Chapter 15
Formative Assessment in Inquiry-Based Elementary Mathematics

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Abstract The chapter presents findings related to Czech teachers' and pupils' difficulties with, opinions on, and needs associated with formative assessment, namely, peer assessment, in inquiry-based lessons. The research was conducted within the EU-funded Assess Inquiry in Science, Technology, and Mathematics Education project (ASSIST-ME). Six teachers of primary mathematics worked with researchers on inquiry tasks and methods of peer assessment and implemented them in their classrooms. The paper focuses mainly on (a) the interplay of teachers' intentions, subject matter, and learners in inquiry; (b) the teachers' role in supporting learning via (formative) assessment; and (c) the pupils' role in their own learning and the learning of peers. Significant phenomena in implementation of assessment were identified, namely, the importance of formulation of learning objectives; pupils' ability to decide about the correctness, identify the mistakes, and give supporting feedback to their peers; possible (and needed) support; and institutionalization of knowledge.

Keywords Formative assessment · Peer assessment · Self-assessment
Inquiry based mathematics education · Primary school level

15.1 Introduction

School assessment as a feedback tool and an important part of interaction among key actors in school education has for a long time been the subject of discussion in the Czech Republic because it influences the character of the entire system of teaching and learning. School assessment is closely related to school tradition and the culture of education. For this reason, the implementation of inquiry based approaches in mathematics education at school includes, among other problems, the challenges of having to comply with curriculum requirements, classroom

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management issues in responding to unexpected and uncertain situations, and the problem of conducting assessment. The objective of the chapter on the general level is to show what changes occurred in classrooms of teachers developing inquiry-based approaches and assessing their pupils' results and achievement, in other words, what the relationship between assessment and inquiry-based education in mathematics is.

The chapter reports the results of a research study on peer assessment carried out in the Czech Republic under the framework of the EU-funded Assess Inquiry in Science, Technology, and Mathematics Education project (ASSIST-ME). The project develops and studies formative and summative methods of assessment that support inquiry-based approaches in teaching science, technical disciplines, and mathematics. Based on analysis of what is known about summative and formative assessments of knowledge, skills and attitudes, the project team proposed a variety of combined methods of assessment. These methods were tested on primary and secondary schools in different countries across Europe (the Czech Republic, Denmark, Finland, France, Cyprus, Germany, Switzerland, and the United Kingdom). The research carried out was focused on creation of formative assessment methods that (1) fit into everyday classroom practice, (2) provide qualitatively oriented feedback of competence-oriented, inquiry-based learning processes, and (3) can be combined with existing summative assessment requirements and methods used in different educational systems.

The chapter is based on data from primary mathematics classrooms collected within the framework of the ASSIST-ME project in the Czech Republic and aims to answer the questions: What difficulties are faced in introducing formative assessment in primary mathematics classroom?

15.2 Background of the Study

15.2.1 Assessment

The issue of assessment has a long tradition at ICME, e.g., Topic Study Group "Assessment and Testing in Mathematics Education" (Suurtamm and Neubrand 2015) at ICME 12 focused on:

- Issues connected to the development of teachers' professional knowledge of assessment and their use of assessment in the mathematics classroom.
- Issues and examples related to the enactment of classroom practices that reflect current thinking in assessment and mathematics education (e.g., the use of assessment for learning, as learning, and of learning in mathematics classrooms).

This chapter contributes to debate the latter of the two topics.

Our study focused on *classroom-based assessment of pupils* (in the sense of definition published in OECD 2013). This assessment requires specification of the purpose for which the data are collected and interpreted, i.e., the purpose of assessing. This affects a number of decisions the teacher makes about (a) the data to be collected (e.g., whether systematically or occasionally), (b) interpretation of them, (c) communication about them, and (d) building further decisions on them (Black et al. 2004). With respect to how assessment is used and with what purpose it is carried out, two approaches are distinguished:

– Summative assessment in which the evaluator checks and summarizes what the pupil has learned. It may concern individual pupils, groups of pupils, or the whole population (for example, large-scale external tests and examinations).
– Formative assessment that supports the pupils' learning process. This involves the processes of data collection and interpretation that learners and their teachers use to make decision about the following: What have the pupils learned so far? Where is their learning aim? How can they be supported and assisted on their way to learning?

Black and Wiliam (2009) explained formative assessment in these words:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

From this definition, the main characteristics of formative assessment can be summarized (see ARG 2002):

– Pupils play active roles in making decisions about their own learning. They can be expected to be able to channel their effort more efficiently if they know the objectives of their learning.
– The teachers' feedback includes advice on how to advance; it does not compare pupils with each other.
– Teachers use the information to make adaptations to their lessons in a way that gives their pupils more opportunity to learn.
– Dialogues between teachers and the pupils support the pupils' reflection on the learning process.
– Pupils develop self-assessment by taking part in determination of what will make them advance.

