GraphAEL: Graph Animations with Evolving Layouts*

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Abstract. GraphAEL extracts three types of evolving graphs from the Graph Drawing literature and creates 2D and 3D animations of the evolutions. We study citation graphs, topic graphs, and collaboration graphs. We also create difference graphs which capture the nature of change between two given time periods. GraphAEL can be accessed online at http://graphael.cs.arizona.edu.

1 Introduction

While static graphs arise in many applications, dynamic processes give rise to graphs that evolve through time. Such dynamic processes can be found in software engineering, internet/telecommunications traffic, and social networks, among others. Typically, the underlying graph structures are large, and depending on the time-granularity, may contain from a few to a large number of timeslices. The graph representing a particular timeslice may contain fewer/more vertices/edges than the one representing the previous timeslice, or it may differ in some other graph attributes, such as node-weights, edge-weights, or labels.

We describe algorithms and techniques for visualization of dynamic graph processes, using the evolution of the graph drawing literature as an example of such a process. GraphAEL is composed of three distinct components:

1. a relational database that stores and catalogs the data;
2. a graphical user interface (GUI) for querying the database and for graph creation;
3. a toolkit for interactive visualization of the resulting graphs.

We built a MySQL database and populated it with all the papers from the graph drawing proceedings in the period 1994-02. The database can be queried online via a GUI. In addition to standard queries and responses, we can generate graphs that evolve through time. In particular, we generate *citation graphs*, *co-authorship graphs*, and *topic graphs* at varying time-granularity. Each of these types comes in two flavors, *individual graphs* and *cumulative graphs*. To study

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the underlying graph processes, we also extract *difference graphs* which facilitate visual discovery of new trends and patterns. The evolution of the underlying graphs and the difference graphs can be studied via 2D and 3D animations.

2 Related Work

The visualization techniques for graphs that evolve through time are related to previous work in dynamic graph visualization. A number of papers use modifications of static graph drawing algorithms [4,13,18]. Incremental graph drawing is used in DynaDag [17]. Bayesian decision theory, together with a force-directed method are used in [3] to model dynamic graphs. Sequences of graphs are considered in [7] in order to create smoother transitions. Special classes of graphs such as trees, series-parallel graphs and *st*-graphs have also been studied in dynamic models [5,15].

More recently, several papers consider larger graphs that evolve through time. In [2] a system for visualizing network evolution is presented, in which each modification is shown in a separate layer of 3D representation with vertices common to two layers represented as columns connecting the layers. Along these lines, Collberg *et al* [6] describe a graph-based system for visualization of software evolution, which uses a modification a force-directed algorithm for visualization of large graphs [11]. Using a preliminary version of our system we analyzed the ACM digital library database, focusing on collaboration graphs and category graphs but the lack of citation data prevented us from extracting citation graphs [8].

3 The Relational Database

We gathered data about the graph drawing community, more specifically the nine years of published proceedings from the International Symposium on Graph Drawing. This data came from three sources: an XML file with information from the DBLP Computer Science Bibliography, a GML file used in the 2001 graph drawing contest [1], and the proceedings themselves. We designed a schema that maps the data to all necessary relations for creation of collaboration, topic, and citations graphs; see Fig. 1. The database schema consists of seven tables storing information about articles, authors, proceedings, and publishers. Rectangles represent entities, circles represent attributes, and diamonds represent binary relationships. The primary key for each entity is underlined.

We used MySQL version 4.0.12, a powerful open-source database system, to implement this schema. The XML file obtained from DBLP contains bibliographical information for every paper published in the GD proceedings. It does not, however, contain citation information. We built an XML parser to extract the relevant bibliographical data from DBLP. The citation data for the period 1994-00 was gathered from the the 2001 graph drawing contest GML file. Citation data for 2001 and 2002, was manually entered directly from the proceedings. We intend to update the database with new entries from the latest proceedings

