Chapter 23
Pedagogies of Emergent Learning

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Abstract We distinguish emergent learning from "teleological" learning, which is learning for the sake of passing pre-defined tests and goals. While teleological learning may succeed or fail, emergent learning is always going on in ways that move pass disciplinary boundaries and anticipated results. To advance a perspective on pedagogies of emergent learning we analyze selected episodes from a program for children who volunteered to enroll. The sessions alternated between the after school club they attended and an art museum. The program engaged the children in basket weaving, in the analysis of baskets exhibited at the museum, and with ways in which flat materials can be shaped in 3D space along distinct surface curvatures. These experiences have inspired us to outline two streams of pedagogical ideas that seem to nurture and go along with the unforeseeable paths of emergent learning.

Keywords Informal mathematics learning · Emergent learning
Pedagogy · Museum learning · Crafts and mathematics

23.1 Introduction

We contrast emergent learning with "teleological" learning, which is learning for the sake of passing pre-defined tests and goals. To grasp the nature of emergent learning and how it differs from teleological learning, we review one of the best known and most cited papers in mathematics education: "The case of Benny" (Erlwanger 1973). In sixth grade Benny was regarded as one of the best students in

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his mathematics class. Since second-grade Benny had been using "Individually Prescribed Instruction" (IPI): a structured sequence of exercises punctuated by multiple-choice tests, such that 80% of the answers were required to be correct in order to advance in the sequence. Benny "was making much better than average progress through the IPI program" (p. 7), which indicated that his test responses had largely been correct according to the key provided by the IPI program. However, as Erlwanger interviewed him, he noticed that Benny was computing answers to problems by applying a multitude of self-generated rules many of which were incorrect, even though in particular cases they would lead to answers consistent with the key (e.g. the result of 0.7×0.5 is 0.35 because, on the left side, "there's two points in front of each number" (p. 8), then Benny used the same rule to evaluate: $0.3 + 0.4 = 0.07$). In addition to many idiosyncratic rules—Benny estimated that "in fractions we have 100 different kind of rules" (p. 10)—his five years of experiences with IPI led him to adopt certain views about the nature of mathematics and mathematics learning. The rules, he thought,

> were invented "by a man or someone who was very smart." This was an enormous task because, "it must have took this guy a long time… about 50 years… because to get the rules he had to work all of the problems out" (p. 12)

Applying diverse rules Benny was able to obtain different answers to the same problem, all of which he deemed to be true ones, although the IPI key accepted only one of them. Erlwanger asked Benny why the teachers would mark as wrong all these other true answers: "They mark it wrong because they just go by the key. They don't go by if the answer is true or not" (p. 12). This mismatch between the variety of true answers and the single one chosen by the key, Benny remarked, "is why nowadays we kids get the fractions wrong" (p. 11).

The practices involved in the use of IPI hinged on whether the students obtained adequate scores on its tests. We call this kind of learning "teleological:" learning for the sake of passing pre-defined tests and goals. At the same time, Benny learned many skills, ways of thinking, and forms of social awareness that were not pre-specified or even intended by the program, the school, and the participating teachers, such as the distinction between truth and key selection, his own confidence as a prolific maker of mathematical rules, or mathematics as an invention of a very smart and hard-working man. We will refer to this learning as "emergent." The concept of emergence is currently used in a range of disciplines, from complexity theory and thermodynamics of far-from-equilibrium systems, to system dynamics and organizational theory (Goldstein 1999; Kreps 2015). Characteristics of emergence include that it is largely unpredictable, not reducible to internal components and variables, self-organizing, and creative. Teleological Learning may succeed or fail; in the Case of Benny success was achieved to a certain degree, as he made more than average progress through the IPI program. Other researchers (Jacobson and Kapur 2012; Jacobson et al. 2016; Jacobson and Wilensky 2006) have elaborated on an approach to emergent learning that differs from ours because they base their work on an a computational model of learning.

In contrast to its etymological roots (Young 1987), the word "pedagogy" is nowadays strongly associated with formal teaching and schooling (Hamilton 1999).

This association elicits a paradoxical sense to the phrase "pedagogy of emergent learning" because school teaching is commonly seen as inherently teleological, as if, without explicit behavioral goals, teaching were to dissolve into a mass of incoherent and random interactions devoid of purpose. Emergent learning, being elusive to anticipated aims and predicted outcomes, appears, for the most part, to be an unintended byproduct of schooling practices that are bound to the achievement of testable results. Thus, clarifying what we mean by pedagogies of emergent learning is a critical matter.

