

# Optimize the Coupon Face Value for Online Sellers

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**Abstract.** The impact of online coupons is well recognized, but few studies have attempted to model the optimal coupon face value. This research examines how the online coupon affects the payoff of a seller and analyzes the optimal coupon face value for online seller. We find that both the fixed costs and matching probability play critical roles in whether the seller chooses to offer coupons or not. We also find that, a seller can maximize his/her profit by providing the online coupon with an optimal face value which has been formulated. Finally, we conclude with managerial implications and directions for further studies.

**Keywords:** Online coupon · Coupon face value · Matching probability · Equilibrium analysis · Online seller

## 1 Introduction

The phenomenon of online coupons as a means of increasing sales is well-known and widely used in the electronic business. Online sellers choose to offer coupons to shape the potential buyers' attitudes and motivate their purchase. And, there are some sellers who offer the coupons earning lots of money on the online platforms. In contrast, other online vendors who do not provide the coupons still make great profits. The difference between the two kinds of sellers raises several questions: In what situation can online sellers benefit from choosing to offer coupons? If a seller offers the coupon, how should he/she optimize the coupon face value to maximize his/her payoff? This paper aims to answer these two questions.

Current research on the online coupons emphasizes the coupon proneness, coupon redemption [1, 2, 14] and online pricing with coupon [10, 11, 18], but few researchers have attempted to model the optimal coupon face value. Ben-Zion [3] has modeled the optimal face value of a discount coupon in traditional economics. But online environment brings some new characteristics, and there is almost no analytic models being developed for online sellers to maximize their profits. This study will fill this gap and sheds some light on the choice of online coupons for sellers.

To do so, we develop a game-theoretic model in which online sellers face choosing to offer coupons or not. We consider an online platform with a group of potential buyers on the one side and a group of potential sellers on the other. We assume the mass of buyers as a fixed value. Besides, the platform offers basic services for free and,

meanwhile, sellers have fixed costs to provide products and can choose to offer coupons to increase their exposure. And, each online seller on the basic platform has a unique trading partner and gets the same likelihood noticed by prospective buyers. We define the likelihood as matching probability. For ease of exposition, we simply assume that the trade occurs when the seller is noticed by the trading buyer. Using this framework, we compare the revenues of sellers' payoffs under the two situations in which the sellers offer the coupons or not.

We identify both the matching probability and fixed costs as the key factors in comparing the two situations. Not surprisingly, the matching probability of the seller plays an important role in determining whether to offer the coupon or not. In addition, it is found that when the matching probability is high, the seller with a larger fixed cost can benefit from offering the coupon. Furthermore, we find that a seller can optimize the coupon face value to maximize his/her payoff, and the optimal coupon face value varies according to the matching probability and the fixed costs.

The contribution of this paper is that we identify the circumstances that an online sellers can benefit from offering coupons and optimize coupon face value for the online seller to maximize his/her payoff. And the paper is organized as follows. We give a brief literature review first, and then we formulate a model and make the equilibrium analysis to optimize the coupon face value for online sellers. Finally, we conclude with managerial implications and further research issues.

## 2 Literature Review

Our study is mainly related to the research about coupons. Firstly, coupon proneness has been widely documented in the literature that buyers with different characteristics, such as age, income, education, brand loyalty, have distinct coupon proneness [9, 17]. Tang and Zhao [22] identified the impact of coupon proneness on mobile coupon sharing in social network sites. Then, the coupon redemption was also studied deeply. Kapil [1] integrated the coupon attraction and coupon proneness as the key factors influencing the coupon redemption. Leone [13] analyzed the impact of coupon face value on the coupon redemption and brand probability. Besides, many scholars associated the coupons with the price discrimination. Narasimhan [18] showed that coupons can be taken as a price discrimination device and be provided to a particular segment of consumers. Moraga-González [16] analyzed different types of coupon under imperfect price information. Furthermore, lots of researchers paid attention to the coupon face value. For instance, Garretson [7] examined the influence of coupon face value on service quality expectations, perceived purchase risks, and purchase intentions in the dental industry. Price [20] modeled the coupon values for ready-to-eat breakfast cereals.

