Chapter 12
How to Teach and Assess Whole Number Arithmetic: A Commentary on Chapter 11

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12.1 Preliminary Considerations About Teaching and Assessing

Teachers are accountable for classroom interactions and pupils' work assessment. However, those visible actions (for instance, during the lesson observed by working group 4 participants in Macao) are only a part of teachers' work, and include also:

- *Planning* not only a single lesson but also a sequence of lessons and more generally thinking about and designing the entire mathematical theme (for instance, WNA as a whole). This usually depends on use of resources that are selected by the teacher.
- *Selecting* the physical objects to be used during the lesson (or the sequence of lessons), the textbook which can be used by the pupils and/or by the teacher as a source of inspiration, the tasks that might be designed by others and are available (by sharing with colleagues, by browsing the Internet, etc.), the items for the assessment, etc.

These aspects of teaching require teacher knowledge, which is not easily observed, since it is accessible only by means of what the teacher might say about her activity, which is always a reconstruction on her part, and what the teacher is doing in the classroom, which is subject to diverse interpretations.

Various aspects of teacher knowledge have been considered within different frameworks that all take into account *pedagogical content knowledge* (Shulman 1986). This model has been refined by Deborah Ball and her colleagues (Ball et al. 2008), who have examined the impact *mathematical knowledge for teaching* on the quality of instruction (Hill et al. 2008). It is thus my purpose to highlight some

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aspects of whole number arithmetic knowledge for teaching which seem important to promote interest-dense situations (Bikner-Ahsbahs et al. 2014) and thus the development of pupils' metacognitive strategies.

12.2 Whole Number Arithmetic and Mathematical Knowledge for Teaching

Chapter 11 and working group 4 of the conference refer to 'teaching and assessing whole number arithmetic'; this formulation leads to consider WNA as a homogeneous domain. However, some very different aspects have been investigated in the conference papers:

- Understanding of numbers
- Place value and written numbers
- Understanding of operations
- Written operations (standard and non-standard)
- Memorisation of numerical facts: additive or multiplicative

In the book, the main topic has been the one chosen for the observed lesson: understanding of addition with carrying.

However, making coherent choices about WNA teaching requires organised knowledge of WNA (Askew 2015) and specific knowledge of concepts (see Barry et al. 2015 for a study about additive problems):

- Which sub-matters are related?
- How these sub-matters are linked together?
- What are the vital choices for teaching numbers?

In order to approach those questions related to the teaching and assessing of WNA, I will take three examples and then return to the Macao lesson.

12.3 Memorising Numerical Facts

During early primary school, pupils are engaged in various memorisation activities: they memorise nursery rhymes and poems, the number name file, the days of the week, the name of the months, the names of their friends, etc. Some years later, they will memorise a lot of facts and rules: grammatical, historical, mathematical rules and facts, etc.

What specific knowledge do teachers need in order to help pupils to memorise numerical facts? Is this different from the memorisation of nursery rhymes? What about grammatical rules? Are all numerical facts alike in terms of memorisation?

Let's begin with the first numerical memorisation: oral number names. This sequence of words share some properties with all songs and rhymes: some parts are made from words without links (one, two, like Humpty Dumpty); some parts are similar (twenty-one is similar to thirty-one, like a chorus) and you have to say it in the right order. However, number names are special: because of their use for counting, in particular, the words have to be clearly separated (one/two/three and not onetwothree), and the exact words and order of the words are crucial. If the oral number file is a base-ten language (a lot of languages across the world are in base twenty, also known as vigesimal[1]), you have thus at least ten different terms to memorise. Those terms are not different from any other list of terms (days of the week, song, etc.). The following names depend on the language you use (see this volume, Chap. 3). If you are very lucky, you may live in a country where the oral numeration is regular: ten-one, ten-two, etc. If you are not so lucky, you will have to memorise other terms, for instance, eleven and twelve in English (and up to the name for 16 in French, etc.). The rest of the list will have some regularity and irregularity, thirteen instead of third-ten, for instance, or a vigesimal system at some point (for instance, from 60 up to 99 in France). Teachers have to be aware of nuances of language in order to understand when they have to treat the number name file exactly like a song and when they might help the children understand how it is built. *This is clearly mathematical knowledge for teaching: it is not mathematical common knowledge*. For instance, a majority of persons in France are not aware of a base twenty (it is different in Belgium, Canada or Switzerland), but for teachers it is vital knowledge to understand that it is quite strange to say *soixante-dix* (sixty-ten), but if you do, it is normal to proceed and say *soixante-et-onze* (sixty-and-eleven).

