1999

Problems 1–9 for 1–2-Years Students and Problems 5–11 for 3–4-Years Students

1. See Problem 4, 1997.

2. Find the global maximum of a function $2^{\sin x} + 2^{\cos x}$.

3. See William Lowell Putnam Mathematical Competition, 1998, Problem A3.

4. See William Lowell Putnam Mathematical Competition, 1988, Problem A6.

5. See William Lowell Putnam Mathematical Competition, 1998, Problem B5.

6. See William Lowell Putnam Mathematical Competition, 1962, Morning Session, Problem 6.

7. See Problem 5, 1997.

8. See William Lowell Putnam Mathematical Competition, 1961, Morning Session, Problem 7.

9. Let $\{S_n,\ n \geq 1\}$ be a sequence of $m \times m$ matrices such that $S_n S_n^{\mathrm{T}}$ tends to the identity matrix. Prove that there exists a sequence $\{U_n,\ n \geq 1\}$ of orthogonal matrices such that $S_n - U_n \to O$, as $n \to \infty$.

10. Let ξ and η be independent random variables such that $\mathsf{P}(\xi = \eta) > 0$. Prove that there exists a real number a such that $\mathsf{P}(\xi = a) > 0$ and $\mathsf{P}(\eta = a) > 0$.

11. Find a set of linearly independent elements $\mathscr{M} = \{e_i,\ i \geq 1\}$ in an infinite-dimensional separable Hilbert space H, such that the closed linear hull of $\mathscr{M} \setminus \{e_i\}$ coincides with H for every $i \geq 1$.

PROBLEM 2 IS PROPOSED BY A. G. Kukush.

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