We conceive of pedagogy of emergent learning as one that drifts and moves along unanticipated flows of emergent learning traversing educands and educators, one in which spontaneous memories, speculations, and projects of the participants may take center stage regardless of whether they accord with pre-conceived end-points. While a pedagogy of emergent learning seeks to instigate collective improvisation, it does preserve the asymmetry between educators and educands, although treating the axis of this asymmetry as, in the words of Rancière (1991 p. 13), will to will and not intelligence to intelligence. "Will to will" entails that educators plan, facilitate, and orchestrate the activities the group engages in; "not intelligence to intelligence" implicates that participants share, make sense and pursue these activities in their own ways nurtured by their desires, histories and contexts of life. In other words, while there is an inequality educand/educator in that the latter regulates and sets up the stage for their joint work, there is a primordial equality educand/educator in their autonomy for expression, recollection, conceptualization, initiative, and insight.

Pedagogy of the Oppressed (Freire 1970) and *The Ignorant Schoolmaster* (Rancière 1991) seem to us inspiring for the development of a pedagogy of emergent learning. Freire saw the dialogue between educators and educands as necessitating humility and the sense that they are all learners: "At the point of encounter there are neither utter ignoramuses nor perfect sages; there are only people who are attempting, together, to learn more than they now know" (Freire 1970). Educators are also learners who struggle against their own assumptions and expectations, pursuing to "learn more than they now know." While it is said that the main goal of the pedagogy of the oppressed is "conscientização" (i.e. approximately, to become aware), we think it is more accurate to say that its goal is to elucidate, to some extent, what "conscientização" amounts to in the context of the circumstances of the educands and educators, as well as the histories of their lives. What makes the pedagogy of the oppressed non-teleological is not the absence of goals, but coping with the ongoing persistent challenge of what the goals are, as well as the openness to their being constantly transformed into new, unanticipated, and often surprising and provisional ends. In other words, the goals themselves are emergent, which entails that they are diverse, shifting, ephemeral, situated, and co-generated.

We suggest that case studies are main sources to elaborate on pedagogies of emergent learning. An important example is the case of "SeanNumbers-Ofala" (2010, Online) focused on interactions in a third grade classroom taught by Deborah L. Ball. This paper is a case study based on a program entitled "Basket

Weaving and Curvature" that was conducted during the fall of 2015. The program was part of the InforMath project, one of whose main goals is to investigate/design informal learning environments amenable to the creation of new social images of mathematics—images that are more inclusive and inspiring than the prevalent ones in our society. Note that this is a goal without finish line and goalposts, not unlike the one of "conscientização." It is a goal irremediably recursive, the pursuit of which entails an ongoing questioning, hopefully insightful, of what it is about and where it comes from.

23.2 Basket Weaving and Curvature

This program is one of several that have been designed and conducted in the context of the InforMath project, which is a collaborative initiative including museum educators from three museums located in Balboa Park, San Diego, as well as faculty members and graduate students from San Diego State University. The children, all members of an after school program at the Boys and Girls Club of Southeast San Diego, volunteered to participate. A recruitment session was held at the Chula Vista clubhouse, where all students in grades 5–8 attended a brief presentation about Mingei museum and had the opportunity to engage with a weaving activity composed of yarn and a cardboard loom. The program consisted of six sessions that took place every other week and alternated locations between the museum and the Chula Vista clubhouse. Eleven students signed up, they were 9–12 years old, with four girls and seven boys, of which eight completed the program. To record each session, two stationary video cameras were used as well as head cameras worn by several of the kids. The Basket Weaving and Curvature program was designed and conducted by two museum educators, Lucera and Johanna, and two math educators, Ricardo and Cierra. Lucera led the activities during the sessions themselves. We will refer to the four of them as the "educators." The educators met in between sessions to design the ensuing ones and to prepare materials accordingly.

The springboard of the program was an exhibition hosted by the Mingei International Museum called "Made in America," which included outstanding craft products from each of the 50 US states. *Made In America* was in the process of installation when we began to envision the program. Lucera and Johanna had produced educational materials to accompany the exhibition. Apprehending the upcoming exhibition as a suitable arena for a program intermingling mathematics and crafts, to be attended by children from Southeast San Diego, were the initial issues we worked on. While the collection encompassed a wide variety of techniques and materials, during our preliminary visits we were particularly lured by several handcrafted baskets (see Fig. 23.1), as well as encouraged by Lucera's past workshop experiences, to engage children in basket weaving. We held several preparatory sessions. In one of them Lucera taught Ricardo and Cierra to create round baskets using two different techniques: coiling and weaving. In parallel to

Fig. 23.1 Some of the baskets included in the *Made in America* exhibition

this preparatory work, Ricardo and Cierra were participating in an online seminar with a mathematician, John McCleary, on topics of differential geometry. This seminar was one of the professional development initiatives held by the InforMath project. At the time, topics discussed in this seminar included surface curvature and geodesics. This overlap of activities evoked the idea of basket weaving as a set of techniques to create, out of flat materials, a shape in three-dimension space. Since 3D shapes can be characterized by the local curvature for each point of the surface, basket weaving appeared to be a suitable maker's context to encounter and use ideas about surface curvature.

