A Logical Framework for Imprecise and Conflicting Knowledge Representation for Multi-agent Systems

Jair Minoro Abe(✉), Nelio Fernando dos Reis, Cristina Corrêa de Oliveira, and Avelino Palma Pimenta Jr.

Graduate Program in Production Engineering, Paulista University, R. Dr. Bacelar 1212, São Paulo, SP 04026-002, Brazil
jairabe@uol.com.br,
{nelio.reis.phd,appimenta}@gmail.com,
crisolive@ig.com.br

Abstract. Nowadays multi-agents has established as one of the most important areas of research and development in information technology. Agents are normally involved in cooperative distributed problem and they face frequently with incomplete and/or conflicting information or task. Since more and more concern is attached to agents' teamwork and agents' dialogue, conflicts naturally arise as a key issue to be dealt with, not only with application dedicated techniques, but also with more formal and generic tools. In this semi-expository paper we show that a formal treatment for multi-agent knowledge representation that can represent conflicts and incomplete information is possible through new logical system, namely the paraconsistente logics. We discuss one of such system adding suitable modal operators for knowledge.

Keywords: Multi-agent systems · Conflicts in distributed systems · Multi-agents and logical representation · Paraconsistente logics

1 Introduction

Multi-agent systems have emerged as one of the most important areas of research and development in information technology in the 1990s. Since the theme has received attention of specialists and nowadays a number of research topics has been considered such as cooperative distributed problem solving, mechanism design, auctions, game theory, multi-agent planning, negotiation protocols, multi-agent learning, conflict resolution, agent-oriented software engineering, including implementation languages and frameworks, E-business agents, novel computing paradigms (autonomic, grid, P2P, ubiquitous computing), among innumerous themes.

In this paper we are focused in the problem of conflict resolution among agents. For this task we need a suitable language for represent agent's communication and moreover inference rules to get interest results; in other words we need an underlying logical system to represent agent's interaction.

S. Umeda et al. (Eds.): APMS 2015, Part I, IFIP AICT 459, pp. 202–210, 2015.
DOI: 10.1007/978-3-319-22756-6_25

Agents' conflicts arise for different reasons, involve different concepts, and are dealt with in different ways, depending on the kind of agents and on the domain where they are considered.

For example,

- incompleteness and uncertainty of the agents' knowledge or beliefs: in dynamic contexts, an agent may have more recent or more complete information than the others, and the differences in the agents' knowledge create knowledge conflicts;
- limited or unavailable resources: not all agents have access to the same resources, thus resulting in resource conflicts;
- differences in the agents' skills and points of view: autonomous and heterogeneous agents have different abilities, or even different preferences, which can cause conflicts if the agents' pieces of information are not comparable, if they come up with different answers to the same questions, or if they are strongly committed to their own preferences.

Up till now, the focus has been much on how to avoid, solve or get rid of conflicts. However, recent research has shown that conflicts have positive effects in so far as they can generate original solutions and be a basis for a global enrichment of the knowledge within a multi-agent system.

Since more and more concern is attached to agents' teamwork and agents' dialogue, conflicts naturally arise as a key issue to be dealt with, not only with application dedicated techniques, but also with more formal and generic tools.

2 Paraconsistent, Paracomplete, and Non-alethic Logics

In what follows, we sketch the non-classical logics discussed in the paper, establishing some conventions and definitions. Let *T* be a theory whose underlying logic is *L*. *T* is called inconsistent when it contains theorems of the form *A* and ¬*A* (the negation of *A*). If *T* is not inconsistent, it is called *consistent.* *T* is said to be *trivial* if all formulas of the language of *T* are also theorems of *T*. Otherwise, *T* is called *non-trivial.*

When *L* is classical logic (or one of several others, such as intuitionistic logic), *T* is inconsistent iff *T* is trivial. So, in trivial theories the extensions of the concepts of formula and theorem coincide. A *paraconsistent logic* is a logic that can be used as the basis for inconsistent but non-trivial theories. A *theory* is called *paraconsistent* if its underlying logic is a paraconsistent logic.

