Structural Brain Mapping

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Abstract. Brain mapping plays an important role in neuroscience and medical imaging fields, which flattens the convoluted brain cortical surface and exposes the hidden geometry details onto a canonical domain. Existing methods such as conformal mappings didn't consider the anatomical atlas network structure, and the anatomical landmarks, e.g., gyri curves, appear highly curvy on the canonical domains. Using such maps, it is difficult to recognize the connecting pattern and compare the atlases. In this work, we present a novel brain mapping method to efficiently visualize the convoluted and partially invisible cortical surface through a well-structured view, called the *structural brain mapping*. In computation, the brain atlas network ("node" - the junction of anatomical cortical regions, "edge" - the connecting curve between cortical regions) is first mapped to a planar straight line graph based on Tutte graph embedding, where all the edges are crossing-free and all the faces are convex polygons; the brain surface is then mapped to the convex shape domain based on harmonic map with linear constraints. Experiments on two brain MRI databases, including 250 scans with automatic atlases processed by FreeSurfer and 40 scans with manual atlases from LPBA40, demonstrate the efficiency and efficacy of the algorithm and the practicability for visualizing and comparing brain cortical anatomical structures.

1 Introduction

Brain mapping was introduced to map the genus zero 3D brain cortical surface (usually brain hemisphere) onto a unit sphere or a planar canonical domain (e.g., a unit disk, a rectangle domain), so that the convoluted and invisible cortical folds are flattened and the geometric details are fully exposed onto the canonical domain. A plausible category of methods is conformal mapping, which preserves angles (local shapes) and therefore is highly desired for brain morphometry study in neuroscience and medical imaging fields. It has been well studied in recent works using spherical harmonic mapping [1], Ricci curvature flow [2] and other methods [3]. Another commonly used category of methods is area-preserving brain mapping [4,5], computed based on optimal mass transportation theory.

Brain anatomical landmarks including gyri and sulci curves are used to help shape registration and analysis applications. One method [2] is to slice the brain surface open along these curves, and map the new surface to a unit disk with circular holes or a hyperbolic polygon; the curves are mapped to circular holes or hyperbolic lines for generating intrinsic shape signatures. The other method [6]

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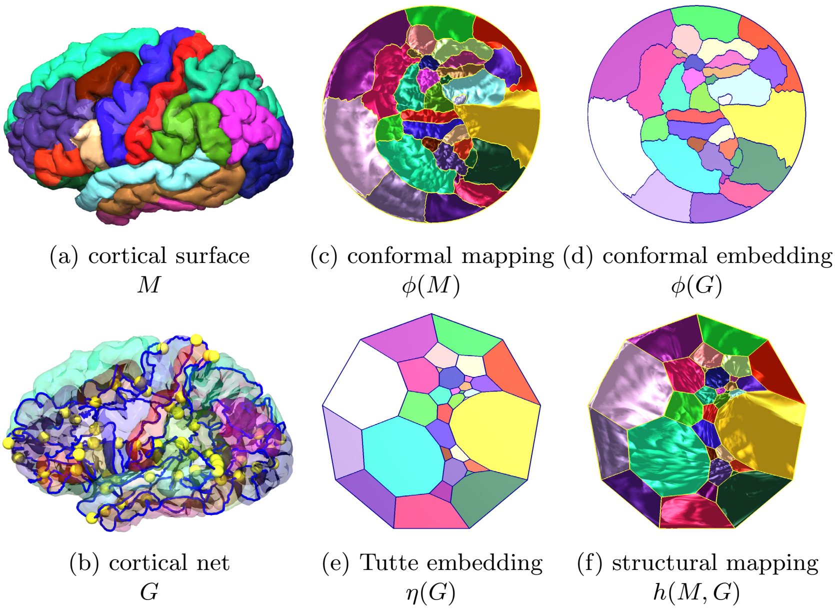
is to map the whole brain surface with interior curve straightening constraints based on holomorphic 1-form method, without changing surface topology; the interior curves are mapped to canonically-shaped segments, e.g., straight lines in a rectangle or circular arcs in a unit disk. Brain anatomical connectivity between cortical regions (atlas) is also one significant anatomical feature. To date, none of the existing methods integrate the anatomical atlas structure into the mapping, and makes the atlas well-shaped in the mapping.

