

# Possible Interpretations for Game Refinement Measure

Shuo Xiong<sup>(✉)</sup>, Long Zuo<sup>(✉)</sup>, and Hiroyuki Iida<sup>(✉)</sup>

Japan Advanced Institute of Science and Technology,  
1-1 Asahidai, Nomi, Ishikawa 923-1211, Japan  
{xiongshuo,zuolong,iida}@jaist.ac.jp

**Abstract.** This paper explores possible interpretations for game refinement measure which has been successfully used to quantify the game sophistication of various types of games such as boardgames and sports. It presents a brief sketch of game refinement theory with a focus on its early works with boardgames, expansion into continuous movement games such as sports, and a bridge between sports and boardgames. It then highlights the bridging idea while considering possible interpretations for game refinement measure, and the meaning of game refinement measure is discussed with a focus on the skill and chance aspects in game playing. It enables to have a new perspective of game refinement theory. Moreover, an example of interpretation for game refinement measure from boardgames and continuous movement games such as MOBA game is shown. The interpretation is well fitting to our intuition as game players and spectators.

**Keywords:** Game refinement measure · Game progress model · Boardgame · Continuous movement game, sports, MOBA game

## 1 Introduction

Game theory is a discipline which stands from the game player's point of view with a focus on how to win a game. However, game designers would consider another important aspect: how to make a game more attractive. With such motivation, a new game theory from the game designer's point of view, called *game refinement theory* [2,3] was proposed in the early 2000s. von Neumann [4] was a pioneer who formed the foundation for the modern game theory, which has widely been applied in various fields. For example, Shannon [5] and Turing [6] proposed the basic framework for computer chess that is the minimax game-tree search, being inspired by the concept of minimax equilibrium [4], typical framework of computer chess called game-tree search was proposed by Shannon [5] and Turing [6], respectively.

One direction with game theory was to find the best move in a game or to ensure the possibility of winning the game based on the understanding of

current positions. Another direction with game refinement theory was to assess the attractiveness or sophistication of a game. In particular, game refinement theory gives a measure to quantify the sophistication of a game. This enables to obtain the deep insight into the current game and improve the quality of the game [7, 8].

The measure of game refinement can also be used to obtain the deep insight into the history of games. For example, it is observed that the evolution of chess has two different directions: one is to increase the search-space complexity and another one is to shift to the comfortable degree of game refinement measure [9]. Hence, it gives a reasonable look on the evolution of specific game variants.

In another way, game refinement theory provides us with another viewpoint of games from the entertainment aspect while game theory helps us understand about the game's mechanism itself. From that viewpoint, we can extend the idea of game refinement into other domains in human life such as sports games, video games, education or business. The possibility of extension comes from the core idea of game refinement theory that is quantifying the engagement. In many activities of human, the engagement is usually used as one of the important standards to evaluate the effectiveness of those activities.

Game refinement theory has been widely applied to many different types of games with the promising results. However, the theory has just one decade history, which may not be established yet. This paper explores possible interpretations for game refinement measure. It highlights the bridging idea between boardgames and continuous movement games like sports. Thus the meaning of game refinement measure is discussed with a focus on the skill and chance aspects in game playing. It will enable to have a new perspective of game refinement theory. Moreover, an example of interpretation for game refinement measure from boardgames and continuous movement games such as MOBA game is shown.

## 2 An Overview of Game Refinement Theory

In this section an overview of game refinement theory is presented. The model of game refinement was first investigated in the domain of boardgames such as chess, later expanded into continuous movement games such as sports games and video games while considering the gap between boardgames and continuous movement games.

### 2.1 Original Model

We review the early work of game refinement theory from [2]. The decision space is the minimal search space without forecasting. It provides the common measures for almost all boardgames. The dynamics of decision options in the decision space has been investigated and it is observed that this dynamics is a key factor for game entertainment. Thus a measure of the refinement in games was proposed [3].

Later, the following works are sketched from [7,10] that expands the model of game refinement which was cultivated in the domain of boardgames into continuous movement games such as sports games and video games.