The idea of formative assessment has been discussed in Czech educational context since the 1990s, but it has only been slowly introduced in school practice (for more details, see Žlábkova and Rokos 2013). Slavík (2003) summarized that although several examples of good practice were offered, empirical research findings focused on problems in using the formative assessment methods were almost absent. Some forms of formative assessment are seen as embedded in common Czech teaching culture, such as immediate teacher feedback as a response to a pupil's problem solving (on-the-fly assessment) or less frequent written comments from teachers. Others are seen as difficult in mathematics on primary school level (e.g., structured classroom dialog or formative peer assessment).

Teaching/learning processes including formative assessment were depicted by Harlen (2014) as a repetitive cyclic process the learner in the center (see Fig. 15.1).

In inquiry-based education, formative assessment naturally penetrates the process of inquiry:

- The teacher formulates a short-term goal of a lesson or several lessons, i.e., the norm the pupil is expected to achieve. In mathematics education, the goal is often operationalized in the form of a problem or a set of problems (see Samková et al. 2015).
- In discussions with peers and the teachers, pupils communicate about how they understand the problems in question, how they solve them, and how they understand their classmates' solutions. If teachers do not find these discussions meaningful, they stimulates them by asking open questions; i.e., the teacher collects data during the activities being carried out, interprets them, and intervenes if necessary. In school mathematics, the teacher makes often conjectures about how the learner thinks.
- The teacher then formulates some recommendation, which Harlen (2014) refers to as "judgement," about the next steps. These steps are then carried out by the pupil in the subsequent process of learning.

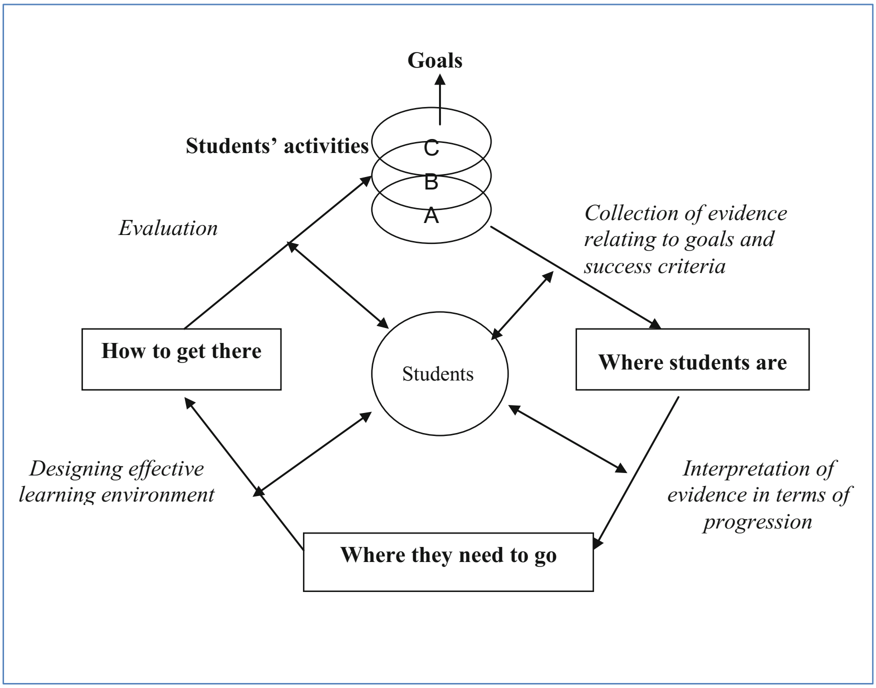

Fig. 15.1 Assessment for formative purposes (Harlen 2014, p. 6)

– Information the teacher gets about a pupil's activity is related to the short-term objectives of a lesson, or several lessons. This information constitutes the basis for making decisions about the next steps or about how to help the pupil carry out these steps. The goal is to support pupils' learning and to provide feedback on the progress they have made in their understanding or skills. It is at the same time feedback for the teacher, who can adapt tasks to the pupil and maximize the opportunity to learn. Empirical evidence has verified that formative assessment increases efficiency of learning, e.g., Black and Wiliam (1998) mentioned studies that stated that "improved formative assessment helps the (so-called) low attainers more than the rest, and so reduces the spread of attainment whilst raising it overall" (p. 3).

Peer assessment, which is the focus of our attention in this study, is understood here in accordance with Boud and Falchikov (2007, p. 132):

> Peer assessment requires students to provide either feedback or grades (or both) to their peers on a product or a performance, based on the criteria of excellence for that product or event which students may have been involved in determining.