Brain networks, the so-called brain graphs [7], have been intensively studied in neuroscience field. Bullmore et al. [7] gave thorough reviews and methodological guide on both structural and functional brain network analysis. In this work, we focus on brain structural network on cortical surface, i.e., cortical network. It has been used to discover the relation of its disorganization to diseases such as Alzheimer's disease [8]. One important task within this is brain network visualization and comparison. To date, it still needs a lot of efforts to explore a more perceptively straightforward and visually plausible graph drawing.

In summary, the motivation of this work is to provide a well-structured convex shape view for convoluted atlas structure, which is more accessible for reading than pure surface mapping (e.g. conformal) views with curvy landmarks and more efficient for anatomical visualization and comparison.

Brain Net. The cortical network studied in this work is *different* from the definition of structural "brain graph" in [7], where the node denotes the cortical region, the edge denotes the connectivity between two cortical regions, and it is completely a topological graph. In this work, we define the node as the junction of anatomical cortical regions and the edge as the common curvy boundary of two neighboring cortical regions. This anatomical graph (see Fig. 1 (b)) is embedded on the 3D genus zero cortical surface, has physically positioned nodes and *crossing-free curvy edges*, and therefore is planar in theory [9]. For simplicity and differentiation, we call it "brain net". In terms of topology, it is the "dual graph" of the brain graph in [7]. We have verified this in our experiments. In this work, brain net is used to drive a canonical surface mapping (the regions and the whole domain are convex). We call this technique *brain-net mapper*.

Approach. This work presents a novel method for brain cortical anatomical structure mapping. It employs the special properties of the anatomical brain net: 1) planar and 2) 3-connected (after testing and minor filtering). The computational strategy is to employ the planar graph embedding as guidance for *structural* brain surface mapping using constrained harmonic map. In detail, first, the 3-connected planar brain net graph is embedded onto the Euclidean plane without crossing graph edges and every face is convex based on Tutte embedding theorem [10]; then, using the obtained convex target domain with convex subdivision as constraints, a harmonic map of the brain surface is computed. The mapping is unique and diffeomorphic, which can be proved by generalizing Radó theorem [11]. The algorithm solves sparse linear systems, therefore is efficient and robust to topology and geometry noises. The resulting mapping exposes invisible topological connectivity and also details cortical surface geometry.



(a) cortical surface M (c) conformal mapping $\phi(M)$ (d) conformal embedding $\phi(G)$

(b) cortical net G (e) Tutte embedding $\eta(G)$ (f) structural mapping $h(M, G)$

Fig. 1. Brain net embeddings for brain A_1 (left hemisphere).

Figure 1 gives an example where regions are denoted in different colors (a). The brain net G (b) is mapped to a planar polygonal mesh (e), where each face is convex and assigned with the corresponding region's color. The planar representation (f) is the final map guided by (e), with *visually plausible structure*, i.e., planar straight lines and convex faces with interior surface harmonically flattened (stretches minimized). It illustrates the cortical anatomical structure (a). We call this mapping *structural brain mapping*. In contrast, conformal map (c) generates the planar graph embedding but with curvy graph edges (d).

To our best knowledge, this is the *first* work to present a structural view of brain cortical surface associated with anatomical atlas by making all anatomical regions in convex polygonal shapes and minimizing stretches. Experiments were performed on 250 brains with automatic parcellations and 40 brains with manual atlas labels to verify the 3-connected property of brain nets (anatomical connectivity) and test the efficiency and efficacy of our algorithm for brain cortical anatomical atlas visualization and further cortical structure comparison.

2 Theoretic Background

This section briefly introduces the theoretic background. For details, we refer readers to [9] for graph embedding and [11] for harmonic mappings.

Graph Embedding. In graph theory, a graph G is k-connected if it requires at least k vertices to be removed to disconnect the graph, i.e., the vertex degree of the graph $deg(G) \geq k$. A planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. Such a drawing is called the planar embedding of a graph, which maps the nodes to points on a plane and the edges to straight lines or curves on that plane without crossings.

A 3-connected planar graph has special property that it has planar crossing-free straight line embedding. Tutte (1963) [10] gave a computational solution, the classical *Tutte embedding*, where the outer face is prescribed to a convex polygon and each interior vertex is at the average (barycenter) of its neighboring positions. Tutte's spring theorem [10] guarantees that the resulting planar embedding is unique and always crossing-free, and specially, every face is convex.