Issues such as those described above have been appreciated by many logicians. In 1910, the Russian logician Nikolaj A. Vasil'év (1880–1940) and the Polish logician Jan Lukasiewicz (1878–1956) independently glimpsed the possibility of developing such logics. Nevertheless, Stanislaw Jaskowski (1996–1965) was in 1948 effectively the first logician to develop a paraconsistent system, at the propositional level. His system is known as 'discussive' (or discursive) propositional calculus'. Independently, some years later, the Brazilian logician Newton C.A. da Costa (1929–) constructed for the first time hierarchies of paraconsistent propositional calculi C_i, $1 \leq i \leq \omega$ of paraconsistent first-order predicate calculi (with and without equality), of paraconsistent description calculi, and paraconsistent higher-order logics (systems NF_i, $1 \leq i \leq \omega$).

Another important class of non-classical logics are the paracomplete logics. A logical system is called *paracomplete* if it can function as the underlying logic of theories in which there are formulas such that these formulas and their negations are simultaneously false. Intuitionistic logic and several systems of many-valued logics are paracomplete in this sense (and the dual of intuitionistic logic, Brouwerian logic, is therefore paraconsistent).

As a consequence, paraconsistent theories do not satisfy the principle of non-contradiction, which can be stated as follows: of two contradictory propositions, i.e., one of which is the negation of the other, one must be false. And, paracomplete theories do not satisfy the principle of the excluded middle, formulated in the following form: of two contradictory propositions, one must be true.

Finally, logics which are simultaneously paraconsistent and paracomplete are called *non-alethic logics*.

3 A Logical Framework for Representing Impreciseness, Conflicts and Paracompleteness

We present, in this section, the multimodal predicate calculi Mτ, based on annotated logics extensively studied by Abe [1, 4, 6, 13] and multimodal systems considered in [7, 8, 9, 10, 11, 12].

The symbol $\tau = <|\tau|, \leq, \sim>$ indicates some finite lattice with operator called the *lattice of truth-values*. We use the symbol ≤ to denote the ordering under which τ is a complete lattice, ⊥ and ⊤ to denote, respectively, the bottom element and the top element of τ. Also, ∧ and ∨ denote, respectively, the greatest lower bound and least upper bound operators with respect to subsets of |τ|. The operator $\sim : |\tau| \rightarrow |\tau|$ will work as the "meaning" of the negation of the system Mτ.

The language of Mτ has the following primitive symbols:

1. Individual variables: a denumerable infinite set of variable symbols: x_1, x_2, …
2. Logical connectives: ¬ (negation), ∧ (conjunction), ∨ (disjunction), and → (implication).
3. For each *n*, zero or more *n*-ary function symbols (*n* is a natural number).
4. For each $n \neq 0$, *n*-ary predicate symbols.
5. The equality symbol: =
6. Annotational constants: each member of τ is called an annotational constant.
7. Modal operators: $[]_1$, $[]_2$, …, $[]_n$, ($n \geq 1$), $[]_G$, $[]_G^C$, $[]_G^D$ (for every nonempty subset *G* of $\{1, \ldots, n\}$).
8. Quantifiers: ∀ (for all) and ∃ (there exists).
9. Auxiliary symbols: parentheses and comma.

A 0-ary function symbol is called a *constant*. We suppose that Mτ possesses at least one predicate symbol.

We define the notion of *term* as usual. Given a predicate symbol *p* of arity *n* and *n* terms $t_1, \ldots, t_n$, a *basic formula* is an expression of the form $p(t_1, \ldots, t_n)$. An *annotated atomic formula* is an expression of the form $p_\lambda(t_1, \ldots, t_n)$, where λ is an annotational constant. We introduce the general concept of (*annotated*) *formula* in the standard way. For instance, if A is a formula, then $[]_1A$, $[]_2A$, …, $[]_nA$, $[]_GA$, $[]_G^CA$, and $[]_G^DA$ are also formulas [7].

Among several intuitive readings, an atomic annotated formula $p_\lambda(t_1, \ldots, t_n)$ can be read: *it is believed that* $p(t_1, \ldots, t_n)$*'s truth-value is at least* λ.