If we now take the memorisation of numerical facts about addition and subtraction, it is first to be noted that it is not obvious to know when you really enter 'addition'. For instance, you have to teach very early that if you say six after five in the oral number file, in consequence if you have five objects and another one, you will have six in all. This will be related later to $5 + 1 = 6$. However, it means that those additive facts (+1) are learned in a totally different way as, for instance, $5 + 3 = 8$.

If we proceed on our reflection about the memorisation of additive facts, there are different ways to give the answer promptly (see Cao et al. 2015 about memorisation of multiplication table). The first is to memorise every additive fact. For numbers less that ten, you have 10×10 facts to memorise, but we have already stated that perhaps you may not have to memorise +1 as an additive fact, which takes off ten results to memorise. If you know $6 + 1 = 7$, do you have to learn that $1 + 6 = 7$, or do you have to know that you can exchange the numbers in addition (commutativity)? There is a balance to be found between the memorisation of facts and the memorisation of properties (which is the very first element of algebraic thinking; see Wong et al. 2015 for other algebraic problems).

If we return to $6 + 7$, you may have memorised the answer among the 90 or 45 facts that remain in your list, or you can think that 'six and four, ten and three: thirteen'. In order to obtain 13 very rapidly, you should memorise the ten comple-

[1] https://en.wikipedia.org/wiki/Vigesimal

ments (five facts only if you consider commutativity) and the procedure to count with ten or multiples of ten. There is, thus, also a balance to be found between the memorisation of facts and the memorisation of procedure.

Furthermore, the choice of procedures is crucial. For the same addition, if you have learnt to calculate using 5 as a step, you will think 'five and five and one and two: thirteen' and not 'six and four and ten and three'.

Those mathematics reflections have a big impact on the teaching and assessing of additive facts. For instance, if you want to induce 5 as a step, you will teach all the decomposition of numbers using 5 as a step from the beginning of the teaching of number: 6 will be considered as 5 and 1, 7 as 5 and 2, etc., and those relationships to 5 will be memorised. To assess the memorisation of additive results will be considered with those results and subsequent procedures in mind. For instance, it would be considered as a really basic task to give the answer to 12 + 13, but more difficult to give the answer to 16 + 14 and even more 17 + 18. On the other hand, if you have memorised additive results using 10 as a step, 16 + 14 should be the easiest.

The role of researchers in mathematics education may have a great impact on teaching and assessing if they help teachers to understand how special mathematics considerations will impact their decisions when they plan their teaching and select their materials and also highlight the different aspects of 'memorisation'.

12.4 Writing Numbers and Numerical Sentences

Some aspects of mathematical writing are specific, and some are shared with all writing language experiences (see Sensevy et al. 2015 for a design which is based on writing mathematics). The aspect which is present in both cases is the possibility that writing offers to avoid painstaking memorisation of facts. Writing is thus always in concurrence with oral memorisation. Another common aspect of writing is the possibility to communicate to others with a spatial or temporal gap. Yet another is the bureaucratic function of writing: when you write, you can organise the objects, for instance, in columns and lines, in ways you cannot reproduce orally (Goody 1986). What is totally different is the lack of connection between sounds and writing: 216 is not read two-one-six, etc. Furthermore, 21 is read twenty-one but when you read 216 you do not hear twenty-one (it is not easy to understand that there are 21 somethings in this number: 21 tens). Another notable disparity is that between the quite universal understanding of written numbers and that of mathematical writing in general. Thus the use of writing in mathematics should be a specific part of teaching mathematics (and not only written standard algorithms; see Zhao et al. 2015).

For instance, suppose that you want to associate eggs with egg cups with a temporal gap (see Alafaleq et al. 2015 for equality problems in textbooks). You have the eggs one day and the egg cups the day after. It is difficult to memorise the number of eggs, and you might use writing for this task. When you implement this situation for pupils (5–6 years old), the use of numerical writing is required in this situation.

Depending on the number of eggs and pupils' knowledge of writing numbers with digits, they may struggle to find a suitable way to use writing.