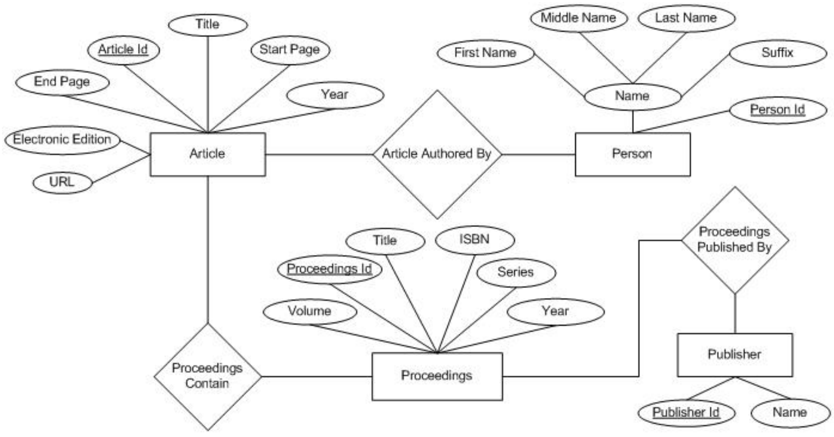


Fig. 1. The bibliographical database schema in Extended Entity-Relationship format.

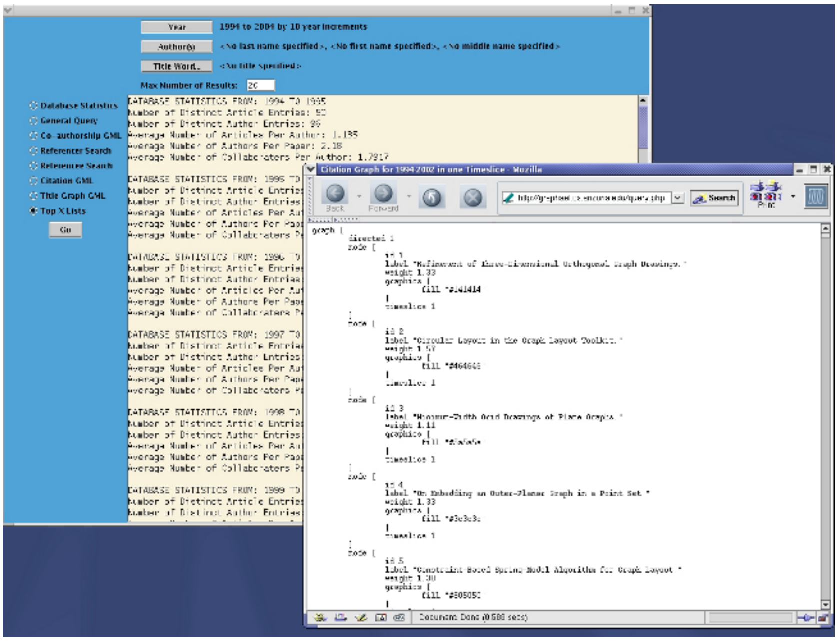
of the symposium on graph drawing, on an yearly basis. The database can be accessed and queried online at `http://graphael.cs.arizona.edu`.

4 Graphical User Interface

To interact with the database we created a graphical user interface; see Fig. 2. The GUI, written in Java, is available as both a web-based Java applet and a downloadable Java application. The applet version makes use of a PHP script as a servlet to maintain the security of the database server. The GUI allows users to query general dataset statistics, search for specific authors and papers, and query citation data by referencer and referencee. In addition, users can generate collaboration, topic, and citation graphs on the fly in the form of GML files with varying time-granularity. The applet opens a new browser window displaying the GML file, which can be saved locally. All of these queries can be limited in scope by date and the amount of results requested.

The top panel of the GUI contains buttons that allow the user to change the querying parameters. The left panel of the GUI contains buttons representing the different types of results and GML files that can be obtained. The 'Go' button, activates a servlet that will then query the database located on the server. The server returns the results to the servlet which then hands them to the applet where they are displayed. The following queries are supported by the GUI:

- Database Statistics: lists data statistics, including number of articles and authors;
- General Query: allows for search by author name, or title words;
- Co-authorship GML: creates a co-authorship graph, for the given parameters;
- Referencer Search: finds all papers that reference a particular paper;



