



# Probabilistic Extensions: A One-Step Framework for Finding Rectangle Attacks and Beyond

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**Abstract.** In differential-like attacks, the process typically involves extending a distinguisher forward and backward with probability 1 for some rounds and recovering the key involved in the extended part. Particularly in rectangle attacks, a holistic key recovery strategy can be employed to yield the most efficient attacks tailored to a given distinguisher. In this paper, we treat the distinguisher and the extended part as an integrated entity and give a one-step framework for finding rectangle attacks with the purpose of reducing the overall complexity or attacking more rounds. In this framework, we propose to allow probabilistic differential propagations in the extended part and incorporate the holistic recovery strategy. Additionally, we introduce the “split-and-bunch technique” to further reduce the time complexity. Beyond rectangle attacks, we extend these foundational concepts to encompass differential attacks as well. To demonstrate the efficiency of our framework, we apply it to *Deoxys-BC-384*, *SKINNY*, *ForkSkinny*, and *CRAFT*, achieving a series of refined and improved rectangle attacks and differential attacks. Notably, we obtain the first 15-round attack on *Deoxys-BC-384*, narrowing its security margin to only one round. Furthermore, our differential attack on *CRAFT* extends to 23 rounds, covering two more rounds than the previous best attacks.

**Keywords:** Rectangle attack · Differential attack · Key recovery attack · *Deoxys-BC* · *SKINNY* · *ForkSkinny* · *CRAFT*

## 1 Introduction

Differential cryptanalysis, proposed by Biham and Shamir [BS91] in 1991, is one of the most efficient and powerful cryptanalysis for symmetric ciphers. In

differential cryptanalysis, the adversary aims to discover the in-homogeneity in high-probability occurrences of plaintext and ciphertext differences, *i.e.*, high-probability differentials. For a certain block cipher, if an  $r$ -round high-probability differential is found, one could add some outer rounds and extract the information of the key in the outer rounds using the differential.

However, it is a challenging task to find a long differential trail with high probability for most ciphers. To this, Wanger proposed the boomerang attack [Wag99] in 1999 as an extension of differential cryptanalysis. In a boomerang attack, two short differential trails are combined to form a long one. The basic boomerang attack requires adaptive chosen plaintexts and ciphertexts. In [KKS00], Kelsey *et al.* converted it into a chosen-plaintext variant, which was named the amplified boomerang attack. Later, Biham *et al.* [BDK01] made further improvements on the amplified boomerang attack by proposing the rectangle attack, which takes into account as many differences as possible in the middle of the distinguisher to estimate the probability more accurately. Since then, the boomerang and rectangle attacks have been extensively studied and applied to many block ciphers. For example, boomerang attacks on full AES-192 and AES-256 in the related-key setting were proposed in [BK09, DEFN22]; the best cryptanalysis results so far on Deoxys [BL23], SKINNY [SZY+22], and GIFT [DQSW22] were all based on boomerang attacks or rectangle attacks.

In either differential attacks or rectangle attacks, it is common to take two steps to mount key recovery attacks. The first step is to find a high-probability distinguisher covering a large number of rounds. To this end, many approaches have been proposed in the literature [Mat94, MP13, SHW+14, HBS21, DDV21]. Once a high-probability distinguisher is obtained, then in the second step, the distinguisher is extended backward and forward over some rounds with probability 1 for the key recovery attack. In recent years, many studies focused on achieving key recovery attacks as efficiently as possible for a given distinguisher. For differential attacks, Boura *et al.* [BDD+23] introduced a novel method that recovers the key in a meet-in-the-middle manner, which brought out improved results on block ciphers SKINNY-128-384 and AES-256. For rectangle attacks, various algorithms have been proposed for the key recovery [BDK01, BDK02, ZDJ19, DQSW21, SZY+22], each of which follows a different strategy for guessing the key. Among them, the unified key recovery algorithm proposed in [SZY+22] supports any strategy for guessing the key and covers all the previous algorithms. Most notably, this algorithm is able to produce the most efficient key recovery attack for a given distinguisher. As a result, it led to the state-of-the-art results of rectangle key recovery attack on *Serpent* [ABK98], *CRAFT* [BLMR19], *SKINNY* [BJK+16] and *Deoxys* [JNPS16].

However, prior research has demonstrated that the best distinguisher does not necessarily yield the most effective key recovery attack [LGS17, ZDC+21, QDW+21]. This phenomenon is even not limited to differential attacks and rectangle attacks [SSD+18, HSE23]. These studies have revealed that distinguishers are more conducive to key recovery attacks when their input and output differences exhibit sparsity and slow diffusion in the extended rounds. In [ZDC+21],

the authors specifically sought differentials that were more favorable for key recovery, while [QDW+21] incorporated both the rectangle distinguisher and the extended part in their modeling to find better attacks.

*Motivation.* Previous research offers valuable insights that inform our work.

- The effectiveness of a key recovery attack hinges not only on the probability of the distinguisher but also on the differential propagation in the outer rounds. Slower diffusion of differences in the outer rounds tends to yield better results.
- The key recovery is likely to be more efficient when the outer rounds involve fewer key bits.
- The choice of key guessing strategy significantly influences the complexity of the key recovery attack.

Despite the key recovery algorithm in [SZY+22] offering the best rectangle attacks for a given distinguisher, we are prompted to explore whether there is room for improving rectangle attacks further when we treat the distinguisher and the extended part as a whole and meanwhile incorporate the lessons mentioned above. Additionally, we are curious whether similar advancements can be applied to differential attacks.

