Mathematics Teachers Working and Learning Through Collaboration

Barbara Jaworski, Olive Chapman, Alison Clark-Wilson, Annalisa Cusi, Cristina Esteley, Merrilyn Goos, Masami Isoda, Marie Joubert and Ornella Robutti

Abstract The authors of this paper were tasked by ICME-13 organisers with conducting a survey on the topic "Mathematics Teachers Working and Learning through Collaboration". Four research questions guided the survey, concerned with: the *nature* of collaborative working; the *people* who engage collaboratively; the *methodological* and *theoretical perspectives* used; what *learning* could be observed and how it related to collaboration? The resulting survey drew from a wide range of sources, identifying papers relevant to the topic—316 papers were identified, analysed against a set of criteria and organised into three major themes, each relating to one or more of our research questions: *Different contexts and features of mathematics teachers working in collaboration*; *Theories and methodologies framing the studies; Outcomes of collaborations*. In addition to the papers revealed by the survey, the team sought contributions from projects around the world which are not represented in the published literature. Members from these projects offered 'narratives' from the work of teachers in the projects. This paper reports on the nature of the projects revealed by the survey and the narratives, their theoretical and methodological focuses, and the range of findings they expressed. While we offer a

B. Jaworski (✉)
Loughborough University, Loughborough, England, UK
e-mail: b.jaworski@lboro.ac.uk

O. Chapman
University of Calgary, Calgary, Canada
e-mail: chapman@ucalgary.ca

A. Clark-Wilson
UCL Institute of Education, University College London, London, UK
e-mail: a.clark-wilson@ioe.ac.uk

A. Cusi · O. Robutti
University of Turin, Turin, Italy
e-mail: annalo@tin.it

O. Robutti
e-mail: ornella.robutti@unito.it

G. Kaiser (ed.), *Proceedings of the 13th International Congress on Mathematical Education*, ICME-13 Monographs, DOI 10.1007/978-3-319-62597-3_17

significant range of factors and findings, resulting from a very considerable work, we are aware of limitations in our study: we missed relevant papers in journals outside our range; papers reviewed were usually not authored by teachers so the teachers' voice was often missing; narratives came from projects with which we were familiar, so we missed others. The survey team is in the process of initiating an ICMI study which can take this work into these missing areas. This paper follows closely the presentation made by the survey team at the ICME-13 congress. In presenting findings from the survey, we have tried to provide examples from and make reference to the survey papers. Because the set of references would be too large to fit within our word limit, we have had to reduce the number of references made. However, readers can find a full set of references in a more detailed paper, Robutti et al. in (ZDM Mathematics Education, 48(5), 651–690, 2016).

ICME-13 Theme—Mathematics Teachers Working and Learning Through Collaboration

Introduction

This paper, produced for the Proceedings of ICME-13, follows as closely as possible our presentation at the congress. It is limited by a necessary imposed length restriction. This means that we have not been able to include all of the citations from our surveyed sources that exemplified our findings. For these we refer the reader to our full paper, Robutti et al. (2016).

Our theme zooms in on the wider professional development scene to focus on *the learning* that occurs when *teachers of mathematics work together collaboratively*, for the mathematical learning of students, which motivates their teaching. We were tasked by ICME to identify and characterise important new knowledge, recent developments, new perspectives, and emergent issues with respect to our theme.

C. Esteley
National University of Cordoba, Córdoba, Argentina
e-mail: esteley@famaf.unc.edu.ar

M. Goos
The University of Queensland, Brisbane, Australia
e-mail: m.goos@uq.edu.au

M. Isoda
University of Tsukuba, Tsukuba, Japan
e-mail: isoda@criced.tsukuba.ac.jp

M. Joubert
African Institute of Mathematical Sciences, Cape Town, South Africa
e-mail: marie@aimssec.ac.za

The Latin word "collaborāre" means "To work in conjunction with another or others, to co-operate" (OED). Collaboration involves mathematics teachers engaging in joint activity, common purpose, critical dialogue and inquiry, mutual support; addressing issues that challenge teachers professionally and reflecting on their role in school and in society. We address *what* co-learning occurs and *how* it occurs.

