Aircraft Runway Acceleration in the Presence of Severe Wind Gusts

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Abstract. This paper concerns the problem of aircraft control during the takeoff roll in the presence of severe wind gusts. It is assumed that the aircraft moves on the runway with a constant axial acceleration from a stationary position up to a specific speed at which the aircraft can go into flight. The lateral motion is controlled by the steering wheel and the rudder and affected by side wind. The aim of control is to prevent rolling out of the aircraft from the runway strip. Additionally, the lateral deviation, lateral speed, yaw angle, and yaw rate should remain in certain thresholds during the whole takeoff roll. The problem is stated as a differential game with state constraints. A grid method for computing the value function and optimal feedback strategies for the control and disturbance is used. The paper deals both with a nonlinear and linearized models of an aircraft on the ground. Simulations of the trajectories are presented.

Keywords: Aircraft runway · Lateral runway model · Differential game · Grid method

1 Introduction

Control of aircraft on the ground is a very complicated problem because of nonlinear effects playing a significant role in the dynamics of aircraft. Moreover, severe wind gusts may lead to rolling out from the runway, especially during high-speed roll.

The following investigations are devoted to the enhancement of aircraft-on-ground models and to the development of controllers providing safe ground operations, including taxing and takeoff run.

In the report [1], a detailed explanation of essential requirements and basic assumptions for aircraft modeling is given, including a description of various elements needed in the model structure. The main focus lies on the description of the interface between the aircraft and the runway pavement.

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In paper [2] a bifurcation analysis of steady-state solutions and a transient analysis are applied to the study of the behavior of aircraft on the ground. A general approach to assess an aircraft's performance during taxiway manoeuvres is introduced. This allows to the author to find maximal loads during taxiway manoeuvres, which is important for assessing existing regulations for the certification of aircraft.

The work [3] presents results and interpretations from the analytical analysis aimed to uncover the dominant directional characteristics of the aircraft. Three mathematical models, of growing complexity, of the aircraft on the ground are used. Some fundamental dynamic characteristics such as e.g. the yaw rate to steering command transfer function are determined.

Paper [4] presents the study of a yaw rate control of the aircraft on the ground. A highly nonlinear realistic model of the aircraft is used, and the control design is based on the feedback linearization technique aimed to design a non-linear controller that forces the system output to follow a linear reference behavior. This approach supposes that the linear reference model perfectly corresponds to the real system. It should be noted that wind disturbances are not included into the study.

Paper [5] uses a simplified LFT (Linear Fractional Transformation) model of an aircraft on the ground. In particular, the nonlinear lateral ground forces are reduced to saturation-type nonlinearities. A robust anti-windup control technique is applied to the simplified model to improve lateral control laws to exclude oversteer when working against lateral wind step inputs.

The works [6,7] are devoted to modeling of the takeoff and landing phases for an unmanned aerial vehicle. The investigation is aimed to the development of an automatic takeoff and landing control system reducing effects of human pilot errors. The main attention is concentrated on the takeoff phase and, in particular, on aircraft's lateral motion during the takeoff roll. The authors apply transfer function techniques to a linearized model of the aircraft on the ground to design a controller. This approach does not provide safety against worst-case disturbances.

Paper [8] concerns the application of differential game theory (see e.g. [9]) to the aircraft takeoff roll. A linearized model of aircraft's lateral motion on the runway is considered there, and a conflict control problem, differential game, is formulated. It is assumed that the first player, autopilot, uses feedback strategies to minimize the objective functional of the form $J = \sigma\big(y(T), \dot{y}(T)\big)$, where y is the lateral deviation, and T is a fixed termination time. The second player, side wind, strives to maximize the objective functional using all possible constrained non-anticipative strategies. Thus, the lateral position and velocity of the aircraft are evaluated only at the termination time T, which is insufficient from the technical point of view. The reason to use such a simplified functional is that the authors could solve only two-dimensional games that time, and this simplification allowed the authors to reduce the original differential game to a two-dimensional one using a variable transformation. The main result of this paper is the construction of optimal feedback strategies of the autopilot in the form of switch lines that divide the reduced two-dimensional state space into components where certain constant values of control are prescribed. A similar representation of optimal strategies of side wind is also found.

