Chapter 5
Cosmic Ray Particle Transport in the Earth's Magnetosphere

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Abstract The transport of the cosmic ray particles in the Earth's magnetic field must be considered for cosmic ray investigations based on cosmic ray measurements in the geomagnetosphere. The motion of charged particles in a magnetic field is defined by the Lorentz force. The trajectories of cosmic ray particles are curved by the Earth's magnetic field. In a first approximation the geomagnetic field can be described by a dipole magnetic field. For a more accurate description the geomagnetic magnetic field is divided into two parts: the inner part generated by an internal dynamo and the outer part induced by different current systems in the ionosphere and the magnetosphere. Models have been developed that describe the inner and the outer magnetic field. The computations of the propagation of the cosmic ray particles in the Earth's magnetosphere are made with computer programs based on numerical integration of the equation of motion. For the specification of geomagnetic effects on cosmic ray particles the concept of cutoff rigidities and of asymptotic directions have been introduced.

5.1 Introduction

Charged particles moving in a magnetic field are deflected. Investigations of cosmic ray observations based on ground-based and on space-based detectors inside the geomagnetosphere require therefore a detailed knowledge of the propagation of the cosmic ray particles in the Earth's magnetic field. The conditions under which a cosmic ray particle has access to a specific point of observation are defined by the Earth's magnetic field, the energy of the particle as well as the direction of incidence.

The implementation of the quantity "magnetic rigidity" is useful as particles with the same rigidity R, charge sign and initial conditions have identical trajectories in a

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static magnetic field. The rigidity R is defined as $R = \frac{pc}{Ze}$, where p is the momentum, c is the speed of light and Ze is the charge of the cosmic ray particle. The unit of the rigidity R is volts. A convenient unit is GV (10^9 V).
History:

~1st cent. AD	First known magnetic compass was invented in China
13th cent. AD	First theories about geomagnetism
In early 1890s	Hendrik Lorentz derived the equation that describes the forces on a charged particle in an electromagnetic field, the so-called Lorentz force
1912	Discovery of the cosmic ray by the Austrian Victor Hess
1930	First works were started by Störmer to understand the geomagnetic effects on cosmic rays (Störmer 1930)
1957/58	International Geophysical Year Systematic investigations of the effects of the Earth's magnetic field on the cosmic rays as observed on the ground

For a more detailed historical overview, see e.g. Smart et al. (2000).
The forces that act on moving charged particles in a electromagnetic field and their effects on the particles' trajectories are described in Sect. 5.2. The Earth's magnetic field, i.e. the inner and the outer part of the Earth's magnetic field as well as the models describing these fields, is addressed in Sect. 5.3. In Sect. 5.4 the differential equations that describe the motion of the charged cosmic ray particles in the Earth's magnetic field and their numerical computation are summarised. The concepts of "cutoff rigidities" and of "asymptotic directions" that have been introduced to quantify the geomagnetic field effect are described in Sect. 5.5.

5.2 Motion of Charged Particles in a Magnetic Field: Lorentz Force

The combination of electric and magnetic forces on a charged particle due to electromagnetic fields is described by the Lorentz force **F**:

$$\mathbf{F} = Ze\,\mathbf{E} + Ze\,[\mathbf{v} \times \mathbf{B}] \tag{5.1}$$

where

- Ze charge of the moving particle, e is the elementary charge
- **E** electric field
- **v** velocity of the particle
- **B** magnetic field

The effect of electric fields can be neglected in the geomagnetosphere due to its high electric conductivity.

The force equation becomes:

$$\mathbf{F} = m \cdot \frac{d\mathbf{v}}{dt} = Ze\,[\mathbf{v} \times \mathbf{B}] \tag{5.2}$$

A charged particle is accelerated perpendicularly to the speed **v**, it follows that the absolute value of the momentum $|m \cdot \mathbf{v}|$ and its kinetic energy are conserved.
For relativistic particles with mass $m = \gamma m_0$, where γ is the Lorentz factor ($\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$) and m_0 is the rest mass of the particle, it follows:

$$\frac{d\mathbf{v}}{dt} = -\frac{Ze}{\gamma m_0}\,[\mathbf{v} \times \mathbf{B}] \tag{5.3}$$