In contrast to the traditional coupons in the offline environment, online environment offers online sellers an opportunity to issue coupons with different face values to potential buyers for immediate online use. Ben-Zion [3] tried to optimize the face value of coupons by analyzing the structure of the market and separating apart the loyal customers. Chiou [6] used panel data to investigate the factors that influence the preference of shoppers for online coupons. Cheng and Dogan [5] emphasized the importance of customer-centric marketing when using Internet coupons. Reichhart [21]

compared the effectiveness of e-mail coupons with mobile coupons and the results indicated that e-mail coupons got a higher response rate. Georgia [8] considered the customer loyalty and analyzed the coupon trading. Martín-Herrán [15] combined coupons with the trade deals with on-package coupons and concluded the optimal strategy. Navdeep [19] examined the discount offers can improve the average expenditure, and the coupons can be seen as one type of discount offers. Jiang [10, 11] focused on the online pricing with the discount coupons and modeled to maximize the profits of firms or sellers.

We focus on the coupon face value in the online environment. Garretson [7] and Price [20] analyze the coupon face value in the traditional economics. Ben-Zion [3] optimizes the coupon face value by separating apart the market and still does not refer to the online characteristics. Different from the above research, we consider an online seller facing whether to offer the coupons or not and take the matching probability and fixed costs as the key factors. Then, we optimal the coupon face value for online sellers aiming to maximize their profits.

### 3 Model

We consider an online two-sided market with multiple sellers and buyers based on a platform. And the platform provides matching between potential sellers and buyers to facilitate transactions. In addition, potential buyers participate in the platform aiming to find their ideal products provided by online venders. We assume that buyers and sellers can participate in the platform without any costs, which is consistent with the popular online platform, [Taobao.com](http://Taobao.com), practices. And the platform can make the profit by offering advertising.

We assume that sellers are listed without differentiation which means that each buyer has a unique trading partner on the platform, and each seller will get the same likelihood noticed by buyers. We denote  $p$  as the likelihood, also called the matching probability, that the seller's product is noticed by buyers. For ease of exposition, we simply assume that the trading occurs when the seller's product is noticed by the potential buyer, which is often used in the prior literature [4]. So, sellers will try their best to improve the matching probability to increase sales, and they can choose to offer online coupons. Undoubtedly, the coupon face value  $\eta$ , determines the matching probability increased, and we denote  $p_1$  as the matching probability the seller offering the online coupon with value  $\eta$  receives. We easily assume all buyers have coupon proneness.

We assume that each seller in the platform is seen as selling a different product and the competition among sellers is not considered, which is consistent the with exited researches on two-sided markets [4, 12]. Then, each seller will have a fixed cost,  $k$ , to provide the product on the platform. We assume that the fixed cost  $k$  is distributed on  $[0, 1]$ . And the participating buyers and sellers will also obtain profits by occurring transactions. We let  $s$  as the expected surplus that a buyer derives from trading and  $\pi$  as the expected revenue that a seller derives. Similarly, we assume  $s \leq 1$  and  $\pi \leq 1$ .

Sellers choose whether to participate in the platform depending on their costs and profits. And we assume a fixed value as the mass of participating sellers which means that the choice of a seller to offer the online coupon will not influence the total mass of

participating buyers. The assumption is reasonable because there are vast buyers in online platforms, and a seller's choice can be ignored. Let  $m$  be the mass of buyers and  $n$  be the mass of sellers participating in the platform. Similarly, we denote  $n_1$  as the mass of participating sellers offering online coupons. As has mentioned above, each buyer has a unique trading partner on the platform, and each seller lists differently. So, the more mass of participating sellers, the more likelihood that a buyer's ideal trading partner is on the platform. And we can assume that the probability that a buyer's trading partner is on the platform is equal to the mass of participating sellers  $n$ . Table 1 summarizes the main notations referred above.

