Skeletonization Algorithm Using Discrete Contour Map

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Abstract. The skeleton of a binary object can be considered as an alternative to the object itself; it describes the object in a simple and compact manner that preserves the object topology. In this paper, we introduce a new definition for discrete contour curves, and we propose a new approach for extracting a well-shaped and connected skeleton of two-dimensional binary objects using a transformation of the distance map into contour map, which allows us to disregard the nature of the distance metric used. Indeed, our algorithm can support various distances such as the city-block distance, the chessboard distance, the chamfer distance or the Euclidean distance. To evaluate the proposed technique, experiments are conducted on shape benchmark dataset.

Keywords: Image analysis · Digital topology · Distance map · Discrete Contour Map · Skeletonization

1 Introduction

The skeleton is a representation widely used for shape description and shape interpretation in several applications of image processing. There are several equivalent definitions in continuous space. For example the skeleton is defined by the set of centers of maximal disks contained in the object, by the set of ridges in the distance map, or by analogy with the fire front propagation as introduced early by Blum [9]. The skeleton of an object can also be defined as the set of centers of the disks that touch the boundary of the object in two or more locations. Skeletonization algorithms proposed in the literature are grouped into three categories: 1) approximation of the fire front propagation [7,20], 2) approximation of the continuous skeleton [10,12] or 3) extraction and interconnection of the centers of maximal disks in the distance map [11,14,19,29]. There are several algorithms that use the distance map to calculate the skeleton of an object, in common cases they involve the following steps: generating the distance map from a binary image, extracting the centers of maximal inscribed disks from the distance map and linking the centers of maximal disks to produce a connected skeleton. Algorithms using approximate distance metrics such as d_4 and d_8 are intensively considered and discussed by researchers and their theory

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is well established. However, these algorithms are not efficient for applications requiring greater precision. Using the Euclidean distance may be considered as a solution to this problem; however, it has topological disadvantages that may directly influence the resulting skeleton. In this paper, we present a new approach to extract the skeleton of binary objects based on the notion of contour map. In fact, we extended the algorithm proposed in [21] - originally developed to support only distances d_4 and d_8 - to be capable of disregarding the nature of distance metric used. Our work is based on the results of Andres and Jacob on the discrete analytical hypersphere [3] to propose a new definition of discrete contour curves, independent of the distance metric used. Based on this definition we introduce the notion of the contour map, which is a transformation of the distance map.

The main features of the proposed skeletonization approach are:

- The Contour map is an abstraction layer between the skeletonization algorithm and the distance function used.
- The algorithm is based on a new definition of discrete contour curves which is valid for several discrete distances.
- The algorithm can be, readily, extended to other distances by adapting the definition of discrete contour curves to such distances.

The remainder of the paper is organized as follows. Section 2 gives insights on the notations and some elementary definitions used in this paper. Sections 3 and 4 introduce the definition of *Discrete Contour Map* and our approach to skeletonization process. Experimental results are presented in section 5. In section 6 we present a historical walk through the skeletonization techniques. Section 7 provides some final conclusions and the impact of the suggested approach.

2 Preliminaries and Notations

We denote $\mathbb{R}$ the set of reel numbers, $\mathbb{Z}$ the set of integers and $\mathbb{N}^*$ the set of strictly positive integers. A discrete point is an element of $\mathbb{Z}^n$ denoted p, an object X is a set of discrete points. Two discrete points $p\left(x_p, y_p\right)$ and $q\left(x_q, y_q\right)$ are 4-neighbor (or 4-adjacent) if: $|x_p - x_q| + |y_p - y_q| = 1$. Similarly, they are 8-neighbor (or 8-adjacent) if: $max\left(|x_p - x_q|, |y_p - y_q|\right) = 1$. A binary image I is a function $\mathbb{Z}^2 \rightarrow \{0, 1\}$: each element of I can have the following values: 1 for object points and 0 for non-object points. Functions d_4, d_8, $d_{\langle a,b,\cdots\rangle}$ and d_E refer respectively to the city-block distance, chessboard distance, chamfer distance and the Euclidean distance. The distance of a point $p{\in}X$ to the border, denoted $d\left(p, \overline{X}\right)$, is the minimal distance of p to the complementary of X. The distance map of an object X relative to the distance metric d, denoted DM_d is the set of points labeled with their minimal distance to the boundary of X. A point $p{\in}DM_d$ is a local maximum in a 8-neighborhood if all its neighbors are at a distance from the boundary lower or equal to $d\left(p, \overline{X}\right)$.