Teachers *working* includes all the dimensions of teaching in and beyond face-to-face activity with students in the classroom:

- the didactics and pedagogy of creating the classroom environment;
- the evaluation of students' mathematical learning;
- the professional development activity through which teachers learn to teach;
- the institutional demands of school and educational system;
- the societal demands of parents, employers and politicians.

Many of the papers that address collaborations involving teachers also speak about *communities* of teachers. Community can be an informal term or be defined theoretically as, for example, *Community of Practice*, or *Community of Inquiry*. Communities of teachers working together can include various 'others'; teacher educators, researchers, didacticians, school leaders, parents and so on.

Research questions guiding the survey are:

1. What is the *nature* of collaborative working (to include the different roles that teachers can play) and how does this relate to situation, culture and context?
2. Who are the *people* who engage collaboratively to promote the effective learning and teaching of mathematics, what are their roles, and how do they relate to each other within the different communities?
3. What *methodological* and *theoretical perspectives* are used to guide and inform collaborative working and learning?
4. What *learning* can be observed and how does it relate to collaboration?

Methodology Adopted for This Survey

The team searched the mathematics education literature for *journals, books, handbooks*, and *proceedings* relevant to the topic, from 2005 to 2015. They first located *articles/chapters/reports* in which *collaboration* is an explicit part of the research design and its influence on mathematics teachers' learning/working practices. Their strategies to select the papers involved manual and automated processes applied to titles, indexes, abstracts and full-texts. A second selection focussed on collaboration, its processes and products. 316 sources were identified that concerned research from across the world including 23% from North America,

9% from South America, 31% from Europe, 4% from Africa, and 24% from Australasia. Eighteen papers reflected cross-continent collaborations, for example between the Netherlands and Indonesia, between Taiwan and Portugal, or between Germany and Chile. The data were entered into a spreadsheet in which we captured both demographic information and a range of factors relating to projects, teachers, researchers, forms of collaboration, impact on students, nature of learning, and so on. Overall we had 8500 cells of data. For more detail see (Robutti et al., 2016).

Analysis of the spreadsheet data led to identification of *fundamental themes* that could frame the topic of collaboration:

- Theme 1—*Different contexts and features of mathematics teachers working in collaboration* [Addressing RQs 1 and 2]
- Theme 2—*Theories and methodologies framing the studies* [Addressing RQ 3]
- Theme 3—*Outcomes of collaborations* [Addressing RQs 1, 2, 3, 4].

The comparison and contrast between the different sources led to the identification of the following dimensions and related sub-dimensions connected to the themes.

Theme 1	1. Initiation, foci and aims 2. How the collaboration was conceived and organised 3. Scale of collaborations 4. Composition and roles
Theme 2	1. Theories that frame the studies 2. Methodologies of work with teachers
Theme 3	1. Reflections on collaborating 2. Impacts on teachers' knowledge, thinking, and practice

Theme 1: Different Contexts and Features

Initiation, Foci and Aims

We found a diversity of forms of initiation, foci and aims, including:

- Initiatives mandated by ministries and national/regional institutions;
- Collaborations supported by ministries and national/regional institutions;
- Research collaborations initiated by researchers;
- Professional development initiated by researchers/didacticians;
- Within-school collaborations without involvement of 'others'.

The foci for projects fall into two broad categories. The first refers to innovation about mathematical content, the development of new curricula, different pedagogical approaches, and the integration of new tools and resources—with aims to promote the development of teachers':

(a) awareness of students' different learning trajectories;
(b) necessary competencies to foster students' learning;
(c) understanding how teaching and learning resources could support/inhibit learning.

As an example of the first kind of aim (a), Fried and Amit (2005) report from a project, in Israel, which involved 31 schools and 82 teachers, who were "encouraged to discuss teaching approaches required by the students at each grade level and the relationships between the different stages of the development" (p. 419).

The paper from Chen and Chang (2012) reports an example of the second kind of aim (b). They describe the development of a small professional learning community in China, focused on improving teachers' discourse-based assessment practice from convergent formative assessment to more divergent formative assessment.

The project presented by Lin, Chen, Hsu, and yang (2013) exemplifies the third kind of aim (c). They report how a group of teachers, in Taiwan, was involved in the design of instruction materials to make them aware of the ways in which these materials could support/inhibit learning.

