$. The reader should observe the convergence and periodicity of the plot.

Since we have clearly argued that no neuron will be trapped at either fixed point '0' or '1', the question which has to be answered is that of knowing whether the system will be chaotic or periodic? Here, we believe that the critical parameter is the refractory parameter, α, in Equation (5), which completely describes the quasi-energy function of the dynamical AdNN.

Unfortunately, at this juncture, a formal analysis is unavailable due to the complexity of the overall network. Rather, we resort to an experimental analysis which shows that the system's behavior is periodic after a short transient phase, and that α is crucial to the length of this transient phase. As α increases, we observe that the transient phase becomes increasingly shorter. For example, if we set $\alpha \leq 15$, the transient phase is about $21,000$. However, the length of this phase reduces sharply to about $5,300$ when $\alpha = 16$. Thereafter, as α is increased (even incrementally, to $\alpha = 16.0001$), the quasi-energy function converges even faster, displaying a transient phase of only approximately $1,400$ iterations, which further reduces to only about 120 iterations when $\alpha = 17$. Indeed, the plots of the quasi-energy function (see for example, Figure 2) demonstrates that the AdNN is both convergent and periodic.

3.2 *Quasi*-Chaotic and PR Properties of the AdNN

We now report the PR properties of the AdNN. Although, in the ideal setting we would have preferred the AdNN to be chaotic (when the value of α does not lead to a periodic output), it turns out that, "unfortunately", its output is not really chaotic. Rather the AdNN demonstrates a phenomenon which we refer to as being *quasi*-chaotic since it is neither completely chaotic nor simply periodic. Indeed, this *quasi*-chaotic phenomenon implies that the system *appears* chaotic if the output is merely compared to the input. But if, on the other hand, the output is examined as a stream independently (i.e., by itself), *it is periodic all the same.* This unexplained behavior was previously unknown.

Table 2. The properties of the AdNN obtained for different values of α. The data set used is the one used by Adachi *et al* (see Figure 1), and the values of the parameters are $k_f = 0.2$, $k_r = 0.9$, and $\varepsilon = 0.00015$. The legend for the table is the following: AM — Associative Memory; QC — *Quasi*-Chaotic; C* — Periodic for a short time and then *quasi*-chaotic; P — Periodic. Observe the PR properties of the AdNN for $\alpha = 26$.

	P1	P2	P3	P4	P5	P6
$\alpha = 10$	A	A	A	A	A	A
$\alpha = 11$	A	A	A	A	A	A
$\alpha = 17$	QC	QC	QC	QC	QC	QC
$\alpha = 18$	QC	QC	QC	QC	QC	QC
$\alpha = 19$	QC	P	QC	QC	QC	QC
$\alpha = 25$	C*	C*	C*	C*	C*	C*
$\alpha = 26$	P	P	P	P	P	QC
$\alpha = 70$	P	P	P	P	P	P

Within this expanded view of perceiving things, we believe that it is still possible to design, develop and implement chaotic PR systems[6]. First of all, unlike the analysis done in [4,5] which concentrates on the Lyapunov analysis, it turns out that the parameters k_r and k_f do not play a crucial role to determine the overall phenomena – as was previously anticipated. Rather, it should be emphasized that the parameter of interest to achieve this, namely α, is one that has not been examined in the literature when it concerns a PR phenomena.

The highlights of the results obtained for the AdNN (for $a_i = 2$) is tabulated in Table 2. We summarize the results by stating that by increasing the value of α, the AdNN demonstrates the following amazing results:

1. **Associate Memory:** When $\alpha = 10$, the AdNN is merely an AM during its long transient phase. It can retrieve *all* the trained patterns non-periodically during its transient phase.
2. **Periodic but neither AM nor PR:** As α increases to 17, the AdNN begins to display some interesting properties. Initially, for $\alpha = 17$ or 18, the AdNN reproduces unrecognized patterns as can be seen from the detailed experimental figures included in [10] (omitted here in the interest of brevity). Observe that none of the trained pattern will be retrieved since the Hamming distance is never 0. Thus, amazingly, the AdNN is neither an AM nor a PR system when $\alpha = 17$!
3. **Periodic and *Quasi*-Chaotic:** If $\alpha = 19$ or 20, the output is a repeated version of unrecognized patterns for certain input patterns, while for other patterns, the system possesses the capability of recognizing them. This can also be observed from from the above-mentioned detailed experimental figures included in [10]. In this figure, the input is Pattern2, and the system yields as output, the same pattern (Pattern2) with a periodicity of 7.

