

Energy-Efficient Power Control and Clustering in Underlay Device to Multi-device Communications

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Abstract. Underlay device to device multicast communication brings great benefits to cellular networks in term of energy and spectral efficiency, since a group of users can communicate directly by reusing cellular resources. In this paper, we propose a scheme to maximize network energy efficiency (EE) through transmission power control. In addition, to get a compromise between fairness and the overall EE, we analyse a second scheme with fairness factors to maximize minimum individual EE. The problem is formulated as a fractional programming optimization and solved with iterative algorithm. Two different EE metrics are analysed via extensive numerical simulations with a spatial Poisson process for the users' locations, and applying different clustering algorithms as K-nearest neighbour, distance limit and DBSCAN.

Keywords: Optimization · Fractional programming · D2D multicasting communication · 5G wireless networks

1 Introduction

The emergence of mobile multimedia rich services increases the number of connected devices and the requested data rate regularly. Consequently, the fifth generation (5G) supposes to have $1000\times$ higher data volume, up to $100\times$ more throughput and connected devices, $10\times$ longer battery life, and low latency [8]. Underlay device to device communication (D2D) is a new paradigm introduced to realize 5G. It allows users in proximity to communicate directly by using resource allocated to cellular user (CUE). This has advantages as [9]: high data rate, energy and spectral efficiency, increase network capacity, low delay. However, it can cause harmful co-channel interference to CUE. The main techniques to mitigate interference are power control and resource allocation. Currently, they are joint to propose a more EE solutions. Several works proposed EE power and resource allocation scheme subject to QoS. In [3], a centralized solution had been conducted to minimize transmission power. The problem was formulated as

mixed integer linear programming (MILP) and solved through Hungarian algorithm. While in [14], the authors proposed a distributed solution to maximize users EE. The problem was formulated as a non cooperative game and solved by an iterative optimization algorithm. However, these works mainly consider unicast D2D communication where single transmitter and receiver communicate. Another interesting approach is to allow users with common content interest to form clusters where one device transmits to multiple receivers. This is called multicast D2D communication (D2MD). Compared to unicast, this mode reduces overhead signals and saves resources yet it is more challenging hence data rate is determined according to the weakest receiver so non fails to decode any of the transmitted data. Most of the available works target interference issue in D2MD communication focus on system throughput maximization subject to QoS constraints. The work in [5], investigate power control for D2D multicast communication. The authors first proposed a distributed scheme for group formation. Then, they studied single and multiple CUE channels re-usage and developed power allocation scheme based on swarm algorithm (PSO). The authors in [10] proposed a heuristic algorithm to solve mixed integer non-linear programming (MINLP) resource and power allocation problem. Moreover, they extended their work in [11] where D2D clusters can reuse the channel of multiple CUE instead of reusing single orthogonal channels and several D2D groups can share the same channel. They proposed greedy and heuristic algorithms and compared their performance. To this end, the paramount importance of green communication concept and the high potential of D2MD [12] motivated us to analysis spectral and energy efficiency in the scenario where underlay D2MD is applied in a cellular network. In this paper, we propose joint power and resource allocation model to maximize system and users EE. However, for analytical purpose we focus on a single channel sharing which lead us to EE power control schemes powered with stochastic geometry and clustering techniques. The rest of the paper is organized as follows. In Sect. 2, we introduce system model. Power control algorithm and problem formulation is discussed in Sect. 3. Finally, we discuss simulation results in 4 and conclude in Sect. 5.

2 System Model

We consider a single cell environment with one base station (BS) located in the centre and multiple uniformly distributed users. On the uplink, M CUE transmit on M orthogonal communication channels. Reusing downlink resources requires sophisticated coordination between UEs and the BS, and has been shown to be less effective than uplink resource sharing. Therefore, we assume the coexistence of K D2MD clusters $k = 1, \dots, K$, that reuse the same channels allocated to CUEs to communicate directly. Each group has head cluster (transmitter) and $|\mathcal{D}_k|$ receivers where $|\mathcal{D}_k| = 1$ is a unicast case [11]. Here, the BS suffers from interference caused by the co-channel D2MD transmitters. Similarly, the $|\mathcal{D}_k|$ receivers of group k suffer from interference caused by the CUE and other transmitters of D2MD groups sharing the same resource block. The SINR $\gamma_{k,m,r}(\{\mathbf{p}_k\}_m, p_m)$ of a D2D receiver r in group k and channel m is given by