The second category of focus refers to the different practices designed to foster teachers' professional learning. The corresponding aims are to evaluate the implementation of specific processes and tools as professional development programmes for mathematics teachers. Krammer et al. (2006), for example, report on a project, developed in Germany and Switzerland, which aimed to examine the conditions and effectiveness of web-based professional development with classroom videos to support mutual exchange, shared reflection and reciprocal analyses of instruction.

How the Collaboration Was Conceived and Organised

We analysed this dimension focusing on two main questions:

- How is collaborative work within the community conceived?
- How is collaborative work within the community activated?

As regards the first question, the team found two ways in which communities developed:

(i) as a declared objective of the collaborations
(ii) as a methodological approach for teacher education.

In the project reported by Potari (2013), developed in Greece, the creation of a community is an object of collaboration (i). The objectives of the mathematics educators were "not to transmit knowledge from the research to the teachers but to form a community of inquiry where the teachers use research as a tool for their inquiry" (p. 509). Collaboration is the central methodological approach of the study

by Martins and Santos (2012), who report from a project in Portugal. The described collaborative work was aimed at evolving teachers' abilities to reflect over time, through the stimuli from other people involved in these reflections (mentor, tutor, supervisor, critical friend).

As regards the second question, the creation of collaborative contexts for the teachers within a community occurred though a number of approaches from which common characteristics were

- Cycles of activities, such as study, design, implementation, analysis, re-design, re-implementation;
- Fundamental roles played by expert figures such as other teachers, teacher educators, mathematicians, and researchers;
- Teachers' engagement: in terms of challenge, solidarity, accountability, trust, respect;
- Activation of processes of reflection, sometimes with reference to theoretical lenses.

One project which stood out in demonstrating all of these characteristics was Brodie and Shalem (2011).

The Scale of Collaborations

Scale varied considerably across the papers reviewed as shown in Fig. 1.

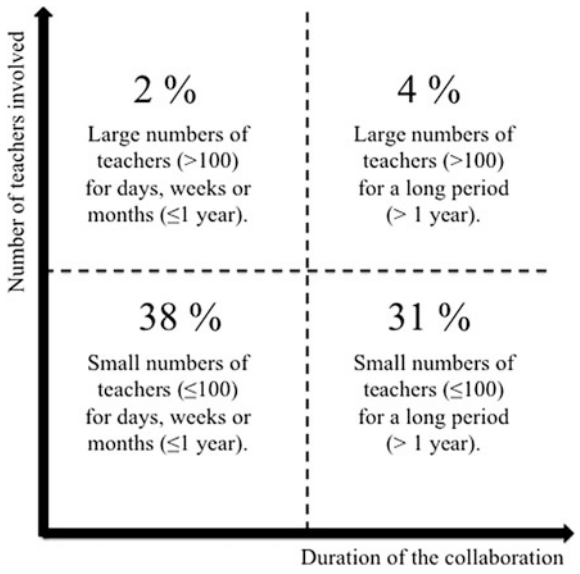

Fig. 1 Number of teachers versus time

It can be seen that large-scale projects in both teacher numbers and length of project were far fewer than those at much smaller scale.

Composition of Collaborative Groups and Participants' Roles

Given the sources of the reviewed papers, participants almost always included mathematics education researchers working with pre- or in-service teachers (who may also be researchers); sometimes, also included were school principals, education assistants and officers; curriculum leaders masters and doctoral students; community leaders; members of specific cultural communities.

Participants' roles were not always clearly described—where they were, key features included teachers designing and piloting curriculum revisions in collaboration with researchers; aiming for 'equal status' between participants; the principle of 'co-learners in partnership' for all participants, and, for teachers, a shift from 'teacher-participants' to 'teacher-researchers'; teachers as mentors to other teachers; dynamic evolution of participants' roles over time. For example, Hospesovà, Machàckovà, and Tichà (2006), in the Czech Republic, aimed for equal status between different participants; Goodchild (2008), in Norway, aimed to develop the principle of 'co-learners in partnership' for all participants.