Harmonic Map. Suppose a metric surface $(S, \mathbf{g})$ is a topology disk, a genus zero surface with a single boundary. By Riemann mapping theorem, S can be conformally mapped onto the complex plane, $\mathbb{D} = \{z \in \mathbb{C} | |z| < 1\}$, $\phi : S \to \mathbb{D}$, which implies $\mathbf{g} = e^{2\lambda(z)} dz d\bar{z}$, where λ is the conformal factor.

Let $f : (\mathbb{D}, |dz|^2) \to (\mathbb{D}, |dw|^2)$ be a Lipschitz map between two disks, $z = x+iy$ and $w = u + iv$ are complex parameters. The *harmonic energy* of the map is defined as $E(f) = \int_{\mathbb{D}} (|w_z|^2 + |w_{\bar{z}}|^2) dxdy$. A critical point of the harmonic energy is called a *harmonic map*, which satisfies the Laplace equation $w_{z\bar{z}} = 0$. In general, harmonic mapping is unnecessarily diffeomorphic. Radó theorem [11] states that if the restriction on the boundary is a homeomorphism, then the map from a topological disk to a convex domain is a diffeomorphism and unique.

3 Computational Algorithms

The computation steps include: 1) compute graph embedding; and 2) compute harmonic map using graph embedding constraints (see Algorithm 1).

The brain cortical surface is represented as a triangular mesh of genus zero with a single boundary (the black region is cut off), denoted as $M = (V, E, F)$, where V, E, F represent vertex, edge and face sets, respectively. Similarly, the brain net is denoted as a graph $G = (V_G, E_G, F_G)$ (3-connected and planar, embedded on M) (see Fig. 1(b)). Thus, we use (M, G) as the input.

Step 1: Isomorphic Graph Embedding. The first step is to compute a straight line convex graph embedding of G, $\eta : G \to \hat{G}$ by Tutte embedding [10]. We first place the graph nodes on boundary ∂M onto the unit circle uniformly, and then compute the mapping positions $\eta(v_i)$ for interior nodes v_i as the barycenters of the mapping positions of neighboring nodes v_j, $\{\eta(\hat{v}_i) = \Sigma_{(v_i,v_j)\in E_G} \lambda_{ij} \eta(\hat{v}_j)\}$. We use $\lambda_{ij} = 1/deg(v_i)$, where $deg(v_i)$ denotes the degree of node v_i in G. Solving the sparse linear system, we obtain the Tutte embedding result $\hat{G}$, which defines a convex planar domain Ω (see Fig. 1(e)).

Step 2: Constrained Harmonic Mapping. The second step is to compute a surface mapping $h : (M, G) \to (\Omega, \hat{G})$ to restrict graph G to the planar Tutte embedding result $\hat{G}$ by a constrained harmonic map (see Fig. 1(f)). We map the whole surface M onto the convex planar domain Ω by minimizing the discrete harmonic energy under graph constraints, formulated as $\min\{E(\phi(v_i)) = \Sigma_{[v_i,v_j]\in E} w_{ij} (\phi(v_i) -$

Algorithm 1. Graph Embedding for Surface Mapping

Input: A triangular mesh with decorative graph (M, G)

Output: A planar triangular mesh with straight line decorative graph $(\Omega, \hat{G})$

1: Compute Tutte embedding $\eta : G \to \hat{G}$

2: Compute harmonic map $\phi : (M, G) \to (\Omega, \hat{G})$ with constraints $\phi(G) = \hat{G}$

$\phi(v_j))^2, \forall v_i \in V\}$, s.t., $\phi(l_k) = \hat{l}_k, \forall l_k \in G, \hat{l}_k = \eta(l_k)$, i.e., l_k is the curvy edge of graph G, and $\hat{l}_k$ is the corresponding edge on the planar graph embedding $\hat{G}$. The solution to the harmonic energy minimization problem is equivalent to solving the linear system $\Delta\phi = 0$ (Δ is the Laplacian operator), descreterized as the linear equations $\{\Sigma_{[v_i,v_j]\in E} w_{ij}(\phi(v_i) - \phi(v_j)) = 0, \forall v_i \in V\}$.

