Teaching and Learning of Calculus

Victor Martinez-Luaces and Sunsook Noh

Aims

This Topic Study Group was a forum for discussions about the research and development in the teaching and learning of Calculus, both at upper secondary and tertiary level. Long and short presentations as well as the posters, showed advances, new trends, and an important work done in recent years on the teaching and learning processes of Calculus.

Organization

At ICME-12, TSG-13 had four one and a half hour timeslots and two general posters sessions. On the website of ICME-12 it is possible to access to all relevant documents including long presentations, short presentations and posters.

The accepted papers were organized as follows:

V. Martinez-Luaces (✉)
University of the Republic, Montevideo, Uruguay
e-mail: victoreml@gmail.com

S. Noh
Ewha Womans University, Seodaemun, Korea
e-mail: noh@ewha.ac.kr

- 4 papers were accepted for long presentations.
- A group of 13 papers were accepted for short presentations. Finally, only 1 of these papers was not presented in the group sessions.
- In each session 3 short presentations and a long one were delivered.
- Each session was devoted to an important topic in Calculus teaching and learning.
- Posters were presented in the general poster session which was common for all TSGs.

The structure for each of the four 90-min session included some brief opening remarks by the co-chairs of the committee, followed by a long presentation (20 min) and 3 short paper presentations (10 min each). After the long and short presentations of each session, the whole group had at least half an hour for questions, comments and general discussion.

The following paragraph provides details on the 4 oral sessions and the 2 poster presentations related to TSG-13.

Long and Short Presentations Delivered

Tuesday, July 10

This morning session was devoted to an important topic in Calculus teaching: the derivative concept. The long presentation was delivered by William Crombie, from U.S.A, who proposed an alternative architecture of Calculus, in order to allow the access to advanced concepts from an elementary standpoint to a larger group of learners. An example of this approach is given by the idea of "transition line" that can be used even before developing limits and derivatives.

After that, the first short presentation was given by Jungeun Park, from U.S.A., who studied the student's discourses on the derivative using a communicational approach to cognition. Particularly, she focused on students' descriptions about the derivative and the relationships among a function, the derivative function, and the derivative at a point.

The next speaker, Miguel Diaz, from Mexico, documented the understanding of the derivative and its meaning on the part of 12 teachers, who teach Calculus in a high school in Mexico, using for this purpose several questionnaires specifically designed.

Finally, Hyang Im Kang from Korea reported how 11th grade students went through in reinventing derivatives on their own via a context problem involving the concept of velocity.

Wednesday, July 11

This second morning session was devoted to modelling, applications and other topics and it started with Victor Martinez-Luaces, from Uruguay, who described teaching experiences with inverse problems—of both causation and specification types—and modelling in Engineering Calculus courses.

The first short presentation was delivered by Mohammad Pourkazemi from Iran. He showed how by giving applied examples of Economics and Management in each section of Calculus, it is possible to increase the interest in Mathematics among students.

Next speaker, Anne D'Arcy-Warmington, from Australia suggested a reversal of the order, showing Calculus applications first and then the rules as a consequence in a semi-modelling style approach.

Finally, Greg Oates from New Zealand reported on 11 contemporary studies selected from the last Delta conference, which presented direct applications to, or important implications for, current practice in the teaching of undergraduate Calculus.

Friday, July 13

The third session was devoted to several important concepts in Calculus, like integrals, series, etc. The long presentation was delivered by Anatoly Kouropatov, from Israel. In his paper, he discussed the idea of accumulation as a core concept for a high school integral Calculus curriculum.

Short presentations started with Maria Teresa Gonzalez, from Spain. In her paper, she described the growth of mathematical understanding in university students, engaged in mathematics classroom tasks about the concept of numerical series.

The second short talk was given by Rafael Martinez-Planell, from Puerto Rico. His paper focused on student graphical understanding of two variable functions. His study—which applies APOS and Semiotic Representation theories—was based on semi-structured interviews with 15 students.

This Friday session finished with Jennifer Czocher, from U.S.A., who investigated about topics in introductory differential equations and their relation with the knowledge that students are expected to retain from their Calculus courses.

Saturday, July 14

The last session of TSG 13 was about pre-Calculus and first Calculus courses, and started with the long presentation delivered by Dong-Joong Kim, from Korea. In his paper, Kim investigates characteristics of the limit concept through the simultaneous use of historical and experimental analyses.

David Bressoud, from U.S.A. was in charge of the first short presentation. He showed the preliminary report of results from a large-scale survey of Calculus I students in the United States. The analysis highlights students' mathematical background as well as aspects of instruction that contribute to successful programs.

Another large scale survey—in this case, carried out in China—was the starting point of the following talk delivered by Xuefen Gao. Her study, involving 256 college-level Calculus students and 3 teachers, investigated the problems and misunderstanding of concepts in Calculus and designed concept-based instruction to help students to understand concepts.

Finally, Jose Antonio Fernandez, from Spain presented results of an exploratory study performed with students of ages 16–17. He investigated the different uses that these students make of terms such as "to approach", "to tend toward", "to reach" and "to exceed", terms that describe some properties of the concept of finite limit.

Poster Sessions

10 posters corresponding to TSG 13 were presented in 2 general poster sessions.

In poster 13-1, Young Gon Bae, from Korea studied how university students matched graphs and functions. In the next poster (13-2) Rie Mizukami, studied the main changes in the Calculus content at senior high schools in Japan. The third poster (13-3) explained by Jacinto Eloy Puig, from Colombia, analyses the important interconnections between infinity and infinitesimal quantities. In the next one—13-4—Youngcook Jun, from Korea, explored how to use CAS to develop a step-by-step solver for Calculus learning. In poster 13-5, Kazuki Chida, from Japan, proposed how to obtain laws about trigonometric functions from a very simple differential equation, without any reference to either an angle or a triangle. The next poster—13-6—showed by Kanna Shoji, from Japan, is aimed for the development of teaching materials, in order to make the students understand the relation between real-life and mathematics. In poster 13-7, Allan Tarp, from Denmark explored Calculus roots in primary and middle school. The next poster, i.e., the 13-9, expounded by Abolfazi Gatabi, shows how Iranian students participate in class-room discussion about infinite and infinitesimal concepts. Poster 13-11, presented by Mikie Takahashi, from Japan, focuses on approximate value calculation and its relation with practical high school mathematics. Finally, in the last poster (13-13), Misfer AlSalouli, from Saudi Arabia, investigates mathematics high school teachers' conceptual knowledge regarding the topics on Calculus.

At the end of the second poster session, the authors had the opportunity for oral presentation of their posters, having the benefit of an audience related to the TSG.

Conclusions

Several issues related to teaching and learning of Calculus regularly appeared in the general discussions located at the end of the oral sessions. The main themes in those discussions were: technology, visualisation, problem-solving, modelling and applications, and assessment, among others. TSG-13 papers also featured learning theories, construction of Calculus concepts and ideas (limits, integrals, derivatives, etc.), roots of Calculus concepts and other important topics in Calculus teaching and learning.

Most of the papers (long and short presentations and posters) showed an interest for innovative approaches to different topics, in order to help students to improve their knowledge and comprehension of Calculus. In several cases, these innovations were directly related to the use of technology, whereas in others, they were more involved in teaching approaches, courses materials, or specific tasks to be carried out by students of different educative levels and careers.

It is hoped that this interesting discussions and interaction between teachers and researchers of different countries will stimulate innovative ideas that will progress the advancement of mathematics education—particularly, in Calculus teaching and learning—into the following years of this new century.