Theme 2: Theories and Methodologies Framing the Studies

Theories

Many papers did not declare explicitly the theoretical perspectives behind a project. Of those that did, four perspectives were evident: *Community* (of *Practice* or of *Inquiry*) (69%), *Activity Theory* (20%), *Metadidactical Transposition* (6%) and *Valsiner's Zone Theory* (5%). These fell into two pairs with characteristics in common.

The first pair of frameworks is concerned with **knowledge growth** promoted by mediation of tools, signs, artefacts and/or tensions and contradictions between different elements of mediation.

1. **Communities**, involving
    - Communities of *Practice* (e.g., Wenger, 1998): with constructs of *engagement*, *imagination*, *alignment.* For example, Goos and Bennison (2008) showed how a community of practice, focused on "becoming a teacher of secondary school mathematics" emerged during a pre-service teacher education program and was sustained after students graduated (p. 43).

- Communities of *Inquiry* (e.g., Jaworski, 2006): expanding on Wenger with a construct of *critical alignment*. For example, Jaworski's (2008) research on creating a community of inquiry with teachers led to critical alignment in practice, treating issues and tensions as central to teachers' lives and work.

2. **Activity Theory** (e.g., Engeström, 1999) with a construct of *expansive learning*. For example, Sakonidis and Potari (2014) used Activity Theory to analyse their joint activity as teacher educators collaborating with teachers, and thus became aware of not only the transformative nature of teachers' professional learning but also aspects of their own practice that allowed further development to occur.

The second pair of frameworks is concerned with evolution of **teacher identities** promoted/constrained by professional development programs and/or institutional contexts.

3. **Valsiner's Zone Theory** (from Vygotsky e.g., Valsiner, 1997):The Teacher's Zone of Proximal Development is expanded by the Zone of Free Movement and Zone of Promoted Action. For example, Goos's (2013) analysis of two case studies of teacher learning and development identified the significance of productive tensions within a teacher's zone system as a potential trigger for change.
4. **Metadidactical Transposition** (Aldon et al., 2013) from the Anthropological Theory of Didactics (Chevallard, 1985) that traces the evolution of teaching practices as teachers and researchers collaborate.

Methodologies

We found here two sorts of methodologies: the *research methodologies* which framed the research in the studies (these were mainly qualitative e.g., classroom observation, interviews); *developmental methodologies* which framed the nature of development in the study (for example, Collaboration, Professional Development, Change and Communities of inquiry). Within the latter, the following were most evident: Lesson Study; Action/Design/Developmental Research; Narrative Analysis; and Other (including Professional Learning Communities, Video Clubs, Online communities). The most commonly used of these were:

- *Lesson Study* (LS)—very few papers reported on LS *in Japan* (more on this below). However, 20% of surveyed papers reported on lesson study from USA, Australia, UK, countries in Asia, Africa, Europe, South America. Often LS involved a cycle of plan, teach/observe, post-lesson discussion and reflection, and promoted the role of "knowledgeable others". Takahashi (2014) showed how knowledgeable others are crucial for bringing new knowledge from research and the curriculum to the lesson study team and for making the connection between research and practice.

- The challenge of embedding Japanese Lesson Study in other cultural contexts was evident: for example, Kusangi (2014) argues that lesson study in Indonesia has been bureaucratized because of the centralized regulation of teaching and the emphasis placed on preparing students for national examinations.
- *Action research, Design Research, Learning Study* and *Developmental Research* all involved iterative cyclic processes, which varied according to the nature and context of the project.
- *Narrative analysis*: a tool for both research and development in which teacher narratives often revealed evidence of learning. For example, Ponte, Segurado, and Oliveira (2003) link the uses of narrative, as a tool, to understanding mathematics teachers' learning through collaboration.

Theme 3: Research Findings and Knowledge Generated

Here we focus on data from survey articles and from project narratives.

Data from Survey Articles

Almost all the articles revealed by the survey reported on research projects. This theme draws together the findings from these research projects, also drawing on the reports of about 50 'teacher enquiries' funded by the National Centre for Excellence in Teaching Mathematics (NCETM) in the UK. The NCETM enquiries were initiated and coordinated by teachers, and when they finished, a report was produced by the teachers.