Definition 1. Let A and B be formulas. We put $A \leftrightarrow B =_{\text{Def.}} (A \rightarrow B) \wedge (B \rightarrow A)$ and $\neg^*A =_{\text{Def.}} A \rightarrow ((A \rightarrow A) \wedge \neg(A \rightarrow A))$. The symbol '$\leftrightarrow$' is called *biconditional* and '$\neg^*$' is called *strong negation.*

Let A be a formula. Then: $\neg^0 A =_{\text{Def.}} A$, $\neg^1 A =_{\text{Def.}} \neg A$, and $\neg^k A =_{\text{Def.}} \neg(\neg^{k-1}A)$, $(k \in \mathrm{N}, k > 0)$. Also, if $\mu \in \tau$, $\sim^0\mu =_{\text{Def.}} \mu$, $\sim^1\mu =_{\text{Def.}} \sim\mu$, and $\sim^k\mu =_{\text{Def.}} \sim(\sim^{k-1}\mu)$, $(k \in \mathrm{N}, k > 0)$. If A is an atomic formula $p_\lambda(t_1, \ldots, t_n)$, then a formula of the form $\neg^k p_\lambda(t_1, \ldots, t_n)$ $(k \geq 0)$ is called a *hyper-literal*. A formula other than hyper-literals is called a *complex* formula.

The postulates (axiom schemata and primitive rules of inference) of Mτ are the same of the logics Qτ [1] plus the following listed below [7], where A, B, and C are any formulas whatsoever, $p(t_1, \ldots, t_n)$ is a basic formula, and λ, μ, μ_j are annotational constants.

(M1) $[]_i(A \rightarrow B) \rightarrow ([]_iA \rightarrow []_iB)$, $i = 1, 2, \ldots, n$
(M2) $[]_iA \rightarrow []_i[]_iA$, $i = 1, 2, \ldots, n$
(M3) $\neg^*[]_iA \rightarrow []_i\neg^*[]_iA$, $i = 1, 2, \ldots, n$
(M4) $[]_iA \rightarrow A$, $i = 1, 2, \ldots, n$
(M5) $\frac{A}{[]_iA}$, $i = 1, 2, \ldots, n$
(M6) $[]_GA \leftrightarrow \wedge_{i\in G}[]_iA$
(M7) $[]_G^CA \rightarrow []_G(A \wedge []_G^CA)$
(M8) $[]_{\{i\}}^DA \leftrightarrow []_iA$, $i = 1, 2, \ldots, n$
(M9) $[]_G^DA \rightarrow []_{G'}^DA$ if $G' \subseteq G$
(M10) $\frac{A \rightarrow []_G(B \wedge A)}{A \rightarrow []_G^BB}$
(M11) $\forall x[]_iA \rightarrow []_i\forall xA$, $i = 1, 2, \ldots, n$
(M12) $\neg^*(x = y) \rightarrow []_i\neg^*(x = y)$, $i = 1, 2, \ldots, n$

Mτ is an extension of the logic Qτ. As Qτ contains classical predicate logic, Mτ contains classical modal logic S5, as well as the multimodal system studied in [7] in at least two directions. So, usual all valid schemes and rules of classical positive propositional logic are true. In particular, the deduction theorem is valid in Mτ ant it contains intuitionistic positive logic.

Theorem 1. Mτ is non-trivial.

Now we introduce a semantical analysis by using Kripke models [3, 5].

Definition 2. A Kripke model for Mτ (or Mτ structure) is a set theoretical structure $K = [W, R_1, R_2, \ldots, R_n, I]$ where W is a nonempty set of elements called 'worlds'; R_i $(i = 1, 2, \ldots, n)$ is a binary relation on W such that it is an equivalence relation. I is an interpretation function with the usual properties with the exception that for each n-ary predicate symbol p we associate a function $p_I: W^n \rightarrow |\tau|$.

Given a Kripke model K for the language L of Mτ, the *diagram* language $L(K)$ is obtained as usual.

Definition 3. If A is a closed formula of Mτ, and $w \in W$, we define the relation $K,w \Vdash A$ (K,w *force* A) by recursion on A:

1. If A is atomic of the form $p_\lambda(t_1, \ldots, t_n)$, then $K,w \Vdash A$ iff $p_I(K(t_1), \ldots, K(t_n)) \geq \lambda$.
2. If A is of the form $\neg^k p_\lambda(t_1, \ldots, t_n)$ ($k \geq 1$), $K, w \Vdash A$ iff $K,w \Vdash \neg^{k-1} p_{\sim\lambda}(t_1, \ldots, t_n)$.
3. Let A and B formulas. Then, $K,w \Vdash (A \wedge B)$ iff $K,w \Vdash A$; $K,w \Vdash B$; $K,w \Vdash (A \vee B)$ iff $K,w \Vdash A$ or $K,w \Vdash B$; $K,w \Vdash (A \rightarrow B)$ iff it is not the case that $K, w \Vdash A$ or $K,w \Vdash B$;
4. If F is a complex formula, then $K,w \Vdash (\neg F)$ iff it is not the case that $K,w \Vdash F$.
5. If A is of the form $(\exists x)B$, then $K,w \Vdash A$ iff $K,w \Vdash B_x[i]$ for some i in $L(K)$.
6. If A is of the form $(\forall x)B$, then $K,w \Vdash A$ iff $K,w \Vdash B_x[i]$ for all i in $L(K)$.
7. If A is of the form $[]_i B$ then $K,w \Vdash A$ iff $K, w' \Vdash B$ for each $w' \in W$ such that $wR_i w'$, $i = 1, 2, \ldots, n$

