The Color Logarithmic Image Processing (CoLIP) Antagonist Space and Chromaticity Diagram

Yann Gavet[1,2(✉)], Johan Debayle[1,2], and Jean-Charles Pinoli[1,2]

[1] École des Mines, Saint-Étienne, France
[2] LGF, UMR CNRS 5307, Lyon, France
yann.gavet@mines-stetienne.fr

Abstract. The CoLIP framework defines a vectorial space for color images. It is in accordance with the theories of visual perception (Weber, Fechner) as well as Hering's trichromacy theory. It is mathematically well defined and computationally usable. This article recalls the fundamentals of the LIP framework for graytone images, and introduces the elementary operations of the vectorial structure for color images. It illutrates the representation of the chromaticity diagram with color modification application, namely white balance correction and color transfer. The results show that the hull of the diagram are not modified, but the colors are.

Keywords: Color logarithmic image processing · Chromaticity diagram · Image processing

1 Introduction

Regarding the fact that the current color spaces (e.g., RGB, CIE XYZ, CIE $L^*a^*b^*$, CIECAM02) do not follow an additive law, and therefore fail to obey the linear concept that is of a very high theoretical and practical interest in mathematics and their applications, the Color Logarithmic Image Processing (CoLIP) framework was developped, in accordance with the main laws and characteristics of the human color visual perception.

The trichromacy theory [13] states that humans have three different receptors sensitive to color stimuli. Indeed, the color photoreceptors in the retina, namely the cones, are sensitive to 3 different wavelength ranges: long, medium and short wavelengths, and thus classified into 3 types of cones: L, M and S, respectively.

The CoLIP theory is based on the Logarithmic Image Processing (LIP) theory that was developed for the representation and processing of images valued in a bounded intensity range [6]. As for the LIP theory [8], the CoLIP theory is physically and psychophysically well justified since it is consistent with the multiplicative image formation model and is consistent with several laws and characteristics of human brightness perception (e.g., Weber's law, Fechner's law, saturation effect, brightness range inversion). The Weber's law [12], and its generalization with Fechner's law [2], are translated into a logarithmic relationship

A. Trémeau et al. (Eds.): CCIW 2015, LNCS 9016, pp. 131–138, 2015.
DOI: 10.1007/978-3-319-15979-9_13

between the perceived brightness versus the stimulus. For each channel LMS, a Weber's fraction does exist, yielding a LMS Fechner's law, respectively [9].

Another important theory is called opponent-process theory. Hering [5], and later Svaetichin [10] noticed that particular colors like reddish-green or yellowish-blue would never be observed. Schematically, the color informations are coded in opponent red-green (denoted rg) and yellow-blue (denoted yb) channels to improve the efficiency of the transmission (reducing the noise) and to decorrelate the LMS channels [1].

2 LIP Theory

The LIP theory (Logarithmic Image processing) has been introduced in the middle of the 1980s [8]. It defines an algebraic framework that allows operations on images in a bounded range [6]. This model is mathematically well defined as well as physically consistent with the transmitted light imaging process.

2.1 Gray Tone Functions

In the LIP theory, a graytone function f is associated to an intensity image F. f is defined on a spatial support $D \subset \mathbb{R}^2$ and has its values in the real-number range $[0; M_0[$, with M_0 being a strictly positive real number. In the context of transmitted light imaging, the value 0 corresponds to the total transparency and M_0 to the total opacity. Thus, the gray tone function f is defined, for F_{max} being the saturating light intensity level (glare limit), by:

$$f = M_0 \left(1 - \frac{F}{F_{max}} \right). \tag{1}$$

2.2 The Vectorial Structure

The vectorial space S of gray tone functions is algebraically and topologically isomorphic to the classical vector space of real-valued functions, defined through the following isomorphism φ and the inverse isomorphism φ^{-1}:

$$\varphi(f) = -M_0 \ln \left(1 - \frac{f}{M_0} \right), f = \varphi^{-1}(\varphi(f)) = M_0 \left(1 - \exp \left(-\frac{\varphi(f)}{M_0} \right) \right) \tag{2}$$

This isomorphism φ allows the introduction of notions and structures outcoming from Functional Analysis, like the Euclidean norm:

$$\forall f \in S, \quad \| f \|_{\triangle} = |\varphi(f)|_{\mathbb{R}},$$

with $| \cdot |_{\mathbb{R}}$ being the usual absolute value.