**Table 1.** Summary of Notations

Notation	Definition and Comments
$\eta$	coupon face value
$k$	seller's cost of proving a product
$m$	mass of participating buyers
$n$	mass of participating sellers
$n_1$	mass of participating sellers offering online coupons
$p$	exposure each participating seller receives, also called "matching probability"
$p_1$	exposure the participating seller offers the coupon with face value $\eta$ receives
$s$	buyer's expected surplus from trading
$\pi$	seller's expected revenue from trading

Then, a seller's expected payoff from participating in the platform:

$$mp(\pi - k) \quad (1)$$

Now consider that when a buyer's ideal trading partner offers the online coupon with face value  $\eta$ , the buyer finds the trading partner and will get profit  $s + \eta$ . And the probability that the buyer can find the partner is  $p$ , so her/his profit derived from this trading can be described as  $p(s + \eta)$ . This transaction can also be interpreted as that the probability that the buyer find the trading partner increases to  $p_1$  ( $p_1 < 1$ ), but the profit still is the expected surplus  $s$ , and then the buyer's profit derived from this trading can be described as  $p_1s$ . Hence, the buyer's profit derived from this trading can be described as  $p(s + \eta)$  and  $p_1s$  simultaneously when  $p_1 < 1$ . And we can get:

$$p(s + \eta) = p_1s \quad \text{if } p_1 < 1 \quad (2)$$

A seller's expected payoff  $U_s$  from participating in the platform with offering the online coupon with face value  $\eta$

$$U_s = \begin{cases} mp_1(\pi - k - \eta) & \text{if } \eta < \frac{1-p}{p}s \\ m(\pi - k - \eta) & \text{if } \eta \geq \frac{1-p}{p}s \end{cases} \quad (3)$$

## 4 Equilibrium Analysis

In this section, we will analyze the participation of sellers. And then, we formulate the coupon face value in order to maximize the profit for the online seller.

### *Participation of Sellers*

We have monotonicity in sellers' participation decisions [4]. In particular, if a seller with a certain cost participates in the platform, the sellers with lower costs also participate in. In addition, if a seller with a certain cost benefits from choosing to offer the online coupon, so does the seller with the lower cost. We denote  $k_A$  as the cost of the marginal seller who is indifferent about participating in the platform,  $k_{A1}$  as the cost of the marginal participating seller who is indifferent about offering the online coupon with value  $\eta$ , if a seller derives a positive payoff from offering the coupon, her/his payoff from participating in the basic platform service without cost should be positive, which implies  $k_{A1} < k_A$ . Therefore, the sellers with costs lower than  $k_A$  participate in the platform, and the mass of participating sellers is  $n = k_A$ .

Based on the above denotation and Eq. (2), we can derive the relationship of a marginal seller:

$$mp(\pi - k_A) = 0 \quad (4)$$

And from Eq. (4), we can derive  $n = k_A = \pi$

Now we consider the situation that  $p_1 < 1$ . As the marginal seller offering the coupon with value  $\eta$ , we can derive the relationship based Eqs. (1) and (3):

$$mp_1(\pi - k_{A1} - \eta) = mp(\pi - k_{A1}) \quad (5)$$

From Eqs. (5) and (2), we can get:

$$k_{A1} = \pi - s - \eta \quad (6)$$

When  $p_1 \geq 1$ . As the marginal seller offering the coupon with value  $\eta$ , we can derive the relationship based on Eqs. (1) and (3):

$$m(\pi - k_{A1} - \eta) = mp(\pi - k_{A1}) \quad (7)$$

From Eqs. (7) and (2), we can get:

$$k_{A1} = \pi - \frac{1}{1-p} \eta \quad (8)$$

From Eqs. (7) and (8), we can get:

$$k_{A1} = \begin{cases} \pi - s - \eta & \text{if } \eta < \frac{1-p}{p}s \text{ and } \eta \leq \pi - s \\ \pi - \frac{1}{1-p} \eta & \text{if } \eta \geq \frac{1-p}{p}s \end{cases}$$

So, we derive that  $k_{A1_{\max}} = \pi - s$ .