For a moving charged particle in a uniform magnetic field, the speed vector **v** can be split into a component parallel $v_\parallel$ and perpendicular $v_\perp$ to the magnetic field **B**. The motion of the particle is then described by a movement with constant speed along the magnetic field $v_\parallel$ and a circular motion around the magnetic field lines.
The centripetal acceleration is

$$\frac{{v_\perp}^2}{r_c} = \frac{Ze}{\gamma m_0} \cdot v_\perp \cdot |\mathbf{B}| \tag{5.4}$$

where r_c is the cyclotron radius or gyroradius.
For the cyclotron radius r_c follows:

$$r_c = \frac{\gamma m_0 \cdot v_\perp}{Ze \cdot |\mathbf{B}|} \tag{5.5}$$

The above formula for the cyclotron radius can be rearranged to give a more practical expression for an estimation of the cosmic ray trajectory characteristic:

$$r_c\,[\text{meter}] = 3.3 \times \frac{(\gamma m_0 c^2\,[\text{GeV}]) \cdot (v_\perp / c)}{(Z \cdot |\mathbf{B}|\,[\text{Tesla}])} \tag{5.6}$$

where GeV is the unit of Giga-electronVolts.

A proton with kinetic energy $E_{kin} = 10\,\text{GeV}$ in the Earth's magnetic field close to the Earth where $|\mathbf{B}| \approx 30{,}000\,\text{nT}$ (Sect. 5.3.1) and $\mathbf{v} \perp \mathbf{B}$, i.e. $v_\perp = 0.99\,c$, has a gyroradius in the order of 10^6 m or 0.15 Earth radii.

5.3 Earth's Magnetic Field

The geomagnetosphere is the region close to the Earth where the motion of charged particles is mainly determined by the Earth's magnetic field. The size, the shape and the inner structure of the geomagnetosphere is configured by the interaction of the solar wind with the Earth's magnetic field. The extension of the geomagnetosphere in space is therefore determined by the equilibrium between the pressure of the streaming solar wind plasma and the magnetic pressure of the magnetosphere. The front end of the magnetopause is at a standoff distance of 10–12 Earth's radii (Re) from the Earth's centre during quiescent solar wind conditions. The magnetotail has a length of at least 100 Re. During times of disturbed solar wind conditions the characteristics of the geomagnetosphere are changed and are defined by the solar wind speed, the particle density in the solar wind, the strength and the direction of the interplanetary magnetic field. In addition, the relative position of the magnetic dipole inside the Earth defines the characteristics of the geomagnetic field. In the models of the inner Earth's magnetic field these effects must be considered.

The magnetosheath, the space between the magnetopause and the bow shock, is the consequence of the fact that the solar wind can not penetrate the Earth's magnetic field because of its high electric conductivity and that therefore the magnetic field of the solar wind must be swept around the magnetopause. The magnetic field strength in the magnetosheath may be in the order of a few 10 nT (Kobel and Fluckiger 1994). The geomagnetic field line topology in the geomagnetosphere is illustrated in Fig. 5.1.

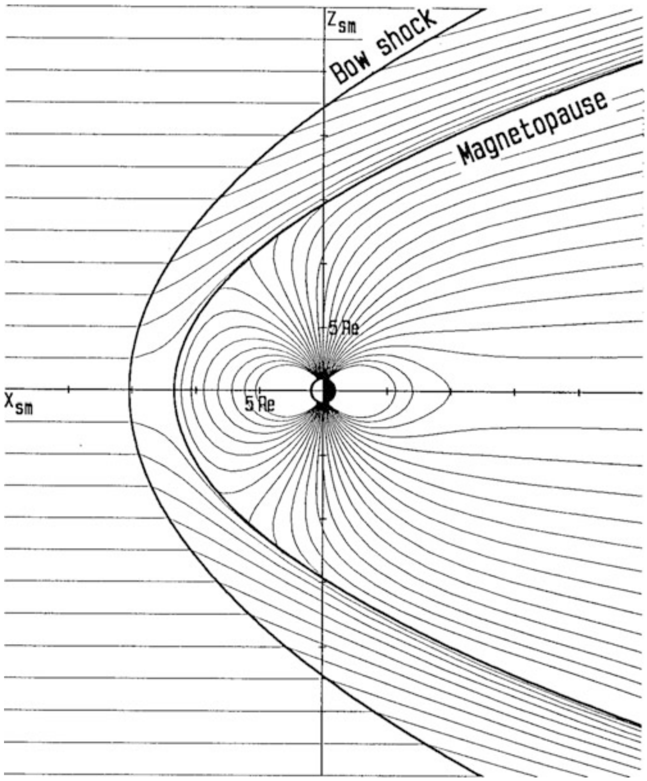

Fig. 5.1 Illustration of the Earth's magnetic field line topology. From Bütikofer et al. (1995)

Close to the Earth the magnetic field can be described in a first approximation by a geocentric dipole (Sect. 5.3.1). In a more accurate description the Earth's magnetic field is divided into an inner and an outer part. The inner magnetic field is produced by sources in the interior of the Earth. It is described by the IGRF model which is based on magnetic field measurements at the Earth's surface (Sect. 5.3.2). The external magnetic field is produced by the different electric current systems in the ionosphere and the inner magnetosphere. There exist different field models that describe the external Earth's magnetic field. The models by Tsyganenko—the models that are mainly used for cosmic ray trajectory computations within the geomagnetosphere—are presented in Sect. 5.3.4.