*Our Contributions.* In this work, we treat the distinguisher and the extended part as a whole and present a one-step framework for finding rectangle attacks. Instead of extending the distinguisher forward and backward with probability 1, we introduce probabilistic differential propagations in the extended part. Traditionally, the extended part only contained deterministic truncated differentials, but now it can include probabilistic (truncated) differential propagations as well. Since the probability of the extended part may be smaller than 1, we refer to the part relevant to the key recovery phase as the *outer part* and the remaining part as the *inner part*.

The effects of probabilistic extensions are multifaceted

- Probabilistic extensions help make differences in the outer part sparse, which potentially reduces the time complexity or covers more rounds.
- The boundaries separating the inner part and the outer part are no longer predefined and not necessarily well-aligned. Instead, they are dynamically determined in conjunction with the probabilistic extensions
- The data complexity is then determined by the overall probability.

To further optimize the time complexity, we propose the *split-and-bunch technique*. When probabilistic extensions are used, differences in the outer part become sparse and certain key bits may form intricate connections with the inner part. In cases where we can identify a compressed representation of these connections, we can isolate and replace these key bits with this compressed version. As a result, the actual number of involved key bits decreases, leading to reduced time complexity.

**Table 1.** Summary of the cryptanalytic results. Rect./ D/ ZC/ ID = rectangle, differential, zero-correlation, impossible differential. RTK/ SK/ ST/ WK/ WT = related-tweakey, single key, single tweak, weak-key, weak-tweak.

Cipher	Rounds	Data	Memory	Time	Approach	Setting	Ref.
Deoxys-BC-384	14	$2^{125.2}$	$2^{140}$	$2^{260}$	Rect.	RTK	[DQSW22]
	14	$2^{115.7}$	$2^{160}$	$2^{260.59}$	Rect.	RTK	Sect. 4.1
	14	$2^{115.7}$	$2^{128}$	$2^{242.7}$	Rect.	RTK	Sect. 4.1
	15	$2^{115.7}$	$2^{128}$	$2^{371.7}$	Rect.	RTK	Sect. 4.1
SKINNY-128-256	26	$2^{126.53}$	$2^{128.44}$	$2^{254.4}$	Rect.	RTK	[DQSW22]
	26	$2^{126.53}$	$2^{136}$	$2^{241.38}$	Rect.	RTK	[SZY+22]
	26	$2^{121.93}$	$2^{136}$	$2^{219.93}$	Rect.	RTK	Full Ver
ForkSkinny-128-256	28	$2^{118.88}$	$2^{118.88}$	$2^{224.76}$	Rect.	RTK	[DQSW22]
	28	$2^{123.89}$	$2^{123.89}$	$2^{212.89}$	Rect.	RTK	Full Ver.
CRAFT	23	$2^{74}$	$2^{51}$	$2^{94}$	D	WK&ST	[LR22]
	26	$2^{73}$	$2^{60}$	$2^{105}$	D	WK&WT	[LR22]
	20	$2^{62.89}$	$2^{49}$	$2^{120.43}$	ZC	SK&ST	[HSE23]
	21	$2^{60.99}$	$2^{100}$	$2^{106.53}$	ID	SK&ST	[HSE23]
	19	$2^{60.99}$	$2^{68}$	$2^{94.59}$	D	SK&WT	[GSS+20]
	21	$2^{60.99}$	$2^{92}$	$2^{87.60}$	D	SK&WT	Sect. 4.2
	23	$2^{60.99}$	$2^{120}$	$2^{111.46}$	D	SK&WT	Sect. 4.2

Moreover, our framework incorporates the holistic key recovery strategy, *i.e.*, the method of finding the best key recovery attack by choosing a proper key guessing strategy.

The main ideas of this framework are not limited to rectangle attacks but can also be applied to differential attacks. We apply the framework to several block ciphers and obtain the following results, which are summarized in Table 1.

- We provide improved 14-round rectangle attacks on Deoxys-BC-384 and introduce the first attack on 15 rounds, reducing the security margin to just one round.
- The data and time complexities of the 26-round rectangle attack on SKINNY-128-256 are reduced. Similar improvements can be made on the 28-round attack on ForkSkinny-128-256. The details can refer to the full version [SYC+24] of the paper.
- We propose differential attacks on CRAFT with up to 23 rounds in the single-key setting, which is two rounds more than the previous best attacks.

*Organization.* The rest of the paper is organized as follows. In Sect. 2, we review differential attacks, rectangle attacks, and the key recovery algorithms. In Sect. 3, probabilistic extensions will be introduced via examples, and the one-step framework will be given. In Sect. 4, we apply our framework to several block ciphers. We conclude this paper in Sect. 5.