Slavík (2003) and others consider peer assessment to be a way to autonomous assessment, i.e., deeper reflection on one's own learning and its results "that learners use on their own, master it, that they understand to the needed extent, that they can explain or defend" (p. 14). Slavík (2003) stated that autonomous assessment partially develops and deepens in relation to self-assessment and partially in relation to assessment of others' performance (most likely of classmates', i.e., peer assessment) through which pupils learn to reflect on their work. Pachler et al. (2010) stressed that learner self-regulation is a core factor in formative assessment and that it is linked to motivation and emotional factors which affect learners' engagement.

Learning benefits are supposed for both pupils acting as assessor and as assessee, as well, because they both can bridge the gaps in their understanding of particular contents and get a more sophisticated grasp on their learning (Topping 2013). In addition, there has been great interest in upscaling formative assessment to change learning/teaching culture (OECD 2015).

15.2.2 Inquiry-Based Approach in Mathematics Education and Assessment

Formative assessment is especially important in the situation of inquiry-based education:

> As is the case in the natural sciences, inquiry-based mathematics education refers to an education which does not present mathematics to pupils and students as a ready-built structure to appropriate. Rather it offers them the opportunity to experience:

- how mathematical knowledge is developed through personal and collective attempts at answering questions emerging in a diversity of fields, from observation of nature as well as the mathematics field itself, and,
- how mathematical concepts and structures can emerge from the organisation of the resulting constructions, and then be exploited for answering new and challenging problem (Artigue et al. 2012, p. 8).

Inquiry-based education in teaching mathematics helps not only to build inquiry-based attitudes in pupils but also to reinforce pupils' understanding of mathematical concepts and procedures. According to Donovan and Bransford (2005), it uses (a) a knowledge-centered lens, focusing attention on "what is taught (learning goals), why it is taught, and what mastery looks like"; (b) an assessment-centered lens, emphasizing the need to provide frequent opportunities "to make students' thinking and learning visible as a guide for both the teacher and the student in learning and instruction"; and (c) a community-centered lens, based on a culture of "questioning, respect, and risk taking," as well as the interaction of learners and teacher as central to the learning process (p. 13). It follows that in the process of inquiry, the roles of the pupil and the teacher and their responsibility for the teaching/learning process change. Primarily, it is the pupil/group of pupils who must be active when looking for information, estimating and guessing, making conjectures, and discovering solutions. When peer assessment is present, pupils must try to understand the solutions of others, comment on them, and give feedback. The teachers' role is to create the right conditions for this. They must create an environment that encourages cooperation, guide their pupils, support them in their search for unknown solving methods, and ask questions, such as "Why?" "How would you explain?" "Is it really so?" and "Do you know any similar problem/task?" The teacher must be proactive, support pupils' efforts, praise pupils' contributions (including giving feedback on mistakes the pupils have made) and must help their pupils advance in learning based on their own independent discoveries and interpretations.

Implementation of inquiry-based approaches brings a radical change to the whole process of education; starting with response to the demands of curricula, to problems that stimulate independent inquiries, to a change in the pupils' and teachers' roles in the teaching/learning process. Pupils and their teacher constitute a complex system with its own dynamics, conditions, and rules. The system can be illustrated by the schema in Fig. 15.2.

Inquiry at school can be depicted as a cycle:

> Like scientific inquiry, mathematical inquiry starts from a question or a problem, and answers are sought through observation and exploration; mental, material or virtual experiments are conducted; connections are made to questions offering interesting similarities with the one in hand and already answered; known mathematical techniques are brought into play and adapted when necessary. This inquiry process is led by, or leads to, hypothetical answers—often called conjectures—that are subject to validation. (Artigue and Baptist 2012, p. 4)

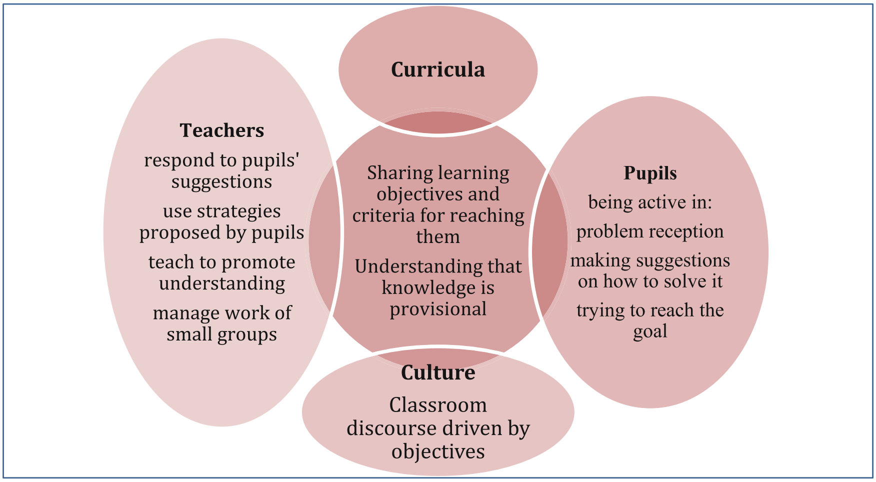

Fig. 15.2 Roles of different agents in the classroom in the course of pupils' independent inquiry (modified according Samková et al. 2015, p. 97)