Then, the following operations of addition, scalar multiplication, opposite and subtraction are defined:

$$\forall f, g \in S,\ f \mathbin{\triangle\!\!\!+} g = f + g - \frac{fg}{M_0}, \tag{3}$$

$$\forall f \in S, \forall \lambda \in \mathbb{R},\ \lambda \mathbin{\triangle\!\!\!\times} f = M_0 - M_0 \left(1 - \frac{f}{M_0}\right)^{\lambda}, \tag{4}$$

$$\forall f \in S,\ \ \triangle\!\!\!-\, f = \frac{-M_0 f}{M_0 - f}, \tag{5}$$

$$\forall f, g \in S,\ f \mathbin{\triangle\!\!\!-} g = M_0 \frac{f - g}{M_0 - g}. \tag{6}$$

The definition of the opposite operation $\triangle\!\!\!-\, f$ extends the gray tone range to the unbounded real-number range $]-\infty; M_0[$.

3 CoLIP Theory

The previous section has introduced the LIP theory for graytone images. This section presents the color space CoLIP, previously defined in [3,4]. It models the different stages of the human color vision, and also defines a vector space for color images.

The starting point for all CoLIP operations is the LMS color space [1]. The numerical applications (conversions from the different color spaces) are performed using the OptProp toolbox[1].

3.1 From Cone Intensities to Achromatic and Chromatic Tones

In the CoLIP framework, the chromatic tones are defined from the cone intensities L, M and S, as:

$$\forall c \in \{l, m, s\}, C \in \{L, M, S\}, c = M_0 \left(1 - \frac{C}{C_0}\right), \tag{7}$$

with C_0 is the maximal transmitted intensity level. M_0 is arbitrarily chosen at normalized value 100. Notice that $C \in]0; C_0]$ and $c \in [0; M_0[$.

The logarithmic response of the cones, as in the LIP theory, is modeled through the isomorphism φ.

$$\text{For } c \in \{l, m, s\},\ \tilde{c} = \varphi(c) = -M_0 \ln\left(1 - \frac{c}{M_0}\right) \tag{8}$$

$(\tilde{l}, \tilde{m}, \tilde{s})$ are called the logarithmic chromatic tones.

To follow the opponent-process theory of Hering, the three logarithmic color channels $(\tilde{l}, \tilde{m}, \tilde{s})$ are represented by a logarithmic achromatic tone $\tilde{a}$, and two

[1] By Jerker Wågberg, *More Research* and *DPC*, www.more.se

logarithmic chromatic tones, $\tilde{rg}$ and $\tilde{yb}$, where $\tilde{rg}$ opposes red and green, and $\tilde{yb}$ opposes yellow and blue. The conversion is obtained by the Eq. 9. The antagonist transformation matrix P is defined as follows according to the CIECAM02 specifications [1]. The achromatic channel is computed considering a ratio 40:20:1 in red, green and blue sensibility of the eye [11].

$$\begin{pmatrix} \tilde{a} \\ \tilde{rg} \\ \tilde{yb} \end{pmatrix} = P \times \begin{pmatrix} \tilde{l} \\ \tilde{m} \\ \tilde{s} \end{pmatrix}, \text{ with } P = \begin{pmatrix} 40/61 & 20/61 & 1/61 \\ 1 & -12/11 & 1/11 \\ 1/9 & 1/9 & -2/9 \end{pmatrix}. \tag{9}$$

3.2 The Trichromatic Antagonist Vectorial Structure

A color tone function, denoted f, is defined on a compact set D, with values in $]-\infty; M_0[^3$, by:

$$x \in D,\ F(x) = \begin{pmatrix} L(x) \\ M(x) \\ S(x) \end{pmatrix} \mapsto f(x) = \begin{pmatrix} a_f(x) \\ rg_f(x) \\ yb_f(x) \end{pmatrix}. \tag{10}$$

Thus, the operations of addition, scalar multiplication and subtraction can be defined in Eq. 11, and 12.