23.3 Episode 1

The first session took place at the Mingei. After a warm up activity, the director of the museum and curator of *Made in America*, Rob Sidner, led a visit to the gallery floor explaining the history of the exhibition and conversing with the children about their impressions and questions. Afterwards the group gathered at the museum's workroom. Lucera initiated a discussion about the differences between straight and curved lines. She introduced the children to a tool we refer to as a "curvature instrument."

The curvature instrument had been designed by the educators over a three-week period before the beginning of the program. After trying out different designs, the final version consisted of a "cross" made out of cardstock with pipe cleaners in between; the two pieces of cardstock were stapled along the edges to keep the pipe cleaners in between. The curvature instrument is used by placing it over an object or certain shape of interest aligning one non-adjacent pair of arms along the orientation of maximal curvature, and contouring the remaining pair of arms to the shape of the object (see Fig. 23.2). The pipe cleaners help maintain the shape of the surface after the tool is detached from the object The idea of the curvature instrument arose from trying to figure out ways to support children to develop an intuitive sense of

Fig. 23.2 Curvature instrument on a sphere (left) and removed (right)

Gaussian curvature at a given point on the surface. Gaussian curvature is obtained by multiplying the maximum and minimum linear curvature around the point of interest. Euler proved in 1760 that on smooth surfaces the maximum and minimum linear curvatures are perpendicular to each other, which necessitated the perpendicularity of the arms of the curvature instrument.

Lucera explained that when the two non-adjacent pairs are bent in the same direction it is said that the curvature is "positive," whereas if each of the two pairs are bent in opposite directions it is "negative"; if one or both non-adjacent pairs are flat the curvature is zero. She showed how the top of her head had a positive curvature whereas the inner side of a bent elbow or knee, has a negative one. The children then used the curvature instrument to ascertain different types of curvature on their bodies. During the last segment of the session Lucera showed how to weave a basket with pipe cleaners and yarn, and then the children selected materials and started to make their first basket.

The second session took place at the Chula Vista clubhouse. They reviewed the activities of the first session, watched and discussed a video showing craftsmen creating blown glass pieces, and continued work on their baskets. During the third session, at the Mingei, the group talked about their baskets and compared techniques (e.g. looping the yarn around each spoke or just alternating inside/outside each spoke). Afterwards they went to the gallery floor to observe and discuss different pieces, particularly woven baskets. Students were encouraged to speculate about what materials and processes went into creating the art pieces. Episode 1 took place during this visit to the gallery floor.

Fig. 23.3 **a** Outline of one of the many vertical reed spokes. **b** Slicing the basket horizontally along equator. **c** Tracing the lower half going outwards. **d** Tracing the upper half going inwards

- Annotated Transcript

Lucera: So, this basket ((see Fig. 23.3a)) also uses spokes. So, it has a bunch of reeds
((which are the spokes, see white outline in Fig. 23.3a)) going up the sides
but I thought this basket was interesting, um, because it started out– if you
just imagine, like, slice it in half ((makes horizontal slicing motion with flat
hand, see Fig. 23.3b)) and the bottom is just like a regular bowl going out
((makes upward swinging gesture following contour of lower half of
basket, see Fig. 23.3c)) but then it started going back in ((uses hand to trace
the contour of the upper half of basket going in, see Fig. 23.3d)). So, how do
you think they did it on this one? Yeah?

Commentary

Lucera started [1–3] by highlighting the vertical reed spokes traversing the basket. Then she imaginarily sliced the basket in half, to mark a difference between the bottom part (i.e. “going out” [5]) and the upper part (i.e. “going back in” [7]). She asked how the basket weaver managed to produce this difference outwards-inwards [8]. Lucera’s question: “So, how do you think they did it on this one?” [8], was an invitation to conjecture the making of a difference. Generally speaking, woven baskets obtain a shape outwards by increasing the distance between spokes or widening the spokes themselves, and likewise turn inwards by decreasing the distance or spoke width—a relationship Lucera was familiar with. Lucera wished to discern, after calling their attention to the vertical spokes, how the children perceived the roles of the spokes in shaping up the basket. It is unlikely that any of the educators knew how to "explain" the relationship between variation of spoke width and curvature. This type of relationship is something that we literally grasp in the context of making, rather than the one of talking.