## 2 Preliminaries

### 2.1 Differential Attacks

Differential cryptanalysis [BS91] is a technique used to analyze the propagation of difference through a cipher  $E : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ . Typically, an attacker aims to find a differential  $(\alpha, \delta) \in \mathbb{F}_2^n \times \mathbb{F}_2^n$  such that the probability  $Pr(\alpha \rightarrow \delta) = Pr[E(x) \oplus E(x \oplus \alpha) = \delta]$  is high. Since  $E$  is a permutation, it follows that  $Pr(\alpha \rightarrow \delta) = Pr(\alpha \leftarrow \delta)$ . A truncated differential [Knu95] is characterized by a set of input differences  $\mathcal{D}_i$  and a set of output differences  $\mathcal{D}_o$  and the probability  $Pr(\mathcal{D}_i \rightarrow \mathcal{D}_o)$  is defined as  $Avg_{\alpha \in \mathcal{D}_i} Pr[E(x) \oplus E(x \oplus \alpha) \in \mathcal{D}_o]$ . Note that  $Pr(\mathcal{D}_i \rightarrow \mathcal{D}_o)$  and  $Pr(\mathcal{D}_i \leftarrow \mathcal{D}_o)$  are usually not equal. Typical sets for truncated differentials often involve patterns where some bits can take on all possible values, while others are constrained to be zero.

In differential-like attacks, a distinguisher is extended backward and forward with probability 1 over some rounds. The goal of these attacks is to recover the key used in the extended rounds. Essentially, the differential in the extended rounds is actually truncated differentials with probability 1. These types of attacks are designed to understand how differences propagate with a high degree of certainty, allowing for the identification of key bits used in those rounds.

**Key Recovery Attacks.** Suppose a differential  $\alpha \rightarrow \delta$  over  $E_d$  is of high probability  $P$ . Suppose  $E_b$  and  $E_f$  are added around  $E_d$ , as shown in Fig. 1.

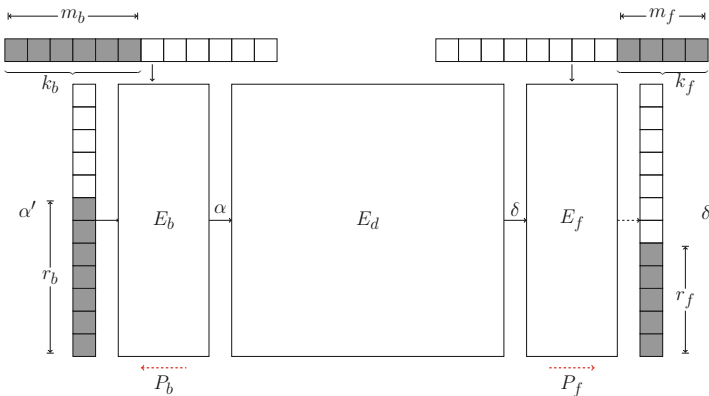


Fig. 1. Outline of the key recovery attack

The input difference of  $E_d$   $\alpha$  propagates back over  $E_b^{-1}$  with probability  $P_b = 1$  to  $\alpha'$ . Let  $V_b$  be the space spanned by all possible  $\alpha'$  where  $r_b = \log_2 |V_b|$ . The output difference  $\delta$  of  $E_d$  propagates forward over  $E_f$  with probability  $P_f = 1$  to  $\delta'$ . Let  $V_f$  be the space spanned by all possible  $\delta'$  where  $r_f = \log_2 |V_f|$ . Let  $k_b$

be the subset of subkey bits that are employed in  $E_b$  and affect the propagation  $\alpha' \rightarrow \alpha$ . Let  $k_f$  be the subset of subkey bits which are employed in  $E_f$  and affect the propagation  $\delta \leftarrow \delta'$ . Let  $m_b = |k_b|$  and  $m_f = |k_f|$  be the number of the bits in  $k_b$  and  $k_f$ .

In a key recovery attack, some key bits may be guessed in advance to sieve the data faster. Suppose a part of  $k_b$  and  $k_f$ , denoted by  $k'_b, k'_f$ , is guessed at first. Let  $m'_b = |k'_b|$  and  $m'_f = |k'_f|$  and  $0 \leq m'_b \leq m_b, 0 \leq m'_f \leq m_f$ . Suppose under the guessed subkey bits a  $r'_b$ -bit condition on the top and a  $r'_f$ -bit condition on the bottom can be verified. Finally, let  $r_b^* = r_b - r'_b$  and  $r_f^* = r_f - r'_f$ . Note that the other parameters are determined when  $k'_b, k'_f$  are chosen.

In [SYL23], a unified key recovery algorithm for differential attack was proposed and the details can refer to Appendix A.1 in the full version [SYC+24] of the paper. With  $k'_b, k'_f$  being guessed at first, the complexities of the differential attack are as follows.

*Complexities.* A plaintext structure takes all possible values for the  $r_b$  bits and chooses a constant for the remaining  $n - r_b$  bits. For one structure, there are  $2^{2r_b-1}$  pairs of plaintext and  $2^{r_b-1}$  of them satisfy  $\alpha$  difference by meeting the  $r_b$ -bit condition. Suppose the number of the structures needed is  $y$  which  $y$  structures can constitute  $y \cdot 2^{r_b-1}$  pairs that satisfy  $\alpha$  difference. Set  $s$  to be the number of right pairs, then  $y \cdot 2^{r_b-1} = s \cdot P^{-1}$  and the data complexity  $D = y \cdot 2^{r_b} = 2s \cdot P^{-1}$ . The memory complexity is  $M = \max\{D, 2^{t+m_b+m_f-m'_b-m'_f}\}$  for storing the data and key counters where  $0 \leq t \leq m'_b + m'_f$ .