The following limitations of this investigation should be mentioned. First, the transformation reducing the original differential game to a two-dimensional one is of course not invertible, and therefore imposing state constraints in the original problem is impossible. Second, the strategies found from the linearized model were not tested in the original nonlinear system. All these reasons give rise to the motivation to investigate the problem with modern tools for solving nonlinear state constrained differential games.

The current paper deals with the problem of aircraft control during the take-off roll and enhances the work [8]. The modification consists in the application of modern grid methods for solving nonlinear differential games (see [10,11]) to a nonlinear lateral motion runway model derived in [6,7]. These methods allow us to solve nonlinear differential games of a relative high dimension with accounting for state constraints. Speaking more certainly, it is now possible to consider the objective functional of the form $J = \max_{\tau\in[0,T]} \sigma\big(x_1(\tau), ..., x_n(\tau)\big)$, $n = 4$ or 5, and therefore to constrain all state variables for all time instants. This allows us to develop a control law that prevents rolling out of the airplane from the runway.

The model parameters are fitted to the characteristics of Boeing-727.

2 Model Equations

Consider an aircraft during the takeoff roll (see Fig. 1).

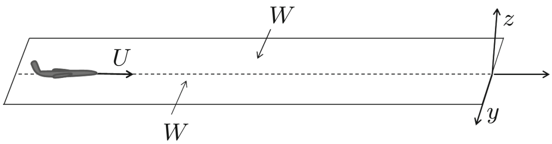

Fig. 1. Aircraft during the takeoff roll under wind gusts.

Let the state variables be defined as follows: y is the lateral deviation, V the lateral velocity, ψ the yaw angle, and R the yaw rate. The model derived in [6] reads:

$$
\begin{aligned}
\dot{y} &= V, \\
\dot{V} &= -UR + (F_u + F_a)/m, \\
\dot{\psi} &= R, \\
\dot{R} &= (M_u + M_a)/I_z.
\end{aligned}
\tag{1}
$$

Here, $U = at$ is the axial velocity increasing linearly with time t according to the acceleration a; F_u and M_u are the undercarriage forces and moments, respectively; F_a and M_a are aerodynamic forces and moments, respectively; m is the aircraft mass, and I_z is the z-axis moment of inertia. The expressions for the forces and moments are given by the formulas

$$
\begin{aligned}
F_u &= N_s C_{\alpha\alpha}\Big[\arctan\frac{V+l_sR}{U}-\delta_s\Big]\cos\delta_s - N_s\mu_f\sin\delta_s \\
&\qquad + N_l C_{\alpha\alpha}\arctan\frac{V-l_mR}{U+l_w/2\,R} + N_r C_{\alpha\alpha}\arctan\frac{V-l_mR}{U-l_w/2\,R}, \\
M_u &= l_s N_s C_{\alpha\alpha}\Big[\arctan\frac{V+l_sR}{U}-\delta_s\Big]\cos\delta_s - l_s N_s\mu_f\sin\delta_s \\
&\qquad - l_m N_l C_{\alpha\alpha}\arctan\frac{V-l_mR}{U+l_w/2\,R} - l_m N_r C_{\alpha\alpha}\arctan\frac{V-l_mR}{U-l_w/2\,R},
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
F_a &= q\cdot S\cdot(C_{y\beta}\beta + b/(2V_a)C_{yr}R + C_{y\delta_r}\delta_r), \\
M_a &= b\cdot q\cdot S\cdot(C_{n\beta}\beta + b/(2V_a)C_{nr}R + C_{n\delta_r}\delta_r).
\end{aligned}
\tag{3}
$$

Here, $V_a = \sqrt{U^2+(V-W)^2}$ is the air speed, W the velocity of side wind; $q = 1/2\rho V_a^2$ the dynamic pressure; $\beta = \arcsin\big((V-W)/V_a\big)$ the sideslip angle; δ_s the steering wheel deflection; and δ_r the rudder deflection. It is assumed that $\delta_s = 1/3\delta_r$ for balanced manoeuvres. The control variable, u, and the disturbance, v, are introduced as follows:

$$u := \delta_r \in [-25, 25]\,\mathrm{deg}, \quad v := W \in [-17, 17]\,\mathrm{m/s}. \tag{4}$$

The following notation for the components of the state vector is used below:

$$x_1 := y, \quad x_2 := V, \quad x_3 := \psi, \quad x_4 := R. \tag{5}$$

The coefficients appearing in (1), (2), and (3) are listed in Table 1. The model is considered on the time interval $t \in [0, T]$, where $T = 34\,\mathrm{s}$.

