

Chapter 6

Hydrodynamics of WECs

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6.1 Introduction

In this chapter we look at the fundamental principles of wave absorption, and of forces on floating bodies. The goal is to build an understanding of the main physical effects involved when trying to extract power from ocean waves.

6.1.1 Wave Energy Absorption is Wave Interference

Imagine a wave travelling in an open ocean area. It carries a certain amount of power. Then put a wave energy absorber in that area under influence of the same incident wave. If there is less energy travelling the ocean after you put the wave energy device there, it means that the device has absorbed energy!

Absorption of energy from gravity waves on the ocean follows the same basic principles as absorption of other types of waves such as electromagnetic waves (for instance radio and telecommunication) and sound. Wave energy absorption should primarily be understood as wave interference: In order to absorb a wave, the wave energy device must generate a “counter-wave” to interfere with the incident wave. If the interference is *destructive* (which in this context is positive!), such that the wave in the ocean is reduced, wave energy is absorbed by the device. This fundamental relation may be formulated in the following way: “A good wave absorber is a good wave maker” [1].

Figure 6.1 illustrates how the wave reflected at a vertical wall (left) may be cancelled by proper wave generation (right). In order to obtain cancellation by destructive

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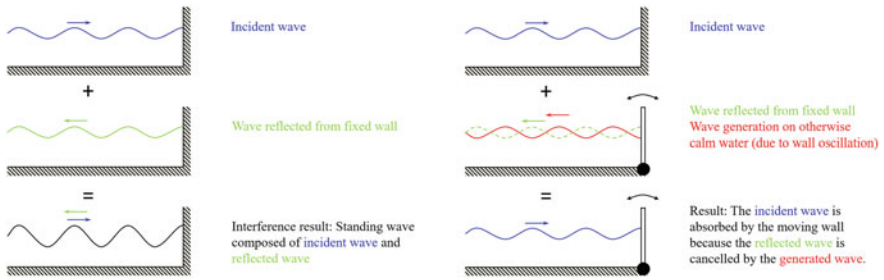


Fig. 6.1 Wave energy absorption as wave interference (Reproduced from lecture notes by Johannes Falnes and Jørgen Hals Todalshaug)

interference between the reflected and the generated wave, it is important that both the phase (timing) and the amplitude (strength) of the generated wave are chosen properly. This is crucial for any wave energy device, but the phase and amplitude may not always be easily controllable. See Sect. 6.1.8 for further treatment of this issue.

6.1.2 Hydrostatics: Buoyancy and Stability

Rigid-body motions are usually decomposed in six modes as illustrated in Fig. 6.2.

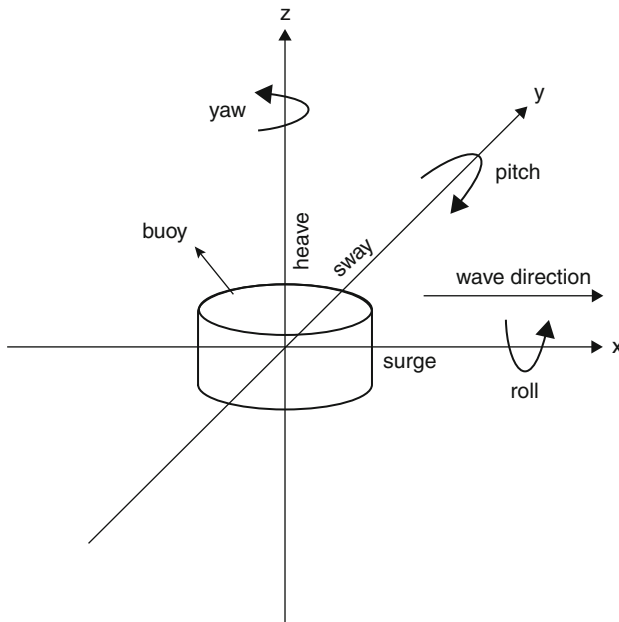


Fig. 6.2 The motion of a floating body is decomposed in translation: surge (1), sway (2) and heave (3); and rotation: roll (4), pitch (5) and yaw (6). The numbers in parenthesis are often used as index for each of these modes

In order to float steadily, a floating body must have large enough volume compared to the mass, and be hydrostatically stable for rotations in roll and pitch. It is hydrostatically stable if the sum of the gravity force and the buoyancy force gives a positive *righting moment*; a net moment working to bring the body back to hydrostatic equilibrium in case of disturbances, see Fig. 6.3. The stability may be characterised by a GZ curve, which gives the moment arm as function of the tilt angle α as shown in Fig. 6.4.

