Reconstruction of Discrete Surfaces from Shading Images by Propagation of Geometric Features

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Abstract. This paper describes two new methods for the reconstruction of discrete surfaces from shading images. Both approaches are based on the reconstruction of a discrete surface by mixing photometric and geometric techniques. The processing of photometric informations is based on reflectance maps which are classic tools of shape from shading. The geometric features are extracted from the discrete surface and propagated along the surface. The propagation is based in one case on equal height discrete contour propagation and in the other case on region propagation. Both methods allow photometric stereo. Results of reconstruction from synthetic and real images are presented.

Keywords: Computer vision; Shape from shading; Discrete surface; Discrete normal.

1 Introduction

Shape recovery is an important domain of computer vision the problematic of which is to reconstruct a surface from 2D images of this surface. In general we consider only topographic surfaces S defined by $z = Z(x, y)$. The human system of vision may combine different informations in order to perform such a reconstruction, like shadings, focus, or stereo informations. But the combination of these informations is not trivial and the methods developed in computer vision are generally based on the processing of one kind of data: shading, shadows, motion, stereo-vision, defocus. In this paper we address the problematic of *shape from shading* which consists in using shading informations to retrieve the normals to the surface and thus its shape. This approach was introduce in 1970 by Horn [3] and many different methods have then been proposed (see Zhang et al. [13] for a comprehensive survey).

The main difficulty of shape from shading is that, for a given light source direction, a gray level may correspond to many different orientations of surface normal. The possible surface orientations for each intensity are usually represented by a map called *reflectance map*. Four approaches have been proposed:

- The global minimization approaches the principle of which is to minimize a global energy function. Usually this energy measures a difference between the image intensity and the intensity calculated from the reconstructed surface. Additional constraints like smoothness or integrability of the surface are often used (see for example Ikeuchi and Horn [5] and Frankot and Chellappa [2]).
- The local derivative approaches the principle of which is to try to recover the shape information from the intensity image and its derivatives. For example Lee and Rosenfeld [7] compute the normal vector to the surface by using the first derivative of the intensity.
- The linear approaches based on the linearization of the reflectance map (see Tsai and Shah [11]).
- The propagation approaches which consist in propagating shape informations from singular points. The first shape from shading technique introduced by Horn was a propagation approache with a reconstruction based on the extension of characteristic strips [3].

In some areas of the image the reconstruction of the surface may be altered by self shadows or intereflection. Self shadows depend both on the orientation of the surface and on the position of the light source. Thus the reconstruction may be improved by considering more than one image of the surface, for instance with several different light source directions. This technique, called photometric stereo, was introduced by Woodam [12] for the reconstruction of the surface gradient from several images and is suited to minimization methods (for example Saito proposed a method based on this principle for recovering the skin surface [8]).

In this work we develop methods based on the processing of geometric features extracted from the explicit reconstruction of the discrete surface. The reconstruction of the surface is based on the propagation of geometrical features related to equal height contours or regions. The propagation of equal height informations called level sets has been introduced by Kimmel and Bruckstein in 1992 [6]. A closed curve was initialized in the areas of singular point and propagated according to the light source direction. The evolution of the parametric curve was solved *via* a Eulerian formulation. The propagation direction in the direction of the light source makes possible to solve the ambiguities and to choose between multiple solutions. But this restriction is not compatible with the use of several light sources.

In the following we propose two methods of shape from shading based on the local estimation of the discrete surface normal and the propagation of height informations. Both methods can be used with several images in order to improve the reconstruction. We have tested these methods both on computer generated images and on real images provided by archaeologists. The real images we use are photos of carved stone and of resin casts of small archaeological objects. In both cases it is possible to make the hypothesis that the surface to reconstruct is a Lambertian surface. A Lambertian surface is a surface with only diffuse

reflectance. The intensity only depends on the normal vector and on the light source direction and thus does not depend on the position of the observer.

In section 2 we briefly recall the definition of reflectance map introduced by Horn in 1977 [4] to describe the luminance of a Lambertian surface in function of the orientation of the surface normal. In section 3 we present a shape from shading method based on the propagation of equal height contours and in section 4 a method based on the propagation of regions. Some results on synthetic and real images are presented in section 5.