$$\frac{h_{k,m,r} p_{k,m}}{\sigma^2 + p_m \beta_{k,m,r} + \sum_{j \neq k} x_{j,m} p_{j,m} h_{j,m,r}} \tag{1}$$

where $h_{k,m,r}$ are the deterministic link gain factors from the transmitter in group k to receiver r on channel m , and $\beta_{k,m,r}$ is similarly the link gain factor from CUE transmitter m to receiver r in group k . Symbol $p_{k,m}$ denotes the transmission power of the D2MD transmitter in group k on channel m , and p_m is the transmission power of CUE user m . For a CUE user m , the SINR is similarly given by

$$\gamma_m(\mathbf{p}_k, p_m) = \frac{h_m p_m}{\sigma^2 + \sum_k x_{k,m} p_{k,m} h_{k,m}}, \quad m = 1, \dots, M \tag{2}$$

where h_m is the link gain from user m to the base station, p_m is the transmitted power, $h_{k,m}$ is the link gain from the transmitter in D2D group k to the cellular base station on channel m and $x_{k,m}$ are binary variables defined for $k = 1, \dots, K$, $m = 1, \dots, M$ as $x_{k,m} = 1$ if D2D group k uses channel m , and 0 otherwise.

The normalized transmission rate (in bits/s/Hz) of CUE m is the channel capacity $r_m = \log_2(1 + \gamma_m)$, for $m = 1, \dots, M$. For D2MD group k , the unique transmission rate is determined by the weakest receiver, i.e., by the receiver with the poorest channel quality. In addition, we account explicitly for the aggregated received rate in group k , which depends on the number of receivers per group $|\mathcal{D}_k|$. Thus,

$$R_k = \sum_{m=1}^M x_{k,m} |\mathcal{D}_k| \min_{r \in \mathcal{D}_k} \log_2(1 + \gamma_{k,m,r}), \quad k = 1, \dots, K. \tag{3}$$

3 Problem Formulation

The energy efficiency (EE) of a given user (in bits/Joule) can be defined as the ratio of the achievable transmission rate and the total consumed power:

$$\eta_m \triangleq \frac{r_m}{s_{c,m} + p_m}, \quad m = 1, \dots, M \tag{4}$$

for a CUE user; and

$$\zeta_k \triangleq \frac{R_k}{s'_{c,k} + \mathbf{p}_k}, \quad k = 1, \dots, K. \tag{5}$$

In (4) and (5), $s_{c,m}$ (respectively, $s'_{c,k}$) is the transmitter circuit power, p_m is the power used by CUE transmitter m , $\mathbf{p}_k = (p_{k,1}, \dots, p_{k,M}) \in \mathbb{R}_+^M$ is the allocated power vector of the head cluster over the M channels, and $\|\cdot\|_1$ denotes the ℓ_1 -norm.¹ We assume that p_m and \mathbf{p}_k satisfy individual power constraints $\mathbf{p}_{k1} \leq \bar{p}_k$

¹ For vectors $\mathbf{x} \in \mathbb{R}^n$, the ℓ_p -norm, for any $1 \leq p \leq \infty$, is $(\sum_{i=1}^n |x_i|^p)^{1/p}$. Hence, the ℓ_1 -norm is $\mathbf{x}_1 = \sum_{i=1}^n |x_i|$, ℓ_2 is the usual Euclidean norm, and $\mathbf{x}_\infty = \max_i |x_i|$.

for $k = 1, \dots, K$, and $p_m \leq P_m$ for all m . Moreover, we explicitly include the assumption that minimum transmission rates have to be satisfied both for the CUE and the D2MD users,

$$r_m \geq \underline{r}_m, \quad \text{and} \quad R_k \geq \underline{R}_k \quad (6)$$

for all indexes k, m , where \underline{r}_m (resp., \underline{R}_k) are the target rates of CUE (D2MD) user m (resp., k). The above definition for energy-efficiency is user-centric. The notation can be extended to present the *global network energy efficiency (GEE)* η by considering the ratio of the aggregated rate and the total consumed powers. More formally, if \mathbf{r} and \mathbf{R} are the vectors of rates for CUE and D2MD groups, respectively, then

$$\eta \triangleq \frac{\|\mathbf{r}\|_1 + \|\mathbf{R}\|_1}{s_c + \sum_k \|\mathbf{p}_k\|_1 + \|\mathbf{p}\|_1} \quad (7)$$