3 Discrete Contour Map

In a topographic map, the contour curves are lines that connect points of equal elevation. Similarly, we consider the contour curves in a distance map d_4 or d_8 as the set of points having the same distance to the border. This definition is meaningless if we are interested in the chamfer distance or Euclidean distance. To give a general definition that characterizes the contour curves, we consider, in the continuous space, circular curves around a point (denoted $C\left(x_c, y_c\right) \in \mathbb{R}^2$) centered in the image. In this case a contour curve corresponds to a circle with center C and radius $r \in R$. The set of points belonging to this circle is defined by: $\left\{P\left(x, y\right) \in \mathbb{R}^2 : \left(x - x_c\right)^2 + \left(y - y_c\right)^2 = r^2\right\}$. In the discrete space several formulations have been proposed for the circle depending on the discretization scheme. In [3] for example, authors proposed a generalized definition in arbitrary dimension, which combines the continuous analytical definition and the properties specific to the discrete space.

Definition 1 (Discrete Analytical Hypersphere [3]). *A discrete analytical Hypersphere $H_n\left(C, r, \omega\right)$ in dimension n, of center $C \in \mathbb{R}^n$, radius $r \in \mathbb{R}$ and thickness $\omega \in \mathbb{R}$, is the set of discrete points $P\left(x_1, \cdots, x_n\right) \in H_n$ such that:*

$$H_n\left(C, r, \omega\right) = \left\{P \in \mathbb{Z}^n : \left(r - \frac{\omega}{2}\right)^2 \leq \sum_{i=1}^{n} \left(C_i - P_i\right)^2 < \left(r + \frac{\omega}{2}\right)^2\right\} \tag{1}$$

This formula defines the points that constitute the discrete circle in an open interval $[-\frac{\omega}{2}, \frac{\omega}{2}[$. For what concerns us - defining circles in dimension 2 with thickness 1 and centered on the origin point $(0, 0)$ - the above inequality is reduced to:

$$H\left(r\right) = \left\{P\left(x, y\right) \in \mathbb{Z}^2 : \left(r - \frac{1}{2}\right)^2 \leq x^2 + y^2 < \left(r + \frac{1}{2}\right)^2\right\} \tag{2}$$

The authors showed that for thicknesses $\omega \geq 1$, the circle is at least 8-connected and the union of circles is a tiling of the discrete space. In other words, each lattice point in $\mathbb{Z}^2$ belongs to one and only one of the concentric circles. Those properties make this definition well-suited for characterizing contour curves in a distance map computed using an arbitrary distance metric. In fact, it solves the topological issues related to the Euclidean distance in discrete space (non-connectedness of curves and the presence of gaps between two successive curves). Thus, we define the contour curve of level k as the set of points that are at a distance d from the object boundary, such that: $d \in [k - \frac{1}{2}, k + \frac{1}{2}[$

Definition 2 (Discrete contour curves). *Given an object X, the contour curve of level $k \in \mathbb{N}^*$ relative to a distance metric d is the set of points $p \in X$ which satisfy the double inequality:*

$$C\left(k\right) = \left\{p \in \mathbb{Z}^2 : \left(m\left(2k - 1\right)\right)^2 \leq 4d^2\left(p, \overline{X}\right) < \left(m\left(2k + 1\right)\right)^2\right\} \tag{3}$$

Where $m \in \mathbb{N}^*$ is the smallest distance to the object boundary, its value is equal to 1 for distances d_4, d_8 and d_E and equal to a for chamfer distances $d_{\langle a,b,\cdots \rangle}$.

This definition is directly applicable in the discrete space and uses only integer operations and has the advantage of being valid for all distance metrics mentioned in this article ($d_4, d_8, d_{\langle a,b,\cdots \rangle}$ and d_E).