The game progress is twofold. One is game speed or scoring rate, while another one is game information progress with a focus on the game outcome. Game information progress presents the degree of certainty of a game's result in time or in steps. Having full information of the game progress, i.e. after its conclusion, game progress  $x(t)$  will be given as a linear function of time  $t$  with  $0 \leq t \leq t_k$  and  $0 \leq x(t) \leq x(t_k)$ , as shown in Eq. (1).

$$x(t) = \frac{x(t_k)}{t_k} t \quad (1)$$

However, the game information progress given by Eq. (1) is unknown during the in-game period. The presence of uncertainty during the game, often until the final moments of a game, reasonably renders game progress as exponential. Hence, a realistic model of game information progress is given by Eq. (2).

$$x(t) = x(t_k) \left(\frac{t}{t_k}\right)^n \quad (2)$$

Here  $n$  stands for a constant parameter which is given based on the perspective of an observer of the game considered. Only a very boring game would progress in a linear function, however, and most of course do not. Therefore, it is reasonable to assume a parameter  $n$ , based on the perception of game progress prior to completion. If the information of the game is completely known (i.e., after the end of the game) and the value of  $n$  is 1, the game progress curve appears as a straight line. In most games, especially in competitive ones, much of the information is incomplete, the value of  $n$  cannot be assumed, and therefore game progress is a steep curve until its completion, along with  $x(t_k)$ ,  $t_k$ ,  $x(t)$  and  $t$ , just prior to game's end.

Then acceleration of game information progress is obtained by deriving Eq. (2) twice. Solving it at  $t = t_k$ , we have Eq. (3).

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n} (t_k)^{n-2} n(n-1) = \frac{x(t_k)}{(t_k)^2} n(n-1) \quad (3)$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our brains. We do not yet know about the physics of information in the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Too little game information acceleration may be easy for human observers and players to compute, and becomes boring. In contrast, too much game information acceleration surpasses the entertaining range and will be frustration, and at some points beyond that could become overwhelming and incomprehensible.

Therefore, we expect that the larger the value  $\frac{x(t_k)}{(t_k)^2}$  is, the more the game becomes exciting, due in part to the uncertainty of game outcome. Thus, we

use its root square,  $\frac{\sqrt{x(t_k)}}{t_k}$ , as a game refinement measure for the game under consideration. We call it  $R$  value for short as shown in Eq. (4).

$$R = \frac{\sqrt{x(t_k)}}{t_k} \sqrt{n(n-1)} = C \frac{\sqrt{x(t_k)}}{t_k} \tag{4}$$

### 2.2 A Bridge Between Sports and Boardgames

Here we consider the gap of game refinement model between boardgames and sports games. We review the observation from [10]. One round in boardgames can be illustrated as decision tree. At each depth of the game tree, one will choose a move and the game will progress. Figure 1 illustrates one level of game tree. The distance  $d$ , which has been shown in Fig. 1, can be found by using simple Pythagoras theorem, thus resulting in  $d = \sqrt{\Delta t^2 + 1}$ .

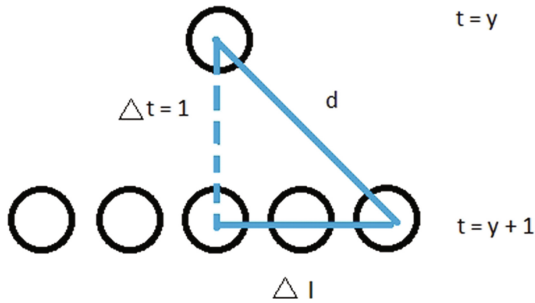


Fig. 1. Illustration of one level of game tree [10]

