Riemann Geometric Color-Weak Compensation for Individual Observers

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Abstract. We extend a method for color weak compensation based on the criterion of preservation of subjective color differences between color normal and color weak observers presented in [2]. We introduce a new algorithm for color weak compensation using local affine maps between color spaces of color normal and color weak observers. We show how to estimate the local affine map and how to determine correspondences between the origins of local coordinates in color spaces of color normal and color weak observers. We also describe a new database of measured color discrimination threshold data. The new measurements are obtained at different lightness levels in CIELUV space. They are measured for color normal and color weak observers. The algorithms are implemented and evaluated using the Semantic Differential method.

Keywords: Universal Design, Color-barrier-free Technology, Color-weak Compensation, Riemann geometry.

1 Introduction

Presenting a color image to observers so that their perception of the image is as similar as possible is a difficult problem. Methods to achieve this goal are important in human computer interface and have received a lot of interest due to recent rapid developments of visual media and wearable display technology. One cause for the problems encountered is the wide variation among observers from those with normal color vision over color-weak to near color blind observers. A second problem is the fact that perception is not directly measurable and there is therefore no objective criterion to measure the differences between the color perception of different observers.

A fundamental information used to characterize color vision properties is color discrimination thresholds and it is thus natural to compensate color-weak vision based on these data. This was described in [2] and [4]. This method characterizes color vision by using the fact that color spaces have a structure that can be described with the help of Riemann geometry [3]. This is used to construct a criterion for color-weak compensation that aims at the preservation of subjective color differences between color-normal and color-weak observers. A map preserving color-differences or Riemannian distance between color spaces is called an

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isometry. Therefore the task to compensate color-weak vision becomes to built a color difference preserving map or an isometry[2].

There are two ways to build an isometry between two color spaces when the Riemann metric tensor in both spaces are available. One is shown in [2][4] to build a set of local isometry maps at neighborhoods of sampling points in the color spaces. The other is shown in [6][5] to build a Riemann normal coordinate system using geodesics in both color spaces. The first one is easier to implement since it only needs linear algebra manipulations at each neighborhood, while the second requires to solve the second order ordinary differential equation to draw geodesics. It also needs a smooth interpolation of the Riemann metric tensor.

However, two problems remained unsolved for the first method. Firstly, estimation of local isometries from observed data could result in ill-conditioned linear equations. One also needs to establish the correspondence between neighborhoods or the origins of local coordinates before estimating the local isometries between them. Both problems are not trivial, in fact, as shown below, the first estimation problem is underdetermined or there is no unique solution to find a local isometry based on Riemann metric tensor information alone. The second problem is directly related with unobservability of color perception. Besides, previously used discrimination threshold data were measured on the chromaticity plane, so only 2D compensation was possible.

In this paper we build on these results and extend them in two directions:

1. The compensation is based on a function that maps the color spaces of the color-weak and the color-normal observer in a way that preserves the color differences as represented by the discrimination ellipsoids. We introduce a new algorithm to determine such a local ellipsoid preserving function f. The construction requires the solution of a nonlinear equation or a singular-value-decomposition of a restricted form of f.
2. The constructed functions f are local and defined on patches. It is therefore necessary to paste these patches together in order to construct a global mapping. We do this by introducing a new algorithm to find correspondences between the origins of local coordinates [1].

All these methods are based on the characterization of the color perception properties in the form of color discrimination data. We also present a new database of threshold data measured at lightness levels $L = 30, 40, 50, 60, 70$ (CIELUV). Previously such data was only available for one lightness level.

We will evaluate the proposed color-weak compensation methods based on the new measurement database in experiments where the performance is evaluated by the Semantic Differential (SD) method [9].

2 Geometry of Color Spaces and Color-Weak Compensation

Color spaces can be modeled as Riemann spaces in which the Riemann metric tensor is defined by the color discrimination threshold (MacAdam ellipsoids).