2   Reflectance Map

It is convenient to choose a coordinate system (O, x, y, z) where the z axis is in the direction of the observer. A direction is represented by a pair of angles (θ, ϕ) where θ is an horizontal component and ϕ a vertical component (see Fig. 1-a). Moreover if we denote by p and q the partial derivatives of the height of the surface $Z(x, y)$ in the direction of x and y $(p = \frac{\partial Z(x,y)}{\partial x}$ and $q = \frac{\partial Z(x,y)}{\partial y})$ we have:

$$p = -\cos\phi\tan\theta \quad \text{and} \quad q = -\sin\phi\tan\theta \tag{1}$$

where (θ, ϕ) is the direction of the normal at the point (x, y). Given a light source of direction (p_s, q_s) and a surface $Z(x, y)$ it is convenient to describe the intensity of the surface by a function $R(p, q)$ of the orientation of the normal. This function is called a *reflectance map*. For a Lambertian surface the reflectance map is given by:

$$R(p, q) = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^2 + q_s^2}} \tag{2}$$

Given an image intensity L_i the possible orientations of the surface at this point are given by the *reflectance equation*:

$$R(p, q) = L_i \tag{3}$$

By substituting p and q by ϕ and θ in Equ. (3) we get:

$$\sqrt{1 + \tan^2\theta} = \frac{1 - \omega\tan\theta}{K} \tag{4}$$

with $K = L_i\sqrt{1 + p_s^2 + q_s^2}$ and $\omega = p_s\cos\phi + q_s\sin\phi$.

The problem of shape from shading is to define strategies to select the better normal orientation among all the possible solutions given by the reflectance map.

3   Contour Based Approach

If we suppose known the horizontal angle ϕ of a normal (p, q) we can derive an expression of the vertical angle from Equ. (4) and we get only two possible solutions:

$$\theta = \arctan\left(\frac{-\omega \pm \sqrt{-K^4 + K^2 + \omega^2 K^2}}{K^2 - \omega^2}\right) \tag{5}$$

The related partial derivatives are given by:

$$\begin{cases} p_1 = -\cos\phi\left(\frac{-\omega+\sqrt{-K^4+K^2+\omega^2 K^2}}{K^2-\omega^2}\right) \\ q_1 = -\sin\phi\left(\frac{-\omega+\sqrt{-K^4+K^2+\omega^2 K^2}}{K^2-\omega^2}\right) \end{cases}$$

$$\begin{cases} p_2 = -\cos\phi\left(\frac{-\omega-\sqrt{-K^4+K^2+\omega^2 K^2}}{K^2-\omega^2}\right) \\ q_2 = -\sin\phi\left(\frac{-\omega-\sqrt{-K^4+K^2+\omega^2 K^2}}{K^2-\omega^2}\right) \end{cases}$$

This result is illustrated by Fig. 1-b which shows a reflectance map as a function of ϕ and θ. The level displayed in black corresponds to the intensity of the point which we want to reconstruct the normal. If we consider a pixel Q of the gray level image for which the horizontal component ϕ_Q of the reconstructed normal is known, the two possible values for θ_Q are given by the points Q' and Q'' of the reflectance map. If we make the hypothesis that the surface to reconstruct is continuous we can choose between the both solutions by selecting the one which is the closest from the normal of a known neighbor. For example, let us consider a pixel P_1 of the gray level image for which the normal of the reconstructed surface is known, and let P_2 be a neighbor of P_1 in the image. If the horizontal component of the normal of the reconstructed surface at P_2 is known, there are only two solutions for the vertical component. We select the solution which is the closest from the normal at $P1$. On the running example the solution for θ_{P_2} is given by the point P_2' of the reflectance map (see Fig. 1-b).
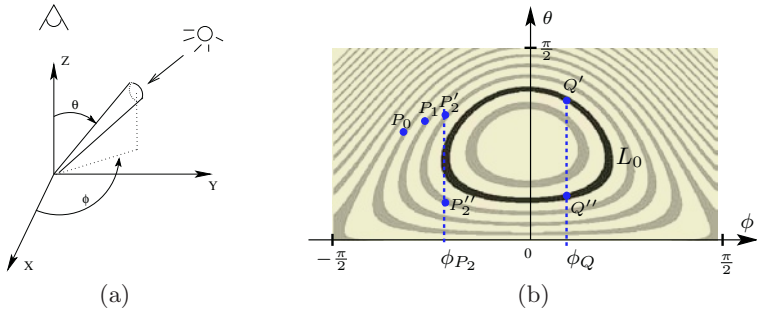
Fig. 1. The angles θ and ϕ associated with a normal (a) (here the normal is also the light source direction), and the two solutions corresponding to a value of the horizontal angle ϕ (b).