Definition 3 (Contour map). *Given an object X, its contour map, relative to a distance metric d, denoted CM_d is an image where each point $p \in X$ is labeled with the level k of the contour curve to which it belongs.*

Note that for discrete distances d_4 and d_8 we have $CM_d = DM_d$, because to each distance value corresponds a separate contour curve. Furthermore, each point at a distance d, which belongs to the contour curve of level k, is always surrounded by points belonging to a contour curve $k' \in \{k-1, k, k+1\}$. This will allow us to limit the check of the inequality 3 in the interval $[k-1, k+1]$. Figure 1 represent respectively the contour map for distances d_8, $d_{\langle 3,4 \rangle}$ and d_E. The set of local maxima points coincides with the skeleton of the object.

(a) CM_8 (b) $CM_{\langle 3,4 \rangle}$ (c) CM_E

Fig. 1. Contour map for distances d_8, $d_{\langle 3,4 \rangle}$ and d_E

4 Skeletonization Algorithm

The set of local maxima points extracted from contour map of an object is not connected in most cases. Therefore, the skeleton defined only by set of local maxima of contour map is not useful for shape analysis applications. To overcome this problem, we need to interconnect all groups of local maxima points and produce a connected skeleton. The skeletonization algorithm proposed in [21] is applicable only for distances d_4 and d_8, it is based on the notion of *multiple points* introduced by Pavlidis [25]. Multiple points are identified in the distance map using local configurations in a 3×3 neighborhood. These points correspond to either a folding of a contour curve on itself near local maxima, or to a shrinkage center in the object. A recursive procedure is applied to perform a steepest ascent from each multiple point until another skeleton point is met. The algorithm produces a correct skeleton with a convenient computational complexity.

We propose a variant of this algorithm, which gives rise to a new skeletonization approach. In fact, we have extended this algorithm to support chamfer distance and the Euclidean distance by replacing the distance map by the contour map introduced in this article. The new algorithm performs the following operations:

1. Generating the distance map from a binary image.
2. Generating the contour map from the distance map.
3. Extracting local maxima points from the contour map.
4. Extracting multiple points from contour map using local configurations.
5. Interconnecting groups of local maxima points by performing a steepest ascent from each multiple point until another skeleton point is met.

The local configurations used in the original algorithm to detect multiple points, do not allow the proposed algorithm to interconnect all groups of local maxima points, it is therefore necessary to introduce other configurations to insure this interconnection. Figure 2 shows the set of additional configurations that we introduced to detect multiple points, the 8 neighbors $p_{i=0,\cdots,7}$ of point p are numbered with respect to the counterclockwise ordering.

·	p_2	·
·	p	p_0
p_5	·	·

·	p_2	·
·	p	·
p_5	·	p_7

·	p_2	·
p_4	p	p_0
·	p_6	·

·	p_2	·
p_4	p	p_0
·	·	·

Fig. 2. Additional configurations used to detect multiple points from contour map, the central point p is a multiple point.

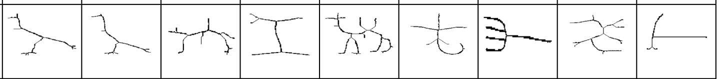
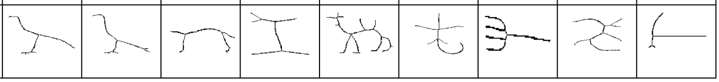
5 Experimental Results

In this section we evaluate the shape topology preservation of the proposed algorithm by conducting experiments on random images from Kimia's shape dataset [28]. We compare results obtained using distances d_8 and $d_{<3,4>}$ with those obtained using the distance d_E. (See table 1). To generate the euclidean distance map we used the Shih's algorithm [30] that achieves the euclidean distance transform in two scans using a 3×3 neighborhood, the algorithm produce a correct distance map in a linear time without iterations.

In real applications, binary images are obtained from cluttered scenes, therefore the boundaries of generated binary shapes contains a lot of noise which affects substantially the resulting skeleton. As an image preprocessing, we apply an edge blurring algorithm to smooth the shape boundary before computing distance map. In fact, rough edges produce local maxima points in the boundary of the shape, these local maxima become endpoints of unwanted branches in the final skeleton.