In respect to this study, we have to better characterize the problems that initiated the inquiries. Openness of problems as the stimuli of inquiry has been requested already by Dewey (1938):

> The original indeterminate situation is not only "open" to inquiry, but it is open in the sense that its constituents do not hang together. A variety of names serves to characterize indeterminate situations. They are disturbed, troubled, ambiguous, confused, full of conflicting tendencies, obscure, etc. (p. 105)

In other words, problems can be interpreted in more ways and there are more correct ways of solving them and sometimes more correct answers. To solve the problem, the pupils discover (or better, rediscover) the ways of its solution. In accordance with their actual knowledge, they make experiments (mental, material, and virtual), observe similarities and differences, and compare them with their current knowledge. Adding more experience and knowledge to their network of knowledge, they restructure the existing. They make mistakes and learn from them (especially their own but also other people's mistakes).

15.3 Empirical Study

15.3.1 Goals and Organization of the Study

The goal of the research study presented here is to identify those phenomena that could be observed in a planned implementation of peer assessment in inquiry-based education in mathematics at the primary school level and, in particular, the difficulties faced in such an implementation.

Four methods of formative assessment were proposed in the frame of the ASSIST-ME project: (a) questions and other interactions between the teacher and their pupils conducted “on the fly”, (b) structured dialogue in the classroom, (c) evaluation (grading and feedback), and (d) peer assessment and self-assessment. These methods were tested in selected primary and secondary schools in the Czech Republic in three rounds in the period from September 2014 to February 2016. Each round took six months. In these six months, the participating teachers assessed their pupils in inquiry-based lessons using methods chosen from the above list.

Six elementary teachers prepared in pairs and individually realized teaching experiments in elementary mathematics in the second, fourth, and fifth grades (pupils mostly aged 7, 9, and 10 years). The teachers together with researchers developed a sequence of 4–6 inquiry-based units, which they implemented mostly in 90-min blocks. The topics were chosen in accordance with the teaching plans of relevant grades: enriching the concept of great numbers and properties of plane geometrical figures and their area.

15.3.2 Preparation of the Educational Experiments

The educational experiments were carried out with the intention of creating a space for independent pupils’ inquiry supported by peer assessment. Lesson planning was always carried out via joint discussion by the local working group (teachers and researchers), during which the goal was formulated and various options for its fulfilment were discussed. The discussion focusing on a priori didactical analysis of content (in the sense formulated by Brousseau 2002) identified the key concepts that became the goal of the teaching experiment.

The goals were operationalized in the form of “the problem of a lesson.” We always were looking for problems that would stimulate inquiry. The main characteristics of these problems were their openness (see Dewey 1938). The following problems are two examples:

Problem 1 Find out how many lentils there are in a half-kilogram package.

Problem 2 Create instruction for your friends on how to determine the number of tiles needed to tile the triangle in Fig. 15.3.

The first problem is indeterminate in terms of the solution procedure. The goal in solving this problem is to cover the topic of great numbers in order to:

- understand that the basis of representing a number in the position system is the grouping of elements according to the base of the numeral system and the notation of the number of these groupings and
- perceive that the value of a figure in the notation of a number depends on its position.

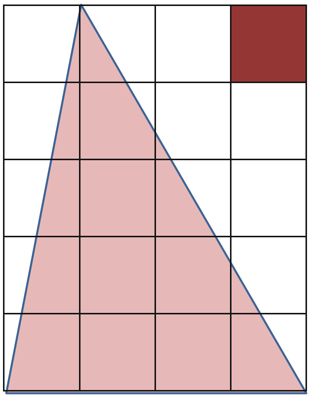

Fig. 15.3 Figure in assignment of tiling problem

Pupils are familiar with grouping into tens and hundreds from counting to 100. It is very likely they have not come across a similar activity with larger sets at school or in their real life. The problem may also be solved by weighing.

The second problem prepares students for the introduction of the concept of the area of a triangle. Pupils may count whole tiles and their parts and may also discover that a triangle is one half of a 4 cm by 5 cm rectangle.

Each of the teachers then individually elaborated a plan for the lesson and realized them with their pupils. The plan was based in general on Polya's model of stages of problem solving (Polya 1945), supplemented by the peer-assessment stage (5) and reflection (6) (see schema in Fig. 15.4).

In most cases, a worksheet was created in which the pupils recorded a solution to the problem and provided peer assessment (an example of the worksheet for the topic of great numbers is included in the Appendix). Pupils described their solution procedure in the worksheets. After that they swapped the worksheets and provided each other written feedback on the solution. Having received comments from peers, each pupil (or a group of pupils) had the chance to revise the original solution with respect to the feedback they had received. In addition, each student also briefly responded in writing to the feedback they had received.

