

17. PROOF OF THE MAIN THEOREM

Theorem 17.1. *Suppose that $\Phi_X : X \rightarrow S$ is weakly prepared, $r \geq 2$ and $\overline{A}_r(X)$ holds. Then there exists a permissible sequence of monoidal transforms $Y \rightarrow X$ such that $\overline{A}_{r-1}(Y)$ holds.*

Proof. The Theorem follows from successive application of Lemma 10.3 and Theorems 13.8, 14.6, 14.7, 15.7 and 16.1 \square

Theorem 17.2. *Suppose that $\Phi_X : X \rightarrow S$ is weakly prepared. Then there exists a sequence of permissible monoidal transforms $Y \rightarrow X$ such that $\Phi_Y : Y \rightarrow S$ is prepared.*

Proof. For $r \gg 0$ $\overline{A}_r(X)$ holds by Zariski's Subspace Theorem (Theorem 10.6 [3]). The theorem then follows from successive application of Theorem 17.1, and the fact that $\overline{A}_1(X)$ holds if and only if $\Phi_X : X \rightarrow S$ is prepared. \square

Theorem 17.3. *Suppose that $\Phi : X \rightarrow S$ is a dominant morphism from a 3 fold to a surface and $D_S \subset S$ is a reduced 1 cycle such that $E_X = \Phi^{-1}(D_S)_{red}$ contains $\text{sing}(X)$ and $\text{sing}(\Phi)$. Then there exist sequences of monoidal transforms with nonsingular centers $\pi_1 : S_1 \rightarrow S$ and $\pi_2 : X_1 \rightarrow X$ such that $\Phi_{X_1} : X_1 \rightarrow S_1$ is prepared with respect to $D_{S_1} = \pi_2^{-1}(D_S)_{red}$.*

Proof. This follows from Lemma 6.2 and Theorem 17.2. \square