Some pupils will try to draw eggs, using the right colour and the right shape, but not the right quantity. They will realise that their writing does not give any information when confronted with the egg cups. A successful procedure might be to use the spatial organisation of the eggs and try to draw a 'map' with the places of the eggs, drawing for instance round shapes. Another procedure is to draw little straight lines: one for each egg. Teachers should consider this procedure as a very interesting attempt at symbolisation and thus encourage this behaviour and not only consider writing with digits. There are different ways to write quantities, and the efficacy of writing depends on the situations you are dealing: in particular situations, even an adult might write ~~IIII~~ ~~IIII~~ ~~IIII~~ in order to keep a record of 15 objects.

It is not enough to acknowledge that writing mathematics is an important part of whole number arithmetic. How this process is approached will vary according to the teacher's interpretation, whether they see writing as a fixed set of rules or as a way to think mathematically. I will illustrate this with an example (Laparra and Margolinas 2009). During a session observed in class 1, pupils were asked to solve the following problem 'There are 12 squares in a box. There are red and blue squares. There are 5 red squares. How many blue squares?' Pupils had some difficulties to solve this problem: they had not studied subtraction previously, and it was the first time they had to solve a word problem. At some point during the lesson, when all pupils were convinced that 7 blue squares was the solution, the teacher asked them to write or draw something in order to explain their solutions. Hamdi (Fig. 12.1) had drawn 12 squares and crossed 5 squares: there are 7 non-crossed squares. The representation of the problem is particularly accurate.

If you read the mathematical sentence, 12 + 5 = 7, you might think that Hamdi has made a big error (that is what the teacher thought), but it is highly improbable that Hamdi thought twelve and five are seven. In writing this wrong sentence, Hamdi demonstrates his current knowledge of written operation. He has written the numbers in the order of the given problem (12, 5, 7) and also in the order he has made use of those numbers in the schema, which is wrong for mathematical sentences but right for linguistic sentences, although the sentence is mathematically well formed. Hamdi certainly knows that the result of calculation is normally after the equality sign, but he doesn't know how to combine the only signs he had already learned (+ and =) in order to explain his reasoning.

On the other hand, if you consider Floriane's production (Fig. 12.2), it is difficult to understand Floriane's solving procedure, but it is very interesting to discover a kind of prefiguration of an equation: $5 + x = 12$.

Fig. 12.1 Hamdi's representation of the squares problem

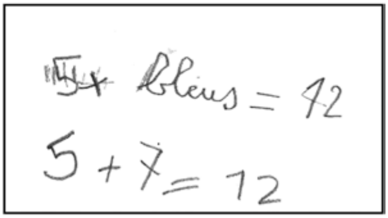

Fig. 12.2 Floriane's representation of the squares problem

Both productions have their own qualities, and they reveal the difficulty to assess the production of written mathematical sentences. Unfortunately, the teacher was only interested in the correctness of the written addition.

There is certainly an important need for collective work in mathematics education in order to convey coherent knowledge about writing numbers and numerical sentences, since this is crucial for teaching and asserting WNA in general, at all levels of teaching.

12.5 The Field of Additive Structures

The expression 'field of additive structures' is taken from Vergnaud (1983, p. 31), whose work is of paramount importance in order to understand together addition and subtraction (for Chinese tradition, see Sect. 11.4.1).

The first comment based on Vergnaud's work is about subtraction – comparison between quantities is only one meaning for subtraction:

> The very first conception of subtraction for a young child is a "decrease" of some initial quantity […]
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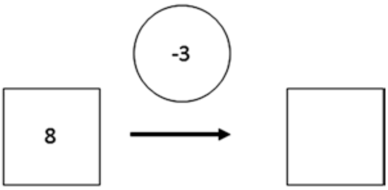
> 
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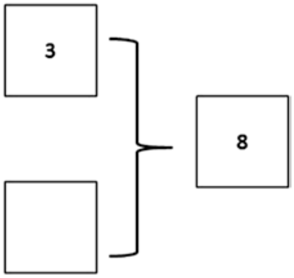
> *Example 1: John had 5 sweets, he eats 3 of them. How many sweets does he have now?* It is not straighforward, with such a conception in mind, to understand –subtraction as a relation of complements.



Example 2: There are 8 children around the table for Dorothy's birthday. 3 of them are girls. How many boys are there? (Vergnaud 1983, pp. 31–32)

In the following pages of the paper, Vergnaud enumerates and exemplifies the other conceptions of subtraction, as the inverse of an increase and as a difference relationship between states, between compared quantities and between transformations, and he concludes with: 'One can easily imagine the difficulties that children may meet in extending the meaning of subtraction from their primitive conception of a 'decrease' to all these different cases' (p. 32). Vergnaud has shown that pupils are able to solve the first problems from a young age, but the more difficult ones only at the end of primary school (even if the calculation, 8–3, remains the same). The same kind of differences of conception exists also for addition, both operations being regrouped in the additive structure field.