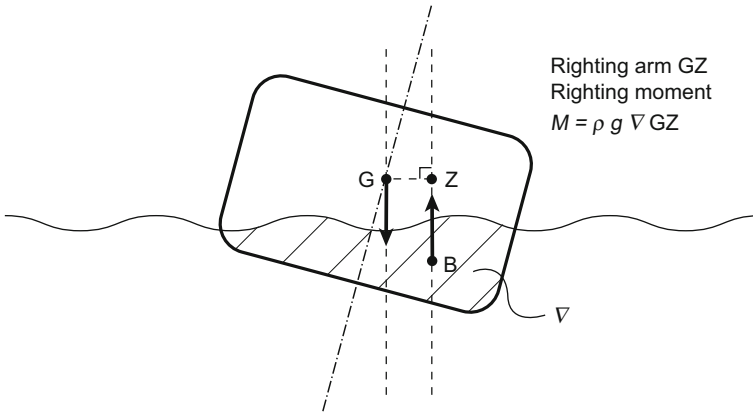


Fig. 6.3 Righting force $\rho g \nabla$

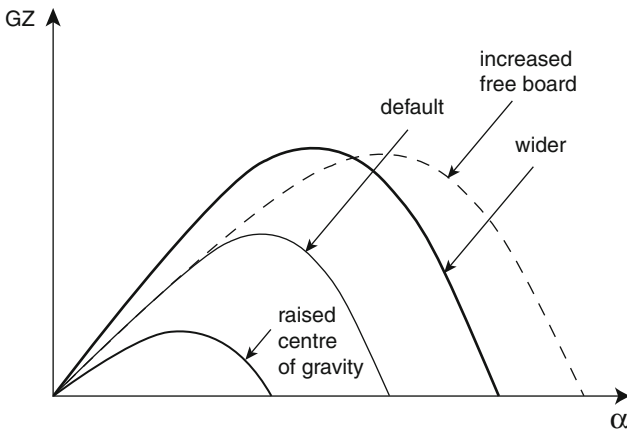


Fig. 6.4 GZ curve and how it is influenced by changes in geometry. Inspired by [2]

The buoyancy force on a partly or fully submerged body is found as

$$F_b = \rho g \nabla$$

the product of water density ρ , acceleration of gravity g and submerged volume ∇ . The centre of buoyancy is found at the centroid of the submerged volume.

For small tilt angles, the righting moment depends on two factors:

- The vertical distance between the centre of mass and centre of buoyancy
- The water-plane area (both its size and distribution)

This means that a floating body may be made more stable by either lowering the centre of mass, raising the centre of buoyancy or by increasing the water-plane area. This will increase the slope of the GZ curve at tilt angle $\alpha = 0$. This slope defines the *hydrostatic stiffness coefficient* for rotation in roll or pitch, which is the increase in righting moment per change in tilt angle:

$$S_{4,5} = \rho g \nabla dGZ_{4,5}/d\alpha$$

If the body is axi-symmetric, the GZ curves for roll (mode 4) and pitch (mode 5) are equal.

The restoring force $F_3 = S_3 \eta_3$ for vertical motion η_3 may be defined by the hydrostatic stiffness coefficient in heave. It only depends on the water plane area A_{wp} :

$$S_3 = \rho g A_{wp}$$

Depending on the shape and how the axis of rotation is chosen, an excursion in heave might induce a rotation in roll or pitch (or both). This may be described by a stiffness coupling term that tells how large the pitching torque will be for a given heave excursion:

$$\tau_5 = S_{53} \eta_3 \text{ with } S_{53} = -\rho g \iint_{A_{wp}} x \cdot dS,$$

where the integral gives the first moment of area about the axis of rotation, and correspondingly for roll, with the only difference that the horizontal off-axis distance x (surge direction) needs to be replaced by $-y$ (negative sway direction). The rotational stiffness property is symmetric, $S_{53} = S_{35}$, such that the following is also then true: $\tau_3 = S_{35} \eta_5 = S_{53} \eta_5$. Further details on hydrostatics for floating bodies may be found in textbooks such as [3, 4].

6.1.3 Hydrodynamic Forces and Body Motions

Hydrodynamics is the theory about forces on and motion of fixed and floating bodies in moving fluids.

A wave energy converter typically experiences the following external loads:

- Gravity
- Buoyancy
- Excitation from incident and diffracted waves
- Wave radiation (forces due to generated waves)
- Machinery forces (PTO force incl. friction)
- Drag: Form drag and skin friction
- Wave drift forces
- Current forces
- Mooring forces

Its inertia governs how the floating body responds to these loads.

The effects of gravity and buoyancy are treated in the above section about hydrostatics.

In principle, the most important forces from the fluid are pressure forces arising due to incident waves and body motions. It is standard practice to divide these pressure forces in *excitation* and *radiation* forces based on a linearised (and thus simplified) description of the problem. Drift forces and form drag must then often be included as corrections in order to yield a sufficiently accurate description of the system behaviour. Skin friction may usually be neglected in wave energy problems.

