Solving the Sophistication-Population Paradox of Game Refinement Theory

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Abstract. A mathematical model of game refinement was proposed based on uncertainty of game outcome. This model has been shown to be useful in measuring the entertainment element in the domains such as boardgames and sport games. However, game refinement theory has not been able to explain the correlation between the popularity of a game and the game refinement value. This paper introduces another aspect in the study of game entertainment, the concept of "attractiveness" to reasonably explain the sophistication-population paradox of game refinement theory.

Keywords: Game refinement theory · Physical model in game · Attractiveness of board like games

1 Introduction

The dynamics of decision options in the decision space has been investigated and we observed that this dynamics was a key factor in gauging game entertainment. Then Iida et al. [1] proposed the measure of the refinement in games. The outcome of interesting games is always uncertain until the very end of the game. Thus, the variation in available options stays nearly constant throughout the game. In contrast to this, one player quickly dominates over the other in uninteresting games. Here options are likely to be diminishing quickly from the decision space. Therefore, the refined games are more likely to be seesaw games. We then recall the principle of seesaw games [3].

Based on the principle of seesaw games, Iida et al. [4] proposed a logistic model of game uncertainty. From the players' viewpoint, the information on the game result is an increasing function of time (the number of moves) t. We further define the information on the game result as the amount of solved uncertainty $x(t)$. Game information progress presents how certain is the result of the game in a certain time or steps. Let B and D be the average branching factor and the average number of the depth of game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be

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given as a linear function of time t with $0 \leq t \leq B$ and $0 \leq x(t) \leq B$, as shown in Eq. (1).

$$x(t) = \frac{B}{D}\,t \tag{1}$$

However, the game information progress given by Eq. (1) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is given by Eq. (2).

$$x(t) = B(\frac{t}{D})^n \tag{2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Then acceleration of game information progress is obtained by deriving Eq. (2) twice. Solving it at $t = T$, the equation becomes:

$$x''(T) = \frac{Bn(n-1)}{D^n}t^{n-2} = \frac{B}{D^2}n(n-1).$$

It is assumed in the current model that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in our minds, but it is likely and we propose that the acceleration of information progress is related to the force in mind. Hence, it is reasonable to expect that the larger the value $\frac{B}{D^2}$ is, the more the game becomes exciting due to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{B}}{D}$, as a game refinement measure for the game considered [4]. We show, in Table 1, a comparison of game refinement measures for traditional board games [4].

Table 1. Measures of game refinement for traditional board games

	B	D	$\frac{\sqrt{B}}{D}$
Chess	35	80	0.074
Go	250	208	0.076
Shogi	80	115	0.078

2 Attractiveness in Board Like Games

One of the limitations of the game refinement value is that it fails in explaining the disparity in the population of fans and players of some of the games with equal or higher game refinement value. For example, GO has a higher game refinement value than soccer, but soccer still attracts a far greater population of fans and players than GO. We define this limitation of game refinement theory as the **sophistication-population** paradox that we will address in the course of this paper. Therefore, this section draws upon the limits of game refinement theory and then proposes a notion of attractiveness in game playing. A mathematical model of attractiveness for board like games is proposed.

Game refinement theory has expressed the relationship between uncertainty of outcome and game progress, a game outcome can be considered based on two factors: skill and chance. In the balanced game, the game outcome is decided by players' skill and chance which incorporates some stochastic events, events which cannot be controlled or predicted. For chess, in the ideal situation, the top player or AI can always select the best moves at any time in the game, and all of players choices or branching factor are decided by players consideration, while we calculate game refinement value by the formula $R = \frac{\sqrt{B}}{D}$.

Similar to the branching factor, we introduce a new concept known as "attractiveness branching factor". If there is an element in the game which can neither be controlled or predicted by audience as well as the players, we refer to that element as the element of attractiveness. Generally, there are three situations which will develop the game uncertainty.

3 Physical Model of Attractiveness in Board Like Games

We take in consideration the human thinking process in the traditional board games, intelligent players would follow such a process as shown in Fig. 1. For example in chess, the average branching factor (say B) is about 35 [6], but out of these choices, many moves are not reasonable to Play. After filtering out some of the existing options based on players' skill, we will be left with a smaller set of plausible moves which we define as "b". Generally, a player is not always aware of the best choice available in b where $b \geq 2$, so he takes a chance, which as a result introduces a chance element in the game. By plausible consideration of available moves depending on the skill of the player and taking some chance, a player decides his final move [2,7].

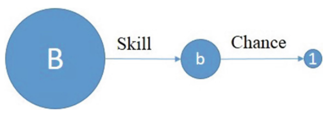

Fig. 1. A model of thinking process in traditional board games

Almost all traditional board games fall in the category of strategic games, and a pre-dominant way of approaching or playing these games is using the concept of **Convergence**. It is a very integral aspect of human thinking process, everyday humans are faced with situations where they have a lot of possible options which can be pursued, but somehow these options are narrowed down to a smaller set based on the existing laws and morality because not all options are plausible.