The time complexity of the differential attack contains four parts:

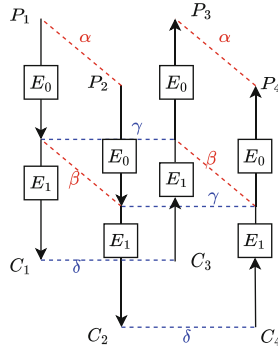
- $T_0 = 2^{m'_b+m'_f} \times D$  for partial encryption and decryption under the guessed key bits;
- $T_1 = 2^{m'_b+m'_f} \times D \times 2^{r_b-1+r_f-n-r'_b-r'_f}$  for getting the pairs that satisfy some filtering conditions;
- $T_2 = D \cdot 2^{m_b+m_f-n-1} \cdot \epsilon, \epsilon \geq 1$  for extracting all the  $D \cdot 2^{m_b+m_f-n-1}$  key candidates where  $\epsilon$  depends on the concrete situation;
- $T_3 = 2^{k-h}$ , where  $h \leq t + m_b + m_f - m'_b - m'_f$  for the exhaustive search.

## 2.2 Rectangle Attacks

In a boomerang attack, the target cipher is treated as a composition of two sub-ciphers  $E_0$  and  $E_1$ , i.e.,  $E = E_1 \circ E_0$ . As illustrated in Fig. 2, the differential trail  $\alpha \rightarrow \beta$  travels in  $E_0$  with probability  $p$ , and the differential trail  $\gamma \rightarrow \delta$  travels in  $E_1$  with probability  $q$ , respectively. Then the probability of the boomerang distinguisher is

$$\Pr[E^{-1}(E(x) \oplus \delta) \oplus E^{-1}(E(x \oplus \alpha) \oplus \delta) = \alpha] = p^2q^2.$$

The basic boomerang attack requires adaptive chosen plaintexts and ciphertexts. As a refinement of the boomerang attack, the rectangle attack requires only chosen plaintexts and considers as many differences as possible in the middle.



**Fig. 2.** Boomerang distinguisher

Since the boomerang attack was proposed, a series of studies have emerged focusing on the analysis of the connection probability. The probability  $p^2q^2$  of a boomerang distinguisher is obtained under the assumption that the two differentials are independent. However, the probability may deviate from  $p^2q^2$  on some concrete ciphers [BK09, Mur11], which demonstrates the dependency between the two differentials for boomerang attack. Instead of splitting the target cipher into two sub-ciphers, Dunkelman *et al.* [DKS10, DKS14] proposed to split it into three sub-ciphers and estimate the probability as  $p^2q^2r$  where  $r$  is the exact probability of the middle part, *i.e.*, the connection probability. That is also known as the sandwich attack. Later in [CHP+18], the dependency issue in boomerang distinguishers was revisited, and a tool named Boomerang Connectivity Table (BCT) was proposed. Immediately after, a generalized framework of BCT is given in [SQH19], which allows the probability  $r$  to be calculated systematically.

### 2.3 Key Recovery of Rectangle Attacks

The key recovery of rectangle attacks has been extensively studied in [BDK01, BDK02, ZDM+20]. Each of these works proposed its own key recovery algorithm that uses a fixed strategy for guessing key bits and the performance varies from cipher to cipher.

Recently in [SZY+22], a unified framework was put forward for finding the best rectangle attack for a given distinguisher. It contains a generic key recovery algorithm and a strategy for finding the best attack. The algorithm covers all the previous key recovery algorithms, as it supports any strategies for guessing key bits or utilizing the filters. Among all the strategies, there will be a certain one leading to an optimal attack. We call this method of finding the best key recovery attack by choosing a proper strategy the *holistic key recovery strategy*.

Suppose we treat a target cipher as  $E = E_f \circ E_d \circ E_b$ , where there is a boomerang distinguisher over  $E_d$  of probability  $P^2$ . As in differential attacks, a part of  $k_b$  and  $k_f$ , denoted by  $k'_b, k'_f$ , is guessed at first, and  $m'_b = |k'_b|$  and  $m'_f = |k'_f|$ . Similarly, with the guessed subkey bits, a  $r'_b$ -bit condition on the top

and a  $r'_f$ -bit condition on the bottom can be verified. Following the generic key recovery algorithm, which can refer to Appendix A.2 in the full version of the paper, the complexities of the rectangle attack are as follows.

*Complexities.* The data complexity is  $D = \sqrt{s}2^{n/2+1}/P$  where  $s$  is the expected number of right quartets. The memory complexity is  $M = f_M(D, k'_b, k'_f) = D + \min\{D \cdot 2^{r_b^*-1}, D^2 \cdot 2^{r_f^*-n-1}\} + 2^{t+m_b+m_f-m'_b-m'_f}$  for storing the data, the pairs and the key counters, where  $0 \leq t \leq m'_b + m'_f$ . The time complexity  $T = f_T(D, k'_b, k'_f)$  is composed of four parts. The time complexity of collecting data is  $T_0 = D$ , the time complexity of doing partial encryption and decryption under guessed key bits is

$$T_1 = 2^{m'_b+m'_f} \cdot D,$$

the time complexity of generating pairs is

$$T_2 = 2^{m'_b+m'_f} \cdot D \cdot \min\{2^{r_b^*-1}, D \cdot 2^{r_f^*-n-1}\},$$

the time complexity of generating and processing quartet candidates is

$$T_3 = 2^{m'_b+m'_f} \cdot D^2 \cdot 2^{2r_b^*} \cdot 2^{2r_f^*} \cdot 2^{-2n-2} \cdot \epsilon,$$

where  $\epsilon$  is a factor that depends on the cipher, and the time complexity of the exhaustive search is  $T_4 = 2^{k-h}$ , where  $h \leq t + m_b + m_f - m'_b - m'_f$ .