Assuming that the approximate value of horizontal difference between nodes is  $\frac{B}{2}$ , then we can make a substitution and get  $d = \sqrt{(\frac{B}{2})^2 + 1}$ . Here  $B$  stands for the average branching factor of a game tree. The game progress for one game is the total level of game tree times  $d$ . For the meantime, we do not consider  $\Delta t^2$  because the value ( $\Delta t^2 = 1$ ) is assumed to be much smaller compared to  $B$ . The game length will be normalized by the average game length  $D$ , then the game progress  $x(t)$  is given by  $x(t) = \frac{t}{D} \cdot d = \frac{t}{D} \sqrt{(\frac{B}{2})^2 + 1} = \frac{Bt}{2D}$ . Then, in general we have Eq. (5).

$$x(t) = c \frac{B}{D} t \tag{5}$$

where  $c$  is a different constant which depends on the game considered. However, we manage to explain how to obtain the game information progress value itself. The game progress in the domain of boardgames forms a linear graph with the maximum value  $x(t)$  of  $B$ . Assuming<sup>1</sup>  $c = 1$ , then we have a realistic game progress model for boardgames, which is given by

<sup>1</sup> In this study we concern about this assumption.

$$x(t) = B\left(\frac{t}{D}\right)^n. \tag{6}$$

We show, in Table 1, measures of game refinement for various games [11–13]. From the results, we conjecture the relation between the measure of game refinement and game sophistication, as stated in Remark 1.

*Remark 1.* Sophisticated games have a common factor (i.e., same degree of informational acceleration value, say 0.07–0.08) to feel engaged or excited regardless of different type of games.

**Table 1.** Measures of game refinement for various types of games

Game	$x(t_k)$	$t_k$	$R$
Chess	35	80	0.074
Shogi	80	115	0.078
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073
Badminton	46.336	79.344	0.086
Table tennis	54.863	96.465	0.077
DotA ver 6.80	68.6	106.2	0.078
StarCraft II Terran	1.64	16	0.081
The king of the fighters 98	14.6	36.7	0.104

### 3 Game Refinement Measure Revisited

It seems that the bridge between boardgame and continuous movement game was successfully built. However, we claim that it is not yet completed. For this purpose we detail the problem while considering the meaning of parameter  $c$  in Eq. (5).

#### 3.1 Possible Interpretations for Game Refinement Measure

For the sports games such as soccer, all the attempted shots or successful shots (goals) are parts of the strategy to win the match, so they are an integral part of the game. In the domain of video games such as StarCraft II, the branching factor was calculated only by reasonable strategies to be considered as part of the winning [12]. This suggests that parameter  $c$  in Eq. (5) is a key factor when considering the gap between boardgames and continuous movement games. It also indicates that the parameter  $c$  can be replaced with  $\sqrt{n(n-1)}$  in Eq. (4).

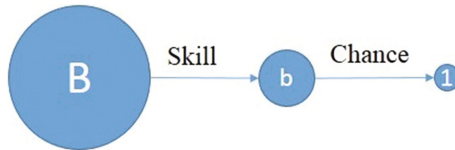
From Eq. (5) we obtain the measure of game refinement for boardgame as shown in Eq. (7).

$$R = \frac{\sqrt{cB}}{D} \quad \left(\frac{1}{B} \leq c \leq 1\right) \quad (7)$$

Where we have  $cB = B$  when  $c = 1$ , and  $cB = 1$  when  $c = \frac{1}{B}$ . Hence, the assumption  $c = 1$  means that we focus on a specific level of players or a certain property of the game under consideration. When we focus on a certain level of players like masters in boardgames, the crucial factor is the game property. If a game is skillful, the parameter  $c$  will decrease, whereas if the game is stochastic,  $c$  will increase. This is because it is usually hard in such a stochastic game to distinguish only fewer good candidates among all possible moves. On the other hands, it would be possible in boardgames like chess for masters to identify a few plausible moves. Note that continuous movement games such as sports games are basically stochastic when compared with boardgames.

*Remark 2.* The parameter  $c = 1$  in Eq. (5) means that the game under consideration is assumed to be insufficiently deterministic to identify plausible candidates.