Year: 1994 to 2004 by 10 year increments
Author(s): <No last name specified>, <No first name specified>, <No middle name specified>
Title Word...: <No title specified>
Max Number of Results: 20

```
DATABASE STATISTICS FROM: 1994 TO 1995
Number of Distinct Article Entries: 50
Number of Distinct Author Entries: 96
Average Number of Articles Per Author: 1.135
Average Number of Authors Per Paper: 2.18
Average Number of Collaborators Per Author: 1.7917

DATABASE STATISTICS FROM: 1995 TO
Number of Distinct Article Entries
Number of Distinct Author Entries
Average Number of Articles Per Au
Average Number of Authors Per Pap
Average Number of Collaborators P

DATABASE STATISTICS FROM: 1996 TO
Number of Distinct Article Entries
Number of Distinct Author Entries
Average Number of Articles Per Au
Average Number of Authors Per Pap
Average Number of Collaborators P

DATABASE STATISTICS FROM: 1997 TO
Number of Distinct Article Entries
Number of Distinct Author Entries
Average Number of Articles Per Au
Average Number of Authors Per Pap
Average Number of Collaborators P

DATABASE STATISTICS FROM: 1998 TO
Number of Distinct Article Entries
Number of Distinct Author Entries
Average Number of Articles Per Au
Average Number of Authors Per Pap
Average Number of Collaborators P

DATABASE STATISTICS FROM: 1999 TO
Number of Distinct Article Entries
Number of Distinct Author Entries
```

http://graphael.cs.arizona.edu/query.php

```
graph [
        directed 1
        node [
                id 1
                label "Refinement of Three-Dimensional Orthogonal Graph Drawings."
                weight 1.33
                graphics [
                        fill "#141414"
                ]
                timeslice 1
        ]
        node [
                id 2
                label "Circular Layout in the Graph Layout Toolkit."
                weight 1.57
                graphics [
                        fill "#464646"
                ]
                timeslice 1
        ]
        node [
                id 3
                label "Minimum-Width Grid Drawings of Plane Graphs."
                weight 1.11
                graphics [
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                ]
                timeslice 1
        ]
        node [
                id 4
                label "On Embedding an Outer-Planar Graph in a Point Set."
                weight 1.33
                graphics [
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                ]
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        ]
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                weight 1.30
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                        fill "#505050"
                ]
```

Document: Done (0.588 secs)

Fig. 2. The Graphical User Interface, shown here displaying a GML file.

- Referencee Search: finds all papers that are referenced from a particular paper;
- Citation GML: creates a citation graph, for the given parameters;
- Topic Graph GML: creates a topic graph, for the given parameters;
- Top X Lists: lists the most productive authors, most collaborative authors, etc.

5 Interactive Visualization of Evolving Graphs

GraphAEL calculates the layout for evolving graphs quickly. The underlying graph drawing algorithm is a modification of the GRIP algorithm [11,12]. Given a graph $G = (V, E)$, GRIP computes a hierarchical sequence of vertex filtrations given by a sequence of m subsets of V. Let V_i, V_j be two filtrations such that $i < j$, then $V_i \subset V_j$ and $V_m = V$. The graphs defined by this filtration sequence are laid out in order, with the layout of the graph at each level i providing a basis to guide the layout of the graph at level $i + 1$. In this way large-scale structures of the graph are quickly captured by the filtration levels with low indices, and the small-scale details are refined in the layout of the levels with high indices.

The layout of each filtration level is calculated with the force-directed placement method of Kamada-Kawai [14]. For a vertex v in G, the displacement of v is calculated by:

$$\boldsymbol{F}_{KK}(v) = \sum_{u \in N_i(v)} \left(\frac{\|pos[u] - pos[v]\|^2}{dist_G(u,v)^2 \cdot edgeLength^2} - 1 \right) (pos[u] - pos[v])$$

This equation is convenient because it calculates the displacement on each vertex based only upon its physical and graph distances from other vertices in the graph. Since in general, vertices in a particular filtration level are not adjacent to any other vertex in the level, but are instead connected to other vertices in the higher levels through a sequence of edges, using graph distances is a reasonable approach. Only in the last level of of filtration does the adjacency make sense, since the last level is the same as the initial graph. Thus at this level, it is possible to use force directed placement methods which depend on a notion of adjacency. The Fruchterman-Reingold method [10] for calculating the displacement of a vertex v, $\boldsymbol{F}_{FR}(v) = \boldsymbol{F}_{a,FR} + \boldsymbol{F}_{r,FR}$, is used at the last filtration level, where the attractive and repulsive force vectors are given by:

$$\boldsymbol{F}_{a,FR} = \sum_{u \in Adj(v)} \frac{\|pos[u] - pos[v]\|^2}{edgeLength^2} (pos[u] - pos[v])$$

$$\boldsymbol{F}_{r,FR} = \sum_{u \in N_i(v)} s \frac{edgeLength^2}{\|pos[u] - pos[v]\|^2} (pos[v] - pos[u])$$

5.1 GraphAEL's Layout Algorithm

While GRIP does a decent job of laying out large static graphs, there is no appropriate mechanism to assign the vertices and edges of a graph attributes, such as weights, which might affect the layout. This is a major drawback in visualizing certain relations and even more so in visualizing evolving graphs. For instance, in visualizing the GD citation graph, it is useful to have a notion of both the weight of a vertex, (which might represent the number of times a paper is cited), and a temporal notion (which might represent the year the paper was written). In contrast, GraphAEL supports the layout of graphs with node-weights and edge-weights, as well as the notion of a *timeslice* which is used to visualize graphs with a temporal component (evolving graphs).