The reported findings fall into two main areas: *Reflections on Collaborating* and *Impacts on teachers' knowledge, thinking and practice*

In *Reflections on Collaborating*:

- *Supporting Factors* fell into three main areas: *roles of participants*; *shared goals; and a feeling of safety.* In terms of the roles of participants, *diversity*, *collaborating with other teachers*, and *shifting roles over time* all seemed to be important. For example, in one of the NCETM reports, Wynne-Jones (2013) explained the value of collaborating with other teachers: "Working alongside a range of colleagues gave a much wider perspective. One person's ideas alone would not have had the depth generated by teachers working in groups." In another, Dowling (2013), a teacher wrote about feeling safe: "You need to risk it and don't get scared".
- *Inhibiting Factors* related to *Ownership*, *Time* and the *Value of Collaboration* (per se). For example, within problems of *ownership*, Besamusca and Drijvers (2013) focused on issues to do with *responsibility*, Hospesovà et al. (2006) focused on teachers feeling *uncomfortable*. To do with problems of *time*, Berg

(2011) focused on shortage of time in day to day activities, Cavanagh and McMaster (2015) focused on the time needed to establish a functional community. Jaworski (2008) focused on the day to day constraints of school life and on the value of collaboration, Campbell (2009) questioned the value of collaboration per se.

In *Impacts on teachers' knowledge, thinking and practice.*

- Changes in *knowledge, thinking and beliefs* referred first to learning to work within a community. Some authors wrote about individuals learning how to participate in the community, others reported learning to work together, describing, for example, how the nature of their collaborative work changed over time (Menezes, 2011). An important aspect of impacts on knowledge and beliefs seemed to be *reflection*. A number of papers reported that the participants had learned to reflect better, with Pires and Martins (2009) claiming that "[t]he program has … allowed the development of their ability to reflect (oral and written) on practices" (p. 47).

Impacts on *practice* mainly referred to better teaching, with a number of papers claiming that participating teachers had improved their teaching in general. It seems that many teachers developed confidence in the classroom, and with this, a willingness to try new approaches (Warren, 2008). In more specific terms, a number of projects reported on *better questioning* in the classroom and a number of others on *better sequencing* of lessons. A large number of the NCETM enquiries reported on specific changes in teaching practice, such as the use of concrete materials and equipment. Riley (2013), for example, reported on the teachers taking part in her project, saying "practical equipment is now being used far more effectively to support the learning and embedding of children's knowledge, skills and understanding of number systems". As a further example, Alldis and Whitney (2013) in their NCETM report quoted a teacher explaining how her teaching had changed: "I spend more time allowing pupils to explore their mathematics observing their discussions with peers and ensuring when pupils come to a solution that they must be able to 'convince' another member of the group and not simply accept an answer." Finally, there appeared to have been an impact on the way many teachers were now *considering their students' thinking*. Posthuma (2012), for example, reported that teachers in her project had learned the importance of attending to student thinking.

Data Directly from the Projects

As our survey progressed and revealed more and more papers, we were increasingly more aware that very much collaborative activity is not (or not yet) reported in published journals. We therefore decided to invite participants from known projects around the world to send us data in the form of narratives. Findings in this section are taken from the narratives we received. Crucially, these narratives highlight the authentic voice of the teachers. Examples include:

The East Africa Mathematics Education and Research Network, with office bearers from Kenya, Tanzania, Rwanda and Uganda, involved collaboration with local partners to support and enable teachers to meet and work together towards improvement of mathematics teaching. Teachers learned collaboratively on issues and topics of significance to them. Asked to identify what was the most significant aspect for them in the process of learning with and from teachers, participants noted the involvement of participants in tackling tricky mathematical topics; freedom in asking and answering questions without being ridiculed; and knowing how to prepare teaching aids for the topic, such as a protractor.

(Contributed by Anjum Halai)

The Focus on Primary Mathematics, a three-year initiative that included teachers from two government schools in Cape Town, which aimed to bring together the two schools to work collaboratively to improve teaching and learning in mathematics in Grades 1–3. Feedback from teachers on *what worked well* included:

- The mathematical learning of the children in their school was better. The children were more confident, enjoyed mathematics more and were achieving better results.
- The initiative had encouraged teachers to discuss, share and collaboratively plan mathematics teaching and through this they had begun to do the same in other subjects.
- They had become much more comfortable with sharing their own lack of mathematical knowledge and understanding and had learnt how to ask questions.
- Their own willingness to open up and be honest about their shortcomings in terms of mathematical understanding had encouraged their colleagues to do the same.