Here, s_c is the total circuit power network devices. The GEE targets the total performance of the cellular network EE. This does not depend on users devices which might not be fair for limited battery life devices. For this reason, we introduce a second performance metric for maximizing the *minimum EE* among all the users in the system. The following definition captures this idea and introduces the notion of fairness between users.

Definition 1. Let $\boldsymbol{\omega} = (\omega_1, \dots, \omega_M) \in \mathbb{R}_+^M$ and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_K) \in \mathbb{R}_+^K$ be two arbitrary weight vectors. The $(\boldsymbol{\omega}, \boldsymbol{\theta})$ -weighted energy efficiency (WEE) is

$$\psi \triangleq \min \left\{ \min_m \omega_m \eta_m, \min_k \theta_k \zeta_k \right\}. \quad (8)$$

The uniform choice $\boldsymbol{\omega}_2 = \boldsymbol{\theta}_2 = 1$, gives equal weight to every user, yields max-min fairness as the optimization criterion. With the above setting, the maximization of the global network energy efficiency can be mathematically formulated as (*problem GEE*): solve $\max_{\mathbf{p}, \mathbf{x}_k \in \mathcal{P}} \eta$, where η is equal to

$$\frac{\sum_m \log_2(1 + \gamma_m) + \sum_k \sum_m x_{k,m} |\mathcal{D}_k| \log_2(1 + \min_r \gamma_{k,m,r})}{s_c + \sum_k \|\mathbf{p}_k\|_1 + \|\mathbf{p}\|_1} \quad (9)$$

where \mathcal{P} is the feasible set of power vectors; namely, the set defined by the constraints

$$\|\mathbf{p}_k\|_1 \leq \bar{p}_k, \quad k = 1, \dots, K; \quad p_m \leq P_m, \quad m = 1, \dots, M \quad (10)$$

$$\underline{R}_k \leq R_k, \quad k = 1, \dots, K; \quad \underline{r}_m \leq r_m, \quad m = 1, \dots, m \quad (11)$$

$$\|\mathbf{x}_k\|_1 \leq s \quad \|\mathbf{x}_{\cdot, m}\| \leq r \quad (12)$$

$$\mathbf{x}_k \in \{0, 1\}^M \quad (\mathbf{p}, \mathbf{p}_1, \dots, \mathbf{p}_K) \in \mathbb{R}_+^{(K+1) \times M} \quad (13)$$

Here, constraints (10) bound the maximum transmission power per user; constraints (11) are the minimum rate conditions, where the rates have been defined previously; constraints (12) enforce the maximum split factor s for every D2MD group and the maximum reuse factor r per subcarrier; and finally (13)

is simply the nonnegativity of all the power vectors. Clearly, (12) are the integer constraints, (11) are the coupling constraints on the transmission powers, and the set $\{\mathbf{p}_k\}$ are the coupling variables of the problem. Finally, note that the QoS constraints (11) are *user-specific*. In the same fashion and subject to the same constraints (10)–(13), the *minimum-EE* optimization problem can be formulated, for fixed weight vectors, as: solve $\max_{\mathbf{p} \in \mathcal{P}} \psi$.

To better understand the influence of D2MD communication on the network performance, we analyze in depth a special case when $M = 1$ or $x_{k,1} = 1$ for all k . This means a single CUE channel is shared among all uses D2D groups. This will give us an idea about maximum capacity of the channel and allows us to clearly see the impact of other system parameters. In such a case the problem turns to a power control problem lies under the fractional optimization class (14) where the decision part is absent, and has canonical form

$$\max_{\mathbf{x} \in \mathcal{C}} \frac{f(\mathbf{x})}{g(\mathbf{x})}. \quad (14)$$

Here, closed-form sufficient and necessary conditions for feasibility can be given, a result adapted from [13]. For single channel case, users are re-indexed such as 0 is the CUE, $|D_0| = 1$ is the BS and $j = 1, \dots, K$ are the D2D transmitters.