Ryan: So, like, what I found right here ((points at bottom of basket)) is like it's going out ((curves hand to mirror contour of lower basket)) and then like on this part ((points near equatorial region where handle reeds depart from wall of basket)) it's going that way ((motions upward with hand following angle of handle reeds, see Fig. 23.4a)) so, like, they can, they can carry it ((points at handle)…
Lucera: Mm-hmm.
Ryan: …like a handle
Lucera: So, okay, so over here ((near equator of basket)) it's like making a turn ((referencing contour of upper basket that curves back in)).
Ryan: Yeah.
Lucera: Can everyone see where he's pointing? Can you point where you're…
Ryan: Like, this point where it goes like that ((uses pointer finger to trace the angle of the handle reeds, see Fig. 23.4b))…
Lucera: Okay, so he's noticed it's making a turn ((curves hand to model curvature of upper basket))

Commentary

From his side, Ryan noticed something different around the equator line: the appearance of a spoke going upwards to hold the handle in position. This spoke is unique because while it appears to be an ordinary spoke on the bottom half, then it breaks free and becomes handle support. Lucera understood his "then like on this

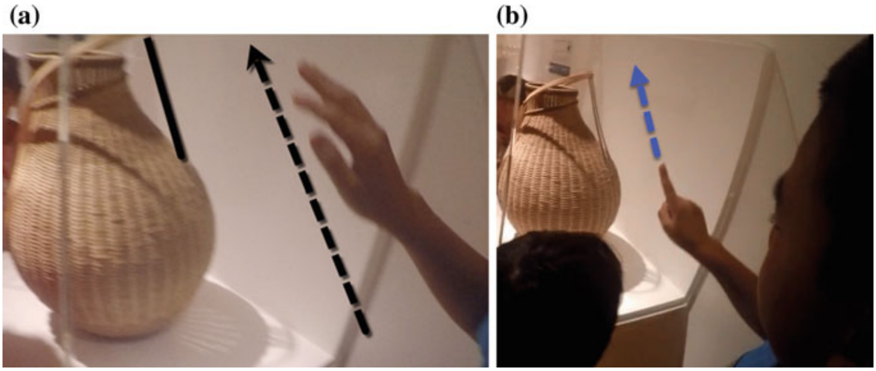

Fig. 23.4 **a** Ryan traces the spoke coming out of the woven reeds to support the handle. **b** Ryan traces the salient spoke again

part ((middle level)) it's going that way ((slanted upwards))" [12–13] as corroborating her "then it started going back in" [7], so that she re-described Ryan's words as "it's like making a turn" [16–17], from the lower to the upper half of the basket. Such mutual unawareness of differences between their accounts is a natural byproduct of the inherent ambiguity of utterances, exacerbated in this case by the inability to touch the basket, as well as the fact that we tend to be primed to perceive what we expect. Unless ambiguity turns out to be minimal, it is only through the insistence of a difference that we face it.

Lucera: Anybody else have ideas how this might've been made? Omar? Did you have something?
Omar: I think they, they used, um, really, really, um, thin wood so it's easier for them to, um, bend the ((points toward basket)) wood.
Lucera: Oh, okay. Okay. So, it's just a matter of using very thin wood so that they can bend it ((makes upward sweeping, semicircular motion)). So you also agree that there is some bending going on.
Omar: Yeah.
Lucera: Yeah?

Commentary

Omar thought that the basket maker had generated the outward/inward difference by bending the spokes, which required them to be made of an easy-to-bend material, such as thin wood. This might have resonated with his recent experiences weaving yarn around pipe cleaners that can be bent effortlessly. While isolated spokes show to contribute to the shape of the basket by their bent, the weaving reeds, as they bring the spokes into mutual relationships, make them bear and sustain the particular shape of the basket. This does not invalidate Omar's remark: had the spokes been made of rigid material, they would have refused to comply with the hands of the weaver as they interlaced the horizontal reeds. The main point we elicit in this commentary is the ongoing merging of perception and imagination, both materializing from memory: as Omar envisioned the (imaginary) making of this (perceived) basket, the salient feature that came to the present surface of memory—a memory that included the making of pipe cleaner baskets and much else—was the bending of the spokes.

Alexa: I think there is a little bit of bending going on, but like right there ((points near equator of basket)), you can see that they're ((the spokes)) bigger and then as they go up ((traces circles with finger as she raises arm)), they become really small ((makes repeated pinching motions with index finger and thumb)).
Lucera: Oh, okay. So let's look over here on this side ((see Fig. 23.5a)). So, she's saying like kind of in the middle the ((indicates greater width with index finger and thumb, see Fig. 23.5b)), the spokes, they get bigger, they get thicker and then as it goes up ((raises arm, indicates lesser width with index finger and thumb)), it gets smaller.

Fig. 23.5 **a** Other side of basket. Note changing width of reed spoke. **b** Lucera's hand highlights greater width of the spokes in the middle height of the basket

Commentary

Alexa acknowledged that there is “bending going on” [35] but she foregrounded another variation: from wide to narrow width. She was referring to the spoke thickness, decreasing from the middle up. Note that Alexa gestured such movement up by tracing circles with her finger as she raised her arm [37]: Alexa imagined the making of this basket in terms of weaving reeds going around and gradually up. Just as in the making of a coiled basket, each circle of threaded reeds has a shorter and shorter perimeter in order to go inwards; the narrowing of the spokes generates such perimeter shortening. As in our previous commentary on Omar’s remark, Alexa’s utterance merges perception and imagination such that a particular feature came to the present from the depth of vast memories, recently stirred by her work with pipe cleaner basketry: the perimeter’s shortening as circles move upwards. Lucera’s requested the group to watch the basket from another side (see Fig. 23.5b), probably motivated by that one being the side that Alexa was observing, and perhaps also by that side of the basket being color-uniform (compare Fig. 23.5b and a), allowing for a more focused appreciation of the spoke width. From that side, Lucera highlighted the narrowing of the spokes [42–44].