We show from [14], in Fig. 2, a model of move candidate selection based on skill and chance. This illustration shows that skillful players would consider a set (say  $b$ ) of fewer plausible candidates among all possible moves (say  $B$ ) to find a move to play. For example, in chess where  $B = 35$  and  $D = 80$ , when assuming  $c = 1$ , then  $R = 0.074$ . On the other hands, as suggested in [14, 15], masters in sophisticated boardgames would consider a very few moves on average in their look-ahead thinking framework. An estimation of the number of plausible candidates as a function of the strength of players (say  $s$ ) may be given by Eq. (8) [3].



**Fig. 2.** A model of candidate move selection based on skill and chance [14]

$$b = B^{\frac{1}{s}} \quad (1 \leq s \in \mathbb{N}) \quad (8)$$

Let us consider the sports case with consideration on such a parameter. Like the boardgame case, we may have a parameter (say  $C_s$ ), as shown in Eq. (9).

$$R = C_s \frac{\sqrt{G}}{T} \quad (9)$$

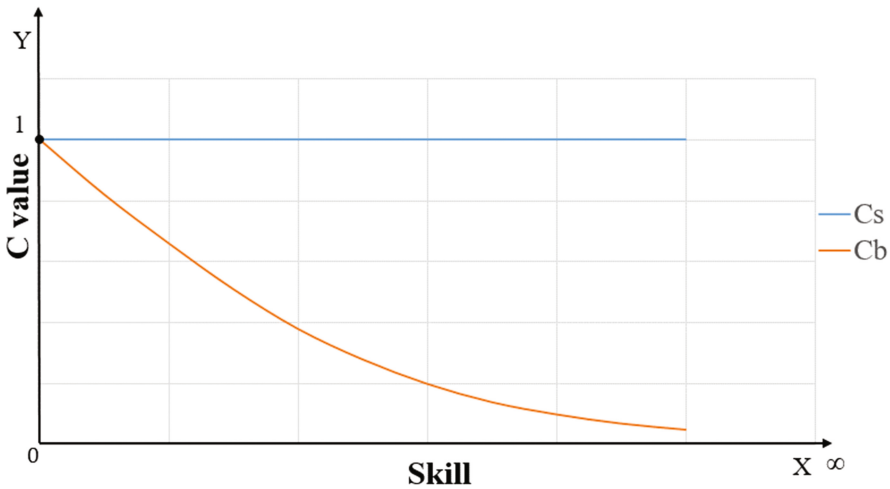
Here we suspect that  $C_s$  may depend on the skill of teams. For the analysis, many data from soccer leagues with different ranking were collected. We show,

in Table 2, measures of game refinement for each soccer league together with the average number of goals ( $G$ ) and shots attempts ( $T$ ) per game. Two typical groups are compared. The first group is clearly stronger than the second groups. England Premier League (EPL), Primera division de Liga (LIGA) and Serie A in the first group are the higher ranking leagues, whereas Chinese Football Association Super League (CSL) in the second group is the lower ranking league [16]. We notice from the results in Table 2 that when two similar-level teams play each other in their leagues  $R$  value is quite similar. Thus, we assume  $C_s = 1$  in this study.

**Table 2.** Measures of game refinement for each league football match

	$G$	$T$	$R$
EPL (2016)	2.84	25.6	0.066
LIGA (2016)	2.75	23.6	0.070
Serie A (2016)	2.77	25.4	0.065
CSL (2014)	2.75	24.6	0.067
CSL (2015)	2.80	24.6	0.068
CSL (2016)	2.67	24.6	0.066

We show, in Fig. 3, the relationship between the parameter  $c$  and chance-and-skill aspect. Note that we assume the estimation of the plausible candidates as described in Eq. (8). From Fig. 3 we conjecture that the parameter  $c$  relates to the strength of players or the difficulty of a game, as stated in Remark 3.



**Fig. 3.** The parameter  $c$  and chance-and-skill aspect of games

*Remark 3.* The value of parameter  $c$  should be lower in the case where the game under consideration is simple to identify fewer plausible candidates or the case where players are very skillful like grandmasters.

Using the estimation of plausible candidates as shown in Eq. (8), we obtain game refinement measures as described in Table 3.