GraphAEL tries to place heavy nodes well away from each other and to place vertices connected by heavy edges, closer to each other. Appropriate modifications to the force-directed algorithm were made to accommodate these characteristics. Each edge in a graph is given an ideal length that it will attempt to attain, although this is not always possible. An edge e of weight w_e connecting vertices u, v of weight w_u, w_v respectively is given an ideal length of $\sqrt{w_u \cdot w_v}/w_e$.

Since the Kamada-Kawai method is used to determine parts of the layout, it is necessary to come up with a modified graph distance for weighted graphs. Because of the computational and space requirements of calculating the effects of all paths between two vertices, or of computing the shortest weighted path between them, GraphAEL uses an approximation. Let $p_1, p_2, \ldots, p_n$ be the sequence of vertices in the shortest unweighted path in G connecting two vertices, u and v. The modified Kamada-Kawai vector is given by:

$$\sum_{u \in N_i(v)} \left(\frac{2\|pos[u] - pos[v]\|^2}{optDist_G(u,v)^2 \cdot edgeLength^2 + \|pos[u] - pos[v]\|^2} - 1 \right) (pos[u] - pos[v]),$$

where $optDist_G(u,v)$ is defined by,

$$optDist_G(u,v) = \sum_{i=2}^{n} \frac{\sqrt{w_{p_i} \cdot w_{p_{i-1}}}}{w_{e_{p_i p_{i-1}}}}.$$

To achieve an aesthetically pleasing layout of the graph, it is also necessary to modify the Fruchterman-Reingold vectors. As in GRIP, the Kamada-Kawai calculation results in a good approximate layout so that the Fruchterman-Reingold calculations can quickly "tidy up" the layout. The modifications needed to support weighted graphs with the ideal edge lengths described earlier are simple:

$$\boldsymbol{F}_{a,w,FR} = \sum_{u \in Adj(v)} \frac{w_e \cdot \|pos[u] - pos[v]\|^2}{edgeLength^2} (pos[u] - pos[v])$$

$$\boldsymbol{F}_{r,w,FR} = \sum_{u \in N_i(v)} s \frac{edgeLength^2 \cdot \sqrt{w_u \cdot w_v}}{\|pos[u] - pos[v]\|^2} (pos[v] - pos[u])$$

5.2 Timeslices and Animation

To visualize a series of graphs, embodying the evolution of a set of relationships over time, we associate another attribute, called a *timeslice*, with each vertex. The timeslice of a vertex is simply a label associated with a vertex. We use the timeslice attribute to partition the vertices of a graph into groups by time. Several simple modifications to the layout algorithm are needed to accommodate timeslice information. Repulsive forces should only exist between vertices in the same timeslice, and thus the optimal distance between vertices in different timeslices which are connected by an edge is zero. In the Kamada-Kawai vectors, the only alteration required is that the the function $optDist_G(u,v)$ be redefined so that for two vertices u, v with timeslice indices of t_u and t_v respectively:

$$optDist_G(u,v) = \delta_{t_u t_v} \cdot \frac{\sqrt{w_u \cdot w_v}}{w_e},$$

where $\delta_{t_u t_v}$ is 1 if $t_u = t_v$ and 0 otherwise. The modifications needed for the Fruchterman-Reingold calculations are similar. Repulsive forces are simply eliminated between vertices in different timeslices, $\boldsymbol{F}_{r,w,t,FR} = \delta_{t_u t_v} \cdot \boldsymbol{F}_{r,w,FR}$ while the attractive forces remain unchanged, $\boldsymbol{F}_{a,w,t,FR} = \boldsymbol{F}_{a,w,FR}$.

