Lifting Filters Adjustment for Lossless Image Compression Applications

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Abstract. A method for adjustment of lifting scheme wavelet filters to achieve a higher image lossless compression is presented. The proposed method analyzes the image spectral characteristics and output the suboptimal coefficients to obtain a higher compression ratio in comparison to the standard lifting filters. The analysis follows by spectral pattern recognition with 1-*NN* classifier. Spectral patterns are of a small fixed length for the entire image permitting thus the optimization of the filter coefficients for different imager sizes. The proposed method was applied to a set of test images obtaining better image compression results in comparison to the standard wavelet lifting filters.

Keywords: lossless image compression, lifting scheme, pattern recognition.

1 Introduction

Actually, the wavelet lifting transform has become a popular and powerful tool for lossless image compression. Nevertheless, a problem to choose or design the wavelet lifting filters optimal to compress a given image is still present.

Lifting scheme first was proposed by W.Sweldens [1] in 1995 permitting integer-to-integer transforms [2]. Normally, wavelet lifting filters are obtained from known biorthogonal wavelet filters factoring the polyphase matrices [1], [3]. For lossless image compression applications, one step lifting filters CDF(2,2) and CDF(4,4) [2] are used.

To improve the compression ratio obtained with lifting scheme, various authors have proposed different techniques for adjustment lifting filters are known [4], [5], [6], [7], [8]. In common, they try to optimize the lifting predictor minimizing the mean square prediction error at the output of the predictor in case of one step lifting, i.e., minimizing the energy of wavelet coefficients. In [4], [7], [8] some positive results are reported (depending on the data) in the combination with other improvements. However, H. Thielemann [5] reported negative results for the least mean square optimization in comparison to CDF(2,2) performance. To obtain better compression, in papers [7], [8] lifting predictor is optimized using the autocorrelation computed on the signal differences instead of the standard autocorrelation.

An approach alternative to prediction error optimization was proposed in [9]. Analyzing the distribution of the wavelet coefficients, the authors suggested the

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minimization of ℓ_1-norm instead of ℓ_2-norm, i.e., minimize the absolute value of error instead of error energy. For the lifting update filter optimization, the minimization of ℓ_2 the difference between the approximation signal and the decimated version of the output of the ideal low-pass filter was proposed. Unfortunately, the minimization of ℓ_1-norm is much more complex and requires sophisticated techniques, so convex analysis by proximity operators and an iterative algorithm were applied for the problem of the weighted ℓ_1-norm lifting prediction filter optimization. The obtained results show a slightly better performance of the proposed method over the standard CDF(4,4) lifting meanwhile the complexity of the proposed adaptive lifting algorithm is very high.

In this work, we present the method to improve the performance of the lifting predictor filters based on the use of pattern recognition when designing the lifing filters for lossless image compression. We consider *k-NN* pattern classification algorithms to classify the image spectrum calculated in the discrete cosine transform domain. As a result, wavelet lifting filters coefficients both of the lifting prediction and lifting update are calculated. The obtained wavelet lifting filters transfor the image more efficiently in terms of compression ratio comparing with the standard wavelet lifting filters CDF(2,2) and CDF(4,4).

The paper is organized as follows. In Section 2, the lifting scheme filters are described and generalized. Section 3 describes the proposed method to obtain the lifting filter coefficients using pattern recognition technique *k-NN*. Section 4 presents simulation results on lossless compression of different standard test grayscale images. Next, the conclusions are given in Section 5.

2 Wavelet Lifting Filtering

The lifting scheme for integer-to-integer transform consists of the following basic operations: splitting, prediction and update [10].