Those distinctions are essential for teacher mathematical knowledge, since they have to be aware of the nature of the problems which are proposed, in order to teach or assess addition and subtraction. The predominance of comparison has to be questioned (Sect. 11.4.1, Kaur 2015, Zhang et al. 2015), in particular in textbooks, because it will impact implicitly on teacher's conception (Sect. 11.7).

12.6 The Macao Lesson: A Commentary

In this last part of this paper, I will give an overview of the points of the Macao lesson on addition, using the different aspects I have introduced earlier.

My first remark is about the subject matter of the lesson: it is well known that addition, even with carrying, is very easy, compared with subtraction (see Pearn 2015), and I would have been very interested to know how the skilful teachers of this school would have taught this challenging subject matter.[2] Even if I understand that the choice was not due to the working group, it is important to reflect on the most favourable environment of a scientific discussion about teaching and assessing WNA.

[2] Another group of participants (from other working groups) actually observed a lesson on subtraction in another school.

However, the lesson is very interesting in itself, and it is in a certain sense a model of mastery of a kind of lesson which can be observed also in other countries:

- The lesson begins with the oral recollection of numerical facts (number combination for ten), during which students are encouraged to give answers very rapidly.
- The teacher has introduced a material (candies and boxes of candies), pupils are mostly working with the material, they express their ideas orally, and they write with the scaffolding of the teacher.
- Different problems of addition of increasing difficulties are introduced with both representation of material and mathematical sentences (40 + 3; 25 + 2; 25 + 20) before the core topic of the lesson, which is to study an addition with carrying (24 + 9).
- The teacher is aware of a variety of possible answers for this problem, she has determined three procedures, and those procedures are represented by numerical sentences which are written in advance and ready to show to the pupils.
- The 'making-ten' strategy is clearly emphasised at the end of the lesson during the 'lesson summary'.

The calculations (Stage 1) in the oral phase represent nearly all the combinations for ten (only 0 + 10 is missing); thus, the answer is always ten. The first five questions are given in order (one number in the addition increases by one at every step). Thus, this first part of the lesson can be considered as a systematic presentation of the combinations for ten, but not as an episode of working on fluency. It is interesting to note that, in my experience, this is very common (in France, at least): oral fluency of number facts is very frequently underestimated. More generally, orality (Goody 1977; Ong 2002), which has its own mode of knowing and organising facts, is not considered as really important. In this lesson, the very fact that the questions were presented as written sentences and organised in two columns, more (first column) or less (second column) organised by increasing one number, is somehow 'transparent' (Margolinas and Laparra 2011). There is frequently little awareness on the part of the teachers that oral mathematical facts and written ones are very different. For instance, in the Macao lesson, there was clearly a choice to be made: working on oral facts (and in this case giving questions orally and choosing questions with results not always ten) or working on organising facts about combinations for ten using writing. For some reason, oral calculation is not seriously considered, even if rapid oral calculation is still useful, whereas written calculation cannot compete with the use of a calculator: you will find more easily your phone in your pocket than paper and pencil.

The use of material (Stages 2–3) is also interesting because it might be found in different countries around the world, where base-ten material is generally used. The material used here has a property which is not always found: there is a ten-place grid composed of five and five places and you can take out or put elements in this grid. In some other classes, groups of ten cannot be decomposed: you thus have units and tens and if you have ten units you have to exchange those for a ten. There are discussions, around the world, on the differences between the two conceptions of place value: a ten is the first group in base-ten numeration (10^1) or a ten is a unit in itself. Choosing this material leads clearly to the first conception, but you can manipulate for at least two very different reasons. The first is purely material: if, for 24 + 9, a pupil shows two complete ten grids, one grid with four candies and one grid with nine candies, the teacher might say: 'you know that you are not allow to do that: you must complete the grid before taking another one'. In this case, pupils are only manipulating objects with a very loose relationship to base-ten operations, which is different if the teacher says: 'In order to organise the candies in base ten, you always have to make a group of ten when it's possible. Can you make another group of ten with your candies?'. The different solutions, which have been shown by the teacher in Stages 2 and 3, were clearly aimed at the second version, because they demonstrate different ways to regroup the candies in tens, using a schematisation. However, we do not know how to consider the relationship between boxes, candies and written numbers on the one hand and the role played by oral numeration on the other. This is particularly important when oral numeration is not congruent to written numeration (which is the case in the major European languages: you say twenty and not two-tens, where in most Asian languages oral numeration is regular). For instance, you can count ten, twenty, twenty-three (see figure in Stage 3) and write 23 as the cultural way to write twenty-three, or you can say two tens and directly write 2 in the left place which is the place value for tens and three units and write 3 in the right place (place value for units). With the same material, both decisions are possible, which are very different from a teaching point of view.