Mathematically, the motion induced by the combination of these forces may be described by the following equation of motion,

$$(m + m_r(\omega))\ddot{\eta}_i + R_r(\omega)\dot{\eta}_i + S\eta_i = F_e + F_{PTO}$$

For simplicity, we have here assumed a regular wave input of angular frequency ω , motion in only one mode i , and disregarded drag, wave drift, current and moorings. The symbols in the equation refer to:

- η_i —position in mode i , with time derivatives $\dot{\eta}_i$ (velocity) and $\ddot{\eta}_i$ (acceleration)
- m —body mass
- $m_r = m_{r,ii}$ —added mass
- $R_r = R_{r,ii}$ —radiation damping (due to generated waves)
- $S = S_{ii}$ —hydrostatic stiffness; the combined effect of gravity and buoyancy
- $F_e = F_{e,i}$ —excitation force
- F_{PTO} —powertake-off (PTO) force, including losses

The equation above is that of a damped harmonic oscillator, such as a mass-spring-dashpot system, forced by an applied load. In our case the applied load is the combination of wave excitation and machinery load. In the following the different forces are explained more in detail without going into mathematical descriptions. Extensive treatment of hydrodynamic theory may be found in textbooks such as [3, 5, 6].

6.1.3.1 Excitation and Radiation Forces—Added Mass and Radiation Resistance

Keeping drag forces apart, the excitation forces are those felt by the body when kept fixed in incoming waves, whereas radiation forces are those felt by the body when moved in otherwise calm water.

The excitation force is found from the hydrodynamic pressure in the sum of incident and scattered waves. If the body is small compared to the wave length, scattering may sometimes be neglected, such that a rough approximation of the excitation force may be found considering only the pressure in the undisturbed incident wave. An improved approximation may be found by use of the so-called *small-body approximation*, or other, that includes a simplified representation of the force produced by the wave scattering.

Forces arising due to body motions are usually referred to as radiation forces. It is common to divide these forces in added-mass forces, proportional to body acceleration, and wave damping forces, proportional to body velocity. The wave damping may also be referred to as radiation resistance.

Physically, the added mass force may be pictured as an inertia force relating to the mass of water entrained with the body motion. It is important to realise that we do not speak of a fixed amount of water—In principle, all of the water is influenced by the motion of a floating body. The added mass coefficient is rather an equivalent quantity telling how large the fluid inertia force becomes when the body is accelerated. When averaged over time, there is no net power flow between the body and the fluid due to the added mass force.

The radiation resistance (or wave damping) force, on the other hand, is closely linked to the average power exchanged between the sea and the body. This force arises due to outgoing waves generated when the body moves. The radiation resistance coefficient tells how large the waves will be. As these are the waves that interfere with the incoming waves, the radiation resistance also indirectly tells how much power we can extract from the incoming waves. As you may understand, this makes the radiation resistance a very important parameter for wave energy extraction.

Unfortunately, in hydrodynamics, both the added mass coefficient and the radiation resistance coefficient depend on the frequency of oscillation, as indicated by the frequency argument in the equation of motion given above. This makes both modelling and optimisation of converters more challenging than it would otherwise

be. The optimal device parameters and machinery settings depend on the wave frequency, which may be constantly changing!

There is a relation between the radiation damping and excitation forces, as both are measures of how strongly linked the body is to the wave field at sea: A body that is able to radiate waves in one direction when moved will experience excitation when acted upon by incident waves coming *from* that direction.

6.1.3.2 Machinery Forces

The machinery (PTO) forces are what separate wave energy converting systems from conventional marine structures. Hydrodynamically it does not matter how the machinery forces are produced. The force must be applied between the wave-absorbing body and a body fixed to the shore or to the sea bottom, or alternatively it may be applied between two floating bodies.

A very common assumption is that the machinery behaves like a linear damper where the force is proportional to velocity. This makes mathematical modelling easy, but there is, however, no general advantage in doing so in practice. What matters is how much and when in the oscillation cycle energy is extracted from the mechanical system by the machinery. See Sect. 6.1.8 for further details.

6.1.3.3 Drag Forces

Drag forces on a floating body mainly stem from vortex shedding when water flows past the body surface, or past mooring lines or other submerged parts of the system. As such, the drag force originates from a loss of kinetic energy. In general, the form drag forces increase quadratically with the relative flow velocity between body and water. If a wave energy converter is made so as to avoid drag losses when operating in normal-sized waves, as suggested in the section on design below, drag forces may still become important in high sea states and storm conditions due to this quadratic relationship.

The scaling of drag forces between model scale and prototype scale is not straight-forward. The flow regime depends on the scale parameter, such that geometric scaling of drag forces may not be applied directly when translating between small-scale experiments and full-scale testing. If, however, it can be established that the drag forces are due to vortex shedding around corners and they are also of secondary importance relative to other loads, geometric scaling may be expected to be a good approximation for the overall system [7].

6.1.3.4 Wave Drift, Current and Mooring Forces

Finally, we have forces that usually give the effect of slow low-frequent excitation and response, namely wave drift forces and mooring forces.

Wave drift forces are due to non-symmetric wave loading on bodies, to interaction between waves of different wave period and to interaction between the wave oscillation and oscillation of the body itself. These effects give a constant or low-frequent excitation of the system. This excitation may become important if the waves are large, or if its period of oscillation coincides with resonance frequencies introduced by the mooring system.

In some locations, tidal and ocean currents give significant forces on the wave energy converters.