**Table 3.** Measures of game refinement for boardgames with different parameters

	R ( $c = 1$ )	R ( $cB = b, s = 2$ )
Chess	0.074	0.030
Shogi	0.078	0.026
Go	0.076	0.019

We here summarize the meaning of game refinement measure.

- In a game where its game refinement measure is higher than the zone value (0.07–0.08), people may feel more entertaining. This is because the game is too stochastic or players are too weak to identify fewer plausible candidates.
- The game with a zone value of game refinement measure has a good balance between chance and skill, in which people may feel comfortable and then the game is sophisticated or fascinating.
- In a game where its game refinement measure is smaller than the zone value, people may feel less entertaining. This is because the game is too simple or players are too strong to experience harmonic uncertainty during the game playing. In this situation the game tends to be competitive [17].

### 3.2 Relative Game Refinement Measure

The game refinement theory is basically used to evaluate the property (sophistication) of games with a focus on the game outcome uncertainty. Let us consider the individual match analysis using game refinement measure [18, 19]. Since each match has an independent game process, game refinement measure can be applied.

We demonstrate an analysis of two extreme conditions and special cases. The first example is The 2014 World Cup semi-final [20]: Germany vs. Brazil, where the number of goals  $G = 8$  and the number of shot attempts  $T = 31$ . When focusing on this match,  $R$  value is given by Eq. (10).

$$R = \frac{\sqrt{G}}{T} = \frac{\sqrt{8}}{31} = 0.091 \quad (10)$$

In fact, this match was not a well balanced. Brazil had 1 goal, whereas Germany had 7 goals. Individually, game refinement measure for Brazil (say  $R_B$ ) and Brazil (say  $R_G$ ) is given in Eqs. (11) and (12), respectively.



$$R_B = \frac{\sqrt{1}}{31} = 0.032 \tag{11}$$

$$R_G = \frac{\sqrt{7}}{31} = 0.085 \tag{12}$$

Apparently, the  $R$  value of Germany is higher than Brazil, which means that Germany had better playing skill. Even more we need to know the psychological meaning of game refinement value for each team’s perception. Then the relative game refinement measure for Brazil (say  $R_r$ ) is given by Eq. (13).

$$R_r = R \times \frac{R_B}{R_G} = 0.091 \times \frac{0.032}{0.085} = 0.034 \tag{13}$$

Similarly, the relative game refinement measure for Germany is given by Eq. (14).

$$R_r = R \times \frac{R_G}{R_B} = 0.091 \times \frac{0.085}{0.032} = 0.242 \tag{14}$$

From Eqs. (13) and (14) we see that from Germany’s perspective, people can enjoy the game for fun. Meanwhile from Brazil’s perspective, people may feel very tough and they must seriously face the game progress. Larger  $R$  value means higher fun, whereas smaller  $R$  value means more serious or competitiveness. Illustration in Fig. 4 shows the relation between  $R$  value and balance between skill and chance in boardgames as well as continuous movement games.

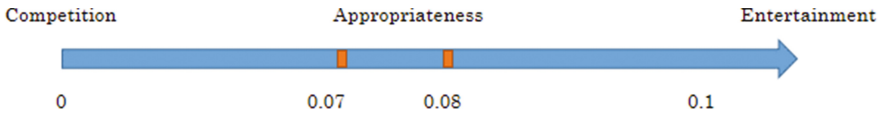


Fig. 4. An illustration of the meaning of game refinement measure

### 3.3 Analysis of MOBA Games

Multi-player On-line Battle Arena (MOBA) [21] is the most popular game type, in which a player controls a single character at one of two teams. MOBA game is a typical continuous movement game. The objective is to destroy the opponent team’s main structure with the assistance of periodically spawned computer controlled units. Player characters typically have various abilities and advantages that improve over the course of a game and that contribute to a team’s overall strategy. Mainly in the world market, it was followed by three spiritual successors: “League of Legends” (LOL), “Defense of the Ancients” (DotA) and “Heroes of the Storm” (HotS) [22].