At a point x in a Riemann space C, the length of the deviation Δx from x is computed as

$$\|\Delta x\|^2 = \Delta x^T G(x) \Delta x. \quad (1)$$

$G(x)$ is a smoothly-varying positive-definite matrix, the Riemann metric tensor. Color differences are distances in the color space and the color discrimination threshold at x is the unit sphere at x. $G(x)$ is determined by color matching psychophysical experiments. The distance between color vectors x_1, x_2 is defined as the length of the shortest curve connecting the two points.

$$d(x_1, x_2) = \int_{\gamma_{12}} \|\Delta x\| = \int_{\gamma_{12}} \sqrt{\Delta x^T G(x) \Delta x} \quad (2)$$

For color spaces C_k with Riemann metric $G_k(x), (k = 1, 2)$ a map f from C_1 to C_2 is a local isometry if it preserves local distance and map discrimination ellipsoid at every x onto ellipsoid at $y = f(x)$:

$$G_1(x) = (D_f(x))^T G_2(y) D_f(x) \quad (3)$$

with D_f the Jacobian of f [3].

A map preserving large color-differences is called a global isometry, which means that the distance between any pair of points in one space is equal to the distance between the corresponding pair of points or their images in the other space. In fact a global isometry is also local isometry and vice versa[1].

If C_n, C_w are the color spaces of a color-normal observer and a color-weak observer, and if we can match the thresholds at every corresponding pair of points in the color spaces, such that the small color differences are adjusted to be always the same everywhere, then the large color difference between any corresponding pair of colors is also identical. The criterion of color-weak compensation is therefore proposed to transform the color space of the color-weak observer by an isometry so that it has the same geometry and therefore the same color differences everywhere as in the color space of color-normal observers[2].

Until now, two ways are proposed to construct an isometry either as a local isometry by discrimination threshold matching at every point [2] or as a global isometry by construct the Riemann normal coordinates at each color spaces[3][6][5]. In the following we will use only the first approach.

3 Compensation Algorithms

3.1 Compensation in 1D Spaces

The colorweak compensation algorithms [2,4] work in the 1D lightness compensation case as follows.

Denote the color spaces of a color-weak and a color-normal observer by C_w, C_n, and the isometry $y = f(x)$ with $f : C_w \longrightarrow C_n$. The discrimination thresholds at $x \in C_w$ and $y \in C_n$ are $\alpha_w(x)$, $\alpha_n(y)$. Denote the common reference point in both C_w and C_n as Q' .

In this case we have: $G_1(x) = 1/\alpha_w^2(x), G_2(y) = 1/\alpha_n^2(y)$ and then we find from the local isometry condition (3) that $1/\alpha_w^2(x) = D_f^2(x)/\alpha_n^2(y)$. Therefore, the isometry f from C_w to C_n has Jacobian

$$D_f(x) = \frac{\alpha_n(y)}{\alpha_w(x)} =: 1 - \omega(x) \quad (0 \leq \omega < 1) \tag{4}$$

Here ω describes the degree of color-weakness: e.g. $\omega = 1$ is color-blind, $\omega = 0$ is color-normal.

Then the color-weak simulation map f can be uniquely obtained from the integral of its Jacobian in C_w:

$$Q'' = f(Q) = \int_{Q'}^{Q} (1 - \omega(x))dx \tag{5}$$

On the other hand, the inverse f^{-1} of f, or the color-weak compensation map, can be obtained from the integral in C_n:

$$P = f^{-1}(Q) = \int_{Q'}^{Q} \frac{1}{(1 - \omega(y))} dy \tag{6}$$

Assuming piecewise constant thresholds or $\alpha_w(x), x$ in the k-th interval $[x_{k-1}, x_k]$ of C_w is a constant equal to $\alpha_w^{(k)} := \alpha_w(x_k)$ on the right end of the interval (and $\alpha_n(y)$ is a constant in k-th interval in C_n equal to $\alpha_n^{(k)} := \alpha_n(y_k)$), the color-weak map (and the compensation map) can be realized by a sum of the discrimination thresholds on direction of lightness:

$$Q'' = \sum_{i=0}^{I} (1 - \omega_i)(x_{i+1} - x_i) = \sum_{i=0}^{I} \alpha_n^{(i)} \tag{7}$$

$$P = \sum_{j=0}^{J} \frac{1}{1 - \omega_j} (y_{j+1} - y_j) = \sum_{j=0}^{J} \alpha_w^{(j)} \tag{8}$$

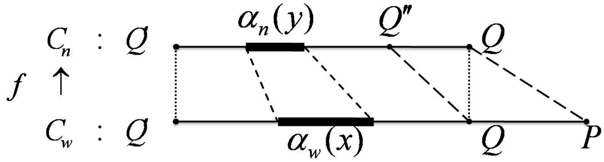

Fig. 1. Local isometry: 1D colorweak map

3.2 Color Weak Compensation in Higher Dimensions

Below, we show how to build the local isometry between color spaces. Such a map from the color space of a color-weak observer to that of color-normal observers is called the color-weak map w in the sense that it shows to color normal observers what the color-weak observer actually sees, which therefore will serve as color-weak simulation map. The inverse map of w, also an isometry, will serve as the compensation map which shows to the color weak observer what the color-normal observers see.