5.3.1 The Magnetic Field of the Earth as a Dipole Field

In a first approximation the Earth's magnetic field can be described by a geocentric magnetic dipole.

The magnetic moment **m** of a circular current is:

$$|\mathbf{m}| = I \cdot S \tag{5.7}$$

where

I electric current
S area that is spanned by the circular current

For the Earth dipole: $|\mathbf{m}| = 8.1 \cdot 10^{22}\ \mathrm{A\,m^2}$
The geomagnetic moment is defined as:

$$\mathbf{M} = \frac{\mu_0}{4\pi}\,\mathbf{m} \tag{5.8}$$

where the vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7}$ Vs / Am

$$\begin{aligned}|\mathbf{M}| &= 8.1 \cdot 10^{25}\ \text{Gauss cm}^3\\ &= 8.1 \cdot 10^{15}\ \text{V s m}\end{aligned}$$

In polar coordinates the components of the magnetic field $\mathbf{B}(r, \lambda, \varphi)$ are:

$$\begin{aligned}B_r &= -\frac{2\,|\mathbf{M}|\,\sin\lambda}{r^3}\\ B_\lambda &= -\frac{|\mathbf{M}|\,\cos\lambda}{r^3}\\ B_\varphi &= 0\end{aligned} \tag{5.9}$$

where

r distance from Earth's center
λ geomagnetic latitude
φ geomagnetic longitude

Magnetic field strength at the geomagnetic equator: $B_\lambda = \frac{M}{r^3}$, $B_r = 0$, i.e. at Earth's surface ($r = 6.4 \cdot 10^6$ m) $B_\lambda \approx 30{,}000$ nT.
Magnetic field strength at the poles: $B_\lambda = 0$, $B_r = \frac{2M}{r^3}$, i.e. at Earth's surface $B_r \approx 60{,}000$ nT.
The magnetic strength as function of λ and r is:

$$|\mathbf{B}| = \frac{|\mathbf{M}|}{r^3}\sqrt{1 + 3\sin\lambda^2} \sim \frac{1}{r^3} \tag{5.10}$$

Units:

$$1\ \text{Gauss}\,(G) = 10^{-4}\ \text{V s/m}^2 = 10^{-4}\ \text{Tesla (T)}$$
$$1\ \text{Gamma}\,(\gamma) = 1\ \text{nT}$$

5.3.2 *Magnetic Field Model Due to Internal Sources: IGRF*

The main geomagnetic field is generated primarily by a hydrodynamic geodynamo in the Earth's fluid outer core which varies slowly with time. The geodynamo has an underlying offset dipolar configuration which is currently tilted at an angle of about 10° with respect to the Earth's rotational axis. The time dependence in the magnetic field models is usually approached by a sequence of static configurations. For a static magnetic field the equations of the magnetostatics must be fulfilled as a special case of the Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \mu_0\,\mathbf{j} \qquad \mathbf{j}\text{: current density}$$

There are no currents to the center of the Earth:

$$\mathbf{j} = 0,\ \text{i.e.}\ \nabla \times \mathbf{B} = 0$$

From this it follows that there exists a potential V, so that

$$\mathbf{B} = -\nabla V$$

Since the magnetic field **B** is divergence-free (Gauss's law for magnetism) it follows for $r \geq$ Re:

$$\nabla^2 V = 0 \text{ or } \Delta V = 0 \quad \text{(Laplace's equation)}$$

Assumption for the solution of the Laplace equation: only sources from the Earth's interior are considered.