We only specify the target positions for the two end vertices of l_k. Other interior vertices on l_k are constrained to $\hat{l}_k$ through a linear combination of two neighbors on $\hat{l}_k$. The linear constraints between coordinates x, y on straight line $\hat{l}_k$ can be plugged into the above system. We employ the mean value coordinates to guarantee the edge weight w_{ij} to be positive. Then in our construction, for each vertex there is a convex combination of neighbors. According to Tutte's spring theorem [10], Radó theorem [11] and generalized Tutte embedding [12], the solution achieves a unique and diffeomorphic surface mapping.

4 Experiments

The proposed algorithms were validated on two databases with different atlas types: 1) the own captured 250 brain MRI scans, we use FreeSurfer to automatically extract triangular cortical surfaces and anatomical atlas (see Figs. 1, 2(a)); and 2) the public 40 brains with manual atlas labels provided by LPBA40 [13] (see Fig. 2(b-c)), we use BrainSuite to correlate the triangular cortical surface with manual labels. All the brains come from human volunteers.

Brain Net Extraction. We extract the brain nets from cortical surface using anatomical region id or color assigned. To employ Tutte embedding, we then test the 3-connected property of all the brain nets using two conditions: (a) every node has ≥ 3 neighboring regions; (b) every region has ≥ 3 boundary nodes. If both are satisfied, then the brain net is 3-connected, a "good" one.

Our tests show that all brain nets satisfy condition (a). All the "bad" regions detected contain 2 nodes, i.e., 2 boundary edges, which contradicts (b). There may be (i) 1, (ii) 2, or (iii) 3 bad regions. Table 1 gives the number of brains for each above case. We use (lh, rh) to denote the percentage of left and right hemisphere brain nets satisfying both (a-b): FreeSurfer $(22.8\%, 100\%)$, LPBA40 $(12.5\%, 82.5\%)$, both $(21.4\%, 97.6\%)$. The tests give that most exception cases are with $1 \sim 2$ "bad" regions, for which we only map one boundary edge to straight line and ignore the other in next structural brain mapping procedure. If the region is totally interior, we randomly select one; if it is adjacent to the brain surface boundary, then we select the boundary one, as shown in Fig. 2(b-c).

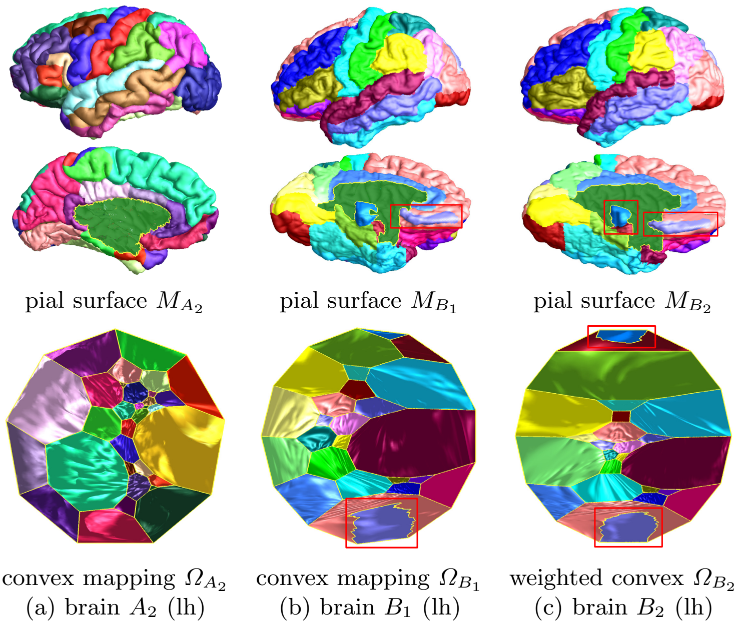
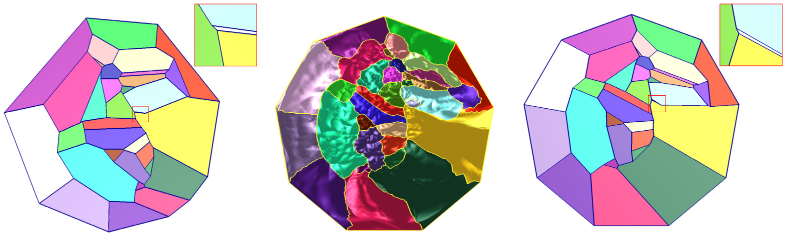


Fig. 2. Structural brain mappings driven by graph embedding.