Assume we have a set of sampling points in the, three-dimensional, color space C_w of a color-weak observer: $\{x_i = (x_1^i, x_2^i, x_3^i)^T\}, i = 1, 2, \ldots$. They correspond to the set of the images of the sampling points in the, three-dimensional, color space C_n of a color-normal observers: $y = (x_2, y_2, z_2)^T = w(x) \in C_n$, $\{y_i = (y_1^i, y_2^i, y_3^i)^T\}, i = 1, ..., N$.

The colorweak map $w : C_w \longmapsto C_n$ is linearly approximated by the Jacobian matrix $D_w^{(k)} = D_w(x_k)$ in the neighborhood of each sampling point and its image neighborhood.

This defines the local affine map between the neighborhood of x_k and the neighborhood of its image $y_k = w(x_k)$ given by

$$y - y_k = D_w^{(k)}(x - x_k) \tag{9}$$

The Jacobian matrix $D_w^{(k)}$ of w is determined again by the local isometry or threshold matching condition (3):

$$G_n^{(k)} = (D_w^{(k)})^T G_w^{(k)} D_w^{(k)} \tag{10}$$

and we will combine the above 1D algorithm in the direction of L with a 2D isometry which compensates chromaticity differences.

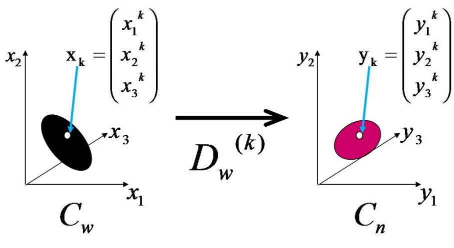

Fig. 2. Local isometry: 3D colorweak map

4 Estimation of the Local Affine Isometry

4.1 Local Linear Isometry

We assume first that a pair of color stimuli x, y of two color spaces C_1 and C_2 corresponding to each other under a (global) isometry is given. The metric tensors at these two points are $G_1(x)$ and $G_2(y)$. We show a method to determine a local linear isometry which maps $x \in C_1$ to $y \in C_2$ which preserves the local geometry or local color difference between the neighborhoods of $x \in C_1$ and $y \in C_2$.

The local linear isometry is the Jacobian of the global isometry at x or a matrix D_f which preserves the Riemann metric $G_1(x), G_2(y)$. It is given as a solution of the following equation:

$$G_2(y) = D_f^T G_1(x) D_f \tag{11}$$

This local linear isometry is not unique as can be easily seen. In the 2D case, this equation involves symmetric matrices of size 2×2 which gives three independent scalar equations. The matrix of the local isometry has however four entries and thus multiple solutions. In 3D case one has six equations and a local isometry defined by nine entries.

In the following we consider a restricted form of f which consists of scalings of the long and short axes of the ellipsoids and a rotation.

$$D_f = R\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a\cos\theta & b\sin\theta \\ -a\sin\theta & b\cos\theta \end{pmatrix}$$

Now we denote $X = a\cos\theta, Y = b\sin\theta, Z = -a\sin\theta, W = b\cos\theta$ to obtain a nonlinear equations in X, Y, Z, W as

$$XY + ZW = 0. \tag{12}$$

Further by choice of the local coordinates as the eigen vectors of $G_1(x)$ one can assume that $G_1(x)$ is a diagonal matrix

$$G_1(x) := \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \ G_2(y) := \begin{pmatrix} g_{11}^{(2)} & g_{12}^{(2)} \\ g_{12}^{(2)} & g_{22}^{(2)} \end{pmatrix}.$$

Therefore the following nonlinear equation can be solved to obtain the entries of D_f.

$$\begin{aligned} \lambda_1 &= g_{11}^{(2)} X^{(2)} + 2g_{12}^{(2)} XZ + g_{22}^{(2)} Z^2 \\ 0 &= g_{11}^{(2)} XY + 2g_{12}^{(2)} (XW + YZ) + g_{22}^{(2)} ZW \\ \lambda_2 &= g_{11}^{(2)} Y^2 + 2g_{12}^{(2)} YW + g_{22}^{(2)} W^2 \\ 0 &= XY + ZW \end{aligned}$$