The selection of the introductory additions (40 + 3; 25 + 2; 25 + 20) highlights the teacher's choices and the mathematical knowledge of the team: in the first, a number with only tens and a number with only units are dealt with independently, in the second, you have to combine the units of the second number with those of the first and, in the last one, this is the same but with tens. Thus, the environment of the last problem, which is the core of the lesson (24 + 9), is not only material; it is also made of mathematical knowledge, which has been carefully introduced by the teacher. In focusing on more general considerations (existence of material, familiarity with the material, etc.), those calculations are components of the *milieu* (Brousseau 1997; Brousseau et al. 2014), which is never only material.

The teacher has determined in advance the possible procedures for 24 + 9. What is striking is that she has written everything in advance (Sect. 11.6). In this case, mathematical writing cannot emerge as a way to understand a solution, and there is no place for false solutions (see Ekdahl and Runesson 2015). Pupils might know the answer, either because other pupils have said it was 33 or because they have counted the candies one by one. Therefore, they might have the mathematical sentence right (24 + 9 = 33), but not the right base-ten properties. Wrong solutions might trigger the occasion to recall what base-ten is about: when you have ten, you regroup (which is true for units but will be true also for tens and so on). For example, with 4 and 9 you can make a ten, either with 4 + 6 (and leave 3) or with 9 + 1 (and leave 3), or you can know that 4 + 9 = 13, which is a ten and 3 units. Thus, it is an opportunity for the teacher to state the reasons for the three different solutions. This demonstrates the downside to having everything written in advance: the reasoning that underpins these solutions might remain unexplained.

The last remark relates to the conclusive part of the lesson. Task designers (Watson and Ohtani 2015) usually carefully describe the 'active' part of the task: the problem to solve and the environment of the problem. However, they usually avoid to enter into considerations about what you might tell pupils regarding what they have learned and what they have to memorise. If we use Brousseau's words (Sect. 11.5), task designers are usually more concerned by the devolution process than by the institutionalisation process (Brousseau 1992; Margolinas 2005; Margolinas and Laparra 2008). The conclusive part in the Macao lesson shows clearly what the teacher expects of the pupils in the future: to learn the ten complements and to learn how to use them. The whole lesson appears, at this moment, as a whole, for students and for the observers.

12.7 Some Concluding Comments

Chapter 11 and working group 4 have taken into consideration some important processes in teacher work. In an attempt to complement this work, I have focused on mathematical knowledge for teaching, in order to stress the need to consider our own conception of whole number arithmetic and the way it impacts our research and our analysis of teacher work.

If we take seriously the very interesting suggestion made in Sect. 11.3.2 to transform a closed question into an open one (Sullivan and Lilburn 2004), we have thus to consider not only the shift in role it implies, but also the mathematical knowledge which might be learned by pupil difficulties and the mathematical knowledge necessary for the teacher. The intent of the Macao lesson, as clearly revealed in the concluding part, was to teach the use of the ten complements in order to give the result of any addition with carrying, which is useful either for mental or written calculations. The purpose of the study of the open problem proposed is completely different: it is true that it involves pupils doing additions and reflecting upon addition as an operation (and even as a function, since it can be modelled using the linear function $y = 33 - x$). The challenge for researchers might also be to find better problems with the *same* purpose, which is a very different question: that is, to focus also on 'daily routine' (see Brombacher 2015).

In general, I think that we often underestimate teacher knowledge required not only for selecting challenging and dense tasks but also, within a determined task, for responding to the diverse needs of individuals (Sect. 11.3.3) and to assess this need (see Gervasoni and Parish 2015). It is certainly not a little challenge for mathematics education research to describe the knowledge at stake, even within the field of WNA and even if we take a single lesson (see Lin 2015 for a development about the algorithm for multiplication). This book is certainly a very important step in this direction.

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