Theorem 1. Define $\gamma_k = 2^{r_k} - 1$ for $k = 0, \dots, K$, and the matrix

$$W = [W]_{k,j} \triangleq \begin{cases} 0, & j = k \\ \frac{h_{k,j} \gamma_k}{h_k}, & j \neq k. \end{cases} \quad (15)$$

A solution exists iff the spectral radius of W is less than one and $(I - W)^{-1} \underline{\mathbf{s}} \leq \bar{\mathbf{p}}$ where $\bar{\mathbf{p}} = [\bar{p}_0, \dots, \bar{p}_K]^T$ and $\underline{\mathbf{s}}$ has elements $s_j = \sigma^2 \gamma_j / h_j$.

If the problem is feasible then it can be solved by finding the unique zero of $F(\lambda)$ where a point $\mathbf{x} \in \mathcal{C}$ solves (14) if and only if $\mathbf{x}^* = \arg \max_{\mathbf{x} \in \mathcal{C}} \{f(\mathbf{x}^*) - \lambda^* g(\mathbf{x}^*)\}$, with λ^* being the unique zero of $F(\lambda) = \max_{\mathbf{x} \in \mathcal{C}} \{f(\mathbf{x}) - \lambda g(\mathbf{x})\}$. To do this, we use Dinkelbach's algorithm [4] (presented in Algorithm 2 jointly for GEE and MEE) to solve one convex problem in each iteration, where $f(\mathbf{x})$ and $g(\mathbf{x})$ are concave and convex respectively. However, in our case the feasible set is convex yet the numerator of (9) is non-concave thus we use a sequential convex programming approach given by $\log_2(1 + \gamma) \geq a \log_2 \gamma + b$, where

$$a = \frac{\bar{\gamma}}{1 + \bar{\gamma}}, \quad b = \log_2(1 + \bar{\gamma}) - \frac{\bar{\gamma}}{1 + \bar{\gamma}} \log_2 \bar{\gamma} \quad (16)$$

Putting all pieces together, we resume the solving procedure in Algorithm 1.

4 Simulations and Results

Stochastic geometry models have been commonly used to map the network into a collection of points over a spatial area so that its actual performance (spectral

Algorithm 1. EE maximization for $M = 1$

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1: if Problem feasible (Theorem 1) then
2:    $i = 0$ . Pick any  $\mathbf{p}^{(0)} \in \mathcal{P}$ . If  $\mathbf{p}^{(0)}$  is feasible then
3:   repeat
4:      $i = i + 1$ 
5:     Maximize (9) or (8) with  $a_k^{(i)}$  and  $b_k^{(i)}$ 
6:     Set  $p_k^{(i)} = 2^{q_k^{(i)}}$ , where  $q_k^{(i)} = \arg \max \tilde{\psi}_i$ 
7:     Set  $\tilde{\gamma}_k^{(i)} = \gamma_k(\mathbf{p}^{(i)})$ 
8:     Update  $a_k^{(i)}$  and  $b_k^{(i)}$  with (16)
9:   until convergence
10: end if

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Algorithm 2. Generalized Dinkelbach's algorithm. The case $I = 1$ is the simple Dinkelbach's algorithm.

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1:  $\epsilon > 0, \lambda = 0$ 
2: repeat
3:    $\mathbf{x}^* = \arg \max_{\mathbf{x} \in C} \min_{1 \leq i \leq I} \{f_i(\mathbf{x}) - \lambda g_i(\mathbf{x})\}$ 
4:    $F = \min_i f_i(\mathbf{x}^*) - \lambda g_i(\mathbf{x}^*)$ 
5:    $\lambda = \min_i f_i(\mathbf{x}^*) / g_i(\mathbf{x}^*)$ 
6: until  $F \leq \epsilon$ 

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efficiency, coverage, etc.) can be analyzed and characterized [1, 2]. Here, we use a standard homogeneous Poisson point process (PPP) distribution to determine the number and locations of the cellular users and of the D2MD transmitters and receivers, with density λ . The received signal or interference power is assumed to vary due to the path loss resulting from the random spatial distribution. Specifically, the channel quality between a transmitter at $y \in \mathbb{R}^2$ is $P_r = P_t \cdot (1 + |y/d_0|^\alpha)$ where P_r is the received power, P_t is the transmitted power, d_0 is a reference distance (100 m in our case), α is the path loss exponent. Also, we chose the CUE with the best channel quality to share his RB with D2D groups [7]. This will allow us to focus on the effect of D2MD communications without

Table 1. System parameters.