Splitting splits the original signal $\{x\}$ into odd and even samples:

$$c_i = s_{2i}, \quad d_i = s_{2i+1}. \tag{1}$$

Prediction, or the dual lifting, at the level k calculates the wavelet coefficients or the details $\{d^{(k)}\}$ as the error in predicting $\{d^{(k-1)}\}$ from $\{s^{(k-1)}\}$ [9]:

$$d_i^{(k)} = d_i^{(k-1)} + \left\lfloor \sum_{j=-\tilde{N}/2}^{\tilde{N}/2} p_j c_{i+j}^{(k-1)} \right\rfloor, \tag{2}$$

where $\{p_j\}$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order that corresponds to the number of vanishing moments. $\lfloor \theta \rfloor$ denotes a rounding operation; it truncates the real numbers without a bias: if $\theta \geq 0$, it maps $\theta + 0.5$ to the largest previous integer value; if $\theta < 0$, it maps $\theta - 0.5$ to the smallest following integer value [2].

<u>Update</u>, or the primal lifting, combines $\{s^{(k-1)}\}$ and $\{d^{(k)}\}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\{x\}$ [10]:

$$c_i^{(k)} = c_i^{(k-1)} + \left\lfloor \sum_{j=-N/2}^{N/2} u_j d_{i+j}^{(k-1)} \right\rfloor, \tag{3}$$

where $\{u_j\}$ are coefficients of the wavelet-based low-pass FIR filter and N is the prediction filter order.

For the inverse transform, the signs of FIR filter coefficients $\{u_j\}$ and $\{p_j\}$ are switched, then, the inverse update, inverse prediction is calculated, and the odd and even data samples are merged [10].

The FIR filters that participate in the prediction and update operation can be described in the domain of Z-transform [11]. According to this approach, the transfer functions of the prediction and update FIR filters can be formulated as follows [12]:

$$H_p(z) = 1 + p_0\left(z + z^{-1}\right) + \ldots + p_{\frac{\tilde{N}}{2}-1}\left(z^{\tilde{N}-1} + z^{-\tilde{N}+1}\right), \tag{4}$$

$$H_u(z) = 1 + H_p(z)\left\{u_0\left[(z) + \left(z^{-1}\right)\right] .. + u_{\frac{N}{2}-1}\left[\left(z^{N-1}\right) + \left(z^{-N+1}\right)\right]\right\}. \tag{5}$$

The $H_p(z)$ must have zero at $\omega = 0$, i.e., at $z = 1$, and $H_u(z)$ should have a zero at $\omega = \pi$, i.e., at $z = -1$. These admissibility conditions are satisfied when [11]

$$\sum_{i=0}^{\frac{\tilde{N}}{2}-1} p_i = -\frac{1}{2}, \quad \sum_{i=0}^{\frac{N}{2}-1} u_i = \frac{1}{4}, \tag{6}$$

When the admissibility condition (6) is satisfied, $H_p(-1) = 2$ and $H_p(0) = 1$ that means the prediction filter has gain 2 at $\omega = \pi$ and unit gain at $\omega = \frac{\pi}{2}$; $H_u(1) = 1$ and the update filter has unit gain at $\omega = 0$. The formulae (6) in the case of (4,4) lifting scheme can be converted to reduce the degree of freedom in the predictor and update coefficients [11]. In our terms, the formulas for the wavelet filters coefficients are as follows:

$$p_0 = -\frac{128 + a}{256}, \quad p_1 = \frac{a}{256}, \tag{7}$$

$$u_0 = \frac{64 + b}{256}, \quad u_1 = -\frac{b}{256}, \tag{8}$$

where a and b are the parameters that control the DWT properties. It can be shown that for the standard lifting filters CDF(2,2) having $p_0 = -0.5$, $p_1 = 0.0$, $u_0 = 0.25$, $u_1 = 0.0$, the values of a, b are $a = 0$, $b = 0$; and for the standard lifting filters CDF(4,4) having $p_0 = -0.5625$, $p_1 = 0.0625$, $u_0 = 0.28125$, $u_1 = -0.03125$, the values of a, b are $a = 16$, $b = 8$. Varying the coefficients a, b values, one can control the properties of the resulting wavelet lifting filters; adjusting them to the signal spectral properties, a higher image compression rate can be achieved [12].

