

Determination and Combination of Quantitative Weight Value from Multiple Preference Information

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Abstract. Recently, the concern of software quality increases rapidly. Although there have been many efforts to establish standards for software quality, such as ISO/IEC 9126, they provide only a framework for quality characteristics and evaluation process. They do not provide practical guidance for deriving reasonable weight value criteria for quality evaluation. This paper presents a method to draw the quantitative weight values from evaluator's subjective data in software evaluation in compliance with ISO/IEC 9126 standard. To eliminate evaluators' subjectiveness and uncertainty, the Dempster-Shafer (D-S) theory is improvised and utilized. The D-S theory is improved with merge rule to reduce the bias of weight value when they are merged with other evaluator's weight value. The proposed merge rule has been tested for its effectiveness with actual evaluation data.

1 Introduction

Software quality is defined as a specification of functions of which software performs. High quality software means that it not only satisfies its specification, but also achieves its quality characteristics, such as functionality, reliability, usability, efficiency, maintainability, portability. To evaluate software quality based on these quality characteristics, various standards and techniques can be applied. ISO/IEC 9126 [1], for example, provides a standard for establishing software quality characteristics and metrics. ISO/IEC 14598 [2] provides methods and procedures for performing of quality authentication. These standards, however, does not specify specific evaluation technique such as the calculation of weight value for evaluation items. Consequently, software quality evaluation tends to depend on assessor's subjective judgment and knowledge. That is, the interpretation of evaluation criteria, semantic relation between evaluation items, and measurement depends on evaluators' viewpoint [3], [4], [5]. Existing studies also overlook the problem of producing incorrect result in combining different weight values from multiple assessors [6], [7], [8].

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This paper describes a quantitative method for calculating the weight value of evaluation items by combining different evaluation values which is retrieved from multiple assessors' subjective opinion. Specifically, this paper discusses how to improve the way of aggregating and combining multiple assessors' opinions considering the relations between the evaluation items.

2 Related Works

2.1 Inference Using Bayes' Theory

Bayes' theory [11], [12], [13] is a statistical inference technique to estimate the probability of certain event occurring in which different hypotheses are given for each evidence. Assuming that there exists mutually exclusive hypothesis H_k ($k=1,2,\dots,n$) for a certain evidence E , then Bayes' theory is described as following (1).

$$P(H_n | E) = \frac{(P(E | H_n) \cdot P(H_n))}{\sum_{k=1}^n P(E | H_k) \cdot P(H_k)} \tag{1}$$

From the above expression, H_n and E are related in cause and effect relation. Then, $P(H_n)$ is called as a priori probability and $P(H_n|E)$ as a posteriori probability. Bayes' theory, however, has a few problems in its application. First, if there are n numbers of evidence and m numbers of hypothesis, it is necessary to know in prior $n \cdot m$ numbers of probability. This means that a large number of evidence and hypothesis requires a great number of priori probability calculation. In reality, it is not possible to provide all priori probability values in advance. Second, the probability value in Bayes' theory is assumed to be mutually exclusive. That is, if the probability of a certain event occurring be 0.5, then the probability for the event not occurring is $1-0.5=0.5$. However, in real situation, the probability for an event not occurring isn't always 0.5 since it is not known whether a certain event occurs or not. Finally, there is no rule for successively accruing the probability value in Bayes' theory. That is, if a new hypothesis is introduced, the Bayes theory has to compute the probability from the scratch again.

2.2 Quantitative Translation from Qualitative Relationship

The technique of deriving quantitative value from qualitative relationship information can be achieved by using Dempster Shafer (DS) theory [9], [10], [14], [17]. It uses the qualitative preference relations such as "A is more important than B" or "A is similar to C."