Teachers gave up time to participate in formal training programmes run during school holidays and regularly participated in professional development sessions (Contributed by Marie Joubert).

Two examples from Latin America, produced by teachers or researchers from Brazil and Argentina. The first was written by, Yuriko, Ana, Fabio and Roberta, four members of a collaborative group from Brazil. The Brazilian group was initiated in 2014 and was motivated by an innovative governmental programme spearheaded by a Regional Secretary of Education. Quotes from the participants include:

- *Yuriko (Researcher)* "The group started in 2014 with 12 teachers from 5 public schools … The innovative feature of the meetings is the collective planning of lessons … Then, the participants visit each other's classrooms to observe, to register in audio/video media [the class] …".
- *Ana (Pedagogical coordinator)*: "The theme of the study chosen by the group was Fractions … During the preparation of lessons, the participants realized the need for a didactical sequence with the manipulation of concrete material…".

- *Fabio (Teacher):* "I learned many conceptual interpretations of *Fraction*, the potential of manipulation of concrete material in the learning of my students. Seeing the video of my lessons was important. I learned how to ask questions."
- *Roberta (Teacher)*: "I think that there was not enough orientation about how to use the Learning Situations in SEE Material and to explore them to improve the classroom practices. Doing so is possible only with the learning community with the understanding brought by group discussions with colleagues and different visions of researchers …".

The second was written by María, a teacher who was part of a collaborative group in Argentina initiated in 2003 by three teachers and a researcher with the aim of producing mathematical modelling activities for the teacher's classroom.

- *Teacher María*: "… a collaborative group which brings together mathematics educators and teachers at the secondary level. Strange situation this meeting of people with different backgrounds respective to work and training that, as an experienced teacher, appeared as a novelty to me".

Finally, we present a few details about Lesson Study as it has developed in Japan and which has been emulated in different ways around the world as indicated above.

Japanese Lesson Study

Lesson study developed as a reproducible science of teaching with theory and practice. In Japan, National Level LS for Leading Curriculum Reform and Innovation of Teaching has developed since 1872. Origins of Lesson Study go back to the rapid, top down establishment of a modern education system in Japan. A search for innovative teaching methods led to the ideas of Pestalozzi, a Swiss educator, to foster a dialogical style of classroom communication. A national system of teacher-led professional development helped teachers to master this approach. LS is maturely established throughout Japan, and therefore central to teachers' practice and developmental activity. A limited number of sources, such as Isoda, Stephens, Ohara, and Miyakawa (2007) and Inprasitha, Isoda, Wang-Iverson, and Yeap (2015), report on Japanese LS.

Concluding Discussion

A Rich and Diverse Picture, but Lacking Teachers' Voice

The sources revealed by the survey provided a rich and diverse picture of the collaborative work involving mathematics teachers that is taking place around the world, albeit predominantly reported from the perspective of researchers. The inclusion of writings from non-academic sources, our narratives and reports,

provides evidence for widespread collaborative activity. However, overall we found a lack of the teachers' voice providing insights to teacher learning. We include here one quotation in which a teacher's voice was vividly expressed in relation to the voices of didacticians [mathematics education researchers] working with the teachers:

> At the beginning [of the project] I struggled, had a bit of a problem with this because then I thought very much about you [didacticians] coming and telling us how we should run mathematics teaching. That was what I thought, you are the great teachers … but now I see that my view has gradually changed because I see that you are participants in this as much as we are, even though you are the ones organising this … . And I think it's much better now, I feel much more comfortable, because now I feel that we are more equal than we were in the beginning, from my point of view (Bjuland & Jaworski, 2009, pp. 34–35).

Strengths and Weaknesses of Survey Methodology

Despite extensive work on this survey, locating and analysing relevant papers and narratives, we are aware that there are still limitations to our work.