*Remark.* In essence, the holistic key recovery strategy is to find a proper choice for  $k'_b, k'_f$  such that the four parts of the time complexity are balanced and that the overall time complexity is minimized under constraints for data and memory.

### 3 A One-Step Framework for Finding Rectangle Attacks

In this section, we present our one-step framework for finding rectangle attacks. To begin, we use some examples from a toy cipher to convey the essential notions of the probabilistic extensions and the split-and-bunch technique. Subsequently, we provide a concise overview of the holistic key recovery strategy. By leveraging these concepts, we present a one-step framework for rectangle attacks, which is likewise applicable to differential attacks.

#### 3.1 Probabilistic Extensions

In a classical differential key recovery attack, some rounds are added around the differential distinguisher  $\alpha \rightarrow \delta$ , as shown in Fig. 1. The  $\alpha$  difference propagates to  $\alpha'$  via  $E_b^{-1}$  with probability 1, and the  $\delta$  difference propagates to  $\delta'$  via  $E_f$  with probability 1. It is natural to consider what occurs when difference propagates probabilistically in the outer parts. The following are examples to illustrate the impact of probabilistic extensions on the data and time complexities. For simplicity, none of the key bits are guessed in advance and the analysis of the memory complexity is omitted in the examples.

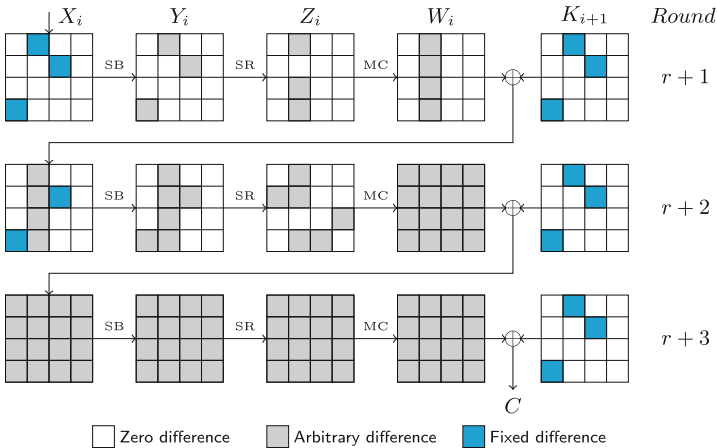


We suppose that the round function of the 128-bit toy cipher is the same as AES and the key schedule is rather simple. Each round uses the 128-bit master key  $K$  as the round key  $RK_i$ . The ordering of the bytes in the state matrix is as follows.

$$\begin{pmatrix} 0 & 4 & 8 & 12 \\ 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \end{pmatrix}$$

*Example 1.* Suppose the probability of the inner part is  $P_d$ , and append 3 rounds. These three rounds are depicted in Fig. 3. The parameters of the attack are  $n = 128, k = 128, m_b = r_b = 0, m_f = r_f = 128$ .

According to Sect. 2.1, the data complexity is  $D = 2s \cdot P_d^{-1}$ . From  $D$  plaintexts, there are  $D \cdot 2^{r_b+r_f-n-1}$  pairs satisfying the plaintext difference and ciphertext difference. Thus the time complexity for constructing pairs is  $T_1 = 2^{-1} \cdot D$ . As there will be  $2^{-1} \cdot D \cdot 2^{m_f-r_f} = 2^{-1} \cdot D$  suggestions for  $k_f$  in total, the time complexity for extracting key candidates is  $T_2 = D \cdot 2^{-1} \cdot \epsilon$ . We precompute several tables as illustrated in Table 2, so that  $\epsilon$  is equivalent to about  $2^{24}$  memory accesses. As the time complexity for the exhaustive search is flexible, we assume it is not dominant. Therefore, the overall time complexity is  $T = T_2 = 2^{-1} \cdot D \cdot 2^{24} = 2^{24} \cdot s \cdot P_d^{-1}$  memory accesses.



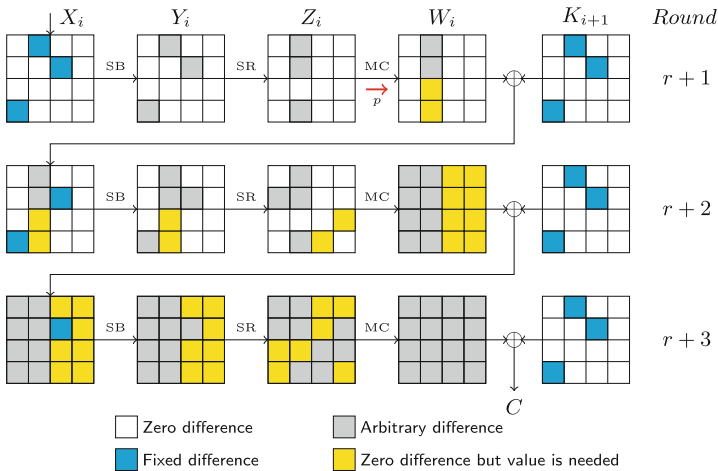
**Fig. 3.** The toy example of differential attack in the related-key model

*Example 2.* Different from Example 1, in this example the  $\delta$  difference propagates to  $\delta'$  via  $E_f$  with probability  $P_f = 2^{-16}$ . As shown in Fig. 4, the probability comes from two inactive bytes of  $W_{r+1}$ , i.e.,  $W_{r+1}[6,7]$ . The effect is that fewer bytes are activated afterward. The parameters of the attack are  $n = 128, k = 128, m_b = r_b = 0, m_f = 128, r_f = 72$ .