$$f \,\triangle\!\!\!\!+\, g = \begin{pmatrix} a_f \,\triangle\!\!\!\!+\, a_g \\ rg_f \,\triangle\!\!\!\!+\, rg_g \\ yb_f \,\triangle\!\!\!\!+\, yb_g \end{pmatrix},\ \lambda \,\triangle\!\!\!\!\times\, f = \begin{pmatrix} \lambda \,\triangle\!\!\!\!\times\, a_f \\ \lambda \,\triangle\!\!\!\!\times\, rg_f \\ \lambda \,\triangle\!\!\!\!\times\, yb_f \end{pmatrix}, \tag{11}$$

$$\triangle\!\!\!\!-\, g = \begin{pmatrix} \triangle\!\!\!\!-\, a_g \\ \triangle\!\!\!\!-\, rg_g \\ \triangle\!\!\!\!-\, yb_g \end{pmatrix},\ f \,\triangle\!\!\!\!-\, g = \begin{pmatrix} a_f \,\triangle\!\!\!\!-\, a_g \\ rg_f \,\triangle\!\!\!\!-\, rg_g \\ yb_f \,\triangle\!\!\!\!-\, yb_g \end{pmatrix}. \tag{12}$$

The set I of color tone functions defined on D and valued in $]-\infty; M_0[^3$, with the operations of multiplication $\triangle\!\!\!\!\times$ and internal addition $\triangle\!\!\!\!+$ is a real vector space. With the logarithmic color tone functions, $\tilde{f} = \varphi(f)$, the classical operations are used $(+, \times, -)$.

3.3 Bounded Vector Space

The vector space I defines a framework for manipulating color tone functions with values in $]-\infty; M_0[^3$. The opponent channels rg and yb are thus not symmetric, which can cause problems for computational manipulation and storage, or for representation. To handle this, it is proposed to introduce the three channels $\hat{f} = (\hat{a}, \hat{rg}, \hat{yb})$ defined by:

$$\hat{a} = a \tag{13}$$

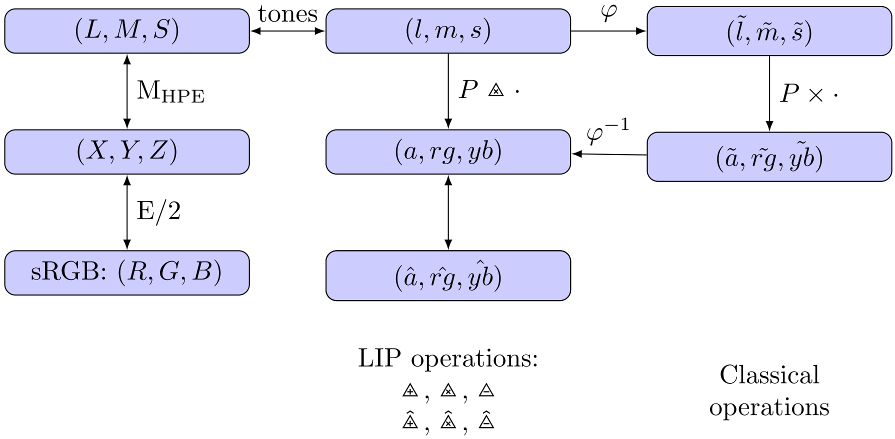
$$\hat{rg} = \begin{cases} rg & \text{if } rg \geq 0 \\ - \,\triangle\!\!\!\!-\, rg & \text{if } rg < 0 \end{cases} \tag{14}$$

$$\hat{yb} = \begin{cases} yb & \text{if } yb \geq 0 \\ - \,\triangle\!\!\!\!-\, yb & \text{if } yb < 0 \end{cases} \tag{15}$$

This representation is illustrated in the next sections.

3.4 Summary

The different conversion operations are presented in the following diagram:



Now that the formal definitions are introduced, the next section will show the connections with the psychophysical theories.

4 Applications

The chromaticity diagram is the representation of colors in a given space, for example (x, y), and in the case of the CoLIP space, $(\hat{rg}, \hat{yb})$. The purple line is a virtual straight line that links extreme values of the spectrum in (x, y). The Maxwell triangle is the representation of all RGB colors in this space (it has a shape of a triangle in (x, y)). The following applications will focus on the effect on the chromaticity diagram.