3 Proposed Method for Image Compression

The proposed method automatically obtains the wavelet filters coefficients from the image data, and can be described by the algorithm steps:

1. At the first step of the proposed algorithm, the image power spectrum is calculated.
2. At the second step, the spectrum is analyzed using artificial intelligence methods to obtain the wavelet lifting filter coefficients.
3. Next, having the lifting filter coefficients, the fast wavelet transform is applied.
4. The transformed image has reduced entropy in wavelet coefficients and can be processed searching the non-zero coefficient trees [13, 14] and then coded by one of the existent entropy coders [15].

At the fourth step of the algorithm, the proper image compression is performed, but in this paper we will concentrate on the steps 1-3.

The first step of the proposed algorithm is apply to the considered image $\mathbf{s}$ of size $M \times N$ the discrete cosine transform (DCT) [16] to obtain the mean power spectrum

$$\mathbf{S}_{DCT} = \left\langle \left(\mathbf{T}_{DCT} \mathbf{s} \right)^2 \right\rangle, \tag{9}$$

where $\mathbf{T}_{DCT}$ is a DCT transform matrix, $\langle \cdot \rangle$ is an averaging operation , $\mathbf{S}_{DCT}$ is a power DCT spectrum.

Subsequently, the resulting vector $\mathbf{x}$ is obtained interpolating the spectrum (9) to have a fixed length of 16 elements:

$$\mathbf{x} = F_{16}^{-1}\left\{ F_M \left\{ \mathbf{S}_{DCT} \right\} \right\}, \tag{10}$$

where $F_M\{\cdot\}$ denotes the direct Fourier transform of size M, and $F_{16}^{-1}\{\cdot\}$ is the inverse Fourier transform of size 16.

Thus, the image characteristic vector $\mathbf{x}$ of the reduced and fixed size is obtained. This vector can be considered as an input pattern. The set of coefficients a, b (8), (9) form the vector $\mathbf{y} = \{a, b\}$, which is the output pattern. Whilst the vector $\mathbf{x}$ is calculated using (9), (10), the vector $\mathbf{y}$ is found by exhaustive search varying the coefficients a, b to obtain those that minimize the transformed image data bitstream.

The patterns $\mathbf{x}, \mathbf{y}$ are associated with each image to form the fundamental set of patterns and perform the supervised learning and classification using 1-*NN* classifier. We choose the *k-NN* technique because it is one of the most efficient and simple existed classification algorithms [17, 18].

With the characteristic vector $\mathbf{x}$ and the filter coefficients $\mathbf{y}$ the associative memory $\mathbf{M}$ is generated and learned. At the learning stage, the empirically obtained coefficients $\mathbf{y}$ are associated with the patterns $\mathbf{x}$ obtained with the proposed spectral model (10), (11). At the retrieval stage, the patterns presented to the memory $\mathbf{M}$ are classified to obtain automatically the wavelet filter coefficients for their use in image compression.

Fig. 1. Standard test images: Lenna, boats, F-16, man, baboon, sailboat, Elaine, couple, Tiffany, peppers, aerial, bike, cafe, tools, Zelda

4 Results

In this Section, the results obtained applying the proposed method are presented. The experiments were accomplished using the fundamental set of the standard gray scale test images of different size (2048x2560, 1524x1200, 1465x1999, 1024x1024, 512 x 512), some of them are shown in Fig.1.

For the classification, the associative memory $\mathbf{M}$ was generated using 1-*NN* classifier. The obtained classification results show that with this classifier all test images from the fundamental set were classified correctly.

The performance in image compression of the proposed method was compared to the performance of the standard wavelet lifting filters, CDF(2,2), CDF(4,4), and the optimized by MSE criteria predictor filter (LPC(4,4)). The quantitative results were calculated in terms of the bitstream of the compressed image using MRWD technique [19] for zero-tree coding, and HENUC entropy coder [20]. For the validation of the proposed method, the Leave-One-Out (LOO) [18] technique was employed for each image from the fundamental image set.