Slack-line mooring systems are usually designed to provide a positioning force to counteract the horizontal wave drift and current forces, whereas taut-line systems may additionally counteract wave-frequent excitation in one or more modes. Unless the mooring lines are used as force reference for the energy conversion, the mooring system is usually designed to give as little influence on the wave absorption process as possible. Low-frequent resonance in the mooring system may be detrimental for the lines in storm conditions or even in normal operating conditions if not properly designed. (See Chap. 7 on mooring)

6.1.4 Resonance

Resonance occurs when a system is forced with an oscillation period close or equal to the system's own natural period of oscillation. Such a resonance period exists if the system has both stiffness and inertia. Thus, for freely floating bodies we have resonance periods for heave, roll and pitch modes. With mooring lines connected we may in addition have stiffness in surge, sway and yaw, giving rise to resonance also in these modes of motion.

The analysis of harmonic oscillators such as described by the equation of motion above tells us that the system will resonate at the period of oscillation where the reactance of the system is zero, $\omega_0(m + m_r) - \frac{S}{\omega_0} = 0$. This happens when the potential energy storage and the kinetic energy storage of the system are of equal size [5]. Solving this equation gives a resonance period of

$$T_0 = 2\pi/\omega_0 = 2\pi\sqrt{(m + m_r)/S}$$

The relative bandwidth of the system is given by

$$\frac{\Delta\omega_{res}}{\omega_0} = \frac{R_r}{\sqrt{S(m + m_r)}}$$

For modes of motion with no or low stiffness ($S \rightarrow 0$) the relative bandwidth automatically becomes very large. The relative bandwidth is a measure of how strongly the system responds to inputs of frequencies other than the resonance

period. Because ocean waves come with varying wave frequency, this is an important property for a wave energy converter. We will return to this subject later.

A useful approximation for the heave resonance period of a freely floating body may be derived if we assume that the cross-section of the buoy is fairly round and relatively constant with depth, such that the heave added mass m_r may be estimated from the width d of the body:

$$T_{0,3} \approx 2\pi\sqrt{(l+d/3)/g}$$

Here l is the draft of the freely floating body.

When a system is resonating with the incident waves, it means that motions tend to be amplified, resulting in large accelerations and forces. For this reason resonance is usually avoided in conventional naval architecture. A wave energy converter, on the other hand, may have to operate at or close to resonance in order to obtain a sufficient power conversion, which is dictated by the phase alignment between excitation force and body velocity. At resonance these are aligned. See also Sect. 6.1.8.

6.1.5 *Oscillating Water Columns—Comments on Resonance Properties and Modelling*

A simplified model to understand the dynamics of the oscillation water column (OWC) would be to think of it as an internal oscillating body of mass equal to the mass of the water in the column. In analogy with the expression for heave buoys, the resonance period may then be estimated roughly as

$$T_{0,owc} \approx 2\pi\sqrt{l_c/g}$$

assuming that the cross-sectional area of the column is fairly constant, and where l_c is measured along the centerline of the column from the inlet to the internal free surface as illustrated in Fig. 6.5. If the column is inclined, as in Fig. 6.6, the denominator must be replaced by $g \cos(\theta)$, where θ is the slope angle relative to the vertical. The absorption bandwidth of water columns may be increased by making a harbour-like construction at the inlet, with side walls reaching out towards the sea [8].

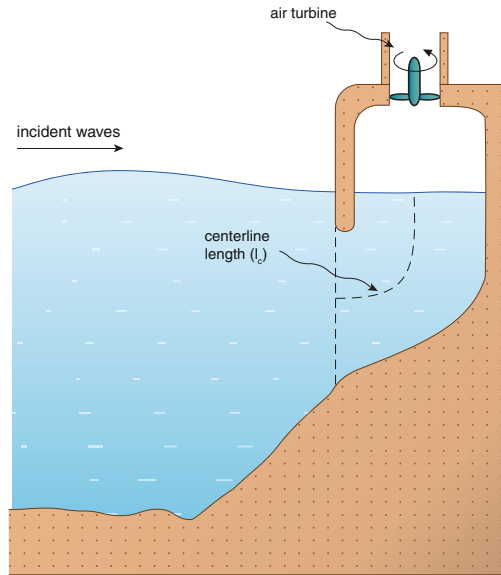


Fig. 6.5 Oscillating water column where the length l_c is measured along the centreline

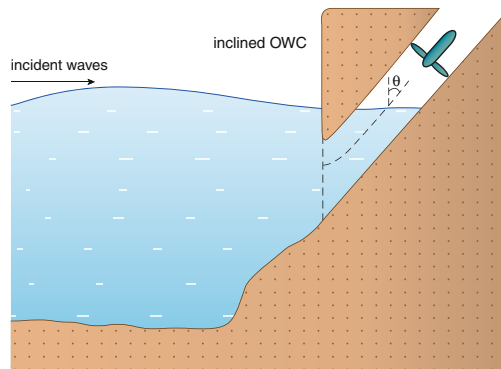


Fig. 6.6 Oscillating water column inclined at an angle θ

For mathematical modelling of OWCs, a simplified model could be established by applying the same rigid-body representation as used for floating bodies, where the free surface of the water column is thought of as a rigid lid [9]. A more accurate representation could be made by taking the free surface and chamber pressure into account [5, 10].