Another way to solve equation (11) is to use the Singular Value Decomposition (SVD) of a matrix. The SVD provides a decomposition of a matrix M as a product $M = UAV$ where U and V are rotation matrices and A is diagonal. We apply this decomposition to the symmetric matrices $G_1 = V_1' A_1 V_1$ and $G_2 = V_2' A_2 V_2$ and the Jacobian $D_f = U_f A_f V_f$ and get

$$V_2' A_2 V_2 = V_f' A_f U_f' V_1' A_1 V_1 U_f A_f V_f \tag{13}$$

We see that we find a solution by first setting $V_2 = V_f, U_f = V_1'$. Using the fact that these matrices are orthogonal we find $A_2 = A_f A_1 A_f = A_1 A_f^2$ and therefore $A_f = \sqrt{A_2 A_1^{-1}}$.

4.2 Estimation of Affine Shifts in the Local Isometries

Now we build the global isometry by stitching local affine isometries. These local isometries are defined in the neighborhoods of every corresponding pair of points in C_w and C_n. Therefore they can be described as linear maps D_i between tangent spaces at $x_i \in C_w$ and $y_i \in C_n$ for $i = 1, ..., N$. So the D_i is defined with x and y as the origins in $T_x C_w$ and $T_y C_n$. However, the determination of the correspondence between x and y is not trivial.

Here we use a method we call neighborhood expansion to estimate the correspondence. We start with a known corresponding pair $O_w \in C_w$ and $O_n \in C_n$ and the Riemann metric $G(O_w)$ and $G(O_n)$ are also given. Such a pair can be chosen as e.g. D65. The two points are used as the origins in the local coordinates given above.

We then build a local linear isometry D between two linear spaces $T_{O_w} C_w$ and $T_{O_n} C_n$ in the way presented in the previous section.

Next we choose the points $x_i, i = 1, .., I$ inside the neighborhood N_{O_w} of O_w which are going to be used as the origins of local coordinates of the second generation in C_w. These neighborhoods then expand from the first neighborhood of O_w. Their images in C_n under the local isometry D can be found as

$$y_i = O_n + D(x_i - O_w) \in T_{O_n} C_n$$

which are used as the origins of the local coordinates in C_n corresponding to the neighborhoods of x_i.

Now for the second generation of these origins one builds local isometries $D_i : C_w \supset N_{x_i} \longrightarrow N_{y_i} \subset C_n, i = 1, .., I$ based on the Riemann metric $G_w(x_i)$ and $G_n(y_i)$. This process is repeated to expand the neighborhoods and for every new generation of origins to build $D_{ji} : C_1 \supset N_{x_{ji}} \longrightarrow N_{y_{ji}} \subset C_2, i = 1, .., N_j$ based on the Riemann metric $G_1(x_{ji})$ and $G_2(y_{ji})$. These local isometries will then eventually define a global isometry from C_w to C_n.

5 Color Discrimination Threshold Data

We used pair comparison experiments to determine the color discrimination thresholds. Measurement methods in psychophysical experiments vary from totally random ordering to adjustments by the observers themselves. While the totally random measurement is precise but time consuming, one wishes to avoid bias due to anticipation and adaptation or learning effects of observers. Therefore, we have chosen a randomized adjustment method as follows.

The observers include a D-type color weak observer and a color-normal observer. The illumination is Panasonic Hf premiere fluorescent light and a SyncMaster XL24 by Samsung is used for display. The Background is neutral grey of N 5.5. The observing distance is 80cm, the two frames on the display are 14×14 cm squares, with the left one as the test color and the right one is compared with the test color.

A session of color-matching starts with the display of a comparison color on the right frame. The observer is asked to use either the mouse wheel or a key touch to adjust the comparison color to the test color as close as possible. An accepted match finishes the session. The comparison color of the test color is randomly chosen on straight lines in 14 directions centered at the test color, with a random distance. The speed of the comparison color changes, responding to the movement of the mouse wheel or the number of key touches are also random. After 4 sessions, neutral gray is shown on the whole display for 7 seconds.