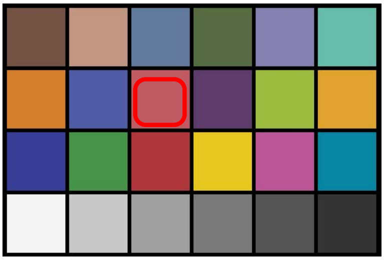
4.1 White Balance Correction

The proposed white balance correction method derives from the Von Kries adaptation model [1,7]. If L, M and S represent the cone responses, and L', M' and S' represent the adapted cone responses, this model can be written as $L' = \frac{L}{L_{\mathrm{White}}}$, $M' = \frac{M}{M_{\mathrm{White}}}$, $S' = \frac{S}{S_{\mathrm{White}}}$.

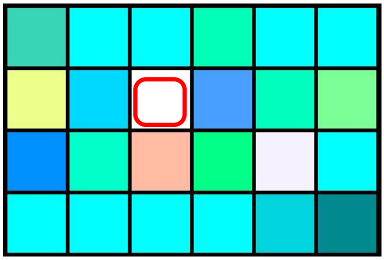
The Figure 1 presents the effects of the correction in the chromaticity diagram. The new white appears at coordinates $(0, 0)$, all colors appear more blue and brighter. This operation is not a translation in the $(\hat{rg}, \hat{yb})$ subspace.

4.2 Color Transfer

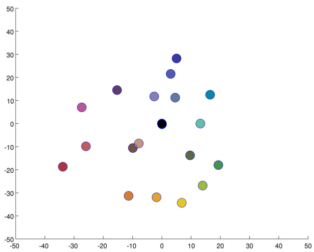
Color transfer is another application of white balance correction. Let us consider two images f_1 and f_2. The following notation is introduced, for a given collection



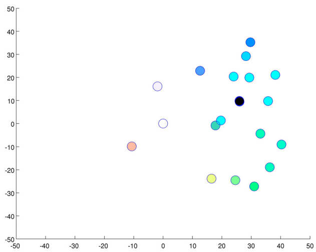
(a) MacBeth colorchecker. The chosen color is delineated with a red square.



(b) White balance correction for one color. The red square designates the white after correction.



(c) Chromaticity diagram of the MacBeth colorchecker. All achromatic tones (gray values from white to black) appear at coordinates $(0, 0)$.



(d) Chromaticity diagram of the MacBeth colorchecker after white balance correction.

Fig. 1: White balance correction with a manual selection of the White, with $\hat{rg}$ in absissa and $\hat{yb}$). in ordinates.

of values c: $\mu(c)$ is the mean of c, and $\sigma(c)$ is the standard deviation of c. For a color image in the CoLIP space,

$$\mu(f) = \begin{pmatrix} \mu(a) \\ \mu(rg) \\ \mu(yb) \end{pmatrix} \text{ and } \sigma(f) = \begin{pmatrix} \sigma(a) \\ \sigma(rg) \\ \sigma(yb) \end{pmatrix} \tag{16}$$

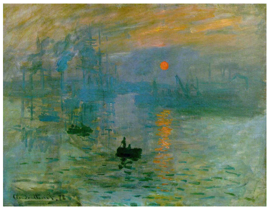
The transfer of colors of image f_1 into image f_2 corresponds to the Eq. 17, that gives the resulting image f_{new}. This formula centers and normalizes the distribution of colors in the original image f_1, and applies the same distribution as in image f_2 to the new image f_{new}.

$$f_{\text{new}} = \left(\frac{\sigma(f_1)}{\sigma(f_2)} \triangle\!\!\!\times (f_2 \triangle\!\!\!- \mu(f_2)) \right) \triangle\!\!\!+ \mu(f_1) \tag{17}$$

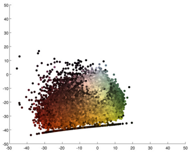
The Figure 2 shows the results of the transfer of the colors of two paintings (from VanGogh and Monet) into the painting of Guillaumin. The hull of the

(a) Guillaumin painting.