Parameter	Values	Parameter	Values
Bandwidth	100 MHz	Max. tx. power	$[-15, 15]$ dBm
Cell size	500 m	Noise power density N_0	-100 dBm/Hz
Number of D2D groups	[1, 11]	Number of iterations	200
Network density (λ)	[50, 250]	Circuit power	10 dBm
Minimum tx. rate	[0.1, 0.4] bit/Hz/s	Path loss exponent	2.5

considering the case of shared channel quality. Concerning the D2MD groups, the head cluster and group formation were done following three different algorithms presented and discussed in bellow. The parameters used for the numerical results reported here are listed in Table 1 and were taken or inspired by similar works [5, 13, 14].

Table 2. Global and minimum EE, rate in KNN (left) and DL (right).

Min rate	GEE	Avg rate	MEE	Min rate	GEE	Avg rate	MEE	Min rate
0.1	226.280	14.538	133.876	1.472	593.807	37.790	207.066	2.277
0.2	222.999	14.352	139.819	1.538	587.773	37.457	209.762	2.307
0.3	218.606	14.093	133.746	1.471	576.695	36.764	211.694	2.328
0.4	216.362	13.980	132.662	1.458	559.525	35.735	204.938	2.254

4.1 *K*-Nearest Neighbour Clustering (KNN)

This is the first algorithm used to classify users into K disjoint groups. The K head clusters are randomly selected among \mathcal{S} , the set of points randomly drawn from a homogeneous Poisson point process with density λ . The remaining users/points in \mathcal{S} are then assigned to the closest group head. Finally, only the groups that reach the target size $|\mathcal{D}_k|$ are retained.

Feasibility was tested for variates of $\lambda = 50, 300$ with 50 as step. The constraints were set to 0.2 b/s/Hz as minimum rate and 0 dBm transmission power for 5 groups of size 2 sharing CUE channel averaged over 200 feasible cases. Clearly, a solution exist for low λ values as 50 users and a relatively large average distance from the head cluster (approx., 120 m) and the weakest receiver, which is naturally decreasing down to 40 m as the density increases up to 300. The percentage of feasibility was high hence 0 infeasible case appeared during test cases. The problem also continuous to be feasible as we increase number of groups per channel yet that has opposite affect on EE. Increasing number of users from 1 up to 11 EE decrease from 303.3774 down to 144.2761 while rate increase from 6.6283 up to 53.5086. The main reason behind low EE is that the average consumed power increase to satisfy minimum rate constraint under high level of interference.

Sum-rate Capacity and EE were analysed while considering a wide range of available transmission power budget $[-15, 15]$ dBm. The minimum rate were fixed to 0.2 b/s/Hz with 5 D2MD per channel of size 2. Figure 1 show global and maximum minimum rate respectively for different λ . Obviously, aggregate and minimum rate increases logarithmically as maximum transmission power increase to saturates to a maximum value hence the aggregate interference level prevents further improvement in data rate. This behaviour is confirmed by global and maximum minimum EE as illustrated in Fig. 2.

Rate Constraints were tested with range of $[0.1, 0.4]$ b/s/Hz. The number of groups per channel was set to 5 of size 2 and maximum transmission power 0 dBm averaged over 200 feasible cases. The results are shown in Table 2. Clearly, GEE and global rate as well as MEE and minimum rate decrease as minimum rate level increase. This is a clear indication that setting a higher value for the minimum transmission rates forces the devices to use proportionally higher power in order to satisfy the constraint. But, since the rate increases only logarithmically with the SINR in the low- or moderate SINR region, these increased rates do not compensate the extra energy expenditure.