The timeslice information alone is not enough to nicely layout evolving graphs; it is also necessary to arrange edges between the timeslices so that the resulting layouts can be used for animation. Perhaps the most straightforward and common case is when one has a series of “snapshots” of a graph taken at some interval over a period of time. When laying out such a graph there are normally two types of constraints which need to be met: Each timeslice should have a pleasing layout, and the layout of consecutive timeslices should be similar, that is, the mental map should be preserved. Another way of formulating this latter constraint is that vertices on one timeslice should tend toward their positions in adjacent timeslices. To meet these constraints the timeslices are combined into a single graph and edges are added between vertices with the same labels in adjacent timeslices.

Since vertices in different timeslices have no repulsive forces between them (but the edges between them retain their attractive forces) these additional edges attract each vertex toward the vertices associated with it in adjacent timeslices. The trade-off between layout within a timeslice and mental map preservation can easily be adjusted by altering the weights of the inter-timeslice edges. Heavier weights lead to better mental map preservation and lighter weights improve the layout of each timeslice. For the visualization of the graphs in this paper the weight of an inter-timeslice edge connecting two vertices of weights w_u and w_v is given a weight of $(w_u + w_v)/2$, causing heavier vertices to stay in the same position over multiple timeslices.

One disadvantage of this method of adding edges is that although the movements of a vertex from one timeslice to the next are minimized, it is still possible that over a span of many timeslices vertices will drift far from their initial positions. If this is an undesirable effect it can be prevented by adding additional edges between vertices in distant timeslices. A straight-forward approach is to create a clique of all the occurrences of each vertex over all timeslices. `GraphAEL` has both options implemented.

Once the layout of the evolving graph has been computed, several visualization options can be used. A static view, capturing all timeslices can be displayed. Each timeslice can be restricted to its own 2D plane or can be drawn in 3D and the individual graphs arranged on top of each other. Edges connecting different timeslices can be shown or hidden. Alternatively, the evolution can be shown via an animation between the timeslices in 2D or 3D. The animation uses an interpolation between timeslices and fading in/out of appearing/disappearing vertices and edges. Edge and vertex coloring is used to convey additional information. For example, in the citation graphs we indicate the direction of an edge by gradually varying the color from light(source) to dark(target) and the vertex color indicates the relative age of a particular paper.

Finally, `GraphAEL` can generate and display *difference graphs*. The difference graph between two adjacent timeslices captures the difference between the two underlying graphs. For example, when visualizing the topic graph we may miss significant growth in an area with a few papers if the corresponding vertices are relatively small. With this in mind, the difference graph captures the percentage

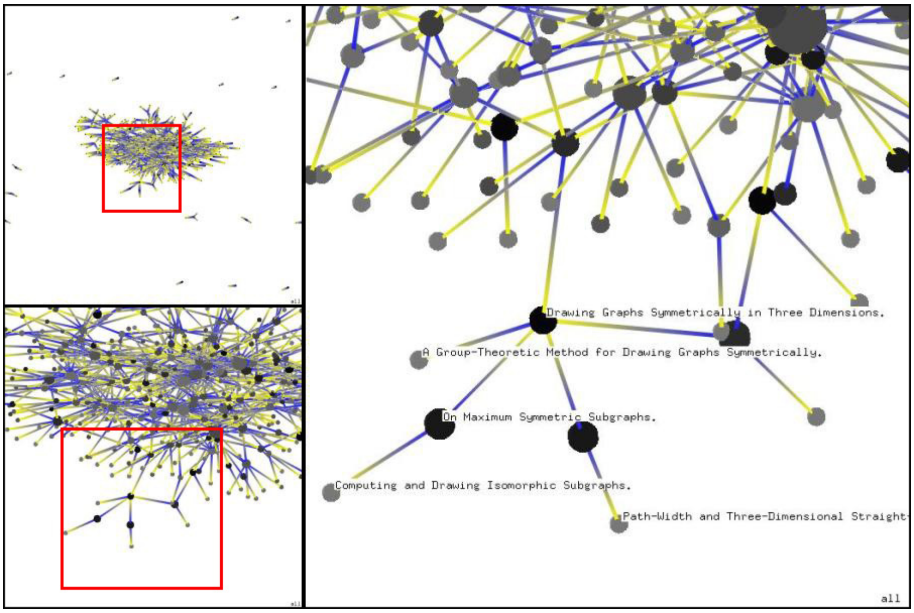
Drawing Graphs Symmetrically in Three Dimensions.
A Group-Theoretic Method for Drawing Graphs Symmetrically.
On Maximum Symmetric Subgraphs.
Computing and Drawing Isomorphic Subgraphs.
Path-Width and Three-Dimensional Straight-
all

Fig. 3. Top left: Cumulative citation graph for the period 1994-02. **Bottom Left:** view of the area enclosed in the red rectangle from top left. **Right:** Labeled vertices in a "symmetry" cluster. Directed edges go from light/source to dark/target. The color of the nodes corresponds to the year it was published: the lighter the node, the older the paper.

change between levels. We create a series of difference graphs and as it is an evolving graph we can use all the tools for evolving graphs.