DS theory compensates the drawbacks of Bayes' theory by defining the belief of hypothesis on H to an interval between $Bel(H)$ and $Pl(H)$. $Bel(H)$ is a belief value of a given evidence and $Pl(H)$ is a plausibility value based on evidence. DS theory also provides a combination rule on merging two random variables that are independent each other (2). A function $m: 2^S \rightarrow [0,1]$ (S is a frame of discernment) is called a basic probability assignment [11], [12], [13], [15], [16].

$$m3 = \frac{\sum_{s1 \cap s2 = s3} m1(s1) \cdot m2(s2)}{1 - \sum_{s1 \cap s2 = \emptyset} m1(s1) \cdot m2(s2)} \tag{2}$$

This combination rule, however, may generate an empty set after intersection of two discernments [7], [8]. This can degrade the correctness of combination results. In addition, none of intending hypothesis can have total value, 1, after normalization process. This means that the lower supporting discernments can produce the higher supporting value after the combination as shown in table 1.

Table 1. Result of normal combination rule in DS theory

	m1	m1({a}) = 0.1	m1({b}) = 0.9
m2			
m2({a}) = 0.1		m3({a}) = 0.01	m3(∅) = 0.09
m2({c}) = 0.9		m3(∅) = 0.09	m3(∅) = 0.81
Normalization		m3({a}) = 0.01 / 0.01 = 1	

3 Improved Combination Rule in DS

As described earlier, the normal combination rule in DS theory can generate empty sets when different discernments are combined. The improved combination rule can reflect assessor’s different opinions by redistributing the empty set value to the discernment. Table 2 shows the result of combining the exclusive discernments using the improved combination rule. Let the basic probability assignment (bpa) assigned by assessor A be m1 and bpa by assessor B be m2. From the table 2, since m1({b})=0.9, m2({c})=0.9, the result of combination of m1 and m2 is m3({∅})=0.81. This means that assessor A assigns the wider opinion to discernment b and assessor B assigns the wider opinion to discernment c. In common, it assigns the narrower opinion to discernment a since the value of m3({∅}) is redistributed to the discernment b and c. In the same way, m3({b})=0.405 and m3({c})=0.405. The final result after the combination shows in table 2.

4 Weight Value Determination in Software Quality Evaluation

This chapter describes an example in calculating the weight value of software quality characteristics such as reliability defined in ISO/IEC 9126. For calculating the weight value, the first step is to determine the focal elements from assessors’ qualitative preference evaluation. After determining the focal elements, the basic probability assignment (bpa) value is computed [9], [10], [14], [17]. If there is more than one assessor, then apply the improved combination rule in combining the bpas. Finally, compute a weight value using the belief function (Bel) and the probability function (Pl).

Table 2. Result of combination about exclusive discernment using improved combination rule in DS theory

m2 \ m1	m1({a}) = 0.1	m1({b}) = 0.9
m2({a}) = 0.1	m3({a}) = 0.01	m3(∅) = 0.09
		m3({a}) = 0.009
		m3({b}) = 0.081
m2({c}) = 0.9	m3(∅) = 0.09	m3(∅) = 0.81
	m3({a}) = 0.009	m3({b}) = 0.405
	m3({c}) = 0.081	m3({c}) = 0.405
Result of improved combination rule	m3({a}) = 0.01 + 0.009 + 0.009 = 0.028	
	m3({b}) = 0.081 + 0.405 = 0.486	
	m3({c}) = 0.801 + 0.405 = 0.486	
	m3({a}) + m3({b}) + m3({c}) = 0.028 + 0.486 + 0.486 = 1	

(1) Determining the Focal Elements and bpa

Given assessor’s qualitative preference relationship, the focal element can be determined using the following definition (3) and (4).

$$A \succ B \leftrightarrow A \text{ is higher weight value than } B. \tag{3}$$

$$A \sim B \leftrightarrow (j(A \succ B) \ \& \ j(B \succ A)) \leftrightarrow A \text{ is similar weight value to } B \tag{4}$$

Let $T = \{a, b, c, d\}$ be the discernment and assume that an assessor has defined preference relationship as in (5).