Let us now consider a discrete surface $Z(x, y)$ and a discrete contour C on this surface. The contour Γ is a sequence of points $(x_1, y_1, z_1) \ldots (x_k, y_k, z_k)$ such that the sequence $\gamma = (x_1, y_1) \ldots (x_k, y_k)$ is a connected path of the discrete plane according to a connectivity relation. Let us also suppose that all the points of Γ have the same height z. Thus we have $\Gamma = (x_1, y_1, z) \ldots (x_k, y_k, z)$ and the horizontal value of the normal vector ϕ at a point P of Γ is also the normal at the related point of the discrete 2D path γ. Thus we have decomposed the problem of determination of the normal into two subproblems:

1. The determination of the horizontal component of the normal by estimation of the discrete normals of a 2D discrete path. We use the discrete tangent estimator of the Euclidean paths model [1].
2. The determination of the vertical component of the normal by selecting in the reflectance map one of the two possible solutions.

Suppose now that we have determined the normal of the reconstructed surface for each point of the equal height contour. Thus we know an estimation of local derivatives (p, q) at each point of the contour and we use them to calculate the height of neighboring points. Fig. 2-a shows how these informations are propagated from an outer contour (the light gray points) for which heights and normals are known to an inner adjacent contour (the dark gray points). Each dark gray point is 4-adjacent to at least one light gray point.

Let P of coordinates (i, j) be a point which the height has to be calculated and $N(P)$ a set of 4-neighbors of P for which the height and the normal are known. The propagation of the height to adjacent points is computed according to the following formula:

$$Z(P) = \frac{1}{|N(P)|} \sum_{P' \in N(P)} (Z(P') + (i' - i)p_{P'} + (j' - j)q_{P'})$$

where i and j are the coordinates of the point P and $(p_{P'}, q_{P'})$ is the direction of the reconstructed normal at P'.

We can now define a strategy to reconstruct the surface corresponding to a shading image. The principle of the reconstruction is to traverse the whole surface by propagating equal height contours. First we initiate an equal height contour for which normals are assumed to be known. The evolution of the contour is done by iterating the following steps:

1. Calculate the normal horizontal components at each contour point of the equal height contour from the related 2D discrete curve.
2. Calculate the normal vertical components and the local derivatives p and q at each contour point of the equal height contour.
3. Propagate the height estimations to the adjacent contour in the direction of the propagation.
4. If some adjacent points have the same height as the contour (the difference between both heights is lower than a threshold), goto step 1.

When the propagation of the contour is achieved a new equal height contour is initialized and propagated.

With this approach it is straightforward to process together several images of a same surface. The images are supposed to have a same point of view and only the light source is changing from an image to another one. According to distribution of the iso-intensity area of the reflectance map we get more precision when the ϕ angle is closest to the light source direction. Thus for each point we consider the image in which its light direction is the closest from the ϕ angle.

4   Region Based Approach

The method of shape from shading described in this section is based on the evolution of patches initialized in plane areas of the source image which are orthogonal to the viewer direction. The principle of the method is to determine the height of the points adjacent to the patch by minimizing the error between the luminance calculated with the normal estimated in the patch and the real image intensity. If R_I is the reflectance map of an image I, if P is a pixel of I, and if (p, q) is the normal of the reconstructed surface at the point P, the error of the reconstruction at P is given by:

$$E(P) = (R_I(p, q) - I(P))^2$$

where $I(P)$ denotes the intensity of P in I.

The normal of a point at the boundary of the patch depends on both points being inside and outside the patch. For the inside points we assume that both the height and the normal is known (the height is a relative height depending on the height of the initial equal height patch). On the other hand no information is known for outside points. The method consists in initializing the height of each point adjacent to the patch with the average of the heights of its neighbors in the patch. Then a point adjacent to the patch, say P, is randomly selected and its height is changed upward or downward by a step Δh. The direction upward or downward is also selected randomly. This change will influence the normal of the points of the patch. Thus it is possible to calculate the error E at each neighbor of P in the patch before and after the vertical move of P. The move is validated only if it makes the error decrease.

Consider the example presented in Fig. 2-b. The initial patch is composed of the dark gray pixels and the pixels connected to the patch are the light gray ones. The white pixels are uninitialized pixels. Any change of the height of the point D involves a modification of the normals of the points A, B and E.

(a)   (b)
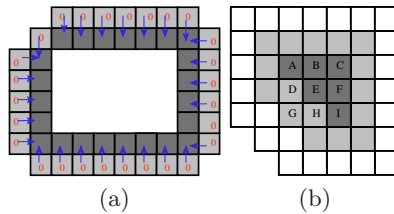
Fig. 2. Example of normal propagation for determining the height of the internal pixels (a) and influence of the height of the pixels connected to the patch (b).