6 Interactive Visualization of the GD Literature

In this section we consider three types of evolving graphs extracted from the MySQL database: citation graphs, topic graphs, and collaboration graphs. We also calculated some general statistics about the data and we include gnuplot charts and tables for the data in the Appendix.

6.1 Citation Graphs

The citation graph for a given time period is a simple vertex-weighted directed graph in which the vertices correspond to distinct articles. A directed edge connects two articles, with the article that cites as the source and the cited article as the target. The weight of a vertex is determined by the number of citations an article received, divided by the number of years since its publication. The citation graphs can reveal information about influential papers in the field, and their relation with the rest of the graph. Fig. 3 shows several views of the citation

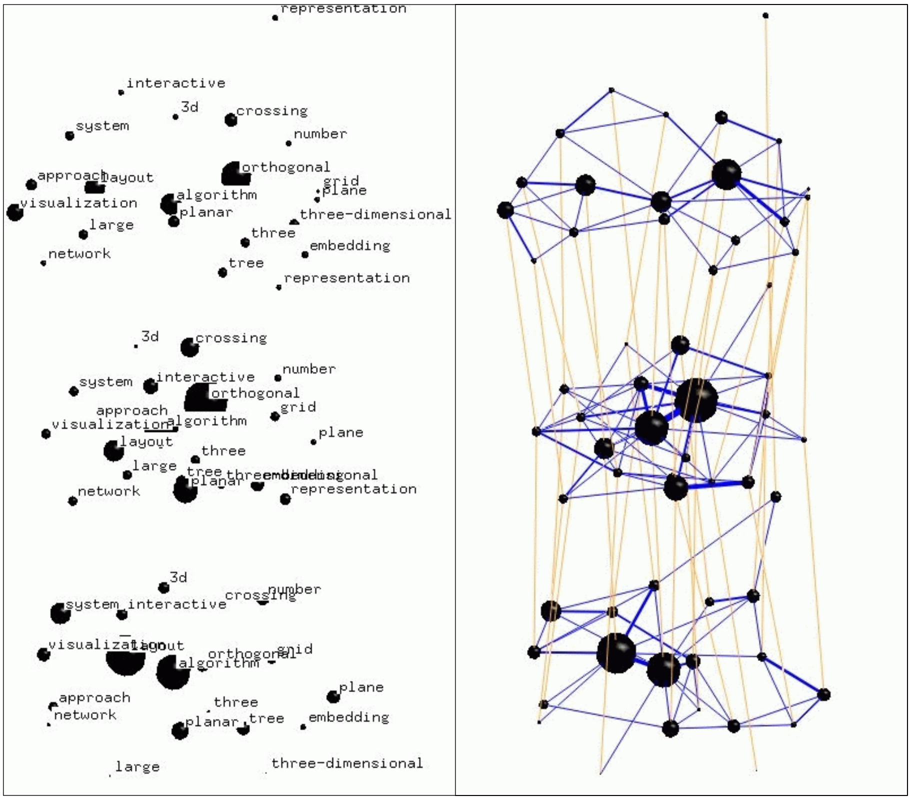

Fig. 4. Individual topic graphs for the period 1994-02, restricted to the top 20 title words, shown in timeslices of 3 consecutive years each: 1994-96 on the bottom, 1997-99 in the middle, and 2000-2002 on the top. On the left are all the labeled nodes in each timeslice with invisible edges.

graph. The size of a vertex reflects its weight, i.e. a large vertex corresponds to an article that is cited many times relative to its age. The vertex color indicates the age of the article, the darker a vertex the more recent the corresponding article is. The edge color changes from yellow to blue to indicate direction, where the blue end of the edge is the target vertex.

