Topic Study Group No. 21: Mathematical Applications and Modelling in the Teaching and Learning of Mathematics

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The Programme

TSG21 was well supported by full papers, orals and posters showcasing research into mathematical applications and modelling education. The structure of the main TSG sessions involved invited plenaries and 20 min research and/or practice presentations. As well as these sessions there were parallel oral communication sessions.

The first main session on Tuesday 26 July: 12.00–13.30 was chaired by Jussara Araújo. Following a welcome and overview of sessions by the chair, Gloria Stillman gave a one hour plenary on the *State of the Art on Modelling in Mathematics Education* which was followed by a lively discussion. Later in the day Oral Communications in three parallel sessions took place from 15:00 to 18:00.

The plenary highlighted the many good arguments already presented in the literature as to why real world applications and modelling should be favoured in curricula. The particular goal of modelling and/applications in curricula underpin the approach taken to modelling in many research studies whether these goals be from a mathematical or an informed citizenry perspective. Uptake of curricula goals and implementations vary widely in practice despite advances in many quarters.

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A recent curriculum document study by Smith and Morgan (2016) in 11 educational jurisdictions identified three orientations to real world contexts in mathematics: as a tool for everyday life, as a vehicle for learning mathematics and as motivation to learn mathematics. Of concern, though, was the provision of alternative pathways in the majority of these jurisdictions with more mathematically advanced pathways having less emphasis on real-world contexts.

In research into the teaching and learning of mathematical modelling there is strong emphasis on developing "home grown theories" where the focus is on particular "local theories" such as modelling competencies rather than general theories from/with application outside the field. As examples of current theoretical lines of inquiry prescriptive modelling, modelling frameworks (e.g., the dual modelling cycle), and modelling competencies (all local theories) and anticipatory metacognition (a general theory) were overviewed to give a flavour of work being done. Some have been the subject of empirical testing or confirmation whilst others await such work. Focuses of empirical lines of inquiry are just as many and varied so examples focused on student modelers, teachers of modelling and task design. Finally, questions for future theoretical and empirical research based on the fore-going examples were outlined. In all, the emphasis was on doing modelling in the classroom so as to be useful!

On Wednesday 27 July: 12.00–13.30 there were two parallel sessions of full paper presentations. The first session was chaired by Morten Blomhøj. Papers were given by France Caron—*Approaches to investigating complex dynamical systems*, Irit Peled—*Shifts in knowledge and participation of children with mathematical difficulties working on modelling tasks*, Dung Tran—*Authenticity of modelling tasks and students' problem solving*, and Miriam Ortega—*Influence of technology on mathematical modelling of a physical phenomenon.*

The other parallel session was chaired by Toshikazu Ikeda. Papers were given by Takashi Kawakami—*Merging of task contexts and mathematics in dual modelling teaching: Case studies in Japan and Australia*, Jill P Brown—*What do we mean by 'context'?* Andreas Busse—*The negative impact of the new German examination tasks on the modelling classroom in Hamburg* and Corinna Hertleif – *Assessing sub-competencies of mathematical modelling in the LIMO project*.

On Friday 29 July 12.00–13.30 again there were two parallel sessions of full paper presentations. The first session was chaired by Dominik Leiss. Papers were given by Xenia Reit—*The potential of cognitive structures in solution approaches of modelling tasks*, Jennifer Czocher—*Making sense of student-generated conditions and assumptions*, Angles Dominguez—*Model application activity: Integration of concepts and models* and Toshikazu Ikeda—*Organizing mathematical modelling in Japanese mathematics curriculum*.

At the same time a second session was chaired by Jill Brown where papers were given by Juhaina Awawdeh Shahbari—*Adapting a cognitive tool for representing teachers' interpretations of students' modelling activities*, Peter Stender—*Heuristic strategies in modelling problems* and Elizabeth W. Fulton—*Teachers as learners: Understanding and valuing mathematical modelling through professional*

development. Again later that day there were Oral Communications in Parallel sessions from 15:00 to 18:00 (see ICME-13 website for details).

The last main session occurred on Saturday 30 August 12:00–13:30 and it was chaired by Gloria Stillman. The session began with two invited half hour plenaries, *Toward a Framework for a Dialectical Relationship between Pedagogical Practice and Research* by Jussara Araújo and *Interplay between Research and Development of Teaching Practices in Mathematical Modelling* by Morten Blomhøj.

Jussara Araújo presented what she saw as the initial steps toward a framework for a dialectical relationship between pedagogical practice and research in the field of modelling in mathematics education. These methodological reflections arose from the development of research on modelling guided by critical mathematics education, and grounded in a socio-political perspective of research. A primary characteristic of the dialectic is that the students|participants are constituted in relation to the teacher|researcher; whereas a second characteristic is that ethical concerns regarding the students|participants help to constitute the methodological rigour of the research that, in turn, is related to the educational quality of the pedagogical practice. Thus, pedagogical practice and research should be seen as part of a single unit, mutually developing and influencing each other; on the other hand, they are different, have different purposes, and may be incompatible, but one presupposes and constitutes the other.

Morten Blomhoej's plenary focussed on the fundamental duality between the teaching aims of (a) developing students' modelling competence and (b) supporting their learning of mathematics through modelling activities. He saw understanding the interplay that results from this duality between research and practice as essential for understanding and furthering the integration of modelling into mathematics teaching. The theories of both teaching and research need to be made concrete and contextualised in relation to teachers' particular modelling projects in order for them to be useful in developmental projects or in-service courses. A learning trajectory through secondary mathematics on the modelling of dynamical phenomena by means of compartment models, difference equations and the use of spreadsheet was used to illustrate how research could and should inform curricula change.

Following a short discussion of these talks, the chair summarised the work of the TSG and closed the session with the expectation that there would be a follow up publication to include at least the main papers and plenaries. Issues related to mathematical applications and modelling in the teaching and learning of mathematics have continued to grow in interest from previous International Congresses on Mathematical Education. This is a very broad field of interest both in terms of educational level range from elementary and primary school to tertiary and from the perspective of mathematics. The TSG thus attracted and catered for a breadth of participants through the individual talks from those interested in the mathematical modelling of primary school students (e.g., Kawakami, Peled), secondary school students (e.g., Busse, Hertleif, Ortega, Tran), and tertiary students (e.g., Dominguez) or both students in schooling and university (e.g., Caron, Czocher,) as well as pre-service and in-service development of their teachers (e.g., Shahbari, Fulton). Others were attracted and catered for by more general issues that surround

the teaching and learning of mathematics through mathematical applications and modelling such as task context (e.g., Brown), assessment and difficulty of modeling tasks (e.g., Busse, Reit), intervention strategies when managing modelling by others (e.g., Stender), authenticity in tasks (e.g., Tran) and the curriculum components and sequences involving modelling-related activities for elementary to high school education (e.g., Ikeda). We thank all the contributors to our TSG whether in the audience or presenting.

Reference

Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum, 27*(1), 24–45.