[6] The detailed mechanics of such a system are, as yet, not fully explained. This is currently being undertaken, and will be, hopefully, reported in a future work.

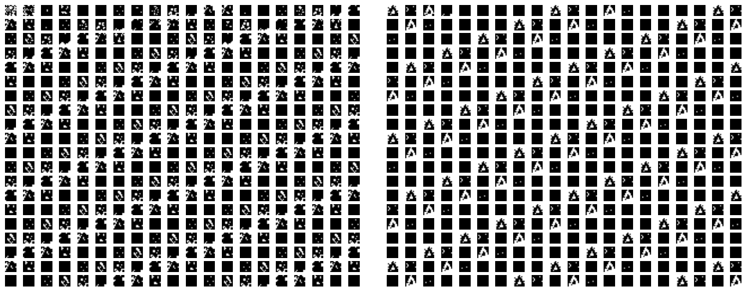
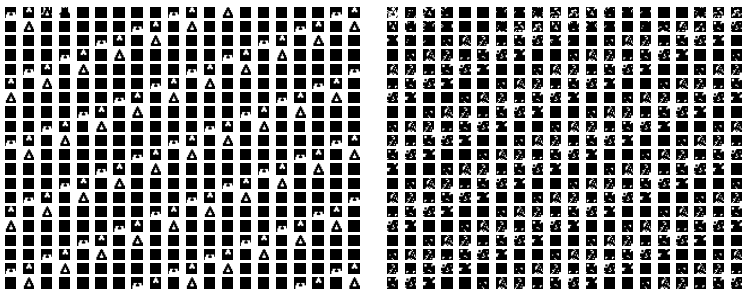


Fig. 3. Visualization of the output obtained from time t=1 to t=400. The input for the figure on the left is an untrained random pattern. The output yielded is neither chaotic nor simply periodic because the input pattern is never repeated, although a completely unknown pattern appears periodically at the output. This sequence of outputs given by the figure on the right demonstrates that the AdNN is capable of recognizing the input pattern correctly even if the percentage of noise is up to 90%.

4. **Short-term Periodic and *Quasi*-Chaotic:** If $\alpha = 21, 25, 29, 30$ or 35, the system has the ability of recalling the input patterns several times, and subsequently these samples from its memory.
5. **Pattern Recognition:** The most interesting scenario, the one which does yield PR, occurs when $\alpha = 26$. In this case, the system's output is periodic (repeating the input) as long as the input is one of the trained patterns. However, on the contrary, if the system is presented with untrained input patterns, it will result in unrecognized *quasi*-chaotic output. This interesting property can be seen from the figures in [10] and 3. The reader should observe the importance of this result: This implies that unlike the earlier claim of [4,5], the AdNN can be used to develop a chaotic PR system. Apart from the case when $\alpha = 26$, this powerful property is also exhibited at other values of α such as $32, 38, 44, 49$ and 61. Thus, it appears that part of the training phase of a chaotic PR system would involve determining the value of α that would lead such a behavior.
6. **Purely Periodic:** Finally, if α is increased to even larger values, for example, $\alpha = 70$, the output is a periodic version of the input independent of what the input is.
7. **Response to Weighted External Inputs:** Another fascinating feature of the AdNN is its behavior when external inputs are applied, that is, when $a_i = 2 + 6x_i$. From our preliminary experimental results it appears as if, in this case, the AdNN behaves like the Mb-AdNN [5].
8. **Recognition of *Inverted* Patterns:** Another property of the AdNN is that it is even capable of recognizing completely inverted patterns. Indeed, our experimental results demonstrate that the AdNN can recognize trained patterns even if the input is almost completely inaccurate. Thus, if the bit-wise percentage of noise is as high as **90**% (implying that the input is almost identical to a totally bit-wise inverted version of the original pattern), the AdNN recognizes the input pattern accurately by a periodic and



(a) (b)

Fig. 4. In the sequence of outputs on the left, (a), the input is a half-masked version of Pattern2. At the output, the AdNN recognizes Pattern2 correctly, by presenting the two halves in *periodic consecutive outputs*. Indeed, even if this masked pattern is garbled with noise up to **30**%, the AdNN continues to produce *quasi*-chaotic outputs, as seen in the sequence of outputs on the right, (b). Again, the parameters are $k_f = 0.2,\ k_r = 0.9$, and $\varepsilon = 0.00015$, with the crucial parameter, α being 26.

quasi-chaotic behavior. Detailed experiments demonstrating this property also are found in [10], omitted here due to space limitations.