The linearized, non-stationary, model reads:

$$
\begin{aligned}
\dot x_1 &= x_2, \\
\dot x_2 &= a_{22}(t)x_2 + a_{23}(t)x_3 + a_{24}(t)x_4 + a_{25}(t)u + c_2(t)v, \\
\dot x_3 &= x_4, \\
\dot x_4 &= a_{42}(t)x_2 + a_{43}(t)x_3 + a_{44}(t)x_4 + a_{45}(t)u + c_4(t)v, \\
\dot u &= -k(u-\bar u).
\end{aligned}
\tag{6}
$$

Here, an artificial control $\bar u$ that may have instantaneous jumps is introduced. The physical control u $(= \delta_r)$ smoothly tracks $\bar u$ with a time lag depending on the parameter k. The artificial control is constrained just as u in (4).

Table 1. Model coefficients approximately corresponding to Boeing-727.

Notation	Name	Value	Units
CG	Center of gravity	-	-
μ_f	Coefficient of kinetic friction	0.5	-
ρ	Air density	1.207	kg/m^3
m	Aircraft mass	288773	kg
S	Wing area	511	m^2
b	Wing span	60	m
I_z	z-axis moment of inertia	67.38e6	$kg \cdot m^2$
l_s	Distance from CG to steering wheel along x	28.36	m
l_m	Distance from CG to main wheels along x	1.64	m
l_l, l_r	Distance from CG to left/right main wheel along y	6	m
l_w	Distance between main wheels ($l_w = l_l + l_r$)	12	m
l_L	Distance from steering to main wheels ($l_L = l_s + l_m$)	30	m
N_s	Normal reactions at steering wheel	154.863	kN
N_l, N_r	Normal reactions at main wheels	1338.99	kN
$C_{\alpha\alpha}$	Tire cornering coefficient	0.25	$1/rad$
$C_{y\beta}$	Output of y-force due to sideslip angle	-0.9	$1/rad$
C_{yr}	Output of y-force due to yaw rate	0	$1/rad$
$C_{y\delta_r}$	Output of y-force due to rudder deflection	0.120	$1/rad$
$C_{n\beta}$	Output of yawing moment due to sideslip angle	0	$1/rad$
C_{nr}	Output of yawing moment due to yaw rate	-0.280	$1/rad$
$C_{n\delta_r}$	Output of yawing moment due to rudder deflection	-0.1	$1/rad$

The coefficients appearing in (6) are defined by the formulas

$$\begin{aligned}
&a_{22}(t) = 0.229(1 - 100/\xi) - 0.345 \cdot 10^{-2}\xi,\\
&a_{23}(t) = 0.12 \cdot 10^{-3}\,\xi^2 - 0.8(1 - 0.01\xi),\\
&a_{24}(t) = -0.138 \cdot 10^{-2}(1 - 100/\xi),\\
&a_{25}(t) = -0.2 \cdot 10^{-4}\,\xi^2 + 0.32 \cdot 10^{-1}(1 - 0.01\xi),\\
&a_{42}(t) = -0.132 \cdot 10^{-1}\xi, \quad a_{43}(t) = -0.464 \cdot 10^{-3}\,\xi^2,\\
&a_{44}(t) = 0.715 \cdot 10^{-1}(1 - 100/\xi),\\
&a_{45}(t) = -0.164 \cdot 10^{-3}\,\xi^2 - 0.3(1 - 0.01\xi),\\
&c_2(t) = 0.345 \cdot 10^{-2}\xi, \quad c_4(t) = 0.132 \cdot 10^{-1}\xi,\ \xi := t + 1,\ k = 4.
\end{aligned} \tag{7}$$

The model is considered on the time interval $t \in [0, T]$, where $T = 34\,\mathrm{s}$.