For oscillating water columns (OWCs), the chamber gas volume gives a compressibility effect that may be important. In model tests of OWC devices, it is

crucial to remember that this compressibility does not scale geometrically: In model scale the chamber must be represented by an increased volume in order to give a representative stiffness force. The scaling factor for the chamber volume should be taken as N_L^2 , where N_L is the length scaling factor. Thus for a model at 1:10 scale the chamber volume should be made equal to 1/100 of the full-scale volume, rather than 1/1000. Although scaled, the compressibility effect of the model will only be correct for small column excursions.

6.1.6 Hydrodynamic Design of a Wave Energy Converter

From the above discussion on waves, wave excitation and radiation we find that, in order to be suitable for absorbing waves, a body must have a shape, size, placement and motion that gives considerable outgoing waves when moved. In the following we will discuss general guidelines for wave energy converter design.

6.1.6.1 Size and Shape

The first rule of thumb for wave-absorbing bodies and water columns is that corners should be rounded. Sharp edges will induce drag and viscous losses that normally subtracts directly from the power available for conversion. If corners can be made with radius of curvature larger than or about equal to the stroke of local water particle motion, the viscous losses are usually negligible [1, 11]. The design should be such that this is the case for average waves at the site of operation.

When we talk about the size of buoys in general, it should be understood as their horizontal extension relative to the predominant wavelength λ unless otherwise specified. The following differentiation may be applied:

- Less than $\lambda/6$: small body
- Between $\lambda/6$ and $\lambda/2$: medium-sized body
- Larger than $\lambda/2$: large body

For small buoys the shape does not matter much as long as viscous losses are avoided: In terms of wave radiation pattern, small buoys will behave similar to an axisymmetric body whatever the shape. This is because its wave radiation may be approximated by that of a point source, or pair of point sources. What matters for such buoys is the available volume stroke. In average, the power that can be absorbed will roughly be proportional to the available volume stroke. The size of the buoy should then preferably be chosen large enough to absorb a substantial part of the available power, but small enough to work on full stroke in normal-sized waves. A Budal diagram can be useful in finding a suitable buoy size for a given location, see the Sect. 1.1.7 for further discussion on these volume stroke and the Budal diagram.

Larger buoys that are not axisymmetric become directional, in the sense that the radiated wave field will differ from that of an axisymmetric body (also discussed below). Increased hydrodynamic efficiency can then be achieved by shaping the buoy such as to generate waves along the predominant wave direction. This is the working principle of a terminator device, which is illustrated in Fig. 6.7. As long as the terminator device is made up of a series of units such as paddles, OWC chambers or similar, there is no obvious limit to its useful size.

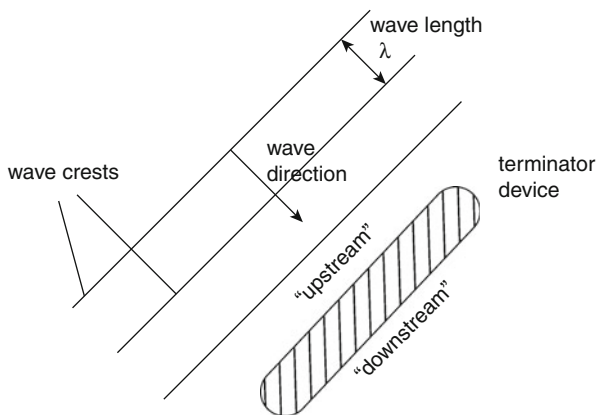


Fig. 6.7 Terminator device. The longest horizontal extension is parallel to the wave crests of waves in the predominant direction

In analogy with the reflecting wall example above (Fig. 6.1): if the body is so large and deep that it reflects most of the incident waves, its motion should induce waves travelling upstream to cancel the reflected waves. On the other side—for bodies that are almost transparent to the incident waves, only waves travelling in the downstream direction need to be generated in order to absorb energy. In practice, we usually have a combination of these two cases.

For axisymmetric bodies, the wave excitation typically increases strongly with width up to an extension of around $\lambda/2$. For bodies beyond this size the increase in size is not paid off by an increase in excitation. This is due to opposing forces over the body surface, making such large bodies less hydrodynamically efficient than smaller bodies. In principle, the same wave radiation pattern as generated by large bodies may be achieved with a number of small bodies placed in a matrix layout, or in a line layout where the each body oscillates in heave and (at least) on more mode of motion.

Whether small or large, the part of the body that is to give the excitation must be found close to the sea surface. Bodies that are placed deep in the water, or which do not have considerable body surface area close to the sea surface, will not be able to absorb much wave energy. This follows directly from how the water moves in a passing wave, with orbital motion (and corresponding dynamic pressure) of

decreasing amplitude as we go deeper in the water, cf. Chap. 3 (The Wave Energy Resource) and Fig. 6.8.