The game progress model of MOBA is given by the average number of successful killing heroes and destroying fortress (say  $K$ ) over the average number of attempts per game (say  $A$ ) [13]. Hence, the game refinement measure of MOBA is given by Eq. (15).

$$R = \frac{\sqrt{K}}{A} \tag{15}$$

The measures of game refinement for various MOBA games are shown in Table 4. Because of the game battle system and macro mechanism, in DotA and LOL one tower equals to 1 kill, and in HotS one castle equals to 4 kills [22]. For killing tendency  $A$ , any tower or castle as 1 attempt is calculated. It is found that  $R$ -value of sophisticated games is located somewhere between 0.07 to 0.08 [2, 10]. Distinctly, we notice that the game refinement value in LOL battle is so high. It means that LOL will be too excited with high entertainment and low competitiveness.

**Table 4.** Measures of game refinement for three MOBA games

	Map or version	K	A	R
HotS	Blackheart's bay	70.90	80.10	0.105
	Sky temple	77.68	79.90	0.110
	Dragon Shire	63.90	88.80	0.090
	Tomb of the SQ	75.00	98.00	0.088
	Infernal shrines	63.08	93.00	0.085
	Cursed hollow	69.55	100.70	0.083
	Battlefield of eternity	99.30	168.8	0.082
	Garden of terror	68.83	88.90	0.093
	Haunted mines	55.68	78.10	0.096
DotA	Version 6.48	69.2	110.8	0.075
	Version 6.51	68.4	110.2	0.074
	Version 6.59	69.8	110.0	0.076
	Version 6.61	70.0	111.6	0.075
	Version 6.64	68.4	110.4	0.075
	Version 6.69	67.8	108.4	0.076
	Version 6.74	62.4	102.6	0.077
	Version 6.77	62.8	102.8	0.077
	Version 6.80	68.6	106.2	0.078
LOL	Version 6.6	37.65	44.26	0.138

Below we summarize the entertaining and competitiveness aspect of MOBA games based on the game refinement values.

**DotA:** DotA is a very stable game, also it is a typical “G-T Model” (continuous movement games), for each version  $R$ -values are all seated between 0.07 to 0.08. Therefore, DotA is a well designed game with a good balance between entertainment and competitiveness, which is suited for competitions. For the activity population, DotA2 has 7.9 million per month all over the world [23]. The measure of game refinement indicates that DotA is the most successful and well balanced MOBA game in the world.

**LOL:** Generally,  $R$ -value in LOL is too high, whereas DotA is almost in the window value. It means that DotA fits for setting as e-sports competition, but LOL is suited to enjoy for entertainment. DotA has powerful skill and more visual impact for each hero, which cares more about management and running. Players need to make a stable and safe environment to carry and develop. Gank usually happens during the whole game. Generally, a DotA game may spend about 50 min but LOL usually takes around 30 min. LOL provides players with a new style of MOBA game that spends less time for each game and forms a fast rhythm. For the activity population, LOL has 67 million per month all over the world [23]. The rhythm of LOL is faster and its game refinement is higher than others. This implies that LOL is able to attract more children, female or beginners who prefer to play it because of the higher entertainment property [1].

**HotS:** For HotS, the most important point is large-scale team combat and the game rhythm is much higher than DotA or LOL. As a new game, HotS still has some insufficient aspects. According to Table 4, the most interesting and exciting map is ‘Sky temple’. ‘Battlefield of eternity’ and ‘Cursed hollow’ have the highest level competitiveness. However, the game refinement measures of HotS are higher than 0.08, which means that compared with DotA, HotS is not so suitable for e-sports competition. Also some serious mechanism issue existed in HotS, DotA focuses on the ana-phase period during the game, but the core mechanism in HotS is wild monster. For this reason, the game depth of HotS is less than DotA and gets a larger  $R$ -value. Therefore, HotS cares more about teamwork than personal operation and game awareness, then we can only find valid data about the population of HotS in US server is 0.13 million, the expected number all over the world will not be larger than DotA2. Nevertheless, the fun of HotS is not derived only from the battle. The various heroes and their talents can provide a lot of enjoyment for Blizzard fans. In addition, they can design maps which become more interesting and well balanced. Also the design group of HotS needs to revise the game mechanism.