$$\{a\} \succ \{d\}, \{b, c\} \succ \{b\}, \{c, d\} \prec \{d\}, \{a, b, c\} \sim \{b, c\}, \{b\} \sim \{d\}, \{b\} > 0, \{d\} > 0 \tag{5}$$

To determine the focal elements, it is necessary to eliminate illogical elements by using the elimination theorem [9], [10], [14], [17]. From the theorem, since $\{c, d\} \prec \{d\}$ and $\{a, b, c\} \sim \{b, c\}$, $\{c, d\}$ and $\{a, b, c\}$ are not focal elements. In this way, the complete focal elements can be achieved: $\{a\}, \{b\}, \{b, c\}, \{c\}, \{d\}$.

After achieving the focal elements, the bpa of $m1$ can be obtained using the equality and perceptor algorithm as following.

$$bpa \text{ of } m1 = \begin{bmatrix} m1(\{a\}) \\ m1(\{b\}) \\ m1(\{b, c\}) \\ m1(\{c\}) \\ m1(\{d\}) \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.125 \\ 0.25 \\ 0.25 \\ 0.125 \end{bmatrix}$$

(2) Application of the Improved Combination Rule

In case there is more than one assessor, then the improved combination rule is applied. Let's assume that other assessor's qualitative preference relationship is given as (6).

$$\{a\} S \{b\}, \{b, c\} > \{a\}, \{b, c\} < \{b\}, \{c\} S \{d\}, \{a\} > 0, \{d\} > 0 \tag{6}$$

The bpa and m2 can be obtained by repeating step (1).

$$bpa \text{ of } m2 = \begin{bmatrix} m2(\{a\}) \\ m2(\{b\}) \\ m2(\{b,c\}) \\ m2(\{c\}) \\ m2(\{d\}) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.125 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

Applying the improved combination rule to the bpa of m1 and m2, the results of the bpa of m3 is shown blow. To evaluate other assessor's value, repeat step (1) and (2).

$$bpa \text{ of } m3 = \begin{bmatrix} m3(\{a\}) \\ m3(\{b\}) \\ m3(\{b,c\}) \\ m3(\{c\}) \\ m3(\{d\}) \end{bmatrix} = \begin{bmatrix} 0.18 \\ 0.15 \\ 0.16 \\ 0.33 \\ 0.18 \end{bmatrix}$$

(3) Computation of a Weight Value Using Bel and PI

The values of Bel and PI function can be computed by using bpa of m3.

$$\begin{aligned} Bel(\{a\}) &= m3(\{a\}) = 0.18, & Bel(\{b\}) &= m3(\{b\}) = 0.15, \\ Bel(\{c\}) &= m3(\{c\}) = 0.33, & Bel(\{d\}) &= m3(\{d\}) = 0.18 \\ PI(\{a\}) &= m3(\{a\}) = 0.18, & PI(\{b\}) &= m3(\{b\}) + m3(\{b,c\}) = 0.15 + 0.16 = 0.31, \\ PI(\{c\}) &= m3(\{c\}) + m3(\{b,c\}) = 0.33 + 0.16 = 0.49, & PI(\{d\}) &= m3(\{d\}) = 0.18 \end{aligned}$$

The interval of focal elements using the values of Bel and PI function is then

$$\{a\}=[0.18, 0.18], \{b\}=[0.15, 0.31], \{c\}=[0.33, 0.49], \{d\}=[0.18, 0.18]$$

In this case, the focal element {b} ranges from 0.15 to 0.31. This means that the belief interval of focal element {b} is between 0.15 and 0.31. Then the weight value of focal element is determined by averaging the values of belief interval. The final weight value is given below.

$$\begin{aligned} \text{Weight value of } \{a\} &= 0.18, & \text{Weight value of } \{b\} &= 0.23, \\ \text{Weight value of } \{c\} &= 0.41, & \text{Weight value of } \{d\} &= 0.18 \end{aligned}$$