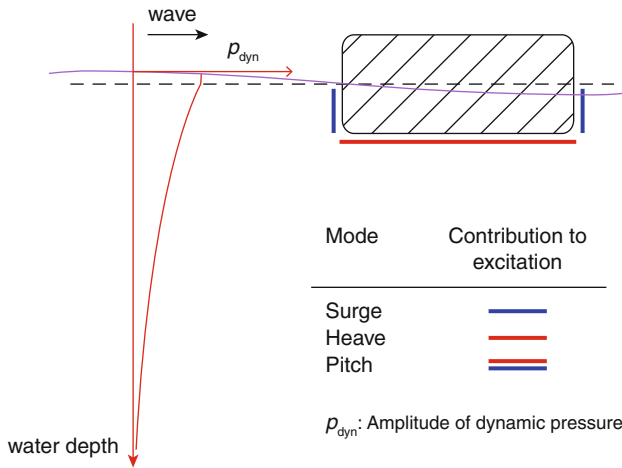


Fig. 6.8 Parts of the body surface area that contributes to surge (*vertical*), heave (*horizontal*) and pitch (both) excitation. The *curve to the left* shows the amplitude of hydrodynamic pressure versus distance to mean surface level

6.1.6.2 Heave, Surge or Pitch?

Although it is possible to convert energy also through other modes of motion, heave, surge and pitch are usually the modes considered in practice.

Heaving buoys and bodies that pitch about an axis close to the mean surface level naturally have high hydrostatic stiffness. Recalling the expression for relative bandwidth given above we see that: Unless provided with some means of reducing the stiffness or controlling the motion, such heaving or pitching systems will have quite narrow response bandwidth, which makes them hydrodynamically inefficient wave absorbers in varying irregular seas. This flaw may be mitigated by active use of the machinery through a proper control strategy, or by including mechanical components to counteract the hydrostatic stiffness.

Pitching about an axis close to the surface is less volumetric efficient than surging when it comes to absorbing power [12]. This is due to the fact that such pitch motion gets its excitation from an area distributed along the direction of propagation for the wave, whereas the surge motion gets its excitation mainly from areas of opposing vertical walls a distance apart. This is illustrated in Figs. 6.8 and 6.9. On the other hand, it may be easier to design a practical machinery for pitching bodies than for surging bodies.

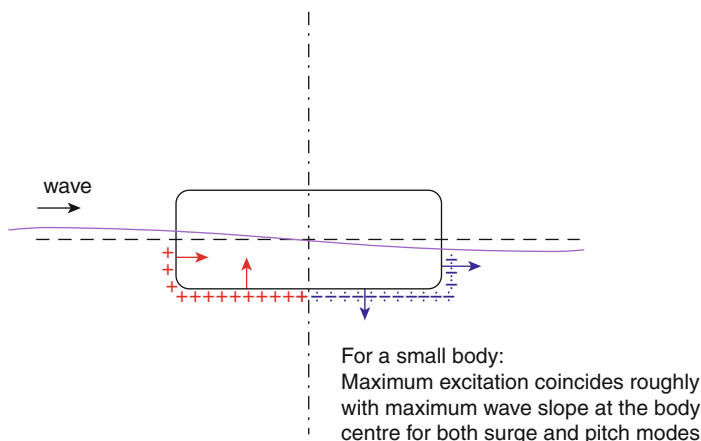


Fig. 6.9 Illustration of the hydrodynamic pressure along a body

For small bodies, heaving motion is the most volumetric efficient. This is because the heave excitation comes from a difference between atmospheric pressure at the top of the body and the full amplitude of hydrodynamic pressure at the bottom, see Fig. 6.8. Surging and pitching take their excitation from a difference in hydrodynamic pressure along the wave, which is quite weak when the body is small. For large bodies the opposite is true, such that pitching and in particular surging motion are favoured over heaving motion.

Systems combining power extraction from two or three modes of motion have the potential of giving a more efficient use of the installed structure [5].

Pitching about an axis close to the bottom is hydrodynamically similar to surging. The surge, sway and yaw modes have no restoring forces, and station-keeping forces must be supplied by moorings or other.

6.1.6.3 Some Examples

Based on the principles explained above, we may think of some examples of systems that would be hydrodynamically good for absorbing wave energy:

- Heaving vertical cylinder of relatively low-draft
- Oscillating volume of small submergence
- Surface-piercing surging flap/bottom-hinged flap
- Large surging bodies.

On the other hand, the following examples should be expected to have poor hydrodynamic performance:

- Heaving deep-draft cylinders: The volume change takes place too far from the water surface to give considerable wave radiation
- Submerged rigid bodies at considerable depth: Same reason as above
- Large heaving bodies, of diameter larger than about $\lambda/3$. Increasing the diameter above this size would increase costs but only weakly increase the excitation.
- Small pitching bodies
- Submerged surging flap

It should be emphasized that the hydrodynamically best-performing system does not necessarily provide the lowest cost of delivered energy.