Then, we can conclude the participation of sellers as follows:

**Corollary 1.** *The sellers who have costs in  $[0, \pi]$  participate in the platform. And sellers with  $[0, \pi - s]$  can benefit from offering the online coupons. In addition, when  $p \geq \frac{s}{\pi}$  the sellers who have costs in  $[0, \pi - \frac{s}{p}]$  benefit from offering the coupons with values in  $[0, (1 - p)(\pi - k)]$ ; the sellers who have costs in  $(\pi - \frac{s}{p}, \pi - s]$  benefit from offering coupons with values in  $[0, \pi - s - k]$ . When  $p < \frac{s}{\pi}$ , the sellers who have costs in  $[0, \pi - s]$  benefit from offering coupons with values in  $[0, \pi - s - k]$ .*

**Proof.** All proofs are in the appendix, unless indicated otherwise.

**The Optimal Coupon Face Value**

The seller maximizes his payoff by choosing the optimal coupon face value  $\eta (0 \leq \eta \leq \pi - s)$ . Increasing the coupon face value will improve the exposure to the potential buyers. However, increasing the coupon face value also means more costs for the seller. So, the seller's maximum profit is the result of the balance between the coupon face value and the cost followed. And, we also assume the choice of a seller whether to offer the coupon does not influence the mass of participating buyers, that means the mass of buyers,  $m$ , is a constant value.

We consider the case that  $\eta = 0$  which means the seller does not offer the coupon. From Corollary 1, we can know that the sellers who have the fixed costs in  $[\pi - s, \pi]$  will not choose to offer the coupons. So, we can draw the conclusion the seller with cost in  $[\pi - s, \pi]$  maximizes his profit by choosing not offering the coupon. Then, we analyze the situation that  $0 < \eta \leq \pi - s$ . And from Eq. (8), our work becomes how to maximize the Equation:

$$\max U_s = \begin{cases} mp_1(\pi - k - \eta) & \text{if } \eta < \frac{1-p}{p}s \\ m(\pi - k - \eta) & \text{if } \eta \geq \frac{1-p}{p}s \end{cases} \quad (9)$$

From Eq. (9), we can conclude the optimal coupon face value for online sellers:

**Proposition 1.** *The optimal coupon face value that the seller should offer is:*

When  $p > \frac{2s}{\pi + s}$ ,

$$\eta^* = \begin{cases} \frac{1-p}{p}s & \text{if } k \leq \pi - \frac{2-p}{p}s \\ \frac{\pi - s - k}{2} & \text{if } \pi - \frac{2-p}{p}s < k \leq \pi - s \end{cases}$$

When  $p \leq \frac{2s}{\pi + s}$ ,

$$\eta^* = \frac{\pi - s - k}{2} \quad \text{if } 0 < k \leq \pi - s$$

## 5 Conclusion

In this paper, we have studied how the choice of offering coupons affects the profits of online sellers. The findings indicate that both the fixed costs and matching probability play essential roles in the choice. We also find that a seller can get profit from providing the online coupon with a face value that depends on the matching probability and fixed cost. Then, we have given the formulation of the optimal coupon face values for online sellers.

### *Managerial Implications*

Our study has several implications. First, we underscore the importance of online sellers to choose the coupon face values according to the matching probability and the fixed costs. This tells online platform managers that matching probability plays a critical role in deciding whether to offer the coupon, and a lower fixed cost always means the seller can offer a higher face value coupon. Second, our analysis indicates that the online seller can maximize his/her payoff by choosing the optimal coupon face value. Moreover, when there is a certain range of choices, a seller with a fixed cost can get profits by providing an online coupon with an optimal face value.

### *Directions for Further Research*

Firstly, in this study we assume that the platform provides the matching services for free, and for further research we can consider the situation when the platform charges a transaction fee. Secondly, this study also assumes all buyers have coupon proneness which is not accurate in the real world, and the various proneness situations should be discussed in the further research. Thirdly, in further studies we can compare online coupons with other promotional services, such as advertisement, and analyze the profits of all participating players.