All property of these three MOBA games can be shown as Fig. 5.

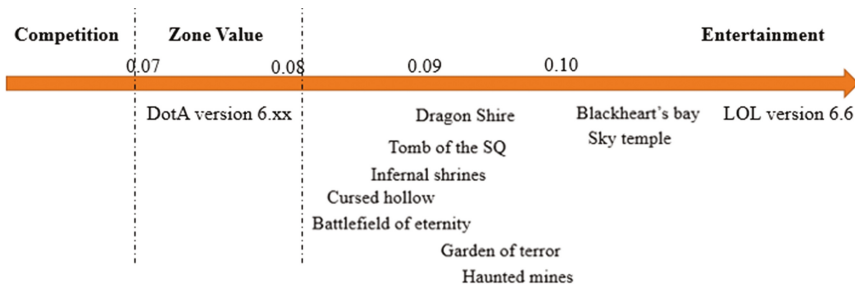


Fig. 5. Entertainment and competition property of three MOBA games

## 4 Concluding Remarks

The notion of game progress and game information progress model for continuous movement games was introduced in the development of game refinement measure. It seemed to be a successful bridge between continuous movement games like sports and boardgames. However, this paper claimed with a focus on the parameter  $c$  in the game progress model for boardgames.

The parameter  $c$  relates to the game balance. The condition  $c = 1$  corresponds to the case where the game is more chance-based one. If the parameter  $c$  becomes lower, the game will be more skill-based one. Moreover, a new perspective of game refinement measure was obtained. Higher (lower)  $R$  value means more entertaining (competitive), whereas 0.07–0.08 should be a comfortable zone due to its good balance between skill and chance in game playing. The analysis of popular MOBA games using game refinement measure supports the observation. The concept of relative game refinement measure was proposed to focus on individual team performance in two team sports such as soccer. The game refinement measure has been used to quantify the game sophistication for the game under consideration. However, we considered the possibility of quantifying the game sophistication from the viewpoint of individual team.

**Acknowledgements.** This research is funded by a grant from the Japan Society for the Promotion of Science (JSPS), within the framework of the Grant-in-Aid for Challenging Exploratory Research (grant number 26540189) and Grant-in-Aid for JSPS Fellow.

## References

1. Gaudiosi, J.: Riot games' league of legends officially becomes most played PC game in the world. *Forbes* (2012)
2. Iida, H., Takahara, K., Nagashima, J., Kajihara, Y., Hashimoto, T.: An application of game-refinement theory to Mah Jong. In: Rauterberg, M. (ed.) ICEC 2004. LNCS, vol. 3166, pp. 333–338. Springer, Heidelberg (2004). doi:[10.1007/978-3-540-28643-1\\_41](https://doi.org/10.1007/978-3-540-28643-1_41)
3. Iida, H., Takeshita, N., Yoshimura, J.: A metric for entertainment of boardgames: its implication for evolution of chess variants. In: Nakatsu, R., Hoshino, J. (eds.) Entertainment Computing. ITIFIP, vol. 112, pp. 65–72. Springer, Boston, MA (2003). doi:[10.1007/978-0-387-35660-0\\_8](https://doi.org/10.1007/978-0-387-35660-0_8)
4. Neumann, J.: Zur theorie der gesellschaftsspiele. *Math. Ann.* **100**(1), 295–320 (1928)
5. Shannon, C.E.: Programming a computer for playing chess. In: Levy, D. (ed.) Computer Chess Compendium, pp. 2–13. Springer, New York (1988). doi:[10.1007/978-1-4757-1968-0\\_1](https://doi.org/10.1007/978-1-4757-1968-0_1)
6. Turing, A.: Chess. Part of the collection Digital Computers Applied to Games. In: Bowden, B.V. (ed.) *Faster Than Thought, a Symposium on Digital Computing Machines* (1953)
7. Panumate, C., Xiong, S., Iida, H.: An approach to quantifying Pokemon's entertainment impact with focus on battle. In: 3rd International Conference on Applied