6.1.6.4 Comments on Alternative Principles of Power Extraction

The discussion in this chapter has focused on the hydrodynamics of oscillating rigid bodies and also touched upon oscillating water columns. There are other ways of extracting power from ocean waves that may show to be worthwhile. These include:

- **Flexible bodies**, for example in the form of flexible bags (cf. the Lancaster flexible bag device) or tubes (cf. the Anaconda device). These interact with the incident waves through an oscillating volume, and in that sense have features in common with heaving semi-submerged buoys.
- **Overtopping devices**, for example designed as ramps or tapered channels. Their relation to oscillating bodies and the wave interference description is somewhat more obscure, although the overtopping may be seen as a local change of fluid volume that would generate waves to interfere with the incident waves. Hydrodynamically, their wave absorption is usually treated as a wave kinematics problem.
- **Hydrofoils**, or other devices that produce pressure differences over slender members. The principle may typically be to install a system of moving hydrofoils that are relatively transparent to the incident waves, but that is used to radiate a wave field to partially or fully cancel the transmitted waves. Such hydrofoils have been successfully used to propel vessels on wave power [13].

These principles of extraction will not be discussed any further here.

6.1.7 Power Estimates and Limits to the Absorbed Power

The gross power absorbed from the sea can generally be estimated in two ways:

1. By considering the incident and resultant wave fields. The difference in wave energy transport by the two tells how much power has been absorbed.
2. By calculating the product of machinery force and velocity.

This requires a quite elaborate modelling of the wave-absorbing system or extensive experimental campaigns. Simplified and rough estimates of the average absorbed power may be found:

- From experience data based on similar systems. A commonly used measure is the *relative absorption width* or *length*, also referred to as the *capture width ratio*, where experience indicates that values between 0.2 and 0.5 are typical across the wide range of converter designs proposed [14]. It is expected that somewhat higher numbers can be reached with improved conversion systems combined with efficient control algorithms as these are in general both still at a low level of maturity. Average capture width ratios higher than 1.0 have been shown to be realistic for operation of point absorbers in irregular waves [15].
- From theoretical upper bounds on the power that can be absorbed by oscillating bodies. These will be treated in the following.

Firstly, the power is *limited by the radiation pattern* for waves generated by the oscillating system. It is useful to look at this limit for two idealised cases: (i) An *axisymmetric* body, which is symmetric about the vertical axis, and (ii) a large body of width comparable to or larger than the wavelength, often referred to as a *terminator* device. The first type will radiate circular waves when oscillating in heave, and dipole-pattern waves when oscillating in surge or pitch. The second type will radiate plane waves over a limited width, see Fig. 6.10 for illustrations. These properties result in the following limits to the power that can be absorbed [5]:

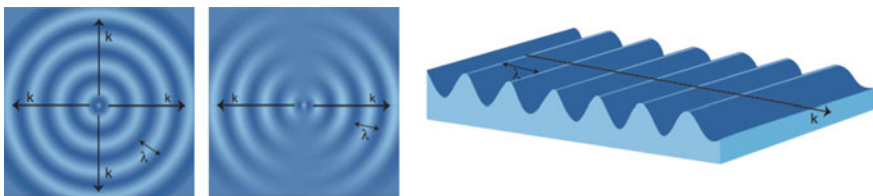


Fig. 6.10 Illustration of radiation patterns: source (*left*), dipole (*middle*) and terminator (*right*) patterns. Reproduced from [16] by courtesy of Elisabet Jansson

- (i) Axisymmetric body: $\bar{P} \leq \alpha J \frac{\lambda}{2\pi}$. This expression implies proportionality to wave period cubed and wave height squared. The parameter α is 1 for heave oscillation and 2 for oscillation in surge or pitch.

- (ii) Terminator body: $\bar{P} \leq Jd$. This expression implies proportionality to wave period wave height squared.

Here, J [W/m] is the wave energy transport, λ is the wave length and d is the width of the terminator body or device.

As mentioned, small bodies behave as axisymmetric bodies over the range of wave periods where the size is small compared to the wavelength.

Secondly, there will always be *limits on the available stroke*. These can be caused by the finite volume of the buoys used, or by limits on the stroke of the machinery, often referred to as *amplitude restrictions*. The stroke limitation leads to an upper bound on the average absorbed that is proportional to the wave height H . The bound further depends on the mode of oscillation [12]:

- For heave mode, $\bar{P} < \frac{\pi}{4} \rho g V_s \frac{H}{T}$, often referred to as *Budal's upper bound*.
- For surge mode, $\bar{P} < 2 \pi^3 \rho V_s \frac{H}{T^3} l$
- For pitch mode, $\bar{P} < \frac{2}{3} \pi^3 \rho V_s \frac{H}{T^3} l$

Here V_s is the available volume stroke (illustrated in Fig. 6.11), T is the wave

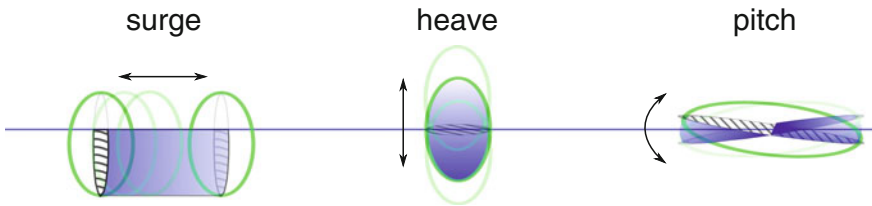


Fig. 6.11 Illustration of the available volume stroke for the different modes of motion. For heave motion, the volume stroke is the body volume itself if not limited by the stroke of the machinery. For surge and pitch motion, the machinery stroke usually sets the limit for the volume stroke

period and l is the length of the device along the direction of wave propagation.