The geomagnetic main field is usually described by spherical harmonics (Chapman and Bartels 1940). Here, the representation of the geomagnetic main field with spherical harmonics according to the method by Gauss with the Schmidt normalization (Chapman and Bartels 1940) is used. In geographic spherical coordinates (r, θ, ϕ) the corresponding geomagnetic potential V can be expressed:

$$V(r, \theta, \phi) = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} (\frac{a}{r})^{n+1} \cdot P_n^m(\cos\theta) \cdot \{g_n^m \cos(m\phi) + h_n^m \sin(m\phi)\} \quad (5.11)$$

where

a	mean Earth's radius, $a = 6371.2$ km
P_n^m	Schmidt normalized (Chapman and Bartels 1940) associated Legendre functions of degree n and of order m
g_n^m, h_n^m	Gauss coefficients

As a consequence of the secular variations in the geomagnetic field, the Gauss coefficients must be determined periodically. The variation of the declination is ∼0.13°/year, the westward drift of the non-dipolar terms has a time period of ∼2000 years, and the change in the dipole moment a period of some 1000 years (Merrill and McElhinny 1983). The International Union of Geodesy and Geophysics (IUGG) and the International Association of Geomagnetism and Aeronomy (IAGA) determine from measurements of the magnetic field **B** on the ground and publish every 5 years the Gauss coefficients g_n^m and h_n^m (Thébault et al. 2015). Before the year 2000, the parameters until degree $n = m = 10$ were determined and since the year 2000 until degree $n = m = 13$, see https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html.
The components of $\mathbf{B}(r, \theta, \phi)$ in spherical coordinates are:

$$\begin{aligned} \mathbf{B_r}(r, \theta, \phi) &= -\frac{\partial V(r, \theta, \phi)}{\partial r} \\ \mathbf{B}_{\text{„}}(r, \theta, \phi) &= -\frac{1}{r} \frac{\partial V(r, \theta, \phi)}{\partial \theta} \\ \mathbf{B}_{\text{ffi}}(r, \theta, \phi) &= -\frac{1}{r\, sin\theta} \frac{\partial V(r, \theta, \phi)}{\partial \Phi} \end{aligned} \quad (5.12)$$

where

r distance from Earth's center
θ geographic co-latitude, $\theta = 90° - \Lambda$
Λ geographic latitude
ϕ geographic length

The strength of the magnetic field at the Earth's surface ranges from less than 30,000 nT or 0.3 Gauss in South America (South Atlantic Anomaly) to over 60,000 nT around the magnetic poles (northern Canada, Siberia and coast of Antarctica, south of Australia). For comparison, the strength of the interplanetary magnetic field near Earth is typically 5 nT, i.e. the Earth's magnetic field at the Earth surface is about four orders of magnitude larger.

5.3.3 Contributions to the Earth's Magnetic Field by Magnetospheric Electric Currents

In addition to the internal sources of the Earth's magnetic field, there is also a contribution of external origin: the electrical current systems in the ionosphere and the magnetosphere. Figure 5.2 shows a schematic view of the different current

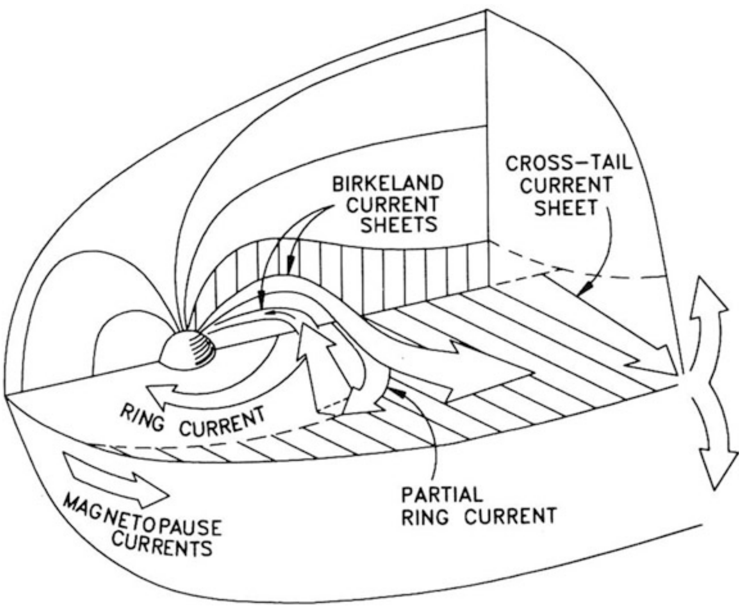

Fig. 5.2 Schematic view of the different current systems which contribute to the Earth's magnetic field.
(From Stern 1994, reproduced with permission from publisher John Wiley and Sons for electronic and press publishing, license number: 4118701268653, license date: May 30, 2017)