Definition 4. Let $K = [W, R_1, R_2, \ldots, R_n, I]$ be a Kripke structure for Mτ. The Kripke structure K *forces* a formula A (in symbols, $K \Vdash A$), if $K,w \Vdash A$ for each $w \in W$. A formula A is called Mτ-*valid* if for any Mτ-structure K, $K \Vdash A$. A formula A is called *valid* if it is Mτ-valid for all Mτ structure. We symbolize this fact by $\Vdash A$.

Theorem 2. Let $K = [W, R_1, R_2, \ldots, R_n, I]$ be a Kripke structure for Mτ. Then

1. If A is an instance of a propositional tautology then, $K \Vdash A$
2. If $K \Vdash A$ and $K \Vdash A \rightarrow B$, then $K \Vdash B$
3. $K \Vdash []_i(A \rightarrow B) \rightarrow ([]_i A \rightarrow []_i B)$, $i = 1, 2, \ldots, n$
4. $K \Vdash []_i A \rightarrow []_i[]_i A$, $i = 1, 2, \ldots, n$
5. $K \Vdash []_i A \rightarrow A$, $i = 1, 2, \ldots, n$
6. If $K \Vdash A$ then $K \Vdash []_i A$, $i = 1, 2, \ldots, n$

Theorem 3. Let K be a Kripke model for Mτ and F a complex formula. Then we have not simultaneously $K,w \Vdash \neg F$ and $K,w \Vdash F$.

Theorem 4. Let $p(t_1, \ldots, t_n)$ be a basic formula and $\lambda, \mu, \rho \in |\tau|$. We have $\Vdash p_\perp(t_1, \ldots, t_n)$; $\Vdash p_\lambda(t_1, \ldots, t_n) \rightarrow p_\mu(t_1, \ldots, t_n)$, if $\lambda \geq \mu$; $\Vdash p_\lambda(t_1, \ldots, t_n) \wedge p_\mu(t_1, \ldots, t_n) \rightarrow p_\rho(t_1, \ldots, t_n)$, where $\rho = \lambda \vee \mu$

Theorem 5. Let A and B be arbitrary formulas and F a complex formula. Then: $\Vdash ((A \rightarrow B) \rightarrow ((A \rightarrow \neg^*B) \rightarrow \neg^*A))$; $\Vdash (A \rightarrow (\neg^*A \rightarrow B))$; $\Vdash (A \vee \neg^*A)$; $\Vdash (\neg F \leftrightarrow \neg^*F)$; $\Vdash A \leftrightarrow \neg^*\neg^*A$; $\Vdash \forall xA \leftrightarrow \exists x\neg^*A$; $\Vdash (A \wedge B) \leftrightarrow \neg^*(\neg^*A \vee \neg^*B)$; $\Vdash \forall A \leftrightarrow \exists x\neg^*A$; $\Vdash \forall xA \vee B \leftrightarrow \exists x(A \vee B)$; $\Vdash A \vee \exists xB \leftrightarrow \exists x(A \vee B)$.

Corollary 5.1. In the same conditions of the preceding theorem, we have not simultaneously $K \Vdash \urcorner A$ and $K \Vdash A$. The set of all formulas together with the connectives $\wedge$, $\vee$, $\rightarrow$, and $\urcorner$ has all properties of the classical logic.

Theorem 6. There are Kripke models K such that for some hyper-literals A and B and some worlds w and $w' \in W$, we have $K,w \Vdash \neg A$ and $K,w \Vdash A$ and it is not the case that $K, w' \Vdash B$.