Jake: Mmm.
Lucera: Does anyone else see that? Do you agree with that? Or are they the same size all the way from the bottom to the top?
Omar: I think they’re the same size.
Lucera: You think they’re the same size? If you look at this little piece up here ((makes measuring gesture with thumb and index finger near top of basket, see Fig. 23.6a))– I wish I could touch it– and then this piece ((makes measuring gesture near bottom)) and the middle ((makes measuring gesture near equator, see Fig. 23.6b)) and down there ((makes measuring gesture near bottom again, see Fig. 23.6c))…it’s the same size? Who thinks it’s the same size? Raise your hand. ((four kids raise their hands)) … Who thinks it’s a different size? ((four kids raise their hands, including one girl who voted again))
Jake: I think it’s just like an optical illusion.
Lucera: I think it’s– Maybe it’s an optical illusion? Okay, then that—yeah, that’s

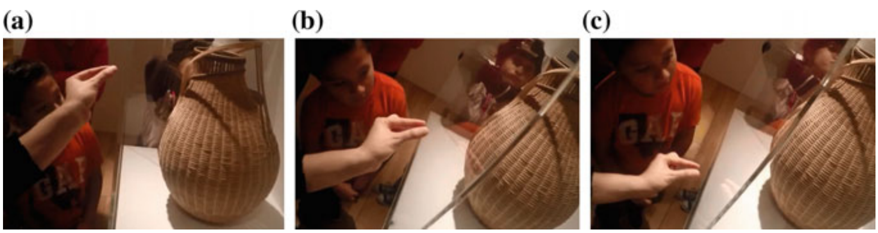

Fig. 23.6 **a** Lucera highlights spoke width at the top of the basket. **b** Lucera highlights spoke width in the middle of the basket. **c** Lucera highlights spoke width at the bottom of the basket

60 making me doubt myself but I think it's slightly different size. I think it, it
61 starts out, um, medium on the bottom ((uses fingers to indicate width near
62 bottom)) and then it gets a little bit thicker ((moves fingers up towards
63 equator to indicate width)) and then it gets thin at the top ((moves fingers
64 near top to indicate width)) Just slightly.
65 Ryan: I wish I could touch it.
66 Jake: Slightly

Commentary

Sensing that some of the children were unconvinced by Alexa's observation [45 and 48], Lucera responded by wanting to show the decrease in width. Limited by her inability to touch the basket [51], she marked the spoke thickness by the separation between the tips of her thumb and index fingers, as they slid vertically over the glass surface. However, as she was enacting the spoke thickness with her fingers, she started to hesitate, to the point of bringing into question the observation itself ("it's the same size?" [54]). Lucera turned to the children asking for a poll of opinions. Following the mixed polled opinions, Jakes remarked that it was "just" an optical illusion [58]. That perception is infused with the imaginary does not mean that the distinction between them vanishes: the question "do we see it or imagine it?", which corresponds to "is it there or is it an illusion?", still makes sense, and emerges with full force when the difference in question is feeble. Lucera accounted for her own doubts by deeming the width difference to be "slight" [60], and yet, she still thought that it was there [60–64]. While the seeing was ambiguous, "thought" brought to her a sense that, in all likelihood, the spoke width varied. Tenuous differences create possibilities for thinking and seeing to reach different conclusions.

23.4 Transition

Woven baskets obtain a shape outwards by increasing the distance between the edges of each vertical spoke and inwards by decreasing it, which can occur with or without a change in spoke width. However, this relationship had not been salient in the practice of weaving yarn with pipe cleaners because, we think, the children worked to regulate the opening of the basket by tightening or loosening the yarn as it went around, rather than by bending the pipe cleaners—the spokes—and keeping their shape and position stable while weaving; this made the tensioning of the yarn the primary method for regulating whether the wall of the basket would curve outward or inward. This observation prompted us to explore alternative crafts in which the separation between successive pairs of spokes becomes the primary manual/material difference engendering shape. While we were seeking alternatives, it happened that a colleague at SDSU, who is a quilter, mentioned fabric bowls and lent us a book about it. This serendipitous event launched us into investigating the

Fig. 23.7 Fabric cutout ready to sew into a bowl

manufacture of fabric bowls and experimenting with different materials and techniques. The kind of fabric bowl we envisioned would be created by sewing the edges of a flat piece of fabric cut with a shape similar to the one shown in Fig. 23.7.