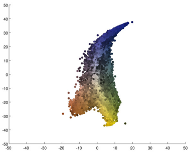
(b) Vangogh painting.



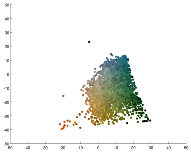
(c) Monet painting.



(d) Chromaticity diagram of 2a.



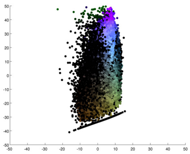
(e) Chromaticity diagram of 2b.



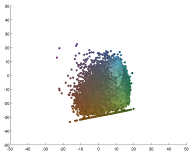
(f) Chromaticity diagram of 2c.

(g) Transfer of color of 2b into 2a.

(h) Transfer of color of 2c into 2a.



(i) Chromaticity diagram of 2g.



(j) Chromaticity diagram of 2h.

Fig. 2: Color transfer of different paintings and the representation of the colors in the $(\hat{rg}, \hat{yb})$ space.

color diagrams in $(\hat{rg}, \hat{yb})$ are similar to the original one, but the colors are now similar to the transfered paintings ones.

5 Concluding Discussion and Perspectives

This article has introduced the CoLIP framework and the representation of the colors of images in the form of a chromaticity diagram in the $(\hat{rg}, \hat{yb})$ space. The effects of basic operations like white balance correction or color transfer are proposed and illustrated, showing that the hull of the diagram is conserved, and the new colors are applied. The CoLIP framework presents two connections with the human visual perception system: it follows the Weber/Fechner law with its logarithmic model, and it also takes into account the opponent-process theory from Hering. Some other color spaces try have psychophysical justifications. For example, the $L^*a^*b^*$ space represents the opponent process with a^* (red-green opposition) and b^* (yellow-blue opposition), and the non linearity follows more or less the Stevens law (power law). In the case of the YC_bC_r color space, the non linearity coming from the gamma correction can be seen as a Stevens law, but the opponent-process theory is not included in this model.

The perspectives in the field of the CoLIP framework are to define color attributes like hue, saturation, and more mathematically, develop a distance between colors and define CoLIP mathematical morphology operators.

References

1. Fairchild, M.D.: Color appearance models. Wiley (2013)
2. Fechner, G.T.: Elemente der Psychophysik. Breitkopf und Härtel, Leipzig (1860)
3. Gouinaud, H., Gavet, Y., Debayle, J., Pinoli, J.C.: Color correction in the framework of color logarithmic image processing. In: Proceedings of the 7th IEEE International Symposium on Image and Signal Processing and Analysis (ISISPA), Dubrovnik, Croatia, pp. 129–133 (2011)
4. Gouinaud, H.: Traitement logarithmique d'images couleur. Ph.D. Thesis, École Nationale Supérieure des Mines de Saint-Etienne (2013)
5. Hering, E.: Outlines of a theory of the light sense. Harvard University Press (1964).(Trans. Hurvich, L.M., Jameson, D.)
6. Jourlin, M., Pinoli, J.C.: Logarithmic image processing. Acta Stereologica **6**, 651–656 (1987)
7. von Kries, J.: Die gesichtsempfindungen. Handbuch der physiologie des menschen **3**, 109–282 (1905)
8. Pinoli, J.C.: The logarithmic image processing model: Connections with human brightness perception and contrast estimators. Journal of Mathematical Imaging and Vision **7**(4), 341–358 (1997)
9. Stockman, A., Mollon, J.: The spectral sensitivities of the middle-and long-wavelength cones: an extension of the two-colour threshold technique of ws stiles. Perception **15**, 729–754 (1986)
10. Svaetichin, G.: Spectral response curves from single cones. Acta Physiol Scand Suppl. **39**(134), 17–46 (1956)
11. Vos, J., Walraven, P.: On the derivation of the foveal receptor primaries. Vision Research **11**(8), 799–818 (1971)
12. Weber, E.: Der Tastsinn und das Gemeingefühl. Handwörterbuch der Physiologie **3**(2), 481–588 (1846)
13. Young, T.: The bakerian lecture: On the theory of light and colours. Philosophical transactions of the Royal Society of London, pp. 12–48 (1802)