systems in the magnetosphere of the Earth. These current systems may vary rapidly, depending on the solar activity. During quiet periods the amplitudes of these external contributions are ~20 nT at mid-latitudes and may increase to more than the ten-fold during geomagnetic storms. The most important current systems are: the ring current in the radiation belts, the Chapman-Ferraro current on the magnetopause (magnetopause current), the field aligned (Birkeland) currents along geomagnetic field lines connecting the Earth's magnetosphere to the Earth's high latitude ionosphere, and the tail currents in the tail of the magnetosphere. The intensities of these currents reach millions of amperes and are related to the solar activity. During times of low solar activity, the standoff distance of the magnetopause currents is at ~10.5 Earth radii and the generated magnetic field close to the Earth is ~25 nT. The ring current, i.e. the longitudinal drift of energetic (10–200 keV) particles that are bouncing along the magnetic field lines between North and South polar regions, has a radius of ~6 R_E during quiet times and its contribution to the magnetic field at the Earth is about ~40 nT.

5.3.4 Magnetic Field Models of the External Sources

Since the 1970s efforts were made to improve the quantitative quality of the magnetospheric magnetic field models, see e.g. the review paper by Walker (1979). Most of the models include currents in the inner magnetosphere in addition to the boundary currents and the magnetotail current system. All the models include the tilt angle of the internal magnetic dipole as an input parameter. With the advent of the space era it became possible to extend the models from low to high altitudes, eventually including even the entire magnetosphere. However, the modeling of the magnetic field in that region is much more difficult, mostly because the magnetic field from external sources (currents in the geomagnetosphere) predominates the magnetic field with growing distance from the Earth. Vector measurements of the magnetic field should be made throughout the entire space where the field should be modeled, i.e. it is necessary to accumulate large amounts of space magnetometer data taken in a wide range of the geomagnetosphere. In contrast to the main geomagnetic field (variations on a timescale of thousands of years), the magnetic field in the outer regions of the geomagnetosphere is a very dynamical system on short time scales and depends on different factors. The first factor is the orientation of the Earth's magnetic axis with respect to the direction of the incoming solar wind flow, which varies with time because of the Earth's diurnal rotation and its yearly orbital motion around the Sun, and the frequent variations of the solar wind characteristics. Another important factor is the state of the solar wind, in particular, the orientation and strength of the interplanetary magnetic field. The interaction between the terrestrial and the interplanetary magnetic fields becomes strongly effective when the interplanetary magnetic field is antiparallel to the Earth's field on the dayside boundary of the magnetosphere. In this case, the geomagnetic and the interplanetary field lines connect across the magnetospheric boundary, which strongly enhances the transfer of the solar wind mass, energy, and electric field inside the magnetosphere.

Different models have been developed to describe the magnetic field in the whole geomagnetosphere (Mead and Fairfield 1975; Olson and Pfitzer 1988; Tsyganenko 1987, 1989, 1996; Ostapenko and Maltsev 1997; Tsyganenko 2002a,b; Tsyganenko and Sitnov 2005). For the determination of cosmic ray particle trajectories mainly the magnetic field models by Tsyganenko (1989, 1996, 2002a,b), Ostapenko and Maltsev (1997) and Tsyganenko and Sitnov (2005) are used.

The Tsyganenko models are semi-empirical best-fit representations for the magnetic field, based on a large number of satellite observations (IMP, HEOS, ISEE, POLAR, Geotail, etc.). The models include the contributions from external magnetospheric sources: ring current, magnetotail current system, magnetopause currents and large-scale system of field-aligned currents.

Tsyganenko model T89 (Tsyganenko 1989) was primarily developed as a tail model. It is based on satellite measurements at distances from the Earth less than 70 R_E, therefore its domain of validity is limited to this region in space. It provides seven different states of the geomagnetosphere corresponding to different levels of the geomagnetic activity represented by the *Kp*-index[1] 0, 1, ..., $\geq$6. The model does not consider the continuous variation of the structure of the magnetosphere as a function of geomagnetic indices like *Dst* and of solar wind parameters. The consideration of these parameters to describe the evolution of the magnetosphere is in particular important during a magnetic storm. Therefore the use of T89 is not reasonable during times when the geomagnetic field is strongly disturbed.

Tsyganenko model T96 (Tsyganenko 1996) considers in contrast to the T89 model the continuous variation of the structure of the magnetosphere as a function of the geomagnetic indices like *Dst* and of the solar wind parameters. In this model the external magnetospheric magnetic field is generated by different current systems where the shape and the strength depend on the dipole tilt angle, on the solar wind dynamic pressure, on the *Dst* index and on the interplanetary magnetic field components B_y^{GSM} and B_z^{GSM} in geocentric solar magnetospheric coordinates (GSM). This model has an explicitly defined realistic magnetopause which is represented by a semi ellipsoid of rotation towards the Sun and by a cylindrical surface in the far tail for $x^{GSM} \leq -60R_E$.