3 Numerical Method

Let us shortly outline the solution method that is applicable both to linear and nonlinear problems. The description will be given in terms of general nonlinear differential games, see [9,12,13].

3.1 Differential Game and Value Function

Consider the differential game

$$\dot{x} = f(t, x, u, v), \quad x \in R^n, \quad u \in P \subset R^p, \quad v \in Q \subset R^q, \tag{8}$$

where u and v are control parameters of the first and second player, respectively. The sets P and Q are given compacts. The game starts at $t_0 \in [0, T]$ and finishes at T. The aim of the first (resp. second) player is to minimize (resp. maximize) an objective functional of the form:

$$J\big(x(\cdot)\big) = \max \left\{ \sigma_0\big(x(T)\big), \max_{\tau \in [t_0, T]} \sigma\big(x(\tau)\big) \right\}, \tag{9}$$

where σ_0 and $\sigma : R^n \to R$ are given functions.

The value function, $\mathcal{W}$, is informally defined by the relation

$$\mathcal{W}(t, x) = \max_{\mathcal{V}^c} \min_{\mathcal{U}} J(x(\cdot)) = \min_{\mathcal{U}} \max_{\mathcal{V}^c} J(x(\cdot)),$$

where the minimum is taken over all admissible feedback strategies of the first player, and the maximum is computed over the so-called feedback counter-strategies of the second player (see [9]). This means that the second player (e.g. wind) can measure the current choice of the first player (e.g. the ruder deflection), which makes the second player more dangerous.

It should be noted that the strong definition of the value function (see [9]) is more complicated than that, because the strategies are in general discontinuous functions of x, and therefore cannot be directly substituted into (8) in place of u and v.

The value function plays a very important role, representing the guaranteed result of the players. For example, let the game starts from a position (t_0, x_0), and $\mathcal{W}(t_0, x_0) \leq 0$. Then, there exists a feedback strategy $\mathcal{U}$ such that, for all trajectories $x(\cdot)$ generated by $\mathcal{U}$ and any $\mathcal{V}^c$, the inequalities $\sigma_0(x(T)) \leq 0$ and $\sigma(x(t)) \leq 0,\ t \in [t_0, T]$, hold. This can be interpreted as obtaining a guaranteed gain at the termination time T and keeping the object inside of prescribed state constraints at any time instant. Moreover, as the Subsect. 3.3 shows, optimal strategies of the players can be constructed in the course of computing the value function. Besides, an optimal feedback counter-strategy of the second player is directly derived from the value function.

It is established, see [10,14,15], that the value function is a viscosity solution of the following Hamilton-Jacobi equation:

$$\mathcal{W}_t + H(t,x,\mathcal{W}_x) = 0, \text{ where } H(t,x,p) = \min_{u\in P}\max_{v\in Q}\langle f(t,x,u,v),p\rangle. \quad (10)$$

This correspondence has given rise to numerical methods for computing value functions. The next subsection describes a grid method developed for computing viscosity solutions of (10) and, therefore, value functions in the differential game (8)–(9).

3.2 Grid Method for Computing the Value Function

To compute the value function, the following finite difference scheme is used, see [10–13].

Let $h_1,...,h_n$, and τ be space and time discretization step lengths. Set $L = T/\tau$, $t_\ell = \ell\tau$, $\ell = 0,1,...,L$, and denote

$$\mathcal{W}^\ell(x_{i_1},\dots,x_{i_n}) = \mathcal{W}(\ell\tau, i_1h_1,\dots,i_nh_n),$$

$$\sigma_0^h(x_{i_1},\dots,x_{i_n}) := \sigma_0(i_1h_1,\dots,i_nh_n), \quad \sigma^h(x_{i_1},\dots,x_{i_n}) := \sigma(i_1h_1,\dots,i_nh_n).$$