**Table 2.** Precomputation tables for Example 1 where  $eqk = SR^{-1} \circ MC^{-1}(K)$

Tables	Involved key	Filters	Remaining pairs
1	$eqk[4, 5, 6, 7]$	$\Delta Z_{r+2}[6] = 0$	$2^{24} \cdot 2^{-1} \cdot D$
2	$eqk[3, 9]$	$\Delta X_{r+2}[3, 9] = \Delta K_{r+1}[3, 9]$	$2^{24} \cdot 2^{-1} \cdot D$
3	$eqk[0, 1, 2]$	$\Delta Z_{r+2}[0, 2, 3] = 0$	$2^{24} \cdot 2^{-1} \cdot D$
4	$eqk[8, 10, 11]$	$\Delta Z_{r+2}[8, 9, 10] = 0$	$2^{24} \cdot 2^{-1} \cdot D$
5	$eqk[12, 13, 14, 15]$	$\Delta Z_{r+2}[12, 13, 15] =$ $\Delta Z_{r+1}[5] = 0$ $\Delta X_{r+1}[3, 4, 9]$	$2^{-1} \cdot D$

The data complexity is  $D = 2s \cdot (P_d P_f)^{-1}$ . The time complexities are  $T_1 = D \cdot 2^{0+72-128-1} = 2^{-57} \cdot D$ ,  $T_2 = D \cdot 2^{-57} \cdot \epsilon$ . We pre-compute several tables as illustrated in Table 3, so that  $\epsilon$  is equivalent to about  $2^{40}$  memory accesses. The overall time complexity is  $T_2 = 2^{-57} \cdot D \cdot 2^{40} = s \cdot P_d^{-1}$  memory accesses.



**Fig. 4.** The toy example of differential attack in the related-key model with probabilistic extension

**Comparison Between Example 1 and Example 2.** Example 1 uses traditional deterministic extensions in which the  $\delta$  difference propagates to  $\delta'$  via  $E_f$  with probability 1. On the contrary, Example 2 uses probabilistic extensions in which the  $\delta$  difference propagates to  $\delta'$  via  $E_f$  with probability  $P_f = 2^{-16}$ .

*Increasing the Data Complexity.* The data complexity for the case with probabilistic extensions is  $P_f^{-1} = 2^{16}$  times the data complexity with deterministic

**Table 3.** Precomputation tables for Example 2 where  $eqk = SR^{-1} \circ MC^{-1}(K)$ 

Tables	Involved key	Filters	Remaining pairs
1	$eqk[9]$	$\Delta X_{r+3}[9] = \Delta K_{r+2}[9]$	$2^{-57} \cdot D$
2	$eqk[0, 1, 2, 3]$	$\Delta Z_{r+2}[0, 2, 3] = 0$	$2^{-49} \cdot D$
3	$eqk[4, 5, 6, 7]$	$\Delta Z_{r+2}[6] = \Delta Z_{r+1}[6] = 0$ $\Delta X_{r+2}[3, 9] = \Delta K_{r+1}[3, 9]$	$2^{-49} \cdot D$
4	$eqk[8, 10 \sim 15]$	$\Delta X_{r+1}[3, 4, 9]$	$2^{-17} \cdot D$

extensions. In other words, the data complexity of the attack is determined by the overall probability  $P_d P_f$  rather than by the probability  $P_d$  only.

*Decreasing the Time Complexity.* The time complexity of Example 1 is  $2^{24} \cdot s \cdot P_d^{-1}$  and it is  $s \cdot P_d^{-1}$  for Example 2. By using the probabilistic extensions, the time complexity is reduced by a factor of  $2^{24}$ . Thus, probabilistic extensions are a technique that not only trades off data against time but may also have a gain as  $2^{24} > 2^{16}$ .

*Flexible Boundaries.* Different from the classical attack in which the boundaries between the inner and outer parts are fixed and well-aligned, the boundaries are flexible when probabilistic extensions are allowed. For a specified round, it is possible that some S-boxes belong to the inner part while others belong to the outer part. More interestingly, probabilistic differential transitions and deterministic differential transitions may occur simultaneously in the outer part and they may be even interleaved, the attack on SKINNY-128-256 can refer to Sect. 4.2 in the full version of the paper. Thus, there are no predefined boundaries. The boundaries will depend on differential transitions.

*Increasing the Number of Filters and Earlier Usage.* On one hand, the utilization of probabilistic extensions leads to an expansion of the total number of filter bits, growing from  $n$  to  $n + \log_2 P_f^{-1}$ . On the other hand, the filters appear earlier than before in the key recovery attack. The ability to employ filters at an earlier stage significantly enhances the efficiency of key candidate extraction, providing a reduction in the time complexity.

**Remarks.** Allowing probabilistic extensions essentially enables probabilistic differential propagations in the outer parts, which is the focus of key recovery.