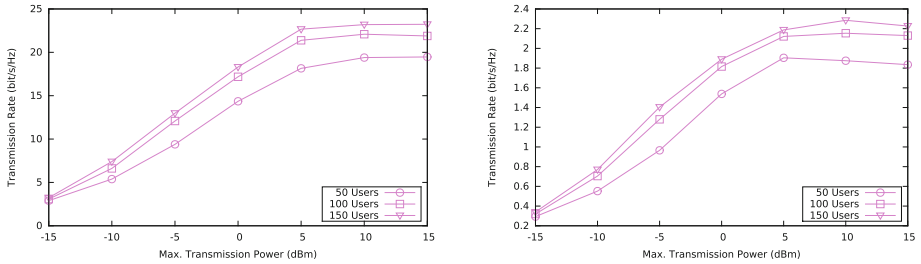


Fig. 1. Global and maximum minimum rate vs. transmission power in KNN.

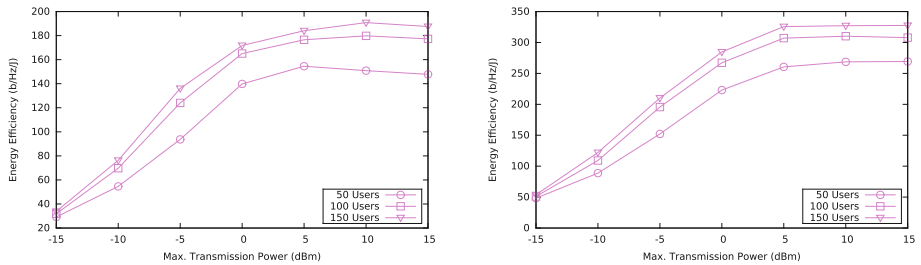


Fig. 2. Global and maximum minimum EE vs. transmission power in KNN.

4.2 Distance Limited (DL)

DL algorithm follows a similar concept to KNN in that we specify the number K of clusters to be formed in advance. Here, we introduce a new parameter called (d_{max}) which is the maximum allowed distance between a transmitter and a receiver, specified as a fraction of the cell radius. This opens the door to form heterogeneous groups with unicast and multicast communication simultaneously.

Feasibility is analysed with different λ and d_{max} values. The minimum rate is set to 0.2 bit/s/Hz and maximum allowed transmission power is 0 dBm with 5 groups. Taking an example $\lambda = 250$ and d_{max} ratios is [1/3, 1/10] average group size goes from 20 down to 2 with distance between 160 m and 40 m. Clearly, having bigger group did not affect problem feasibility as we still have 0 infeasible cases in total. The problem also continuous to be feasible as we increase number of groups per channel yet that has opposite affect on EE. Increasing number of users from 1 up to 11 EE decrease from 591.4391 down to 424.2723 while rate increase from 12.6849 up to 53.5086. The main reason behind low EE is that the average consumed power increase to satisfy minimum rate constraint under high level of interference.

Sum-rate Capacity and EE was tested in DL using similar constraints values as KNN. We set $\lambda = 150, 200$ and 250 to ensure multicast cast communication in minimum. Vales of d_{max} ratio were 1/5, 1/6 and 1/7 respectively with average group size 3. Figure 3 show global and minimum rate respectively for different λ . Obviously, we have similar behaviour as in KNN as aggregate rate or minimum rate increase logarithmically with maximum transmission up to saturates point hence the aggregate interference level prevents further improvement in data rate. This behaviour is confirmed by global and maximum minimum EE as illustrated in Fig. 4. Compared to KNN, higher values were attained here hence we have larger groups. We notice also small difference between values among various λ due to d_{max} control at this level.

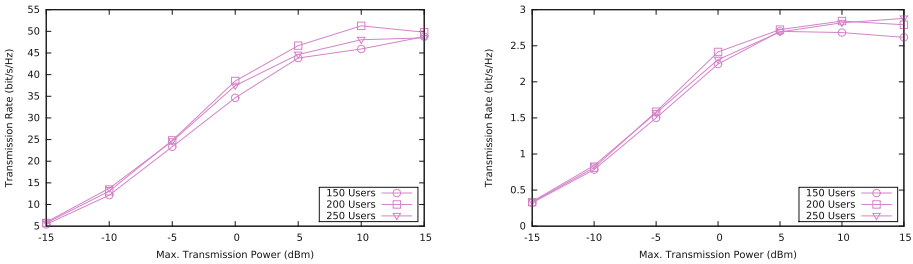


Fig. 3. Global and maximum minimum rate vs. transmission power in DL.