After a lengthy process of repeated experimentation, we ended up using cotton fabric ironed on both sides of a thick stabilizer (see Fig. 23.8). This material was then cut with a laser cutter according to templates generated in Geometer's Sketchpad (see Fig. 23.9). The two control points can be moved to change the radius of curvature of the two arcs of a circle enclosing each petal. The petals can be seen as equivalent to the spokes in a woven basket; the shape of the petals determines the separation between two successive in/out thread shifts at different heights, and regulating accordingly the overall shape of the bowl (Fig. 23.10).

23.5 Episode 2

During the 4th session, at the Chula Vista clubhouse, the children were asked to wrap large balls in paper and discuss the origin of the wrinkles appearing on the wrapping paper. After this initial experience transforming a flat surface into a curved 3D shape, Lucera introduced the materials for the fabric bowls. Each child chose the fabric and the template they wanted to use. Most of the 5th session at the Mingei was spent sewing the fabric bowls. Then they went to the gallery floor with their bowls to discuss and identify ways in which the shape of baskets exhibited in *Made in America* were similar or different than the shape of their fabric bowls. Episode 2 took place during this visit.

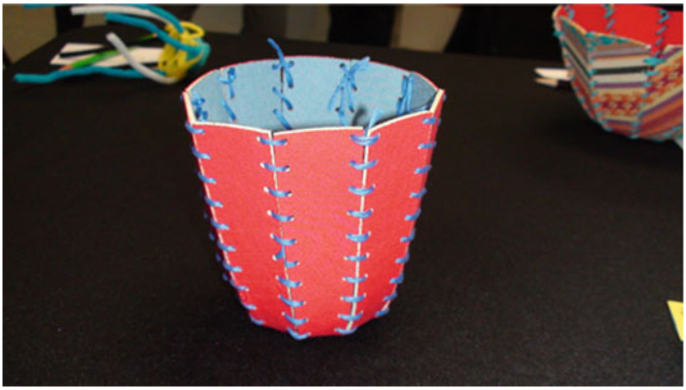

Fig. 23.8 A sewn fabric bowl

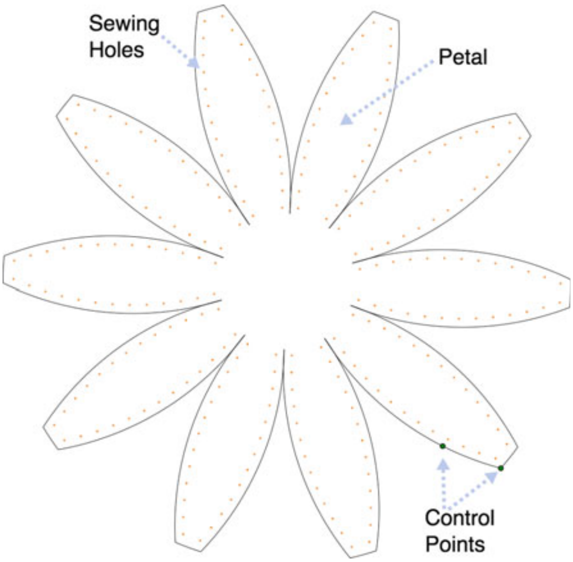

Fig. 23.9 Template generated in GSP with ten "petals"

Fig. 23.10 A child sewing his fabric bowl

- Annotated Transcript

Johanna: How about other people's baskets? Did you have a chance? Which one did you notice? [That looks like yours?

Gabriella: [Umm (.) Well, I… I noticed that THAT ((Gabriela points at a basket on the opposite side of the room, see Fig. 23.11)) one over there… …

Johanna: Oh, so you want to go all the way over here. Let's take a look.

Gabriella: ((while the group is walking towards the basket)) Yeah. If–um, Alexa ((Gabriella looks towards Alexa)) sewed hers on up like, um, a basket, it would look like this ((the one she had pointed at)).

Johanna: Oh, yeah. Okay, so you guys– 'cause yours ((Alexa's bowl)) isn't complete yet ((See Fig. 23.12, Alexa's bowl is not completely sewn)) but we're thinking that if that was sewn all of the way up that it would look really similar to this ((basket selected by Gabriela))?

Allison: [Yeah, 'cause it's like she could pro'ly like bend it a little ((Allison points at Alexa's bowl, on the upper side, see Fig. 23.13))

Gabriella: [Yeah, 'cause it's– it's small ((Gabriella shows how the slices become narrower on the upper side, see right side of Fig. 23.13))

Allison: just to make it like the final thing to look like that ((like the basket they are looking at)).