The sampling points in the CIELUV space are arranged on five planes of $L = 30, 40, 50, 60, 70$. On each plane, a uniform grid of sampling points is selected using the following number of gridpoints within the gamut of the lightness : 9 points in $L = 30$, 13 points in $L = 40$, 19 points in $L = 50$, 20 points in $L = 60$, 16 points in $L = 70$, therefore 77 points in the whole space.

The ellipsoids are then estimated from the observation data using the methods in [2][7][8].

Example threshold ellipsoids measured in 3D and $L = 60$ are shown in Fig. 3 to Fig. 6.

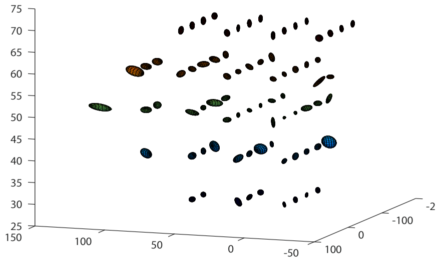

Fig. 3. Ellipsoids of color normal observer

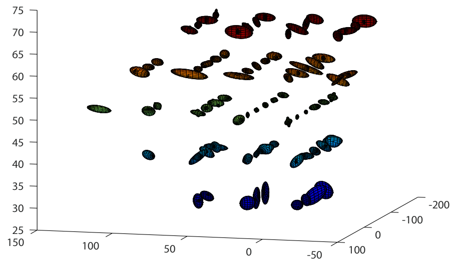

Fig. 4. Ellipsoids of color weak observer

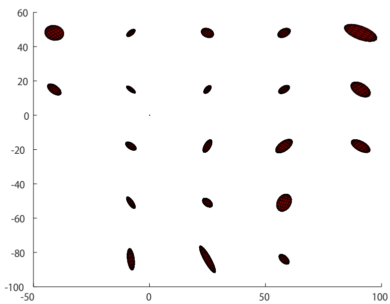

Fig. 5. Color normal discrimination threshold ellipses in L=60

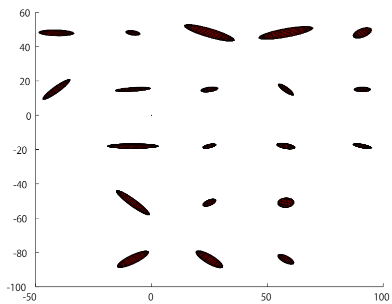

Fig. 6. Color weak discrimination threshold ellipses in L=60

6 Experiments and Evaluation

Compensation and color-weak simulation of an image using the proposed algorithms applied to the new data are shown in Fig.7,8,9.

Fig. 7. "Mountain": Original

Fig. 8. "Mountain": Color-weak simulation

Fig. 9. "Mountain": Compensation

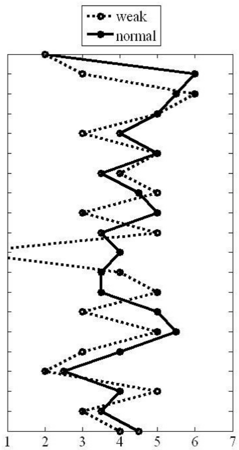

Fig. 10. Color-normal and color-weak view the original "Mountain"

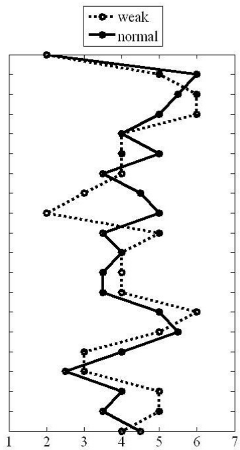

Fig. 11. Color-normal views the original, color-weak views the compensation of "Mountain"

Table 1. "Moutain": SD score

	Correlation	Distance
Before compensation	-0.721866	0.558297
After compensation	0.238643	0.380957

The performance of the color-weak compensation is difficult to evaluate directly. Below we apply the Semantic Differential (SD) method [9] to evaluate the results of the proposed method.

We choose 20 adjective pairs from the 76 pairs used in [9]. Objectives are marked for every question in a seven score scale.

7 Summary and Conclusions

We used approaches from the theory of Riemannian manifolds to develop a new method to construct mappings between color spaces of color weak and color normal observers. We showed how the linear approximation of the local mapping between the color spaces can be found by solving non-linear equations or Singular Value Decomposition. We also presented a method that allows us to stitch together these local solutions. Furthermore we described a new, extended database containing the color discrimination data of color normal and color weak observers. We illustrated the results obtained with the new method and evaluated it with the help of SD-evaluation.

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