Chapter 1
Introduction

I introduce the basic principles of control theory in a concise self-study guide. I wrote this guide because I could not find a simple, brief introduction to the foundational concepts. I needed to understand those key concepts before I could read the standard introductory texts on control or read the more advanced literature. Ultimately, I wanted to achieve sufficient understanding so that I could develop my own line of research on control in biological systems.

This tutorial does not replicate the many excellent introductory texts on control theory. Instead, I present each key principle in a simple and natural progression through the subject.

The principles build on each other to fill out the basic foundation. I leave all the detail to those excellent texts and instead focus on how to think clearly about control. I emphasize why the key principles are important, and how to make them your own to provide a basis on which to develop your own understanding.

I illustrate each principle with examples and graphics that highlight key aspects. I include, in a freely available file, all of the Wolfram Mathematica software code that I used to develop the examples and graphics (see Preface). The code provides the starting point for your own exploration of the concepts and the subsequent development of your own theoretical studies and applications.

1.1 Control Systems and Design

An incoming gust of wind tips a plane. The plane's sensors measure orientation. The measured orientation feeds into the plane's control systems, which send signals to the plane's mechanical components. The mechanics reorient the plane.

S. A. Frank, *Control Theory Tutorial*, SpringerBriefs in Applied Sciences and Technology, https://doi.org/10.1007/978-3-319-91707-8_1

An organism's sensors transform light and temperature into chemical signals. Those chemical signals become inputs for further chemical reactions. The chain of chemical reactions feeds into physical systems that regulate motion.

How should components be designed to modulate system response? Different goals lead to design tradeoffs. For example, a system that responds rapidly to changing input signals may be prone to overshooting design targets. The tradeoff between performance and stability forms one key dimension of design.

Control theory provides rich insights into the inevitable tradeoffs in design. Biologists have long recognized the analogies between engineering design and the analysis of biological systems. Biology is, in essence, the science of reverse engineering the design of organisms.

1.2 Overview

I emphasize the broad themes of feedback, robustness, design tradeoffs, and optimization. I weave those themes through the three parts of the presentation.

1.2.1 Part I: Basic Principles

The first part develops the basic principles of dynamics and control. This part begins with alternative ways in which to study dynamics. A system changes over time, the standard description of dynamics. One can often describe changes over time as a combination of the different frequencies at which those changes occur. The duality between temporal and frequency perspectives sets the classical perspective in the study of control.

The first part continues by applying the tools of temporal and frequency analysis to basic control structures. Open-loop control directly alters how a system transforms inputs into outputs. Prior knowledge of the system's intrinsic dynamics allows one to design a control process that modulates the input–output relation to meet one's goals.

By contrast, closed-loop feedback control allows a system to correct for lack of complete knowledge about intrinsic system dynamics and for unpredictable perturbations to the system. Feedback alters the input to be the error difference between the system's output and the system's desired target output.

By feeding back the error into the system, one can modulate the process to move in the direction that reduces error. Such self-correction by feedback is the single greatest principle of design in both human-engineered systems and naturally evolved biological systems.

I present a full example of feedback control. I emphasize the classic proportional, integral, derivative (PID) controller. A controller is a designed component of the system that modulates the system's intrinsic input–output response dynamics.

In a PID controller, the proportional component reduces or amplifies an input signal to improve the way in which feedback drives a system toward its target. The integral component strengthens error correction when moving toward a fixed target value. The derivative component anticipates how the target moves, providing a more rapid system response to changing conditions.

The PID example illustrates how to use the basic tools of control analysis and design, including the frequency interpretation of dynamics. PID control also introduces key tradeoffs in design. For example, a more rapid response toward the target setpoint often makes a system more susceptible to perturbations and more likely to become unstable.

This first part concludes by introducing essential measures of performance and robustness. Performance can be measured by how quickly a system moves toward its target or, over time, how far the system tends to be from its target. The cost of driving a system toward its target is also a measurable aspect of performance. Robustness can be measured by how likely it is that a system becomes unstable or how sensitive a system is to perturbations. With explicit measures of performance and robustness, one can choose designs that optimally balance tradeoffs.

1.2.2 Part II: Design Tradeoffs

The second part applies measures of performance and robustness to analyze tradeoffs in various design scenarios.

Regulation concerns how quickly a system moves toward a fixed setpoint. I present techniques that optimize controllers for regulation. *Optimal* means the best balance between design tradeoffs. One finds an optimum by minimizing a cost function that combines the various quantitative measures of performance and robustness.

Stabilization considers controller design for robust stability. A robust system maintains its stability even when the intrinsic system dynamics differ significantly from that assumed during analysis. Equivalently, the system maintains stability if the intrinsic dynamics change or if the system experiences various unpredictable perturbations. Changes in system dynamics or unpredicted perturbations can be thought of as uncertainties in intrinsic dynamics.

The stabilization chapter presents a measure of system stability when a controller modulates intrinsic system dynamics. The stability measure provides insight into the set of uncertainties for which the system will remain stable. The stability analysis is based on a measure of the distance between dynamical systems, a powerful way in which to compare performance and robustness between systems.

Tracking concerns the ability of a system to follow a changing environmental setpoint. For example, a system may benefit by altering its response as the environmental temperature changes. How closely can the system track the optimal response to the changing environmental input? Once again, the analysis of performance and robustness may be developed by considering explicit measures of system characteristics. With explicit measures, one can analyze the tradeoffs between competing goals and how alternative assumptions lead to alternative optimal designs.

All of these topics build on the essential benefits of feedback control. The particular information that can be measured and used for feedback plays a key role in control design.

1.2.3 Part III: Common Challenges

The third part presents challenges in control design. Challenges include nonlinearity and uncertainty of system dynamics.

Classical control theory assumes linear dynamics, whereas essentially all processes are nonlinear. One defense of linear theory is that it often works for real problems. Feedback provides powerful error correction, often compensating for unknown nonlinearities. Robust linear design methods gracefully handle uncertainties in system dynamics, including nonlinearities.

One can also consider the nonlinearity explicitly. With assumptions about the form of nonlinearity, one can develop designs for nonlinear control.

Other general design approaches work well for uncertainties in intrinsic system dynamics, including nonlinearity. Adaptive control adjusts estimates for the unknown parameters of intrinsic system dynamics. Feedback gives a measure of error in the current parameter estimates. That error is used to learn better parameter values. Adaptive control can often be used to adjust a controller with respect to nonlinear intrinsic dynamics.

Model predictive control uses the current system state and extrinsic inputs to calculate an optimal sequence of future control steps. Those future control steps ideally move the system toward the desired trajectory at the lowest possible cost. At each control point in time, the first control step in the ideal sequence is applied. Then, at the next update, the ideal control steps are recalculated, and the first new step is applied.

By using multiple lines of information and recalculating the optimal response, the system corrects for perturbations and for uncertainties in system dynamics. Those uncertainties can include nonlinearities, providing another strong approach for nonlinear control.