6.2 Topic Graphs

The topic graph for a given time period is a simple vertex-weighted and edge-weighted undirected graph in which vertices correspond to title words and edges are placed between title words that co-occur in research papers. The weight of a vertex in the topic graph is proportional to the number of papers that contain the corresponding word in their titles. Similarly, the edge weight is proportional to the number of papers in which the two corresponding words co-occur in the title. Topic graphs can reveal information about related topics, the concentration

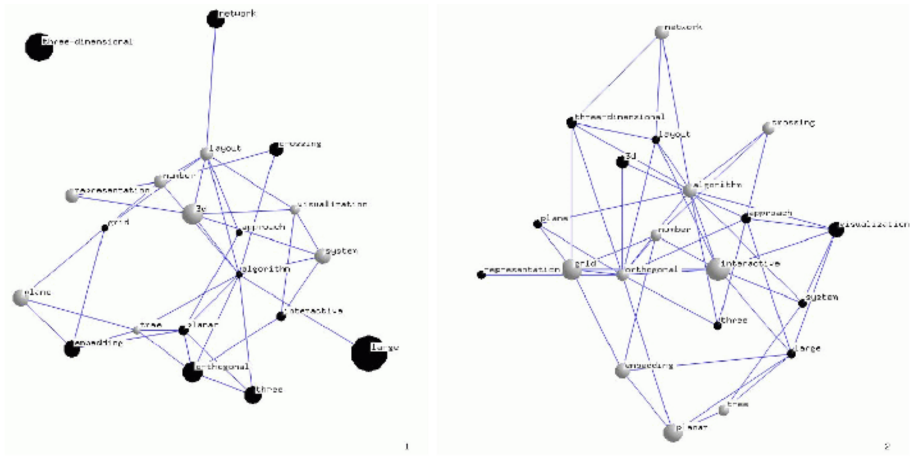

Fig. 5. The individual *difference graphs* capturing the change from 1994– to 1997–99 (left) and 1997–99 to 2000–02 (right). Dark nodes indicate growth and light nodes indicate decline. Note the percentage increase in "large" and decline in "interactive".

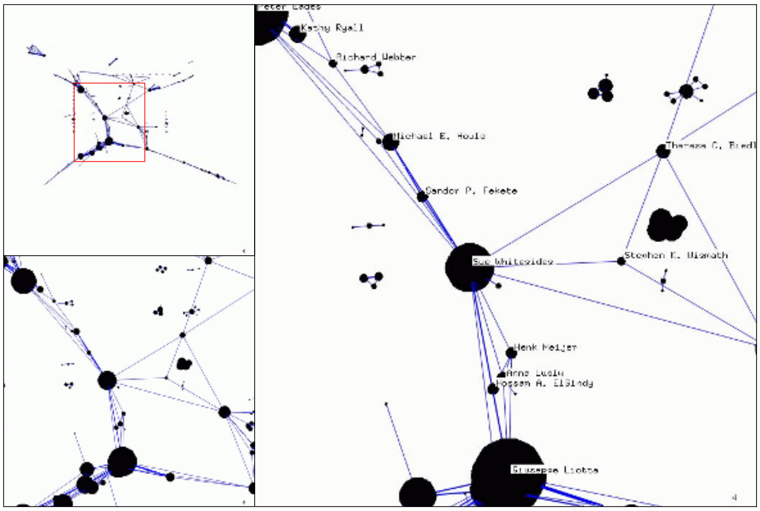

Fig. 6. **Top left:** collaboration graph for the period 1994-97. **Bottom Left:** view of the area enclosed in the red rectangle from top left. **Right:** closer view with more details.

of research on a specific topic and the trends as they evolve through time. Fig. 4 contains several visualizations of the topic graph. The edges connecting same vertices in adjacent timeslices help with mental map preservation and are used to determine the vertex locations in the animation between timeslices.