Tsyganenko model T01 (Tsyganenko 2002a,b) is based on the same principles as the model T96 but has essential improvements. The T01 model considers the variable configuration of the inner and near magnetosphere for different interplanetary conditions and ground disturbance levels.

[1]The *Kp*-index is a quasi-logarithmic quantity for the variation of the magnetic field intensity at the Earth's surface as function of time. The range of the *Kp*-index is 0° (for quite conditions) over 0+, 1−, 1+ to 9° (for extreme disturbed magnetic field (geomagnetic storm)). It is derived from the maximum fluctuations of the horizontal components of the Earth's magnetic field observed by observation stations around the world and is published every 3 h.

Tsyganenko model T04 (Tsyganenko and Sitnov 2005) is a dynamical model of the storm-time geomagnetic field in the inner magnetosphere, using space magnetometer data taken during 37 major events in 1996–2000 and concurrent observations of the solar wind and the interplanetary magnetic field. Therefore, this model is only applicable for times with strong disturbances of the geomagnetic field.

The Tsyganenko model T89 is usually used to compute cosmic ray trajectories in the geomagnetosphere due to the much simpler utilisation with only a few input parameters (date, time, *Kp*-index) and the much less time-consuming computation effort compared to the other Tsyganenko models.

The magnetic field of the magnetosheath is usually not considered in the computation of cosmic ray particle trajectories, although the change in the direction of approach at the border of the geomagnetosphere due to the effect of the magnetosheath may be a few 10° at low rigidities ($R \sim 1$GV) (Bütikofer et al. 1997).

5.4 Computation of the Propagation of Cosmic Ray Particles in the Earth's Magnetosphere

There exists no solution of the equation of motion of a charged particle in the geomagnetosphere magnetic field in a closed form. Therefore the determination of cosmic ray trajectories in the geomagnetosphere is almost exclusively made by numerical integration on computer by using a model of the magnetic field in the geomagnetosphere. The cosmic ray particle trajectories are computed backward, i.e. starting at the location of observation and compute the trajectory away from the Earth. Thereby the effect is used that the path of a negatively charged particle with mass, m, charge, Ze, and speed, $\mathbf{v}$, in a static magnetic field, $\mathbf{B}$, is identical to that of an identical but positively charged particle with reverse sign of the velocity vector. For observation locations at ground the computations start at an altitude of typically 20 km above sea level as the interactions of primary cosmic ray particles with atoms and atomic nuclei in the atmosphere become important below this altitude (Smart et al. 2000).

The equations of motion of charged particles in a known magnetic field $\mathbf{B}(r, \theta, \phi)$ in a spherical coordinate system (r, θ, ϕ) are:

$$\begin{aligned}
\frac{dv_r}{dt} &= \frac{Ze}{m}(v_\theta B_\phi - v_\phi B_\theta) + \frac{v_\theta^2}{r} + \frac{v_\phi^2}{r} \\
\frac{dv_\theta}{dt} &= \frac{Ze}{m}(v_\phi B_r - v_r B_\phi) - \frac{v_r v_\theta}{r} + \frac{v_\phi^2}{r \tan\theta} \\
\frac{dv_\phi}{dt} &= \frac{Ze}{m}(v_r B_\theta - v_\theta B_r) - \frac{v_r v_\phi}{r} - \frac{v_\theta v_\phi}{r \tan\theta}
\end{aligned} \tag{5.13}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = v_r$$
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{v_\theta}{r}$$
$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{v_\phi}{r\sin\theta}$$

where B_r, B_θ, B_ϕ are the known magnetic field components, v_r, v_θ, v_ϕ are the particle velocity components, c is the speed of light, Ze and m are respectively the charge and the mass of the particle, and r is the radial distance of the location of the particle from the center of the Earth.

The statement of the problem of the particle trajectory computation belongs to the category of initial value problems. They start at a selected time t_0 and with a set of known variables r_0, θ_0, ϕ_0, v_{r_0}, v_{θ_0}, v_{ϕ_0}. From this set of initial values the corresponding values after a short time interval Δt, i.e. at time $t_0 + \Delta t$, can be computed.