If P is a point adjacent to the patch we denote by $\Pi(P)$ the points of the patch which are 8-adjacent to P. We define the error $E_\Pi(P)$ by:

$$E_\Pi(P) = \sum_{Q \in \Pi(P)} E(Q)$$

When P is randomly selected and moved along the vertical axis to a point P' the move is validated only if $E_\Pi(P') < E_\Pi(P)$. The processing of a point P of the neighborhood of the patch may be summarized as follows:

1. Select a direction (upward or downward).
2. Let P' be the result of the move of P in the selected direction. Estimate the normals at each point of Π for the positions P and P',
3. Compute $E_\Pi(P)$ and $E_\Pi(P')$.
4. If $E_\Pi(P') < E_\Pi(P)$ then validate the move of P.

The iteration of this process may diverge if the estimation of the normal is not accurate enough. This raises a problem since only the immediate neighborhood of a point P can be used to estimate the normal at this point. This is because the height is defined only for the points which are either inside the patch or adjacent to it. We have experimented the method with two different normal estimations: a discrete estimator and a direct geometric calculation.

When using the discrete estimator the vertical step Δh is set to 1. We used the discrete estimator proposed by Thurmer [10]. Since we only consider a 3D-neighborhood of size 1 the estimation of the normal at a point P is given by $\sum_{Q \in N(P)} \frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||}$ where $N(P)$ is the subset of the 26-neighborhood of P being outside the surface according to the normal direction.

Fig. 3.

With the direct geometric calculation the step Δh may be set to values lower than 1. Let P_0 be the point which the normal is estimated and $P_1, \ldots P_8$ its neighborhood on the reconstructed surface, and $S_1, \ldots S_4$ its interpixel neighborhood (see Figure 3). The normal at S_1 is estimated by the cross-product $\overrightarrow{P_0 P_2} \times \overrightarrow{P_3 P_1}$ and so on for S_2, S_3 and S_4. The normal at P_0 is then the average of normals at S_1, S_2, S_3 and S_4 [9].
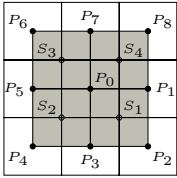
When the height of all the points adjacent to the patch has been determined we select the points (i, j) such that $Z(i, j) \in [h - \Delta h, h + \Delta h]$, where h is the reference height of the patch. The reference height of the initial patch is zero and change with the growing of the patch as described below. The selected points are added to the patch and the process is iterated until saturation. The reference height of the patch is then increased by Δh and the patch is saturated with the points which the estimated height is around the new current patch height. The points of decreasing height are processed in the same way. The process is iterated until the whole image is traversed.

This method has been tested with the both normal estimation methods described above. It appears that with the discrete estimator the method always converge when with the direct geometric calculation the method may diverge. On the other hand the discrete estimator produces rough results because of the smallness of the available neighborhood when the direct geometric calculation gives more accurate results and can be used with threshold lower than one. Thus we use first the discrete estimator to initialize the reconstructed surface

(a)   (b)

(c)   (d)

(e)   (f)
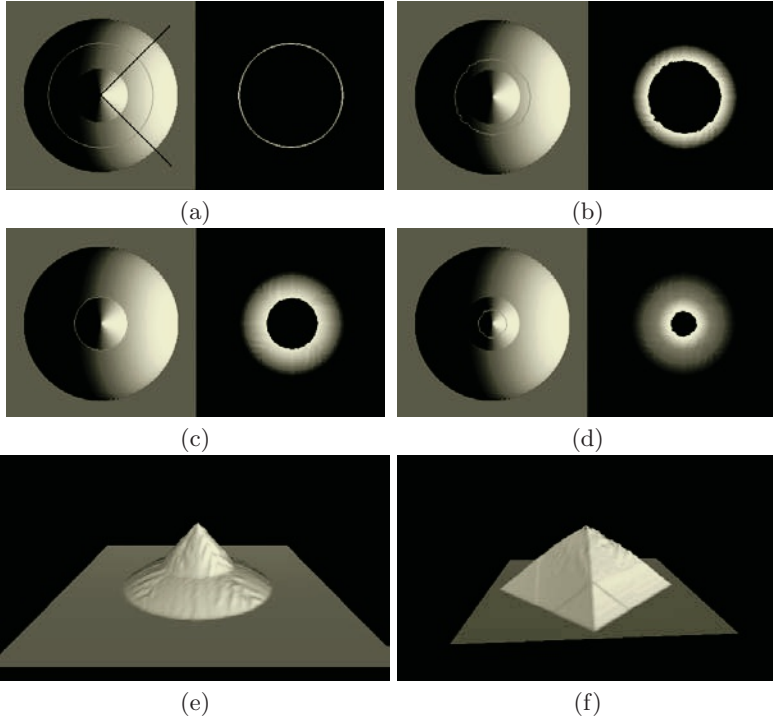
Fig. 4. Contour evolution with the contour based method and related height maps (from (a) to (f)). Reconstruction of a sphere-cone (e) and of a pyramid from synthetic images (f).

and then we refine the construction with the direct geometric calculation and with thresholds lower than one.