- It is clear that the data complexity is related to the overall probability. Probabilistic extensions can strike a harmonious balance between data complexity and time complexity. However, it is important to note that probabilistic extensions do not necessarily increase data complexity in the search for a globally optimal attack, as exemplified by the attack on Deoxys-BC-384 in Sect. 4.1.

- Employing probabilistic extensions has the potential to reduce the density of active cells in the outer parts while simultaneously increasing the number of available filters. Notably, the probabilistic differential transitions are flexibly distributed in the outer parts and they do not have to connect to the differential trail over the inner part directly. This flexibility facilitates the adaptable selection of filters, ultimately resulting in improved time complexity.
- These extensions can be applied to  $E_b$  as well. Setting  $P_b \geq 1$ ,  $P_f \geq 1$  makes the probabilistic extensions cover the traditional deterministic extensions. Different extensions lead to different placements of filters. Among all possible extensions, there may be some that lead to more efficient key recovery attacks, in terms of time complexity or the number of rounds that can be attacked, compared to what deterministic extensions can achieve. Our applications to concrete ciphers will confirm this. Importantly, although our examples in this section primarily pertain to differential attacks, these same principles are applicable to rectangle attacks as well.

### 3.2 The Split-and-Bunch Technique

From Example 2, it is known that using probabilistic extensions brings the filters closer to the ciphertext, but the involved keys are the same. In this example, the last step is to verify  $\Delta X_{r+1}[3, 4, 9]$ . To compute this 3-byte difference, one has to know the values of  $W_{r+1}[6, 7]$ . However, another 56 key bits, *i.e.*,  $eqtk[8, 10 \sim 15]$  are required to compute  $W_{r+1}[6, 7]$  from the ciphertext. In other words, the 56-bit key brings the final 24-bit filter. Therefore, the remaining pairs expand by a factor of  $2^{32}$  as shown in Table 3. A question arises: Does the 56-bit key have to be traversed? To answer this question, we start with an observation on Example 2.

**Observation 1.** *Let  $k_f = k_f^1 || k_f^2$  with  $m_f^1 = 72$  and  $m_f^2 = 56$ . For one pair of ciphertexts, if we traverse all possible values of the 16-bit  $W_{r+1}[6, 7]$  without trying all possible values of the 56-bit  $k_f^2$ , we can find that:*

- For the right  $k_f^1$ , when a right pair takes the right values in the two bytes, it makes the counter plus one. Traversal must cause the right value to be taken, so the number of suggestions for the right subkey is the same.
- For a wrong pair, the number of suggestions for the wrong key is equal to expanding the number of pairs by a factor of  $2^{16}$ .

*We call this technique a split-and-bunch technique. Let  $e$  be the average number of suggestions for a wrong key  $k_f$  when the split-and-bunch technique is not used. When it is used, the number of suggestions for a wrong  $k_f^1$  is expanded by a factor of  $2^{m_e} = 2^{16}$  from  $e$ , *i.e.*, it is  $e \cdot 2^{m_e}$ , while the number of suggestions for the right  $k_f^1$  is  $s + e \cdot 2^{m_e} = s + e \cdot 2^{16}$ .*

In Example 2, the whole key is involved in  $E_f$ , *i.e.*,  $m_f = 128$ . In total we will get  $D \cdot 2^{m_f - n - 1} = D \cdot 2^{-1}$  suggestions for 128-bit  $k_f$ . On average, the number

of suggestions for a wrong subkey is  $e = D \cdot 2^{-129} = P_d^{-1} \cdot s \cdot 2^{-128} \leq 1$ , while it is  $s + e$  for the right subkey.

When we use the split-and-bunch technique and set counters only for the  $m_f^1 = 72$  key bits, we will get  $D \cdot 2^{-57} \cdot 2^{16} = D \cdot 2^{-41}$  suggestions for 72-bit  $k_f$ . Thus the number of suggestions for a wrong  $k_f^1$  is  $e' = 2^{-41-72} \cdot D = P_d^{-1} \cdot s \cdot 2^{-112}$ , while it is  $s + e'$  for the right  $k_f^1$ , where  $e' = 2^{m_e} \cdot e$ . Thus,  $m_e = 16$  is called the number of expansion bits. When  $e$  and  $e'$  are enough small, one could use the split-and-bunch technique to decrease the time complexity. The following example gives a better illustration.

*Example 3.* we use the split-and-bunch technique for this attack, as shown in Fig. 4. The number  $m_f$  of the involved key  $k_f$  in  $E_f$  is 72. The other parameters are  $n = 128, k = 128, m_b = r_b = 0, r_f = 72, m_e = 16$ .

The data complexity is  $D = 2s \cdot (P_d P_f)^{-1}$ . The time complexities are  $T_1 = D \cdot 2^{72-128-1} = 2^{-57} \cdot D, T_2 = D \cdot 2^{-57} \cdot \epsilon$ . We pre-compute several tables as illustrated in Table 4, so that  $\epsilon$  is equivalent to about  $2^8$  memory accesses. The overall time complexity is  $T = T_2 = 2^{-49} \cdot D = 2^{-32} \cdot s \cdot P_d^{-1}$  memory accesses.