The obtained results are presented in the Table 1. From this table, one can conclude that the proposed method outperforms the standard wavelet lifting filters CDF(2,2), CDF (4,4) and MSE-optimized predictor (LPC 4,4) in compression ratio for all tested standard natural images.

Table 1. Results of compression of test images, obtained by the standard wavelet lifting filters CDF(2,2), CDF(4,4), optimized for the minimum MSE predictor (LPC,4,4) and the proposed method, bits/pixel. The best results are marked by bold.

Image	CDF(2,2)	CDF(4,4)	LPC(4,4)	Preoposed method bitstream	LOO bitstream
aerial	5.24066	5.17578	5.19009	**5.16342**	5.17419
aerial2	5.40441	5.3759	5.38447	**5.37385**	5.38103
baboon	6.05795	6.02716	6.06048	**6.02478**	6.05154
baloon	3.0455	2.99612	3.02295	**2.9922**	3.00654
barb2	4.79006	4.73098	4.74082	**4.72457**	4.7434
bike	4.57663	4.55101	4.55337	**4.55101**	4.55138
board	3.82749	3.80355	3.79845	**3.78509**	3.80127
boats	4.10733	4.04614	4.046	**4.03661**	4.18232
café	5.39622	5.37565	5.39241	**5.37094**	5.37296
cmpnd1	2.27545	2.68219	2.39968	**2.27545**	2.64905
couple	4.8731	4.85717	4.85882	**4.85412**	4.85666
ct	4.60305	4.69238	4.62991	**4.57779**	4.64642
elaine	4.92572	4.89178	4.90475	**4.87786**	4.89654
f-16	3.99676	3.94662	3.94525	**3.93924**	3.94436
finger	5.68924	5.54129	5.51876	**5.50491**	5.52554
girl	4.07311	3.96761	4.00397	**3.95411**	3.97384
gold	4.60704	4.59701	4.59911	**4.59336**	4.59654
hotel	4.62189	4.60278	4.60252	**4.59913**	4.59859
lenna	4.27642	4.22351	4.24301	**4.22158**	4.237
man	4.67456	4.63626	4.64535	**4.63614**	4.64019
peppers	4.65405	4.6304	4.64795	**4.62765**	4.63247
sailboat	5.12625	5.08691	5.09573	**5.08102**	5.09973
target	2.4187	2.5133	2.44238	**2.41616**	2.48993
tiffany	4.20437	4.17398	4.19503	**4.17376**	4.20129
tools	5.48917	5.46594	5.46482	**5.46008**	5.46103
txtur2	5.60155	5.59089	5.60213	**5.58855**	5.59666
water	3.34702	3.02778	3.29817	**3.02778**	3.00013
woman	4.51546	4.4769	4.47416	**4.47301**	4.47652
x_ray	6.90568	6.94129	6.92038	**6.90438**	6.92017
zelda	3.87498	3.82118	3.84654	**3.81327**	3.8238

5 Conclusions

In this paper, we have proposed the method for wavelet lifting filter optimization based on the image spectral analysis in the DCT domain and on the use of artificial intelligence model, in particular, 1-*NN* classifier that demonstrated to be competitive versus other known models.

The designed algorithm was tested on different standard test images. The obtained lossless compression results were compared to the results obtained with the standard wavelet lifting filters CDF(2,2) and CDF(4,4). With the proposed method, a higher compression ratio in terms of entropy was obtained for all considered test images.

Though the presented results not reflected the real bitstreams of the compressed data, the optimization of the entropy resulted in less bitstream data. Nevertheless, the future work will be concerned with the implementation of some wavelet sub-band non zero data trees forming technique and entropy codec. As a future work, we consider also the implementation of the proposed method of optimization at each wavelet decomposition level expecting the better results in term of compression ratio.

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