Computing and Information Technology/2nd International Conference on Computational Science and Intelligence, Okayama, pp. 60–66 (2015)

8. Punyawee, A., Panumate, C., Iida, H.: Finding comfortable settings of snake game using game refinement measurement. In: Park, J.J.J.H., Pan, Y., Yi, G., Loia, V. (eds.) *CSA/CUTE/UCAWSN -2016*. LNEE, vol. 421, pp. 66–73. Springer, Singapore (2017). doi:[10.1007/978-981-10-3023-9\\_11](https://doi.org/10.1007/978-981-10-3023-9_11)
9. Cincotti, A., Iida, H., Yoshimura, J.: Refinement and complexity in the evolution of chess. In: *Proceedings of the 10th International Conference on Computer Science and Informatics*, pp. 650–654 (2007)
10. Sutiono, A.P., Purwarianti, A., Iida, H.: A mathematical model of game refinement. In: Reidsma, D., Choi, I., Bargar, R. (eds.) *INTETAIN 2014*. LNICSSITE, vol. 136, pp. 148–151. Springer, Cham (2014). doi:[10.1007/978-3-319-08189-2\\_22](https://doi.org/10.1007/978-3-319-08189-2_22)
11. Nossal, N.: Expansion of game refinement theory into continuous movement games with consideration on functional brain measurement. Ph.D. Thesis, Japan Advanced Institution of Science and Technology (2015)
12. Xiong, S., Iida, H.: Attractiveness of real time strategy games. In: *2nd International Conference on Systems and Informatics (ICSAI)*, pp. 271–276. IEEE (2014)
13. Xiong, S., Zuo, L., Chiewvanichakorn, R., Iida, H.: Quantifying engagement of various games. In: *The 19th Game Programming Workshop*, pp. 101–106. Information Processing Society of Japan (2014)
14. Iida, H.: Fairness, judges and thrill in games. In: *IPSJ-SIG-GI Technical Report*, vol. 28, pp. 61–68 (2008)
15. De Groot, A.D.: *Thought and Choice in Chess*. Mouton Publishers, The Hague (1965)
16. Ranking data of soccer league. <http://goal.sports.163.com>. Accessed 2017
17. Xiong, S., Tiwary, P.P., Iida, H.: Solving the sophistication-population paradox of game refinement theory. In: Wallner, G., Kriglstein, S., Hlavacs, H., Malaka, R., Lugmayr, A., Yang, H.-S. (eds.) *ICEC 2016*. LNCS, vol. 9926, pp. 266–271. Springer, Cham (2016). doi:[10.1007/978-3-319-46100-7\\_28](https://doi.org/10.1007/978-3-319-46100-7_28)
18. Prasertsakul, P., Iida, H., Kondo, T.: Boring game identification: case study using popular sports games. In: *SICE Annual Conference, Society of Instrument and Control Engineers* (2016)
19. Panumate, C., Iida, H.: Quantifying enjoyment of individual match in games. In: *The Annual Conference on Engineering and Applied Science, Higher Education Forum* (2016)
20. Groll, A., Schauburger, G., Tutz, G.: Brazil or Germany—who will win the trophy? Prediction of the FIFA World Cup. 2014 based on team-specific regularized Poisson regression (2014)
21. Johnson, D., Nacke, L.E., Wyeth, P.: All about that base: differing player experiences in video game genres and the unique case of moba games. In: *Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems*, pp. 2265–2274 (2015)
22. Xiong, S., Zahi, H., Zuo, L., Wu, M., Iida, H.: Analysis of the “Heroes of the Storm”. *Int. J. Adv. Comput. Sci.* **4**(6), 79–82 (2015)
23. Minotti, M.: League of Legends vs. DOTA2 vs. Smite vs. Heroes of the Storm. <http://venturebeat.com/2015/07/15/comparing-mobas-league-of-legends-vs-dota-2-vs-smite-vs-heroes-of-the-storm/>