**Acknowledgments.** This research is supported by the Shaanxi Humanities and Social Science Talent Plan (HSSTP) through grant ER42015060002. And this research is also supported by National Natural Science Foundation of China (71502132), Natural Science Basic Research Plan in Shaanxi Province of China (2015JQ7274) and the Fundamental Research Funds for the Central Universities (JB150603).

## Appendix

### **Proof of Corollary 1**

Proof. From Eq. (4), we can get:  $n = k_A = \pi$  which means that the mass of participating sellers is  $\pi$ , and the sellers with  $[0, \pi]$  can participate in the platform. And  $k_{A1_{\max}} = \pi - s$  which means the sellers with  $[0, \pi - s]$  can choose offering the coupons. Then, we analyzed the profitable situation.

When  $p_1 = \frac{s+\eta}{s}p < 1$ , the increased profit of the seller who offers the coupon with value  $\eta$  will be  $\sigma U_s = -\frac{1}{s}mp\eta[\eta - (\pi - k - s)]$ , and when  $\frac{1-p}{p} \leq \pi - s$ , that is equal to

$p < \frac{s}{\pi}$ , the sellers who have costs in  $[0, \pi - s]$  can offer the coupons with values in  $[0, \pi - s - k]$ .

When  $p_1 = \frac{s+\eta}{s}p \geq 1$ , the increased profit becomes  $\sigma U_s = m[\eta - (1 - p)(\pi - k)]$ , and  $\eta < \pi - s$ , so we can get  $p \geq \frac{s}{\pi}$ . Then, we discuss the fixed costs, and we can get the sellers who have costs in  $[0, \pi - \frac{s}{p}]$  can offer the coupons with values in  $[0, (1 - p)(\pi - k)]$ ; the sellers who have costs in  $(\pi - \frac{s}{p}, \pi - s]$  can offer the coupons with values in  $[0, \pi - s - k]$ .

**Proof of Proposition 1**

Proof. Firstly, we do not consider the choice of offering coupons or not and just analyze the Eq. (9). When  $\eta < \frac{1-p}{p}s$ , the profit of the online seller becomes that  $U_s = mp_1(\pi - k - \eta) = m\frac{s+\eta}{s}p(\pi - k - \eta)$ . And we can derive that:

$$U_s = -\frac{mp}{s}\eta^2 + \frac{mp}{s}(\pi - k - s)\eta + mp(\pi - k)$$

From the above Equation, we can get that when  $\frac{1-p}{p}s < \frac{\pi-k-s}{2}$  which means  $p > \frac{2s}{\pi+s}$ , the optimal coupon face value is  $\eta^* = \frac{1-p}{p}s$ .

When  $\frac{1-p}{p}s \geq \frac{\pi-k-s}{2}$ , the optimal coupon face value becomes:  $\eta^* = \frac{\pi-s-k}{2}$ .

When  $\eta \geq \frac{1-p}{p}s$  which means  $p \geq \frac{s}{\pi}$ , the profit of the online seller becomes:  $U_s = m(\pi - k - \eta)$ . We can get that the optimal coupon face value is  $\eta^* = \frac{1-p}{p}s$ .

Then, we add the condition under which buyers can benefit from offering coupons, and it is also provided by Corollary 1. We can conclude the optimal coupon face value for online sellers as follow:

When  $p > \frac{2s}{\pi+s}$ , and if  $k \leq \pi - \frac{2-p}{p}s$  the optimal coupon face value is  $\eta^* = \frac{1-p}{p}s$ , and if  $\pi - \frac{2-p}{p}s < k \leq \pi - s$ , the optimal coupon face value will be  $\eta^* = \frac{\pi-s-k}{2}$ . When  $p \leq \frac{2s}{\pi+s}$  and  $0 < k \leq \pi - s$ , the optimal coupon face value will be  $\eta^* = \frac{\pi-s-k}{2}$ .

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