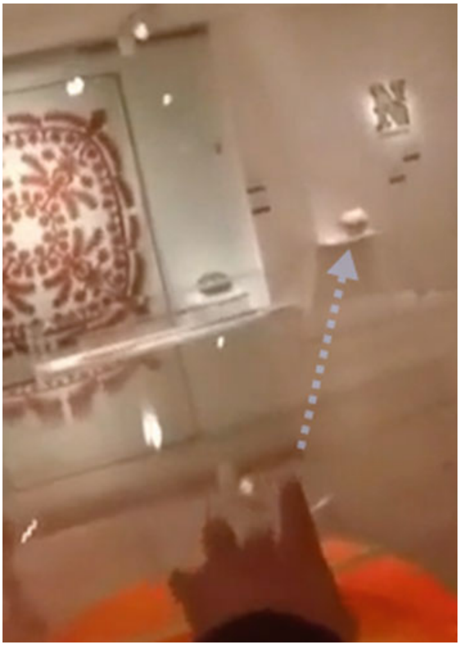

Fig. 23.11 Gabriella points at a basket she had noticed

Fig. 23.12 Alexa holds her bowl, not yet completely sewn, whose overall shape will be similar to the exhibited basket

Fig. 23.13 Allison points at the upper side of Alexa's bowl while Gabriella, on the right side, shows that the upper side of the basket is "small"

Gabriella: 'Cause it goes small and then it gets fat ((Gabriella traces with her thumb and her index finger a width that starts small, gets "fat", and then gets smaller again, see Fig. 23.14))

Commentary

Alexa had chosen her template to be such that the edges of the petals fit an arc with a small radius of curvature—the type shown in Fig. 23.9. Most of the other children's templates were of the kind shown in Fig. 23.7. Her choice made of her bowl one that went conspicuously inwards over the upper half. This is the common feature that Allison indicated by touching Alexa's unfinished bowl and pointing at

Fig. 23.14 Gabriella traces the shape of the "petals" imaginarily forming the basket

the basket. Gabrielle traced on the glass panel the shape of a petal that would generate the contour of the basket see Fig. 23.14. Because this is a coiled basket that has no spokes or woven reeds, there is no physical indications on it of petal-like units; nevertheless, she found compelling, and others plausible, to imagine it split in petals with a certain outline, as if she were overlaying Alexa's bowl on the basket.

Returning to our theme of the imaginary infusing perception, both of them surging from memory, this episode is an occasion to elaborate further on the nature of memory and the significance of making and crafting for the genesis of memory. Memory is much ampler than individual recollections, both in the sense that what Allison and Gabrielle pointed out originated from group activity with a range of materials and tools, and that there was no single event from a past moment reenacted in their analysis of the basket. Furthermore, memory is deeply-rooted in materiality: the basket and Alexa's bowl remember countless versions of their own making, which they open to the grasp of others and things. Joining shaped petals is a "version" of the making of the basket—one that is markedly different from the processes that had been followed by its basket maker, but that was, in non-trivial ways, implicitly conveyed by them. The materiality that grounds memory allows for making and crafting being rich and nuanced sources: they bestow tangibility, texture, and bodily skill upon things, even when those things are out of touch or beyond the creative abilities of the perceivers. Neither Johanna nor the children could touch the basket, and yet, Alexa's bowl, unfinished and made out of other materials and techniques, passed onto the basket features graspable by skin, muscles, and sight. Because of this, making and crafting can be, among genetic sources of memory, exceptionally generous, just as the engagement with playing instruments can give birth to entirely new forms of music appreciation. Each craft gives in its own ways: basket weaving with yarn and pipe cleaners conferred to the basket shown in Fig. 23.3a attributes different from the ones sewing fabric bowls did to the basket shown in Fig. 23.14, such as the latter one being a composition of petals.

23.6 Towards a Pedagogy of Emergent Learning

Far from trying to demonstrate “best practices,” we have shared some of our experiences with the Basket Weaving and Curvature program for the sake of investigating *a kind* of practice, whose main orientation is a quest for pedagogies of emergent learning. For this kind of pedagogy there are no best practices because no concrete attempt can be isolated from the circumstances of its development, the contingencies pervading its daily events, and the life history of the participant individuals and institutions. At most, given historic and contextual aspects, one can discriminate promising or rather-to-be-avoided ways of doing things. Ultimately, the character of a pedagogy of emergent learning is to be expressed by the ongoing outline of an ethics, which is a never-completed moving outline. Freire, for instance, emphasized the importance of “humility” on the part of educators and participants (“there are only people who are attempting, together, to learn more than they now know” Freire 1970). Like any other ethical rule, this is not a talisman. Not only because an act that seems humble to someone may strike as arrogant to someone else, but also because, aside from extreme cases, every judgment of this sort is necessarily bounded by invisible biases and more or less partial grasp of the circumstances. Ethical rules can be pursued not as maxims guiding behavior, but as efforts to keep certain questions alive (e.g. What does it mean, here and now, or there and yesterday, to be humble?). Part of this aliveness is the shared sense that there are no ideal or perfect actions and that, in retrospect, one can always imagine what appears to be more desirable ways of doing things, even though uncertainty about them cannot be dispelled (Nemirovsky et al. 2005). A pedagogy of emergent learning is distinct, we think as of now, by its openness to unanticipated courses of action, freedom from predefined testable outcomes, mostly voluntary participation, and, yes, humility. For the most part, these features make such pedagogies difficult to pursue, other than marginally, in formal education, but they can be central, we propose, in informal mathematics education (Nemirovsky et al. 2016), of which the Basket Weaving and Curvature program is an instance. We will delineate two streams of pedagogical ideas as recently inspired by our experiences in the program: (1) Explorations at the Edge; and, (2) Opening Avenues of Expression.