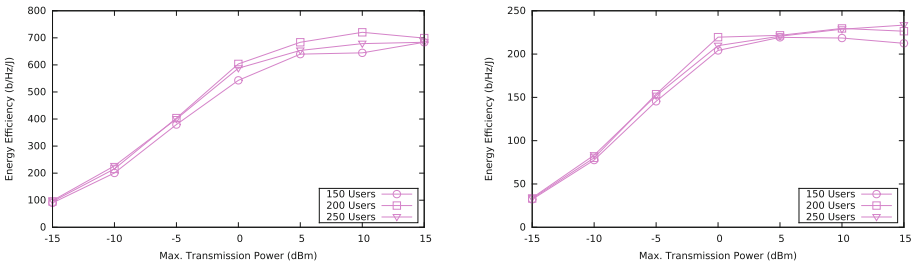


Fig. 4. Global and maximum minimum EE vs. transmission power in DL.

Rate Constraints were testes with $\lambda = 250$ and d_{max} ratio is $1/8$. The maximum transmission power was fixed to 0 dBm and minimum rate is in the range of $[0.1, 0.4]$ b/s/Hz with a step 0.1. The results are shown in Table 2. Again, we notice similar but higher values of GEE, global rate, MEE and minimum rate as in KNN. This confirm that setting a higher value for the minimum transmission rates forces the devices to use higher power to satisfy the constraint.

4.3 Density-Based Spatial Clustering of Applications with Noise

The DBSCAN algorithm [6] is used to identify dense zones by taking a reference point and searching over its neighbourhood within a limited distance d_{max} . If the density of points in this area is above a threshold, the area is identified as a potential group. The same process is applied to neighbours until all points are checked. If some point is not a neighbour of a group, it is regarded as a noise. In the original version of DBSCAN, a minimum number of neighbours $|\mathcal{D}_k|$ is identified to specify the core, border and noise points. Here, we follow a similar process (but varying d_{max}) and declare the core point of the discovered groups as the cluster heads.

Table 3. Global and minimum EE, rate in DBSCAN.

Min rate	GEE	Avg rate	MEE	Min rate
0.1	261.4748	53.2012	89.8905	0.9886
0.2	252.3485	51.0620	83.3020	0.9152
0.3	233.2863	47.0388	79.7355	0.8715
0.4	214.7058	44.1950	67.0430	0.7149

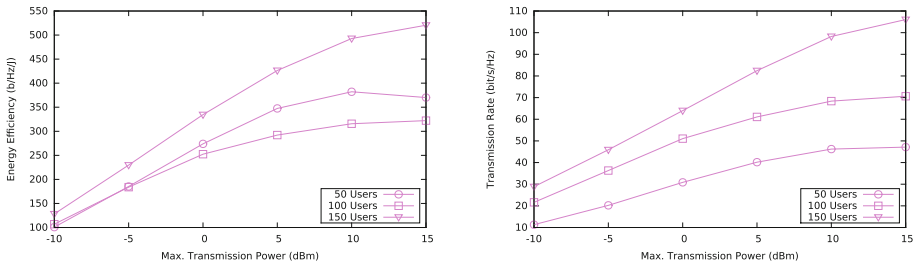


Fig. 5. Global rate and EE vs. transmission power in DBSCAN.