Proof. Let $W = \{\{a\}\}$ and $R = \{(\{a\},\{a\})\}$ (that is $w = \{a\}$) and $p(t_1, \ldots, t_n)$ and $q(t'_1, \ldots, t'_n)$ basic (closed) formulas such that $I(p) \equiv \top$ and $I(q) \equiv \bot$. As $\top \geq \top$, it follows that $p_{\top}(t_1, \ldots, t_n) \geq \top$. Also, $\top \geq \sim\top$. So, $p_I \geq \sim\top$. Therefore, $K,w \Vdash p_{\top}(t_1, \ldots, t_n)$ and $K,w \Vdash p_{\sim\top}(t_1, \ldots, t_n)$. By condition 2 of Definition 3, it follows that $K,w \Vdash \neg p_{\top}(t_1, \ldots, t_n)$. On the other hand, as it is false that $\bot \geq \top$; it follows that it is not the case that $q_I \geq \top$, and so, it is not the case that $K,w \Vdash q_{\bot}(t'_1, \ldots, t'_n)$. □

Theorem 7. For some systems Mτ there are Kripke models K such that for some hyper-literal formula A and some world $w \in W$, we don't have $K,w \Vdash A$ nor $K, w \Vdash \neg A$.

Corollary 7.1. For some systems Mτ there are Kripke models K such that for some hyper-literal formulas A and B, and some worlds $w, w' \in W$, we have $K,w \Vdash \neg A$ and $K, w \Vdash A$ and we don't have $K,w \Vdash B$ nor $K,w \Vdash \neg B$.

The earlier results show us that there are systems Mτ such that we have "inconsistent" worlds, "paracomplete" worlds, or both.

Now we present a strong version these results linking with paraconsistent, paracomplete, and non-alethic logics.

Definition 5. A Kripke model K is called *paraconsistent* if there are basic formulas $p(t_1, \ldots, t_n)$, $q(t_1, \ldots, t_n)$, and annotational constants $\lambda, \mu \in |\tau|$ such that $K,w \Vdash p_{\lambda}(t_1, \ldots, t_n)$, $K,w \Vdash \neg p_{\lambda}(t_1, \ldots, t_n)$, and it is not the case that $K,w \Vdash q_{\mu}(t_1, \ldots, t_n)$.

Definition 6. Asystem Mτ is called *paraconsistent* if there is a Kripke model K for Mτ such that K is paraconsistent.

Theorem 8. Mτ is a paraconsistent system iff $\#|\tau| \geq 2$.

Proof. Define a structure $K = [\{w\}, \{(w, w)\}, I]$ such that $\begin{cases} q_I = \bot \\ p_I = \mathrm{T} \end{cases}$

It is clear that $p_I \geq \top$, and so $K \Vdash p_{\top}(t_1, \ldots, t_n)$. Also, $p_I \geq \sim\top$, and, so $K \Vdash p_{\sim\top}(t_1, \ldots, t_n)$, or $K \Vdash \neg p_{\top}(t_1, \ldots, t_n)$. Also, it is not the case that $q_I(t_1, \ldots, t_n) \geq \bot$, so it is not the case that $K, w \Vdash q_{\bot}(t_1, \ldots, t_n)$. □

Definition 7. A Kripke model K is called *paracomplete* if there are a basic formula $p(t_1, \ldots, t_n)$ and an annotational constant $\lambda \in |\tau|$ such that it is false that $K,w \Vdash p_{\lambda}(t_1, \ldots, t_n)$ and it is false that $K,w \Vdash \neg p_{\lambda}(t_1, \ldots, t_n)$. A system Mτ is called *paracomplete* if there is a Kripke models K for Mτ such that K is paracomplete.

Definition 8. A Kripke model K is called *non-alethic* if K are both paraconsistent and paracomplete. A system Mτ is called *non-alethic* if there is a Kripke model K for Mτ such that K is non-alethic.

Theorem 9. If $\#|\tau| \geq 2$, then there are systems Mτ which are paracomplete and systems Mτ' that are not paracomplete, $\#|\tau'| \geq 2$.

Corollary 9.1. If $\#|\tau| \geq 2$, then there are systems Mτ which are non-alethic and systems Mτ' that are not non-alethic, $\#|\tau'| \geq 2$.