Table 3. Result after applying the combination rule

Assessor	Preference relationship	Focal element	Bpa
A	{a}>{b}	{a}, {b}	{a}=0.7, {b}=0.3
B	{c}>{d}	{c}, {d}	{c}=0.7, {d}=0.3
C	{a}>{c}	{a}, {c}	{a}=0.7, {c}=0.3
Combination of A and B (normal combination)	-	-	-
Combination of A and B (improved combination)	-	a},{b},{c},{d}	{a}=0.392, {b}=0.108, {c}=0.392, {d}=0.108
Combination of A and C (normal combination)	-	{a}	{a}=0.49 (before normalization) {a}=1 (after normalization)
Combination of A and C (improved combination)	-	{a},{b},{c}	{a}=0.784, {b}=0.108, {c}=0.108

Table 4. Combination result of multi evaluation items

Assessor	Preference relationship	Focal element	Bpa
A	{a}>{b},{b}>{c}	{a},{b},{c}	{a}=0.5 , {b}=0.3, {c}=0.2
B	{a}>{b,c}	{a},{b,c}	{a}=0.7, {b,c}=0.3
Combination of A and B (not exclusive combination)	-	{a},{b},{c},{b,c}	{a}=0.701 , {b}=0.152, {c}=0.09, {b,c}=0.056

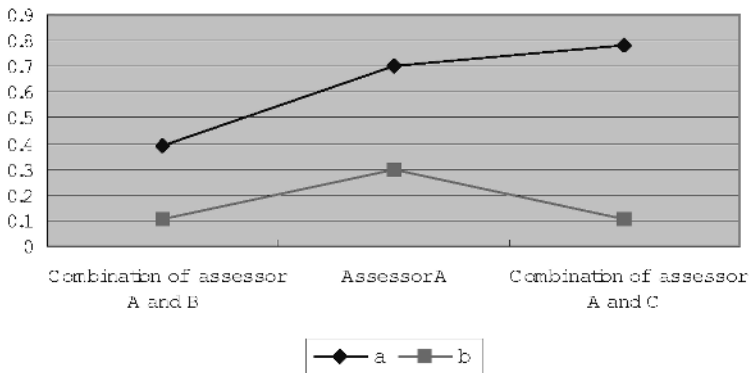


Fig. 1. Change of bpa on the focal element {a} using the improved combination

5 Evaluation Result and Analysis

5.1 Comparison of the Combination Rule

From the table 3, the results of normal combination of A and B are empty for any focal element because it is not possible to compute a weight value. However, the improved combination shows no empty set thanks to redistribution of empty set value.

5.2 Analysis of Improved Combination Rule

As shown in figure 1, the change of bpa on the focal element {a} using the improved combination is reduced from 0.7 to 0.392. This result indicates that assessor A assigns a high value to {a} but assessor B doesn't assigns any value to {a}. The change of bpa on the improved combination for A and C in table 3 is raised from 0.7 to 0.784. This result explains that both assessor A and assessor B assigns high value to {a}.

Two combination results on bpa with focal element {b} are same since the focal element {b} is not common on the two combinations. On the other hand, bpa of focal element {a} is 0.784 because focal element {a} is common on the combination of both assessor A and C. Table 4 illustrates that combination result of multi evaluation items.

The change of bpa on the combination A and B in table 4 is also raised from 0.5 to 0.701. Although assessor B had given preference relationship including multiple evaluation items such as $\{a\} > \{b,c\}$, we can obtain proper combination result.

6 Conclusion

The weight value calculation plays a key role in evaluation and selection of good quality software. This paper describes an quantitative method for calculating the weight value using DS theory. The proposed method eliminates the problem of assessor's subjective opinion and also improves the way of combining multiple assessors' opinion. The effectiveness of the new method has been verified with an example. The improved DS theory, however, still suffers from a great amount of numeric calculation ($O(n^2)$) since it uses every exclusive *bpas*.

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