Most of skeletonization algorithms based on distance map are developed to support only one distance function, their extension to another distance function is not a trivial task. The Contour map defined in this work allow our algorithm to overcome this problem. In fact, extending the algorithm to another distance function involves only extending the definition of the discrete contour curve. As shown in table 1, the skeletons obtained using distances d_8, $d_{\langle 3,4\rangle}$ and d_E are well-connected, centered in the object and contain all significant branches of the shape. Except for distance d_8, where some irrelevant branches appear in the skeleton, no pruning process is required to remove redundant branches. The results obtained using distances $d_{\langle 3,4\rangle}$ and d_E are similar, one can use either $d_{\langle 3,4\rangle}$ or d_E without impacting the performance of the algorithm. For both distances, efficient algorithms exist for computing the distance map with linear run-time complexity.

Table 1. Binary shapes from Kimia's dataset [28] and their skeletons using contour maps CM_8, $CM_{<3,4>}$ and CM_E

Shape		
Skeleton	CM_8	
	$CM_{<3,4>}$	
	CM_E	

6 Related Work

In this section we present the most common ideas and techniques proposed in the literature to extract skeleton of $2D$ shapes. Skeletonization algorithms first appeared in the sixties. Blum [9] presented the process of skeletonization as a transformation of the image - called *Medial Axis Transformation* - to extract a new shape descriptor. Hilditch [15,16] proposed a sequential algorithm based on the notion of the *crossing number*. When the neighbors of a pixel are traversed in sequence, the crossing number is the number of times one crosses over from a white pixel to a black pixel. Pixels are traversed and marked for deletion under conditions that maintain skeleton connectivity and preserve two-pixel thickness. Rosenfeld [27] established the necessary and sufficient conditions for preserving topology while deleting border points in parallel process of skeletonization. Arcelli proposed a parallel algorithm [5] that deletes pixels using two 3×3 masks together with their 90° rotations. Dyer and Rosenfeld [13] worked out an algorithm for extracting skeleton from gray-scale images. It uses a generalized definition of pixel connectivity: two pixels are *connected* if there is a path joining them with no pixel lighter than either of them. Pavlidis [23–26] introduced the definition of *multiple pixels*, pixels that are traversed more than once during contour tracing, points with no neighbors in the interior and points on two-pixel-wide lines. *Multiples pixels* as well as the neighbors of skeletal pixels from a previous iteration are retained to maintain the connectivity of the skeleton. Arcelli [6] proposed another sequential skeletonization algorithm. It performs a contour tracing to detect the pixels for deletion. Arcelli and di Baja [8] presented a necessary definition to satisfactorily detect *multiple pixels* introduced by Pavlidis. Later [4] they developed a sequential algorithm that uses a *4-distance* transform to find a set of skeletal pixels using one scan of the image, followed by a second scan to remove unwanted pixels. Parker et al. [22] introduced the force-based approach for skeletonization. The authors define a *skeletal pixel* as being as far from the object boundary as possible while maintaining connectivity properties. The skeleton is interpreted as a global property of a binary object, and the boundary is used to locate the skeleton pixels. Andreadis et al. [2] presented an algorithm to extract a skeleton using morphological operators on image defined in the HSV color space. Huang et al. [17] proposed another parallel thinning algorithm. Pixel elimination rules are based on 3×3 windows considering all kinds of relations formed by 8 neighbors of the object pixel. Ji and Feng [18] proposed a method that interprets the image as a $2D$ thermal conductor that consists of pixels, where pixel intensity represents the temperature. The skeletonization is considered as an inverse process of heat conduction. Tang et al. [32] proposed a skeletonization algorithm based on a wavelet transform. The algorithm extracts an initial skeleton in a regular region followed by second stage to connect the initial skeletons in the singular region. Wan et al. [31] presented an algorithm that extract a skeleton in 3 stages. In the first stage, the Euclidean distance map of the image is generated. In the second stage, the local maximal disc centers are marked as skeleton points. In the last stage, a connected skeleton is generated by linking isolated skeleton points. Recently, Abu-Ain et al. [1] proposed an algorithm for optical character recognition (OCR) consisting of three main stages; conditional contour selection stage, pixel removing stage, and one pixel width stage.

7 Conclusion

This study presents a new skeletonization approach, which is to use the contour map as an alternative to the distance map. Using this approach, we proposed an algorithm capable of disregarding the nature of distance metric used. Indeed, we were able to generalize an existing algorithm to support more distance metrics such as the chamfer distance or Euclidean distance. The skeleton obtained by this method has the essential characteristics required by applications dealing with shapes description and interpretation in image processing.

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