We also construct topic *difference graphs* in which the weight of a vertex is the percentage of its weight change between adjacent timeslices. The edge weight

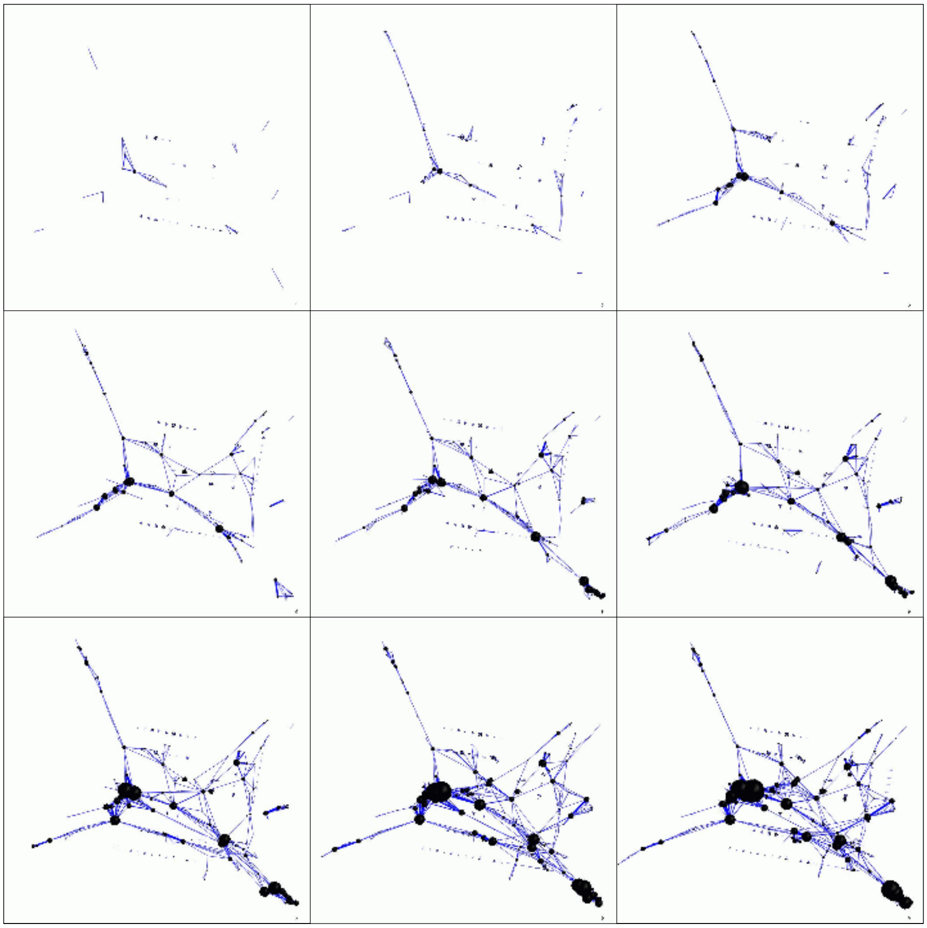

Fig. 7. Screenshots from the animation of the cumulative collaboration graph in the period 1994-02. Each timeslice represents a one year period. A movie of the actual animation can be seen at the `GraphAEL` webpage.

is proportional to the corresponding percent-change between two time periods. Together with the topic graphs, the topic-change graphs can be effectively used to visualize the evolution of GD research focus through time. Fig. 5 shows the topic-change graph corresponding to the topic graph in Fig. 4.

6.3 Collaboration Graphs

Collaboration graphs are simple undirected node-weighted and edge weighted graphs. Vertices represent unique authors and there is an edge between two vertices if the respective authors have collaborated on a research paper. The weight of a vertex is determined by the author's collaborativeness and productivity. The

Table 1. Statistics for GD, ACM [8], and NCSTRL [16].

General	**GD-Value**	**ACM-value**	**NCSTRL-value**
Total papers	413	51503	13169
Total authors	502	81279	11994
Authors per paper	2.54	2.32	2.22
Papers per author	2.09	1.80	2.55
Collaborators per author	3.74	3.36	3.59
Percentage of giant component	49	49	57.2
Percentage of 2^{nd} component	0.02	0.11	0.004
Clustering Coefficient	0.60	0.62	0.50
Average Distance	4.33	9.26	9.7
Maximum Distance	10	30	31

weight of an edge represents the strength of the collaborative ties between two authors. Fig. 7 shows the evolution of the cumulative collaboration graph, i.e., each timeslice adds to the previous timeslice. Once an interesting timeslice to be viewed is found it is easy to get a detailed visualization of that specific timeslice. For example, Fig. 6 shows the collaboration graph for the period 1994-97. For a more detailed view we zoom in one of the areas where there is a concentration of several well-connected components.

6.4 Statistics from the Graph Drawing Literature

We compared the data from the GD database, with the data from two computer science databases, the ACM database [8] and the NCSTRL database [16]. Table 1 shows a summary of the overall statistics and they are examined in detail in the full version of the paper [9].

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