It is possible to make the method running with more than one patch. All the patches are processed in parallel. The initial reference height is the same for all the patches but as soon as the patches grow their reference height may change independently. When two different patches meet they are merged and both their reference heights and the height of their points are adjusted according to an average value of the height of the points of the junction.

Finally we can process several images in parallel by summing the error $E_\Pi(P)$ calculated for each reflectance map. The results are improved by invalidating in each image the self shadows areas.

5   Experiments

The first results presented in this section are reconstruction performed on computer generated images. We have used sets of four images of a same scene made of simple forms such as spheres and pyramids. The vertical orientation of the light source θ_s was set to $70°$ in order to improve the precision of the reflectance map. The Fig. 4 shows different steps of the reconstruction of an object which

(a)   (b)   (c)   (d)
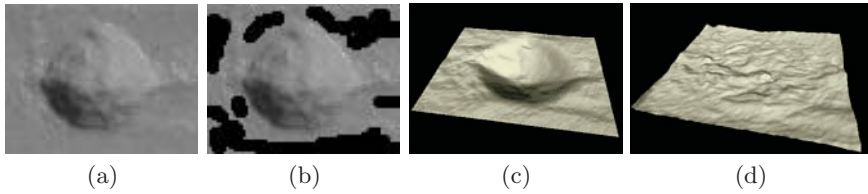
Fig. 5. One of the four real source images (a) and the initial patches used in the reconstruction. The result of the reconstruction (c) and a 3D visualization of the difference between the reconstructed surface and the scanned one.

consists of a sphere and a cone. For each point the image with the higher value is used to compute the vertical component of the normal. We can see on the image in the left part of Fig. 4-a the angular sector of this image which is used in the reconstruction. The other points are reconstructed by using one of the three other images. In Fig. 4-e and f is displayed the result of the reconstructions of the object Fig. 4 (a) and of a pyramid. On real images this method is quite sensitive to the noise and errors may be propagated along the whole reconstruction.

The second method gives very good results with synthetic images. We have also experimented it for the reconstruction of archaeological objects. The first example comes from miscroscope images of a resin cast of a small hole which was also scanned with a 3D scanner. It was thus possible to compare the result of the reconstruction with the real surface. We have used four images of the cast with a vertical angle of the light source θ_s set to $60°$ and an horizontal angle ϕ_s varying by step of approximatively $90°$. The light source direction was determined from the shadow of a small vertical pin. One of the four images is displayed in Fig. 5-a and the black areas in Fig. 5-b are the initial patches. The result of the reconstruction is displayed in Fig. 5-c and the differences between the reconstructed height map and the real one are displayed in Fig. 5-d. We have experimented this method with more complex surfaces such as the surface of the border of the replica of an ivory spatula. The spatula presents on her border a set of notches. One photo of the spatula with the initial patches is displayed in Fig. 6-a. The initial patches (drawn in black in the image) were selected on small plane region located between the notches. The interest of the reconstruction of archaeological objects have been validated by archaeologist expert.

6   Conclusion

In this paper we have proposed two new methods for reconstruct a surface from shading images. Both methods are based on the propagation of geometric features along the reconstructed discrete surface. The first method decomposes the estimation of the normal at a point of the reconstructed surface into a geometric estimation of the horizontal component and a photometric estimation of the vertical one. The second method uses normal reconstructions to minimize a photometric error function. It appears that the second method gives better results than the first one with photo of real objects but it would be interesting to try to use the principle of the first one to improve the results of the second one. More

(a)   (b)
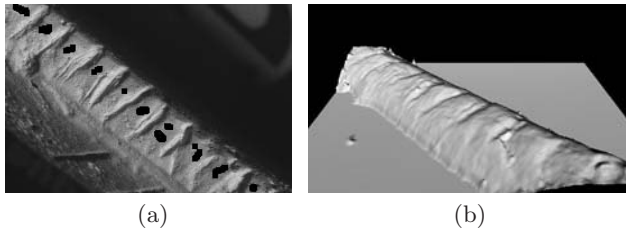
Fig. 6. One of the four real images of a replica of a prehistoric spatula with the initial patches drawn in black (a) and a representation with OpenGL of the reconstructed surface (b).

generally we have proposed an approach to develop shape from shading methods in the context of discrete geometry.

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