- Although our search methodology revealed 316+ papers, we know we missed relevant papers in journals outside our range.
- Papers reviewed were usually not authored by teachers so the teachers' voice was largely missing.
- Our narratives came from projects with which we were familiar and so we missed many more opportunities to include the teacher voice than we accessed.

Questions for Further Research

As we analysed the papers revealed in our search and studied the narratives we received it became clear that the search and its analysis left many questions unaddressed, such as:

- What brings teachers into effective collaborative learning situations?
- How can the teachers' voice be heard in ways that matter?
- What do mathematics teacher educator researchers need to learn?
- How can research support teacher development, as well as recording it?

In recognition of limitations in the survey and the many questions still to address, members of the survey team intend to propose an ICMI study on the topic of the survey: *Mathematics teachers working and learning through collaboration.* If you are interested in participating in such a study, please watch out for an announcement.

References

Aldon, G., Arzarello, F., Cusi, A., Garuti, R., Martignone, F., Robutti, O., et al. (2013). The Meta-Didactical Transposition: a model for analysing teachers education programmes. In L. A. M. & A. Heinze (Eds.), *Proceedings of PME 37* (Vol. 1, pp. 97–124). Kiel, Germany: PME.

Alldis, J., & Whitney, H. (2013). Developing effective continuity in learning from KS1 to 4 in representing mathematically through collaborative research and development (NCETM CTP 4013). https://www.ncetm.org.uk/files/18384131/CTP4013+Final+Report.pdf

Berg, C. V. (2011). In-service teachers' professional development: Which systemic aspects are involved? *Research in Mathematics Education, 13*(2), 223–224.

Besamusca, A., & Drijvers, P. (2013). The impact of participation in a community of practice on teachers' professional development concerning the use of ICT in the classroom. In A. Lindmeier, & A. Heinze (Eds.), *37th Conference of the International Group for the Psychology of Mathematics Education, Kiel, Germany, 2013* (Vol. 2, pp. 81–88).

Bjuland, R., & Jaworski, B. (2009). Teachers' perspectives on collaboration with didacticians to create an inquiry community. *Research in Mathematics Education, 11*(1), 21–38. doi:10.1080/14794800902732209

Brodie, K., & Shalem, Y. (2011). Accountability conversations: Mathematics teachers' learning through challenge and solidarity. *Journal of Mathematics Teacher Education, 14*(6), 419–439. doi:10.1007/s10857-011-9178-8

Campbell, M. P. (2009). Mathematics teachers and professional learning communities: understanding professional development in collaborative settings. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Atlanta, Georgia*, (pp. 956–964).

Cavanagh, M., & McMaster, H. (2015). A professional experience learning community for secondary mathematics: Developing pre-service teachers' reflective practice. *Mathematics Education Research Journal, 27*(4), 471–490. doi:10.1007/s13394-015-0145-z

Chen, C.-H., & Chang, C.-Y. (2012). An exploration of mathematics teachers' discourse in a teacher professional learning community. In T.-Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, Taipei, Taiwan,* (Vol. 2, pp. 123–130).

Chevallard, Y. (1985). *La transposition didactique*. Grenoble: La Pensée Sauvage.

Dowling, D. (2013). Hungary for calculation: developing approaches to calculation in the new curriculum using Hungarian methodology as our inspiration (NCETM CTP4213). https://www.ncetm.org.uk/files/20365123/CTP4213+Final+Report.pdf

Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R.-L. Punamäki (Eds.), *Perspectives on activity theory* (pp. 19–38). Cambridge: Cambridge University Press.

Fried, M. N., & Amit, M. (2005). A Spiral Task as a Model for In-service Teacher Education. *Journal of Mathematics Teacher Education, 8*(5), 419–436. doi:10.1007/s10857-005-3850-9

Goodchild, S. (2008). A quest for 'good' research. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education (Vol 4) the mathematics teacher educator as a developing professional* (pp. 201–220). Netherlands: Sense Publishers.

Goos, M. (2013). Sociocultural perspectives in research on and with mathematics teachers: a zone theory approach. *ZDM Mathematics Education, 45,* 521–533.