As seen, this second upper bound is inversely proportional to the wave period for heave motion, and to the wave period cubed for surge and pitch motion.

The graphs in Fig. 6.12 illustrate these limitations to the maximum absorbed power. We may refer to these as “Budal diagrams”.

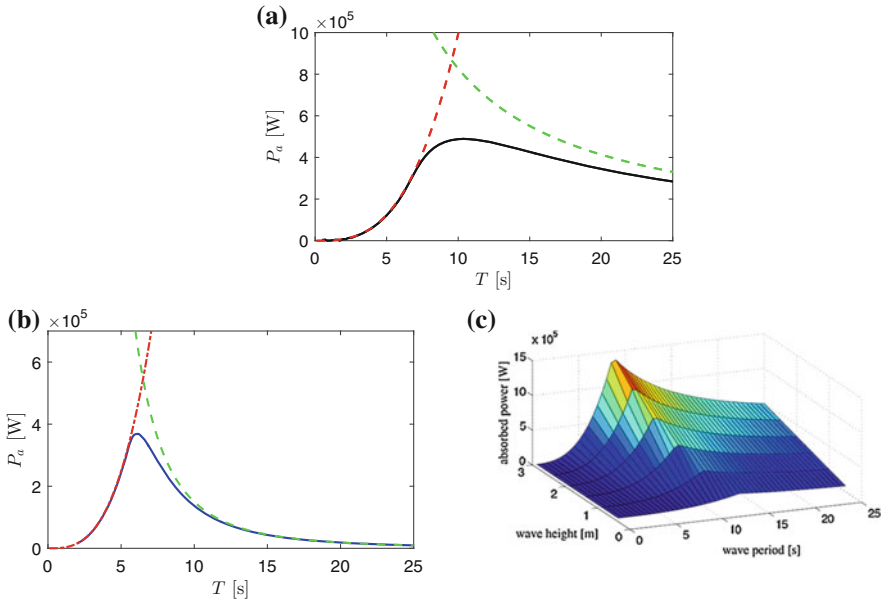


Fig. 6.12 Upper bounds for a heaving semi-submerged sphere (a), and for a surging semisubmerged sphere (b), both of radius 5 m, and with an incident wave amplitude of 1.0 m. Fully drawn lines shows the absorbed power curves for an optimally controlled buoy. Budal diagram for the heaving sphere extended to also include variation in the wave height H (c). All the diagrams are based on a stroke limit of ± 3 m

Estimates for the delivered energy from a power plant should always include losses introduced by the power conversion equipment. These will depend strongly on the type of machinery used. It must also be remembered that, in order to approach the limits described above, the motion of the buoys need to be close to optimal. This is the topic of the next section.

6.1.8 Controlled Motion and Maximisation of Output Power

The energy absorbed from a wave by an oscillating body only depends on the motion of the hull relative to the wave. Imagine that you could force the body to move exactly as you wanted. For maximum power extraction, there would be an optimum motion path (position and velocity) that you should try to follow, and that would obviously depend on the incident wave. We may call this the *optimum trajectory* in space and time given the incoming wave.

When designing the wave energy converter system, we should try to make the system such that its response to incoming waves is close to the optimum trajectory. The response is governed by the combination of inertia, stiffness, damping and

machinery (PTO) forces. This means that the response may be improved either by the design of the buoy (inertia, stiffness and wave damping), or by using the machinery to get closer to the optimum trajectory. In practice we often use a combination of the two.

Speaking in terms of regular waves, the optimum trajectory for a single-mode absorber may be specified in terms of relative phase and relative amplitude between the wave and the buoy oscillation: The velocity should be in phase with the excitation force, and the amplitude should be adjusted to give the correct interference with the incident wave. The amplitude may be adjusted by changing the damping applied by the machinery.

If in resonance with the incident wave, and with a correctly adjusted machinery loading, a single-mode absorber will automatically obtain the optimum trajectory in the unconstrained case. This is because, at resonance, the optimum phase condition is fulfilled. This does not mean, however, that the buoy needs to be in resonance to perform well. What matters is the phase, or timing, of the motion relative to the incident wave. Systems that are designed to have a large response bandwidth will perform well also off the resonance period. Wave energy converters based on hydro-mechanical systems with low stiffness, such as surging buoys and flaps, will inherently have a large response bandwidth. Others must include mechanical solutions or control strategies to widen the response bandwidth.

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