Table 1. Statistics on brain nets, meshes and time. lh (rh) - left (right) hemisphere.

Data	FreeSurfer (lh)	FreeSurfer (rh)	LPBA40 (lh)	LPBA40 (rh)
#region,#node	33~35, 62~70	33~35, 41~52	24~28, 64~72	20~22, 39~55
#triangle,time	277k, 20 secs	279k, 20 secs	131k, 10 secs	131k, 10 secs
#good (a-b)	57	250	5	33
#bad (i/ii/iii)	193/0/0	0/0/0	24/10/1	7/0/0

Structural Brain Mapping. The algorithms were tested on a desktop with 3.7GHz CPU and 16GB RAM. The whole pipeline is automatic, stable and robust for all the tests. Table 1 gives the averaged running time. Figures 1-2 show various results, by which it is visually straightforward to figure out local topology (adjacency of regions) and tell whether two atlases are isomorphic (or topologically same); in contrast, it is hard to do so in a 3D view. Note that the polygonal shape is solely determined by the combinatorial structure of the brain net. Brains with consistent atlases are mapped to the same convex shape (see brains A_1, A_2), which fosters direct comparison. Brains B_1, B_2 from LPBA40 are with different brain nets, especially around the exception regions. Even though the unselected edges of the bad regions appear irregular, the mapping results are visually acceptable and functionally enough for discovering local and global structures and other visualization applications, such brain atlas comparison.



(a) straight line drawing of Fig. 1(c) (b) convex harmonic mapping of Fig. 1(a) (c) straight line drawing of (b)

Fig. 3. Straight line graph drawings induced by conformal and harmonic mappings.

5 Discussion

This work focuses to present a novel brain mapping framework based on Tutte embedding and harmonic map with convex planar graph constraint. For better understanding the method and its potentials, we have the discussions as follows.

Convex Shape Mapping: The cortical surface can be directly mapped to canonical domains such as conformal map to a disk [2] (Fig. 1(c)) and harmonic map to a convex domain [1] (Fig. 3(b)). Each map can define a planar straight line graph embedding (Fig. 3(a,c)) by simply connecting the nodes on the planar domain, but it may generate concave and skinny faces and cannot guarantee "crossing-free" and further the diffeomorphic brain mapping. Our method can solve these, and the diffeomorphism property has been verified in all the tests. If the graph is not 3-connected, we use valid subgraph for guiding the mapping. The unselected part is ignored and won't affect the diffeomorphism.

Topology and Geometric Meanings: This work studies "graph on surface" and its influence to surface mapping. The convex map preserves the topology of the graph on the canonical domain and minimizes the constrained harmonic energy (preserving angles as much as possible under constraints), therefore is more accessible and perceptually easy to capture global and local structures.

Advantages: In theory, the method is rigorous, based on the classical Tutte graph embedding for 3-connected planar graphs (Tutte's spring theorem [10]), and the harmonic map with linear convex constraints with uniqueness and diffeomorphism guarantee (Radó theorem [11], generalized Tutte embedding [12]). *In practice*, all the algorithms solve sparse linear systems and are easy to implement, practical and efficient, and robust to geometry or topology noises. The framework is general for surfaces decorated with graphs.

Extensions: This method is able to reflect more original geometry by introducing weighted graph embeddings, and can be extended to handle high genus cases by using advanced graph embeddings.

Potentials for Brain Mapping and Other Biomedical Research: The structural brain mapping can help understand anatomical structures and monitor anatomy progression, and has potential for brain cortical registration with atlas constraints.

This anatomy-aware framework is general for other convoluted natural shapes decorated with feature graphs (e.g., colons), and can be applied for their anatomy visualization, comparison, registration and morphometry analysis.

6 Conclusion

This work presents a brain cortical surface mapping method considering cortical anatomical structure, such that the complex convoluted brain cortical surface can be visualized in a well-structured view, i.e., a convex domain with convex subdivision. The algorithms based on Tutte embedding and harmonic map are efficient and practical, and are extensible for other applications where 3-connected feature graphs are associated. We will further employ weighted graph embeddings to reflect the original geometry in the mapping, and explore the potentials for solving brain surface registration and analysis problem in future works.

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