There exist different types of numerical methods to solve the initial value problems e.g. Runge–Kutta or Bulirsch-Stoer method (Press et al. 1986). These methods optimize in different ways the step size of the numerical integration, i.e. the interval size Δt, on the one hand to prevent that the error per step exceeds a preset maximum value and on the other hand to reduce the computation time. Different computer codes for the cosmic ray trajectory computations in the Earth's magnetic field have been developed (see e.g. Shea and Smart 1975; Flueckiger and Kobel 1990; Desorgher et al. 2006).

Figure 5.3 shows an illustration of charged particle trajectories with different rigidities entering the Earth at the same location from zenith direction. The cosmic

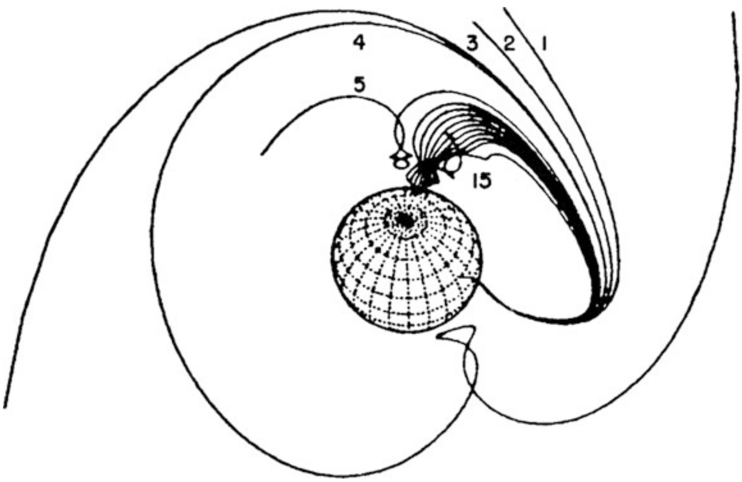

Fig. 5.3 Charged particle trajectories with different rigidities in the Earth's magnetic field. (From Smart et al. 2000, reproduced with permission from publisher Springer for electronic and press publishing, license number: 4118710609018, license date: May 30, 2017)

ray particle trajectories labeled 1–3 have high rigidities and are therefore less bent compared to the trajectories labeled with values >3. The trajectories 4 and 5 show loops, but both can escape the geomagnetosphere, i.e. cosmic ray particles with these rigidities can reach the specified location on Earth from zenith direction ("allowed trajectories"). Particles with even lower rigidities are more bent and the trajectories of these particles penetrate the Earth (re-entrant trajectories), i.e. particles with these rigidities can not reach the location of observation from zenith direction from outside of the geomagnetosphere ("forbidden trajectories").

5.5 The Concept of Cutoff Rigidities and Asymptotic Directions

The "cutoff rigidities" and the "asymptotic directions" have been introduced to specify the geomagnetic effects on cosmic ray particles and to determine the cosmic ray particle spectral characteristics and the anisotropy near Earth but outside the geomagnetosphere from cosmic ray measurements at ground (neutron monitors, muon detectors) or by space based detectors within the geomagnetosphere.

The cutoff rigidity at a selected location and with a specific direction of incidence is defined as the rigidity below which the cosmic ray particles have no access to this location from the given direction of incidence, i.e. trajectories with rigidities larger than the cutoff are "allowed trajectories" whereas trajectories with rigidities below the cutoff rigidity are "forbidden trajectories". The asymptotic direction of cosmic ray particles is used as the particle's trajectory direction of approach at the boundary of the geomagnetosphere.

The cutoff rigidities are determined by trajectory calculations at discrete rigidity intervals starting from a value above the highest possible cutoff rigidity down below the lowest possible allowed trajectory. The trajectories over this rigidity range show different features: first discontinuity in asymptotic direction, first forbidden trajectory, then usually a range of allowed and forbidden trajectories (co-called cosmic ray penumbra), lowest allowed trajectory.

The following parameters are used to describe the cutoff rigidity (Cooke et al. 1991):

- main cutoff rigidity R_M or upper cutoff R_u: rigidity of the last allowed trajectory before the first forbidden. This cutoff rigidity is close to the first discontinuity rigidity R_1, $R_M \approx R_1$
- Störmer cutoff R_S or lower cutoff R_l: rigidity of the last allowed trajectory, i.e. trajectories of particles with rigidities $< R_S$ are forbidden
- R_c: effective cutoff rigidity which is between R_u and R_l taking into account the penumbra, see Eq. (5.15).