Let c be a grid function. Assume that the variable x runs over all grid nodes and define the following upwind operator:

$$F(c;t,\tau,h_1,...,h_n)(x) = c(x) + \tau\min_{u\in P}\max_{v\in Q}\sum_{i=1}^{n}(p_i^R f_i^+ + p_i^L f_i^-),$$

where $f_i = f_i(t,x,u,v)$ are the right hand sides of the control system, and

$$a^+ = \max\{a,0\}, \quad a^- = \min\{a,0\},$$
$$p_i^R = [c(x_1,...,x_i+h_i,...,x_n) - c(x_1,...,x_i,...,x_n)]/h_i,$$
$$p_i^L = [c(x_1,...,x_i,...,x_n) - c(x_1,...,x_i-h_i,...,x_n)]/h_i.$$

An approximate solution is the output of the following backward in time finite-difference scheme:

$$\mathcal{W}^{\ell-1} = \max\left\{F(\mathcal{W}^\ell;t_\ell,\tau,h_1,...,h_n),\sigma^\ell\right\}, \ \mathcal{W}^L = \sigma_0^h, \ \ell = L,L-1,...,0. \quad (11)$$

This algorithm is proposed and analyzed in [10–13]. It was stated there that its convergence rate is of order $\sqrt{\tau}$ if $\tau/h_i = c, i = 1...n$, where c is a small enough constant. This convergence rate is not improvable when applying grid methods to Hamilton-Jacobi equations arising from differential games.

3.3 Control Design

When running the algorithm (11), the minimizing grid values of the control,

$$u^{\ell}_{i_1 i_2 \ldots i_n} = \arg_u \min_{u \in P} \max_{v \in Q} \sum_{i=1}^{n} (p_i^R f_i^+ + p_i^L f_i^-),$$

are stored on a hard disk for each grid multi index $i_1 i_2 \ldots i_n$ and each time sampling index ℓ. The control at a time instant t_ℓ and the current state $x(t_\ell)$ is computed as $\mathcal{L}_h[u^\ell]\big(x(t_\ell)\big)$, where u^ℓ denotes the grid function $u^{\ell}_{i_1 i_2 \ldots i_n}$, and $\mathcal{L}_h$ is an interpolation operator.

A counter-strategy of the second player is defined as follows. Let $\big(t_\ell, x(t_\ell)\big)$ be the current position of the game, and a control u of the first player is chosen. Then the second player chooses its control as

$$v = \arg_v \max_{v \in Q} \mathcal{L}_h[\mathcal{W}^\ell]\big(x(t_\ell) + \tau f(t_\ell, x(t_\ell), u, v)\big),$$

where $\mathcal{W}^\ell$ is the grid approximation of the value function at the time instant t_ℓ, computed by formula (11).

4 Simulation Results

This section describes simulation results for the models (1)–(4) and (6)–(7). In both cases, the objective functional of the form (9) with the functions

$$\sigma_0(x) = \max\left\{\frac{|x_1|}{10}, \frac{|x_2|}{5}, \frac{|x_3|}{10}, \frac{|x_4|}{5}\right\} - 1, \ \sigma(x) = \max\left\{\frac{|x_1|}{15}, \frac{|x_2|}{5}, \frac{|x_3|}{15}, \frac{|x_4|}{5}\right\} - 1$$

is used. Thus, the controls u and $\bar{u}$, see (4) and the last equation of (6), strive to satisfy the conditions

$$\sigma_0(x(T)) \le 0 \text{ and } \sigma(x(t)) \le 0, \ t \in [0, T],$$

for any realization of the disturbance v constrained as in (4). In other words, u (resp. $\bar{u}$) strives to satisfy the conditions

$$|y(T)| \le 10\,\mathrm{m}, \ |V(T)| \le 5\,\mathrm{m/s}, \ |\psi(T)| \le 10\,\mathrm{deg}, \ |R(T)| \le 5\,\mathrm{deg/s}$$

at the termination time $T = 34\,\mathrm{s}$ and to keep the state constraints

$$|y(t)| \le 15\,\mathrm{m}, \ |V(t)| \le 5\,\mathrm{m/s}, \ |\psi(t)| \le 15\,\mathrm{deg}, \ |R(t)| \le 5\,\mathrm{deg/s}$$

for all time instants. According to the problem statement, this is possible if the value function, see Sect. 3.1 and the explanation there, is non-positive at the initial state $\{t = 0, \, y = 0, \, V = 0, \, \psi = 0, \, R = 0\}$.