Theorem 10. Let U be a maximal non-trivial maximal (with respect to inclusion of sets) subset of the set of all formulas. Let A and B formulas whatsoever. Then if A is an axiom of $M\tau$, then $A \in U$; $A \wedge B \in U$ iff $A \in U$ and $B \in U$;$A \vee B \in U$ iff $A \in U$ or $B \in U$; $A \to B \in U$ iff $A \notin U$ or $B \in U$; If $p_\mu(t_1, \ldots, t_n) \in U$ and $p_\lambda(t_1, \ldots, t_n) \in U$, then $p_\rho(t_1, \ldots, t_n) \in U$, where $\rho = \mu \vee \lambda$; $\neg^k p_\lambda(t_1, \ldots, t_n) \in U$ iff $\neg^{k-1} p_{\sim\lambda}(t_1, \ldots, t_n) \in U$. If A and $A \to B \in U$, then $B \in U$; $A \in U$ iff $\neg^* A \notin U$. Moreover $A \in U$ or $\neg^* A \in U$. If A is a complex formula, $A \in U$ iff $\neg A \notin U$. Moreover $A \in U$ or $\neg A \in U$. If $A \in U$, then $[]_i A \in U$.

Proof. Let us show only 3. In fact, if $p_\mu(t_1, \ldots, t_n) \in U$ and $p_\lambda(t_1, \ldots, t_n) \in U$, then $p_\mu(t_1, \ldots, t_n) \wedge p_\lambda(t_1, \ldots, t_n) \in U$ by 2. But it is an axiom $p_\mu(t_1, \ldots, t_n) \wedge p_\lambda(t_1, \ldots, t_n) \to p_\rho(t_1, \ldots, t_n)$, where $\rho = \mu \vee \lambda$. It follows that $p_\mu(t_1, \ldots, t_n) \wedge p_\lambda(t_1, \ldots, t_n) \to p_\rho(t_1, \ldots, t_n) \in U$, and so $p_\rho(t_1, \ldots, t_n) \in U$, by 6.

Given a set U of formulas, define $U/[]_i = \{A | []_i A \in U\}$, $i = 1, 2, \ldots, n$. Let us consider the canonical structure $K = [W, R_i, I]$ where $W = \{U | U$ is a maximal non-trivial set$\}$ and the interpretation function is as usual with the exception that given a n-ary predicate symbol p we associate the function $p_I : W^n \to |\tau|$ defined by $p_I(\overset{\circ}{t}_1, \ldots, \overset{\circ}{t}_n) =_{def.} \vee\{\mu \in |\tau| \, | \, p_\mu(t_1, \ldots, t_n) \in U\}$ (such function is well defined, so $p_\perp(t_1, \ldots, t_n) \in U$). Moreover, define $R_i =_{Def.} \{(U, U') | U/[]_i \subseteq U'\}$. □

Lemma 1. For all propositional variable p and if U is a maximal non-trivial set of formulas, we have $p_{p_I(\overset{\circ}{t}_1, \ldots, \overset{\circ}{t}_n)}(t_1, \ldots, t_n) \in U$.

Proof. It is a simple consequence of the previous theorem, item 5. □

Theorem 11. For any formula A and for any non-trivial maximal set U, we have $(K, U) \Vdash A$ iff $A \in U$.

Proof. Let us suppose that A is $p_\lambda(t_1, \ldots, t_n)$ and $(K, U) \Vdash p_\lambda(t_1, \ldots, t_n)$. It is clear by previous lemma that $p_{p_I(\overset{\circ}{t}_1, \ldots, \overset{\circ}{t}_n)}(t_1, \ldots, t_n) \in U$. It follows also that $p_I(\overset{\circ}{t}_1, \ldots, \overset{\circ}{t}_n) \geq \lambda$. It is an axiom that $p_{p_I(t1, \ldots, tn)}(t_1, \ldots, t_n) \to p_\lambda(t_1, \ldots, t_n)$. Thus, $p_\lambda(t_1, \ldots, t_n) \in U$. Now, let us suppose that $p_\lambda(t_1, \ldots, t_n) \in U$. By previous lemma, $p_{p_I(t1, \ldots, tn)}(t_1, \ldots, t_n) \in U$. It follows that $p_I(t_1, \ldots, t_n) \geq \lambda$. Thus, by definition, $(K, U) \Vdash p_\lambda(t_1, \ldots, t_n)$. By Theorem 10, $\neg^k p_\lambda(t_1, \ldots, t_n) \in U$ iff $\neg^{k-1} p_{\sim\lambda}(t_1, \ldots, t_n) \in U$. Thus, by Definition 3, $(K, U) \Vdash \neg^k p_\lambda(t_1, \ldots, t_n)$ iff $(K, U) \Vdash \neg^{k-1} p_{\sim\lambda}(t_1, \ldots, t_n)$. So, by induction on k the assertion is true for hyper-literals.