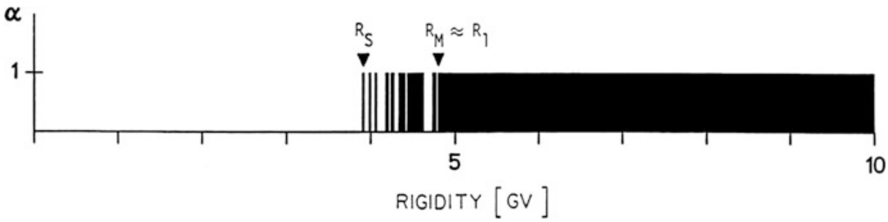

Fig. 5.4 Function $\alpha(R)$ for the station Jungfraujoch for vertical direction of incidence and corresponding cutoff rigidity values: R_S (Störmer cutoff), R_M (main cutoff), and R_1 (first-discontinuity rigidity)

For a location of observation and for a selected direction of incidence the effect of the Earth's magnetic field on the accessibility of cosmic ray particles is described by the filter function $\alpha(R)$:

$$\alpha(R) = \begin{cases} 0 & : \text{ if trajectory is forbidden for rigidity } R \\ 1 & : \text{ if trajectory is allowed for } R \end{cases} \tag{5.14}$$

Figure 5.4 shows the function $\alpha(R)$ for the station Jungfraujoch for vertical direction of incidence.
The effective cutoff rigidity R_c is given by

$$R_c = R_S + \int_{R_S}^{R_M} \alpha(R) dR \tag{5.15}$$

The effective geomagnetic cutoff rigidity R_c depends on the location of the observer, the direction of incidence into the atmosphere, the date and time, and the degree of disturbance of the geomagnetic field. The cutoff rigidities for ground-based cosmic ray stations and for vertical incidence range from $R_c \approx 0$ GV at the magnetic poles to $R_c \approx 15$ GV at the geomagnetic equator.

If one follows the cosmic ray particle's trajectory away from the Earth, the amount of bending per path length caused by the magnetic field is decreasing, i.e. the direction of the particle's trajectory approaches asymptotically its direction with no magnetic field. In the field of cosmic rays the expression asymptotic direction is used for the direction of the cosmic ray particle trajectory when it penetrates the border of the geomagnetosphere (magnetopause). The asymptotic direction of a cosmic ray particle that reaches the location of observation from a selected direction depends on the geographic coordinates of the observer and of the cosmic ray particle's rigidity. Figure 5.5 shows the trajectory of a cosmic ray particle reaching a location on the Earth from a selected direction and its puncture through the magnetopause. The arrow gives the direction of the trajectory at the puncture: the asymptotic direction.

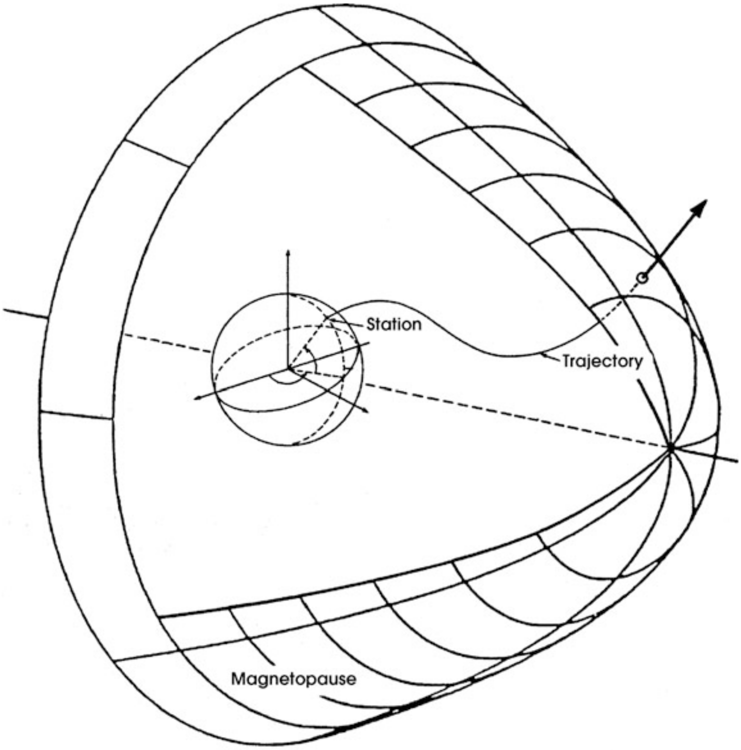

Fig. 5.5 Illustration of a cosmic ray particle's trajectory through the geomagnetosphere reaching a selected location on the Earth from a selected direction and of its related direction of approach at the magnetopause (asymptotic direction)

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