Chapter 17
Concluding Remarks

In this textbook we have highlighted a set of important results on queues with Lévy input, and explicitly drawn the connection with fluctuation theory. An obvious disclaimer is in place here: with this field being large, some relevant contributions may have been overlooked. Also, given the connection between Lévy-driven queues and risk theory in a Lévy environment, compactly reflected by Eqn. (2.5), perhaps not all relations with the vast finance and insurance literature have been fully exploited.

Despite the fact that the field develops rapidly, there are still many open problems; we mention here just a few challenging directions.

(i) In the first place, quite a number of results presented in this book are restricted to spectrally one-sided cases, whereas in practical situations the underlying Lévy process often has *two-sided jumps*; see however [22, 149, 150].
(ii) Another domain in which still only partial results are known is that of Lévy-driven *networks*: hardly any results are available when the underlying network does not satisfy conditions (T_1)–(T_5); see however the novel contribution [166].
(iii) Also, in the area of *numerical evaluation* (by either simulation or numerical inversion) there is still substantial scope for improvement.
(iv) Finally, there are still many open problems related to various *functionals* of the workload process: for instance, one would like to uniquely characterize the full distribution of $V(t, u)$, as defined in Section 4.3, and only partial results are available for the area under the workload graph [14, 48].

The variety of open questions, which emerge from analyzing Lévy-driven queueing systems and Lévy fluctuation theory, stimulates the current research to lie at the interface of such areas as extreme value theory, stochastic geometry, large deviations, stochastic simulation theory, etc. This fuels the expectation that Lévy-driven queueing theory and fluctuation theory will increasingly become a key subdiscipline of applied and theoretical probability.

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