Differential games (1)–(4) and (6)–(7) are solved using numerical methods outlined in Sect. 3. The calculations are performed on a Linux SMP-computer with 8xQuad-Core AMD Opteron processors (Model 8384, 2.7 GHz) and shared 64 Gb memory. The programming language C with OpenMP (Open Multi-processing) support is used. The efficiency of the parallelization is up to 80%.

When solving the differential game related to the linear model (6)–(7), a rectangular $40 \times 20 \times 40 \times 20 \times 30$ grid is chosen. In the case of the nonlinear model (1)–(4), a rectangular $40 \times 20 \times 40 \times 20$ is used.

Figure 2 shows the simulation of the linear model (6)–(7) with an optimal feedback control strategy and the corresponding optimal feedback counter-strategy for wind. The horizontal axes measure the traveled distance in meter, the vertical axes measure the lateral deviation y (meter), the yaw angle ψ (degree), the rudder deflection δ_r (degree), and the velocity of side wind (meter/sec), respectively. The vertical bold bars, drawn to the right in the first two graphs, show the admissible interval for the terminal values of y and ψ, respectively. It is seen that the terminal and state constraints are satisfied for y and ψ. It should also be noted that the other two variables, V and R (not shown here), satisfy their terminal and state constraints too.

Figure 3 presents the simulation of the nonlinear model (1)–(4) using the optimal feedback control strategy found for the linear model (6)–(7), whereas the disturbance is formed using the optimal feedback counter-strategy for wind taken from the nonlinear model. It is seen that the terminal and state constraints are

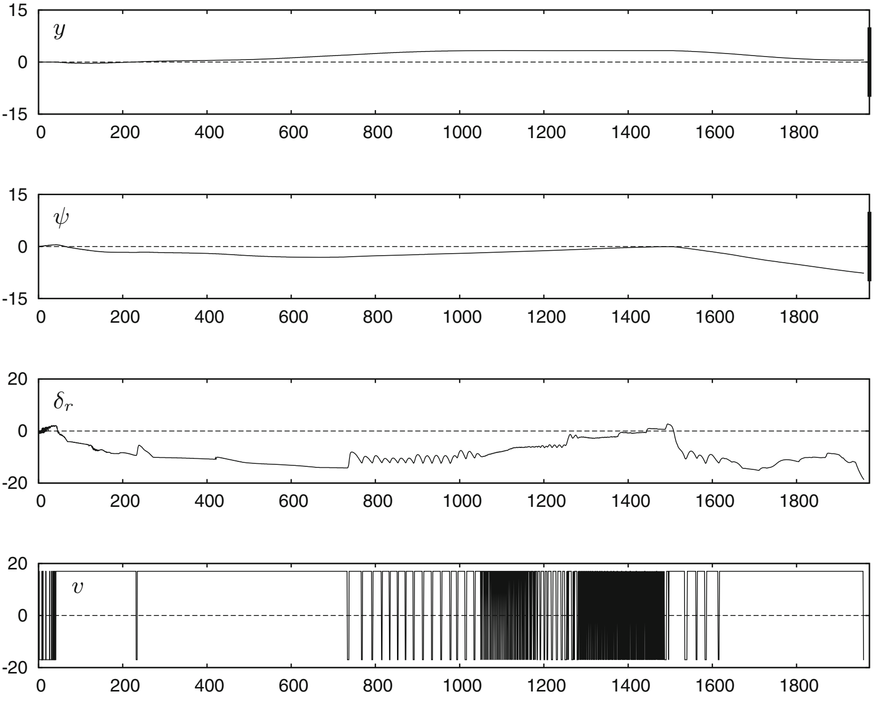

Fig. 2. Simulation of the linear model (6)–(7).

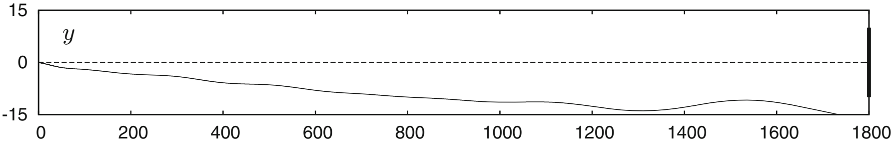

Fig. 3. Simulation of the nonlinear model (1)–(4) with the optimal feedback control strategy found for the linear model (6)–(7), whereas the optimal feedback counter-strategy for wind is taken from the nonlinear model.