15.3.3 Data and Their Analysis

We have acquired a rich source of data from the 16 experiments that were implemented:

- Video recordings (2 cameras: the teacher's and one in the classroom) of lessons and their transcriptions
- Structured classroom observation protocols from the researchers
- Pupils' written productions (worksheets capturing problem-solving and assessment comments by classmates)

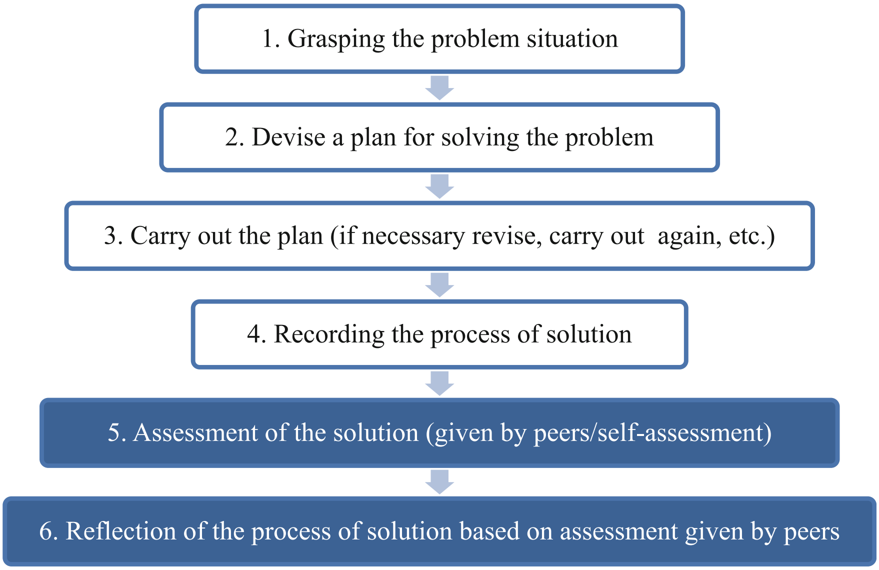

Fig. 15.4 Schema of planning experimental education units

– Audio recordings of group work if it took place
– Semi-structured interviews with the teachers who realized the teaching experiments before the lesson to determine how they understood inquiry-based mathematics education and the role of formative assessment in it
– Audio recordings of discussions in local working groups during the preparation of lessons and their transcripts
– Short interviews with teachers after each lesson.

Data were analyzed qualitatively. Transcripts were coded via open coding. The codes were derived from the characteristics of formative assessment, the inquiry-based education, and perceived actions. Gradually, a list of codes were created, including codes for the activities of both teachers (e.g., assignment of the problem; explaining on the initiative of the teacher or pupil(s); monitoring the activities of pairs or groups of pupils; discussion with pupils; individuals, pairs, or groups giving feedback; questions to individuals, pairs, or groups; individuals requesting clarification; feedback directing solutions for the whole class, feedback directing solutions for individuals; reactions to pupils' explanations; general evaluation of pupils' work; assessment of pupils' work; putting knowledge into context with what has been previously learned; and indication of the importance of knowledge for the future) and pupils (clarification requests, comments on the problem, solving tasks in pairs or groups, a request for clarification during the solution of the problem, a request for equipment, loud comment on the solution of the problem solution summary, and reaction to summarize for the teacher).

Specific issues related to difficulties that teachers and pupils had with introducing and conducting assessment gradually emerged in the process of pupils' independent inquires, such as how to support peer assessment, what the role of the teacher in peer assessment is, and what difficulties pupils have with peer assessment.

15.4 Selected Findings and Discussion

We identified significant phenomena in implementation of assessment in the teaching experiments.

15.4.1 Formulation of Learning Objectives

It is essential for the teachers' planning and decisions in the lesson that the didactical objective to be achieved in the lesson should be clearly defined. This premise is often discussed in the materials for teachers. When teaching mathematics at the primary school level in the Czech Republic, teachers often use materials to support their teaching that define only the topic of the lesson and cover classroom management (what problems will be included in the lesson, how they will be arranged, and what form of classroom organization will be used). Teachers do not plan lessons with respect to what the pupils will learn but with the objective of correctly solving problems provided in textbooks, workbooks, and worksheets.

In inquiry mathematics education, the quality of feedback that pupils may gain from their solutions depends on the accuracy of the definition and the operationalization of the learning objective of the "inquiry." At the beginning of our sequence of lessons, the objectives defined in cooperation with the teachers were quite general (e.g., "get experience," "apply a known procedure in a new environment"). The experiments showed that it is essential for assessors to state the objective in terms of the expected pupil's performance.