Goos, M. E., & Bennison, A. (2008). Developing a communal identity as beginning teachers of mathematics: Emergence of an online community of practice. *Journal of Mathematics Teacher Education, 11*(1), 41–60. doi:10.1007/s10857-007-9061-9

Hospesovà, A., Machàckovà, J., & Tichà, M. (2006). Joint reflection as a way to cooperation between researchers and teachers. In *30th Conference of the International Group for the Psychology of Mathematics Education,* (Vol. 1, pp. 99–103).

Inprasitha, M., Isoda, M., Wang-Iverson, P., & Yeap, B. (Eds.). (2015). *Lesson study: Challenges in mathematics education (Vol. 3, series on mathematics education)*. Singapore: World Scientific.

Isoda, M., Stephens, M., Ohara, Y., & Miyakawa, T. (Eds.). (2007). *Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement*. Singapore: World Scientific.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187–211.

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development: Teachers and didacticians in collaboration. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 3, pp. 309–330). Rotterdam: Sense Publishers.

Krammer, K., Ratzka, N., Klieme, E., Lipowsky, F., Pauli, C., & Reusser, K. (2006). Learning with classroom videos: Conception and first results of an online teacher-training program. *ZDM, 38*(5), 422–432.

Kusanagi, K. N. (2014). The Bureaucratising of Lesson Study: A Javanese Case. *Mathematics Teacher Education and Development, 16,* 84–103.

Lin, F.-L., Chen, J.-C., Hsu, H.-Y., & Yang, K.-L. (2013). Elaborating stages of teacher growth in design-based professional development. *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 3).

Martins, C., & Santos, L. (2012). Development of reflection ability in continuous training in mathematics (PFCM). In T.-Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, 2012* (Vol. 3, pp. 193–200).

Menezes, L. (2011) Collaborative research as a strategy of professional development of teachers. In B. Ubuz (Ed.), *35th Conference of International Group for the Psychology of Mathematics Education, Ankara, Turkey, 2011* (Vol. 3, pp. 252–232). PME.

Pires, M. V., & Martins, C. (2009). *Olhares sobre um plano de formação contínua em Matemática.* Paper presented at the VI CIBEM or Ibero-American Congress on Mathematics Education, Puerto Monte, Chile,

Ponte, J. P., Segurado, I., & Oliveira, H. (2003). A collaborative project using narratives: What happens when pupils work on mathematical investigations? In A. Peter-Koop, V. Santos-Wagner, C. Breen, & A. Begg (Eds.), *Collaboration in teacher education: Examples from the context of mathematics education* (pp. 85–97). Dordrecht: Kluwer.

Posthuma, B. (2012). *Mathematics teachers' reflective practice within the context of adapted lesson study., 33,* 3. doi:10.4102/pythagoras.v33i3.140

Potari, D. (2013). The relationship of theory and practice in mathematics teacher professional development: An activity theory perspective. *ZDM Mathematics Education, 45*(4), 507–519. doi:10.1007/s11858-013-0498-2

Riley, S. (2013). Extending Teacher Pedagogy of Early Mathematics Development to ensure Arithmetic and Mathematical Proficiency across Key Stage 1 (NCETM CTP 2513). https://www.ncetm.org.uk/files/17095392/CTP2513+Final+Report.pdf

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., et al. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM, 48*(5), 651–690. doi:10.1007/s11858-016-0797-5

Sakonidis, C., & Potari, D. (2014). Mathematics teacher educators'/researchers' collaboration with teachers as a context for professional learning. *ZDM Mathematics Education, 46*(2), 293–304. doi:10.1007/s11858-014-0569-z

Takahashi, A. (2014). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development, 16*(1), 4–21.

Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development* (2nd ed.). New York: John Wiley & Sons.

Warren, E. (2008). Early childhood teachers' professional learning in early algebraic thinking: A model that supports new knowledge and pedagogy. *Mathematics Teacher Education and Development, 10*, (30–45).

Wenger, E. (1998). *Communities of practice. Learning, meaning and identity*. Cambridge: Cambridge University Press.

Wynne-Jones, G. (2013). Collaborative development of a calculation policy as an opportunity for teacher CPD (NCETM CTP 2713). https://www.ncetm.org.uk/files/17715281/CTP2713+Coppice+Primary+School+Final+report.pdf