Board games are the same, in order to win the game players need thought to filter their strategy, this thought process is a typical convergence process [7]. Therefore we refer to the thought process as the resistance force in game process and we redefine the formula for force as follows [5].

$$\mathbf{F - f = ma} \tag{3}$$

In Eq. (3), $F = m \times R^2$, $a = {r_0}^2 = (\frac{1}{D})^2$. Consider chance element and skill element, while $B = b$ in Fig. 1, we have $a_b = r^2 = (\frac{b}{D})^2$. By substituting all the values in Eq. (3), separately for skill and chance we obtain the set of Eq. (4).

$$\begin{cases} f_s = F_s - m \times r^2 = m \times R^2 - m \times r^2 \\ f_c = F_c - m \times {r_0}^2 = m \times r^2 - m \times {r_0}^2 \end{cases} \tag{4}$$

Therefore, the total resistance force will be the sum of equations in Eq. (4), which is described as $f = F - m \times a = f_c + f_s$.
By the theorem of impulse, we have the following formula.

$$J = \int_{t1}^{t2} f dt = f \Delta t = mv$$

For game refinement theory we have $v = \frac{f \Delta t}{m}$, while the F is replaced by F_s (the force by skill) and F_c (the force by chance), the formula will be changed as follows.

$$f \Delta t = (f_s + f_c) \Delta t = mv$$

In anytime for one deterministic game, "mass" will not be changed, therefore in any time duration Δt, we have

$$v = \frac{(f_s + f_c)\Delta t}{m} = \frac{f_s \Delta t}{m} + \frac{f_c \Delta t}{m} = v_s + v_c$$

Then according to Fig. 2, for the v_c part, the "attractiveness theory" will be obtained. In game refinement theory, the v is $\frac{B}{D}$; while v replaced by $v_c + v_s$, the refinement theory will be $\frac{B}{D} = \frac{B_c + B_s}{D} = \frac{B_c}{D} + \frac{B_s}{D}$. B_c means the branching factor which was controlled by chance element in game and B_s means the branching factor which was controlled by player's skill. Then we call the B_c as a new parameter character $B' + 1$, which changes the "branching factor" into "the branching factor which players cannot control or predict", "1" means the final determined movement. Then we can apply the likeness game refinement theory as shown in Eq. (5). In Eq. (5), B' means "attractive branching factor", D means "Depth" of game.

$$A = \frac{\sqrt{B'}}{D} = \frac{\sqrt{B_c - 1}}{D} = \frac{\sqrt{b - 1}}{D} \tag{5}$$

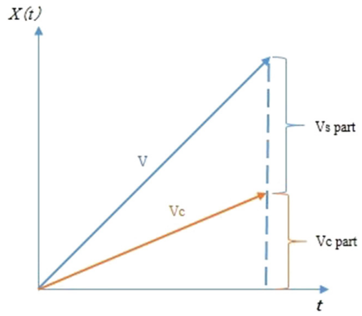

Fig. 2. The speed of attractiveness

4 Application of Attractiveness Theory

We consider three situations: the traditional board games; the games in which information cannot be observed in-game time (non-stochastic incomplete information game); the games in which only stochastic events occur (stochastic complete information game), then we choose Shogi (Japanese Chess) and StarCraft II HOS version as the benchmark for our mathematical model.

From the master's point of view, there are only a few plausible candidates to play at each position in chess [7]. In this study we assume that it may be equal to $\log_3 B$ in the sophisticated boardgames such Shogi and Go. Using this assumption we have the following conclusion. In Shogi the game attractiveness value equals to $A = \frac{\sqrt{B_c - 1}}{D} = \frac{\sqrt{4-1}}{115} = 0.015$, whereas the game refinement value is 0.078.

The attractiveness scaling factor of any game is given by Eq. (6).

$$P_{asf} = \frac{Game\ attractiveness\ value}{Game\ refinement\ value} \tag{6}$$

So, the attractiveness scaling factor of Shogi is 19.3 %. Similarly, we can calculate the attractiveness value for StarCraft II HOS version as shown in Table 2 [8]. The higher the attractiveness value of the game, the larger the population of fans and players the game will attract.

Table 2. The three properties of different game

Game	R value	A value	P_{asf}
Shogi	0.078	0.015	19.3 %
StarCtaft II	0.0695	0.0266–0.0423	38.27 %–60.86 %

5 Conclusion

In this paper, according to the Newton's classical mechanics we have found the meaning of Mass and Force in games. Then based on the theorem of impulse, we educed the property, concept and value of "attractiveness" to explain the sophistication-population paradox put forth by the game refinement value. We introduced the concept of "attractiveness scaling factor", which can be used to show whether a game is compatible for a novice or a weaker player or not. Higher the attractiveness scaling factor more chance a weaker player will have of winning the game. The lower attractiveness factor means the game will shift towards a more skill based game outcome and hence there will be less chance for novice or weaker player to win the game. Usually a high attractiveness scaling factor of a game will attract more fans and players population. This as a result explains as to why Shogi and Go even after having a really high game refinement value and sophistication still attract a much smaller fans and player population when compared to the more popular games of similar game refinement value such as soccer.

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