This can be illustrated on the solution of Problem 2, in which the pupils were asked to formulate comprehensible instructions on how to determine the number of tiles needed to tile the triangle in Fig. 15.3. Assessors decided whether the instructions were clear and could be used for determining the number of tiles needed to tile other triangles. This assessment could have been initiated by a concrete question: Did the solvers determine correctly that 15 tiles are needed? But asking this question would not correspond to the defined learning objectives: Pupils gain pre-concepts that form the basis of measuring the area of a triangle, namely, an experience with filling in a triangle with, for instance, squares (i.e., by a selected unit). For that reason, we asked the assessor to determine the number of tiles in another triangle according to the instructions, and after that we asked questions: Is the instruction correct? Is the instruction clear?

15.4.2 *Supporting Self-Assessment and Formative Peer Assessment*

The pupils who participated in the experiments had before had the opportunity to occasionally do inquiries in mathematics under the guidance of their teachers, but this experimental education was their first experience with peer assessment. As mentioned above, to support the peer assessment we (the members of the local working group) designed the worksheets. The worksheets were used both for recording the solution of the problem and peer assessment. In the process of discussion of the content of the worksheet, we immediately realized the difficulties that peer assessment in inquiry-based education in mathematics create. The problems that the pupils deal with can usually be solved in various ways. Not all of these ways are directed to meet intentional educational goals, and for that reason it is difficult to formulate the rubrics in the worksheet to enable the pupils to assess the work of their classmates.

For example, in the Lentil problem (determine how many lentils there are in a half-kilogram package) we assumed that pupils would determine the weight (or volume) of a certain number of lentils or vice versa and then use a reasoning based on knowledge of proportionality. It would also be possible to use ways based on estimation. The specific solutions can vary in details. The uncertainty of the situation created difficulty in formulating the questions on the worksheet. We decide to distinguish the correct solution and ask the question: What did you like in the solution? In the solutions that the assessors considered erroneous, the instruction was: Recommend to your classmates how to get the correct number.

The pupils solved this problem in groups. The groups then assessed the work of another group using the questions from the worksheet. Table 15.1 presents three

Table 15.1 Several solutions of the lentil problem and its peer assessment

	Solvers (assessees)	Assessors
S1	First we determined that 5 g contains 80 grains. 500 ÷ 5 = 100, 100 × 80 = 8000 There should be 8000 lentil grains in the package	☺ We like that they have the same principle as we have ☹ But weighing needs to be accurate
S2	First we determined the mass of the whole package. Then we calculated how many grains there are in one gram. We then calculated the problem and got 93,258 as the result	☹ We cannot assess this. They do not write what problem they were solving. Therefore we do not know how they calculated it
S3	We poured the grains into a large vessel and weighed it. We subtracted the mass of the vessel from the mass of the grains. We found out that the grains weighed 501 g. Then we found out that 20 grains weigh 1 g Calculation: 501 × 20 = 10,020	☺ The procedure was correct ☹ But our result was different

descriptions of the solution procedure (S1 correct, S2 incomplete, S3 incorrect) and their assessment by classmates.

Although the assessors of S2 expressed the error in the solution quite accurately, the assessees in the final discussion criticized the assessors not to give them supporting feedback.

In the following experiments, we split the solution into stages and asked for separate assessment of each stage. However, our questions still had to be general in order not to lead the pupils to a "correct" solution.

15.4.3 Correctness of Solution of the Problem and Peer Assessment

The greatest advantage of peer assessment lies in the fact that a classmate may often give a problem solver more comprehensible feedback because assessor and assessee have almost the same learning experience and speak the same language. In our study, we experienced two difficulties. The solvers found it very difficult to record their solution procedure (see examples in Table 15.1), and for this reason the assessors sometimes did not fully understand the solution. The second problem was related to the assessors, who were not always able to assess the correctness of the solution. Sometimes it was difficult for assessors to decide whether the problem had been solved correctly, in other words, to assess the individual steps of the solution procedure described by their classmates, and it was equally difficult to communicate this assessment in a comprehensible way. In S2 (Table 15.1), some steps that were not needed for the solution are described (the mass of lentils in the package was given in the assignment), while other steps were not described clearly enough to make a decision on their correctness. The assessors commented on the second part of the solution quite clearly. However, they did not comment on the fact that it was not necessary to determine "what the mass of the package was." Another problem was that number of lentils was approximately 10 times higher than the correct solution. The peers did not comment this fact because they were not sure of the correct answer.

Essential in inquiries are those erroneous contributions that move the solution forward. However, these are not often assessed by an evaluator who is familiar with similar methods of work as the solver. The situation becomes even more complicated if the solution is not described clearly and comprehensibly by the solver.