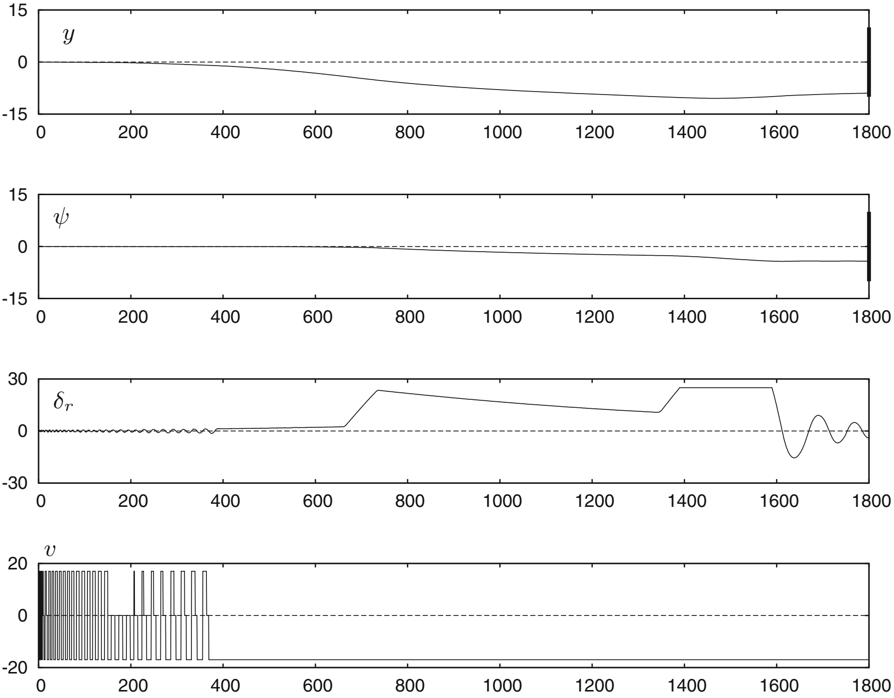

Fig. 4. Simulation of the nonlinear model (1)–(4).

violated. This means that the linearized model (6)–(7) does not properly reflect the dynamical properties of the real nonlinear plant. Thus, the construction of controllers based on linearized models is questionable.

Figure 4 shows the simulation of the nonlinear model (1)–(4) with the optimal feedback control strategy and the corresponding optimal feedback counter-strategy for wind, found from the four-dimensional nonlinear differential game (1)–(4). During the simulation of trajectories, the output, u, of the optimal control strategy is smoothed with the filter $\dot{\delta_r} = -4\,(\delta_r - u)$. It is seen that the terminal and state constraints are satisfied for y and ψ. The other two variables, V and R (not shown here), satisfy the terminal and state constraints too.

The simulation results show that the control strategy found from the linear differential game associated with the models (6)–(7) works perfectly in the linear model, and does not work in the nonlinear one.

The control strategy found from the nonlinear differential game (1)–(4) works perfectly in the real nonlinear model against very severe wind disturbances comparable with hurricane. It should be noted that none conventional control system cannot apparently keep the aircraft on the runway in the presence of smart wind gusts obtained from the nonlinear differential game. However, our control strategies ensure the desired terminal and state constraints (see Figs. 2 and 4). Moreover, the strategies work stable in a wide range of discretization parameters such as time sampling and spatial steps in the algorithm (11), which is checked in numerous test runs. Finally, these strategies can be physically implemented on board, because all state variables used in them are available for measurements.

5 Conclusion

The current investigation shows that methods based on the theory of differential games can be successfully applied to nonlinear conflict control problems related to aircraft's takeoff roll under severe wind gusts. The paper demonstrates the following advantages: A very detailed nonlinear model of aircraft's takeoff roll is used. The corresponding highly nonlinear differential games are solved using a novel grid method, and optimal control strategies ensuring a safe takeoff roll are designed. It is planned to test them on a flight simulator providing a fully realistic model of an aircraft.

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