- Explorations at the Edge

The “edge” that we have in mind is one that adjoins or brings into contact two territories, like the edge of a sea bordering both, an expanse of water and a strip of country land. One side of the edge is a territory that appears firm and amenable to walk through, the other side is to be navigated with caution and wonder, without straying too far from the edge, as it is outpouring with questions and barely seen possibilities extending up to a remote horizon. Husserl thought that every object is located at an edge of that sort, demarcating, as it were, its sides directly perceived and the indefinite anticipations of the unseen sides: “…every object is not a thing isolated in itself but is always already an object in its horizon of typical familiarity and precognizance.” (Husserl 1975, p. 122). He then wrote a crucial idea:

> But this horizon is constantly in motion; with every new step of intuitive apprehension, new delineations of the object result, more precise determinations and corrections of what was anticipated. (Husserl 1975, p. 122)

An exploration at the edge, we suggest, is an activity in which horizons are set in motion. An example of such exploration, as it took place in the Basket Weaving and Curvature program, was the work with fabric bowls. While neither the educators, nor the children, had ever sewn a fabric bowl, we were all acquainted with bowls and fabric. Fabric bowls and the techniques of their making were at an edge separating familiarities with various bowls and types of fabric from expanses circumscribed by a horizon of barely seen possibilities: What shapes can they have? Do they keep their shapes stably? How firmly can they hold content? What are suitable materials allowing for easy sewing? How do template shapes correlate with bowl shapes? and so forth. Certain skills that were for some participants on one side of the edge, were for others on the other side, such as sewing: the children told us that they had never sewn, and that, with few exceptions, they had never seen anyone sewing (a couple of grandmothers were the exception). Stemming from their concurrent participation in a geometry seminar, for Ricardo and Cierra the horizon of fabric bowls encompassed also the creation of flat maps for the rounded earth, as well as the distribution of Gaussian curvature on a 3D surface.

Star and Griesemer (1989) introduced the notion of "boundary object," which are objects, such as architectural drawings or soil samples, that are used and conceived differently by different disciplines and practitioners, while serving to coordinate their collaborative work. Similarly, exploring objects and techniques at the edge can nurture and mesh the diverse horizons of the explorers; an example of which, we think, took place in Line 20 of Episode 2, when Giselle traced on the glass enclosing a coiled basket, the shape of a petal.

The Basket Weaving and Curvature program included other explorations at the edge, some of which reached only an embryonary stage, such as the ones involving the curvature instrument and the paper wrapping of balls. The program has inspired us to propose that explorations at the edge, particularly when they are at the edge for all participants, including the educators, are very significant for pedagogies of emergent learning. Ultimately, it seems fair to say, emergent learning is the collective mobilizing of horizons.

- Opening Avenues of Expression

There is an important difference, particularly in the context of mathematics education, between representing and expressing (Whitacre et al. 2009). Instead of presenting-again, in a different format, what had been present before, an expression is an explosion of meaning without clear boundaries, subject to never-ending interpretations. It matters greatly whether we see a gesture, a diagram, a drawing, or an utterance as a representation or an expression. During the final session in which the children shared their work with parents and other adult attendants, a boy explained that the inside and outside colors he had chosen for his fabric bowl—dark on in the inside, light on the outside—were like some people he knew who looked nice from outside but were bad inside. In one of the individual interviews that Cierra conducted with the children, a girl said that she saw herself as an "art person," and then she pointed, as a mode of evidence, to her fabric bowl held on her other hand. Letting baskets, woven yarn around pipe cleaners, fabric bowls, and craftwork exhibited in *Made in America*, be expressions traversing disciplinary, institutional, and historical boundaries we customarily take for granted, amounts to opening avenues of expression. We propose that this is a major quality for the kind of pedagogy we try to understand. It is through expression that the emergent finds itself, for a gaze seeking a set representation, such as a certain definition or graph, is blind to emergent learning.

It is complex but possible to discern aspects of what has been learned in a program infused with qualities such as Explorations at the Edge and Opening Avenues of Expression. The analysis of videotaped episodes and interviews moves us to reckon that participants in the Basket Weaving and Curvature program learned, with various degrees of subtlety, that there is a relationship between the shape of petals and of sewn bowls, or that an art museum can be a fascinating place. Along the same lines, the authors of this paper sense the burgeoning appearance of seed-ideas about the roles of craftwork in mathematics learning. Had the exhibit been another one, or many of the contingent events populating the program been absent, a different learning would have emerged.

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