In our study, the solvers could react to the peer assessment at the conclusion of the whole solution process. Several answers to the question: "Did your friends' advice help you?" were negative, using a sad emoticon: "☹ It did not because if they write that the measuring should be accurate, we don't know how exactly." "No. Those who were checking our work must have lost their lentils." "☹ Saying the result was different is of no help." As the children grew more experienced with peer assessment, they grew more self-critical in these final comments: "We think it should have been briefer and we did not finish the manual because we had forgotten." "We think it's all quite muddled." "We could do it better to make it comprehensible for everybody. But otherwise we think it's OK."

In the final interview, students answered a question: Do you think you did well in assessing your peer(s)? The majority of pupils' felt relatively competent in assessing their peers (58%), while about a quarter of the pupils responded that they did not work well. Others (17%) were not able to evaluate their work and responded that they did not know. Among reported difficulties, the most frequent was the lack of knowledge or skills necessary for correct assessment, which was associated either with uncertainty about the solution of the inquiry task or the criteria for the assessment. The pupils mentioned, for example, that "it was difficult to decide whether it is correct or not," "I did it differently and I am not sure that this could be realized," etc. Dealing with this uncertainty in the classroom is crucial for implementation of peer feedback in a broader context.

15.4.4 Peer Assessment and Institutionalization of Knowledge

Formulation of the objectives of an inquiry is connected to the issue of institutionalization of the gained knowledge. We found out that at the end of inquiry-based lessons, the pupils expected an unequivocal decision on what had been done correctly. They expected the result of their solution—the discovered knowledge—to be shared by the group, critically discussed, and then accepted. The final summary was in the hands of the teacher. However, if the teacher had not stated the learning objective clearly enough, their summary was very vague ("You worked very nicely," "I am pleased with your work."). Our findings are in agreement with the theory of didactical situations. The need for the inclusion of an institutionalization phase was theoretically grasped by Brousseau and Balacheff (1997) and introduced in the model of the so-called a-didactical situation (a situation in which the teacher let the pupils to discover part of mathematical knowledge). The presence (and necessity) of the institutionalization phase in independent problem-solving situations has been confirmed in the Czech educational context (Novotná and Hošpesová 2013).

15.4.5 Other Methods of Formative Assessment in Our Experiments

Although the Czech part of the project focused on peer assessment, we also monitored the implementation of other forms of formative assessment or action supporting it. On-the-fly assessment (immediate corrective feedback and reinforcement) was present, especially in the inquiry phase. It is a part of the Czech culture of primary mathematics that the teacher corrects pupils' work while they are working. On-the-fly assessment can also be considered peer feedback. During the group discussions, the pupils considered the suggestions for the solutions presented by their members.

15.5 Concluding Remarks

The research question we were focusing on was: What difficulties have to be faced in introducing formative assessment in the primary mathematics classroom? Let us now summarize these difficulties from the point of view of the actors in education: teachers and pupils. The interviews with teachers after the realization of each lesson showed that the peer assessment during the inquiry-based tasks in mathematics is rather difficult and challenging for both teachers and pupils.

15.5.1 Formative Assessment and Teachers

The teachers reflected on their role, which they found even more important and difficult than they foresaw. The main problems they identified were time and resources demands (worksheets, assessment tools, and teacher assistant time). This is related to the issue of appropriate support that teachers need. At the beginning of experimental teaching, the teachers appreciated the worksheets for pupils. During and after the experiments, they reported that they preferred to see a more experienced colleague's teaching, real teaching-situation stories, a databank of tasks, training courses for teachers, and researchers' on-site support. Some training, therefore, should precede the implementation of formative assessment, and assistants in classes would also be helpful, as the implementation would be time consuming and not all teachers have proper readiness.

During the experimental teaching, the teachers realized that pupils frequently were not able to provide effective feedback to their peers, and they felt uncertain how to help them. After the experimental teaching, the teachers mentioned that the pupils were often not able to give meaningful feedback to the recipient that would provide enough hints about how to proceed further. They saw the importance of development of the pupils' assessment skills, which should help pupils not only in learning to assess (both self and peers), but also in mastering the curriculum.

The teachers also reflected that they had difficulties in summing up the lessons in a final whole-class discussion that could institutionalize the new piece of knowledge (Brousseau and Balacheff 1997).

Some teachers saw parents' views on learning as an obstacle to the implementation of formative assessment: "… only working with the pupils' book is seen as sound learning, anything else is seen as entertainment or relaxation," "… I do not know how I could defend the time we have spent on it and explain to parents that we did not practice enough of the tasks in the pupils' book."

Some teachers' comments were directly related to the realization of experimental lessons. In particular, written peer feedback seemed too difficult; the pupils were able to be more precise and detailed when speaking. In introducing formative (peer) assessment, it would be good to start with simpler tasks and with some task for training, e.g., working on a series of similar tasks and only at the end asking the pupils for peer feedback. The teachers also mentioned that the pupils would prefer to have an opportunity to see and discuss more solutions before assessing. Some of the teachers considered this to be a big issue, as they realized that pupils may need more time to think the assessment over.