Feasibility. As in the previous cases, we start the performance analysis by assessing problem feasibility for some typical densities (here, $\lambda = 50, 100, 150$) and several d_{max} ratios $\{1/3, 1/19\}$. The minimum rate is set to 0.2 bit/s/Hz

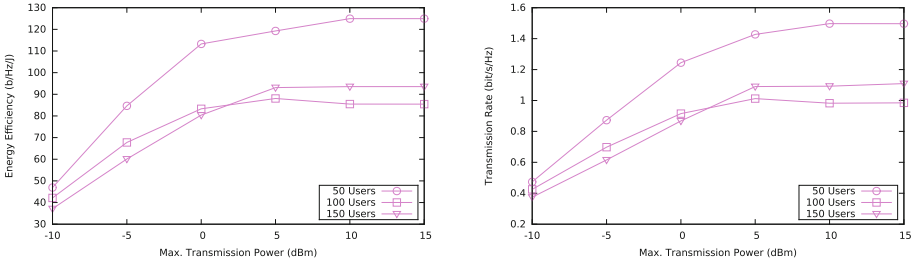


Fig. 6. Minimum rate and EE vs. transmission power in DBSCAN.

and maximum allowed transmission power is 0 dBm. In contrast to the previous clustering algorithm, DBSCAN allows us either to have many groups with moderated sizes, or few groups with large sizes. The unbalance (in the size of its groups) appears due to the indirect control of points number per given dense area. This fact explains why the number of infeasible cases is significantly larger than with the other clustering techniques (e.g., a maximum of 831 not solvable cases were found). For instance, for $\lambda = 100$ and ratio of 1/4, we had 2 D2MD groups of size 52 and a maximum distance inside a cluster of 350 m. Reducing d_{max} to 1/7 yielded more reasonable (and feasible) configurations, with 18 D2MD groups of size 3, a diameter equal to 71 m and without infeasible instances. As the transmission power budget decrease, some infeasible cases appear with -5 dBm and -10 dBm whereas -15 dBm generates extremely low feasibility percentage.

Sum-rate Capacity and EE. Both measures have been tested in DBSCAN using the usual parameters for the power and rate constraints. We tried $\lambda = 50, 100, 150$ to study the effect of density on this algorithm. For the d_{max} ratio, we chose 1/5 and 1/7, with 9 and 18 as the average number of groups and 2, 3 and 6 receivers per group, respectively. Figure 5 shows the global EE and aggregated rate, respectively, for different λ . We notice a similar performance to the results by the previous algorithms for the aggregate and individual rates. Similarly, Fig. 6 illustrates the minimum rate and EE. As proven previously, each channel has limited users capacity such as EE and minimum rate decrease while average rate increase. This can be seen clearly as low λ values as higher MEE and minimum rate are.

Rate Constraints. We repeated the numerical computations for assessing how a stricter rate constraint could change the fundamental trade-off between rate and energy efficiency. To this end, we pick $\lambda = 100$ and a configuration by design with 18 groups and 3 receivers per group. Now, the GEE decreases with the minimum acceptable rate. The same pattern happens for MEE, as detailed by Table 3.

Discussion. The EE and rate curves have similar shape in all the clustering algorithms. This means that the clustering technique is not determinant to the fundamental performance of this type of systems, if one ignores the practical issues of use a particular clustering technique in a real network, and with

a distributed implementation. Nevertheless, the performance can be enhanced under certain conditions. The KNN algorithm is highly sensitive to the users density, λ . Its performance improves as λ increases, and this means a shorter distance between head cluster and the receivers. Clearly, since this distance is explicitly controlled in DL, the performance with DL is only slightly depending on the density. In addition, the flexibility of DL concept makes possible to have heterogeneous groups in area and number of UEs, and particularly the coexistence of unicast and multicast groups. Also, DL clustering offers higher EE and transmission rates, again as a immediate consequence of the closeness between the cluster head and a typical receiver in its group, shorter than the average distance observed with other clustering algorithms. The feasibility test shows that the problem can be solved for a wider distance, yet the fraction of feasible cases is largely influenced by the power budget.

5 Conclusion

In this paper, we proposed a general model to study global and users individual EE in D2MD. The problem is formulated as a joint power and resource allocation, yet for analytical tractability we limited the number of shared channel to one. Then, optimal power control was investigated. We proposed a combined framework of optimization and stochastic geometry which allow us to assess a number of performance measures as D2D coverage, global and users EE and rate, or the capacity of a RB. The clustering algorithms do not have significant impact on EE or rate, yet can be used to slightly improve the fundamental performance. Also, under certain conditions a single resource block has a limited capacity where as more users sharing the channel as the EE decrease.

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