9. **Recognition of *Masked* Patterns:** Perhaps the most fascinating unreported property of the AdNN is its ability to recognize masked or occluded patterns. In this case, the system is trained with a set of training patterns. On testing, however, the system is provided with a noisy version of one-half of a trained pattern, while the other half is completely random. In this case too, the network resonates sympathetically for trained patterns by presenting the two halves in *periodic consecutive outputs*[7], while it is *quasi*-chaotic for untrained patterns. This can be observed from Figure 4 where, in the first case, the masked image is generated as alluded to here, and in the second case, the input is further garbled with **30**% noise. In both of these cases, the system is capable of recognizing the true pattern with with a periodic and *quasi*-chaotic behavior.

4 Conclusions

In this paper we have concentrated on the field of Chaotic Pattern Recognition (PR), which is a relatively new sub-field of PR. Such systems, which have only recently been investigated, demonstrate chaotic behavior under normal conditions, and resonate when it is presented with a pattern that it is trained with. The reported systems work with the Adachi Neural Network (AdNN) [1,2] which has Associative Memory (AM) properties, and which also emanates periodic outputs on being exposed to trained patterns. In this paper, we have presented a collection of previously unreported properties of the AdNN. We have shown that

[7] There is no other expression to describe this phenomenon than *amazing*!

it goes through a spectrum of characteristics as one of its crucial parameters, α, changes. As α increases, it is first an AM, and it then becomes *quasi*-chaotic. The system is subsequently distinguished by two phases where in the former it is *quasi*-chaotic for some patterns and periodic for others, and in the latter, it exhibits PR properties. It is fascinating that the AdNN also possesses the capability to recognize masked or occluded patterns, and even patterns which are completely inverted – which properties, are to the best of our knowledge, novel.

References

1. Adachi, M., Aihara, K.: Associative dynamics in a chaotic neural network. Neural Networks 10(1), 83–98 (1997)
2. Aihara, K., Takabe, T., Toyoda, M.: Chaotic neural networks. Physics Letters A 144(6,7), 333–340 (1990)
3. Bishop, C.: Neural Networks for Pattern Recognition. Oxford University Press, Oxford (1995)
4. Calitoiu, D., Oommen, B., Nussbaum, D.: Desynchronzing of chaotic pattern recognition neural network to model inaccurate parception. IEEE Trans. on Systems, Man, and Cybernetics-part B: Cybernetics 37(3), 692–704 (2007)
5. Calitoiu, D., Oommen, B., Nussbaum, D.: Periodicity and stability issues of a chaotic pattern recognition neural network. Pat. Anal. Applic., 175–188 (October 2007)
6. Fausett, L.: Fundamentals of Neural Networks: Architectures, Algorithms And Applications. Prentice Hall, Upper Saddle River (1994)
7. Freeman, W.: Tutorial on neurobiology: from single neurons to brain chaos. International Journal of Bifurcation and Chaos in Applied Sciences and Engineering 2, 451–482 (1992)
8. Friedman, M., Kandel, A.: Introduction to Pattern Recognition, Statistical, Structural, Neural and Fuzzy Logic Approaches. World Scientific, Singapore (1999)
9. Fukunaga, K.: Introduction to Statistical Pattern Recognition. Academic Press, London (1990)
10. Qin, K., Oommen, B.J.: The Spectrum of Chaotic Pattern Recognition Properties Obtainable Using Adachi's Neural Network. Unabridged version of this paper (Submitted for Publication) (Can be made available if required)
11. Ripley, B.: Pattern Recognition and Neural Networks. Cambridge University Press, Cambridge (1996)
12. Schurman, J.: Pattern Classification, A Unified View of Statistical and Neural Approaches. John Wiley and Sons, New York (1996)
13. Skarda, C., Freeman, W.: How brains make chaos in order to make sense of the world. Behaviorai and Brain Sciences 10(2), 161–195 (1987)