We realized that it is quite essential to elaborate the learning progress structure of the inquiry task, but it is more or less impossible to do it for inquiry that has been initiated using an open problem. Summary of peer feedback and learning accomplished is important for "institutionalization of new knowledge," i.e., deepened understanding of the concept being studied.

15.5.2 Formative Assessment and Pupils

The trend is to make pupils responsible for their learning. Whether they are ready to accept this responsibility largely depends on whether they understand what the meaning of various school activities is and on their ability to recognize when to use which activity. This means pupils need the space to define and accept goals of their learning and the space to see whether they have actually achieved those goals. If pupils are given the chance to speak about their learning and reflect on the process of learning, higher order thinking skills and metacognition are developed.

Our experiments showed that pupils adapted to inquiry-based education and peer assessment very quickly. Pupils' willingness to inquire and to assess each other and their success in these activities developed as they gained experience. For both individuals and groups to become independent while solving problems and develop towards autonomous assessment is a gradual process that must be given enough space at schools. Our experience showed that formative assessment of peers' work is more productive if preceded by a discussion with the whole class on the possible solutions of the problem. This discussion reduces pupils' uncertainty associated with the correctness of the problem's solution, which is the main problem of peer assessment from the pupil's point of view.

15.5.3 Formative Assessment and Culture

The relationship between content (curriculum requirements), how the content is used (pedagogy), and assessment is well known: What is taught is influenced by how it is taught, and what is assessed influences both what is taught and how it is

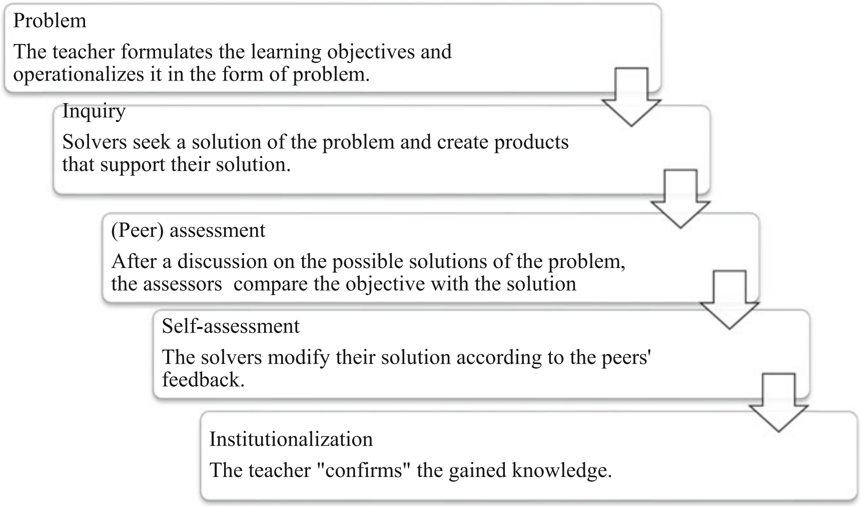

Fig. 15.5 The process of formation of autonomous assessment (*Source* Hošpesová and Žlábková 2016)

taught. It follows that if the curriculum is packed with content and assessment requires memorization of facts instead of conceptual understanding, learner-centered approaches including inquiry-based education will be out of place and there is no need to demand that pupils should be responsible for their learning.

In our project, the course of an inquiry-based lesson including formative assessment is expressed by the schema in Fig. 15.5. Although inquiry and formative assessment are both described as cyclical processes, in the lessons they have the nature of a linear process that starts with the assignment of the problem and ends with a teacher's summary.

Mathematics, with a greater extent of generic skills, is different from the science subjects, as teachers see open inquiry in math as more difficult to prepare and manage. Peer assessors feel somewhat less certain when providing feedback unless the institutionalization of learned knowledge precedes.

For primary-level group and pair activities, including inquiry and peer feedback, seem to be convenient. We recommended starting with formative assessment activities very soon: Even second-grade students can try them and learn a lot (though mostly about the feedback process itself).

The strong tradition of summative assessment in the Czech Republic calls for open discussion in broader contexts, including the general public (media, etc.), in order to slowly change the assessment culture.

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Appendix 1: Assessment Tools Worksheet 1: Find Out How Many Lentils There Are in a Half-Kilogram Package. (Colored Parts Are Intended for Peer Assessment.)

Names of the pupils solving the problem:				

Estimation: ______

How did we proceed?

______

______

______

Result: ______

Names of pupils who assess the solution:				

Is the result correct? ☺ ☹

☺ What did you like in the solution?

______

______

☹ Recommend your classmates how to get the correct number

______

______

Revision: ______

______

______

Did the advice from your classmates help you? How? ☺ ☹

______

______

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