The other cases, the proof is as in the classical case. □

Corollary 11.1. A is a provable formula of $M\tau$ iff $\Vdash A$

4 Concluding Remarks

It is quite interesting to observe the role of conflict within a multiagent system, i.e. how this system may evolve thanks to, despite, or because of conflicts. Such concept receives different 'interpretations' or characterizations depending on of the domain considered. Some considerations regarding to it

- It is easier not to be in conflict than to be in conflict. The former may mean that the agents are not even interacting. The latter supposes that the agents are within the same context.
- incompleteness and uncertainty of the agents' knowledge or beliefs: in dynamic contexts, an agent may have more recent or more complete information than the others, and the differences in the agents' knowledge create knowledge conflicts;
- limited or unavailable resources: not all agents have access to the same resources, thus resulting in resource conflicts;
- differences in the agents' skills and points of view: autonomous and heterogeneous agents have different abilities, or even different preferences, which can cause conflicts if the agents' pieces of information are not comparable, if they come up with different answers to the same questions, or if they are strongly committed to their own preferences.
- When two agents (in this case, e.g. two robots) a conflict does not seem to be necessarily symmetric: conflict (a, b) does not imply conflict (b, a). When two robots roam in a 2D space for instance, the notion of spatial conflict appears only at the time when a robot attempts to move to the location of the other robot. But the latter does not see this conflict.
- Are conflicts useful? The answer depends on the problem. To be useful, a conflict must be observed. For example, if an agent think about a solution and ask for two other agent's (experts) for an opinion and they have contradictory opinions, the former agent can decide, for instance, consult a third agent.
- What we learn from a conflict depends on the situation. Learning is possible if agents are aware of the conflict.
- Conflicts are positive in certain cases, e.g. they may create specific behaviours, create competition, or stimulate inference.

Up till now, the focus has been much on how to avoid, solve or get rid of conflicts. However, recent research has shown that conflicts have positive effects in so far as they can generate original solutions and be a basis for a global enrichment of the knowledge within a multiagent system. Thus this work is a contribution in this direction, showing, for instance that it is unnecessary to try to avoid conflicts; on the contrary with a logical knowledge representation of conflicts we can manage mathematically them and so it is possible to understand better the nature of them [2].

References

1. Abe, J.M.: Fundamentos da lógica anotada (Foundations of annotated logics). Ph.D. thesis, University of São Paulo, São Paulo, p. 135 (1992) (in Portuguese)
2. Abe, J.M.: Some aspects of paraconsistent systems and applications. Logique et Anal. **15**, 83–96 (1997)
3. Cresswell, M.J.: Intensional logics and logical truth. J. Philos. Logic **1**, 2–15 (1972)
4. Akama, S., Abe, J.M.: Many-valued and annotated modal logics. In: Proceedings of IEEE 1998 International Symposium on Multiple-Valued Logic (ISMVL 1998), pp. 114–119 Fukuoka, Japan (1998)

5. Cresswell, M.J.: Logics and Languages. Methuen and Co, London (1973)
6. Da Costa, N.C.A., Abe, J.M., Subrahmanian, V.S.: Remarks on annotated logic. Zeitschr. f Math. Logik und Grundlagen d. Math. **37**, 561–570 (1991)
7. Fagin, R., Halpern, J.Y., Moses, Y., Vardi, M.Y.: Reasoning About Knowledge. The MIT Press, London (1995)
8. Fischer, M.J. and Immerman, N.: Foundation of knowledge for distributed systems. In: Halpern J.Y. (ed.) Proceedings of the Fifth Conference on Theoretical Aspects of Reasoning about Knowledge, pp. 171–186. Morgan Kaufmann, San Francisco (1986)
9. Lipman, B.L.: Decision theory with impossible possible worlds. Technical report working paper, Queen's University (1992)
10. McCarthy, J.: Ascribing mental qualities to machines. Technical report CRL 90/10, DEC-CRL (1979)
11. Parikh, R., Ramamujan, R.: Distributed processing and the logic of knowledge, pp. 256–268. In: Parikh, R. (ed.) Proceedings of Workshop on Logics of Program (1985)
12. Rosenschein, S.J.: Formal theories of AI in knowledge and robotics. New Gener. Comput. **3**, 345–357 (1985)
13. Sylvan, R., Abe, J.M.: On general annotated logics, with an introduction to full accounting logics. Bull. Symbolic Logic **2**, 118–119 (1996)