**Table 4.** Precomputation tables for Example 3 where  $eqk = SR^{-1} \circ MC^{-1}(K)$

Tables	Involved key	Filters	Remaining pairs
1	$eqk[9]$	$\Delta X_{r+3}[9] = \Delta K_{r+2}[9]$	$2^{-57} \cdot D$
2	$eqk[0, 1, 2, 3]$	$\Delta Z_{r+2}[0, 2, 3] = 0$	$2^{-49} \cdot D$
3	$eqk[4, 5, 6, 7]$	$\Delta Z_{r+2}[6] = \Delta Z_{r+1}[6] = 0$ $\Delta X_{r+2}[3, 9] = \Delta K_{r+1}[3, 9]$	$2^{-49} \cdot D$
4	$W_{r+1}[6, 7]$	$\Delta X_{r+1}[3, 4, 9]$	$2^{-57} \cdot D$

**Comparisons Between Example 2 and Example 3.** Example 3 uses the split-and-bunch technique and the time complexity is  $T = 2^{-49} \cdot D = 2^{-32} \cdot s \cdot P_d^{-1}$ , which reduces the time complexity of Example 2 by a factor  $2^{-32}$ .

*Decreasing the Time Complexity.* The split-and-bunch technique reduces the time complexity further. This technique splits the key bits into two distinct categories: friendly key bits and unfriendly key bits. The former facilitates the efficient application of filters, while the latter, albeit intricately connected to some filters, can map to values comprising only a limited number of bits (i.e.,  $m_e$  bits). Rather than directly trying all possibilities of the unfriendly key bits themselves, we streamline the process by traversing the  $m_e$ -bit value, ensuring that the correct key is not overlooked. Consequently, the split-and-bunch technique results in an expansion of the counter value for incorrect keys but simultaneously reduces the overall number of key suggestions. This deliberate trade-off effectively minimizes the overall time complexity.

**When Counters are Not Used.** Differential attacks are possible without using counters in some cases, for example, using enumeration [Din14]. The split-and-bunch can also be used and may provide an improvement. Suppose we split at an  $m_e$ -bit state, after which key extraction becomes two parts. If the complexity of key extraction for each part is reduced by more than  $2^{m_e}$ , then split-and-bunch can provide an improvement.

### 3.3 Holistic Key Recovery Strategy

Probabilistic extensions fit well with the unified key recovery algorithm from [SZY+22]. However, the situation for the holistic key recovery strategy is different. Previously, the holistic key recovery strategy was considered in cases where a boomerang distinguisher was given. Hence, the boundaries between the inner and outer parts are clear and well-aligned. The only task is to find a proper set of key bits to be guessed in advance so that the overall time complexity is optimized.

On the contrary, the boundaries are not predetermined when probabilistic extensions are allowed. Instead, the boundaries and the set of guessed key bits should be determined together. That is, the situation for the holistic key recovery strategy is more generic in the presence of probabilistic extensions. To apply the holistic key recovery strategy, one needs to determine the involved key bits  $k_b, k_f$  from the boundaries, the guessed key bits  $k'_b, k'_f$ , and other parameters affected by  $k'_b, k'_f$ , including the obtained filters.

### 3.4 The Framework for Rectangle Attacks

Previously, one looked for the best rectangle attack for a given distinguisher. In this subsection, we present the one-step framework for rectangle attacks where the distinguisher and the extended part are considered together as a whole. The framework has four components: the unified key recovery algorithm, the holistic key recovery strategy, the core part, and the probabilistic extensions. In practice, we search for a cell-wise active pattern with the overall time complexity of the attack as the objective function and then instantiate it. As the treatment of the core part remains as in previous rectangle attacks, we focus on the upper and lower parts where probabilistic extensions occur and the determination of the boundaries is needed. Next, we present a constraint programming model for these parts.

Suppose the boomerang distinguisher  $\alpha \rightarrow \delta$  over the middle part  $E_d$  has probability  $P_d^2$ . Suppose  $\alpha$  propagates backward to  $\alpha'$  with probability  $P_b \leq 1$  and  $\delta$  propagates backward to  $\delta'$  with probability  $P_f \leq 1$ . The data complexity depends on these probabilities. Besides the probabilities, another important factor is the boundaries between the inner and outer parts since they mark the positions the key recovery phase has to reach and also determine the involved keys  $k_b, k_f$ .

*Data Complexity.* The probability for the whole attack is  $P^2 = P_b^2 P_d^2 P_f^2$ . We will show the formula for the data complexity remains as  $D = \sqrt{s} 2^{n/2+1}/P$ . Each plaintext structure takes all possible values for the  $r_b$  bits and chooses a constant for the remaining  $n - r_b$  bits. For each structure, there are  $2^{2r_b-1}$  pairs of plaintext with difference in  $V_b$  and  $P_b \cdot 2^{r_b-1}$  of them satisfy  $\alpha$  difference. That is equal to a  $(\log_2 P_b^{-1} + r_b)$ -bit condition. The distinguisher over the inner part has probability  $P_d^2$ , the number of quartets satisfying the input difference  $\alpha$  should be at least  $s P_d^{-2} 2^n$  for a rectangle attack, where  $s$  is the expected number of right quartets. Suppose the number of structures needed is  $y$ . These structures can constitute  $2 \cdot \binom{y \cdot P_b \cdot 2^{r_b-1}}{2}$  quartets that satisfy  $\alpha$  difference. Due to the output difference of  $E_d \delta$  propagating forward over  $E_f$  with probability  $P_f$  to  $\delta'$ , a right quartet propagates forward over  $E_f$  with probability  $P_f^2$ . Thus we get  $2 \cdot \binom{y \cdot P_b \cdot 2^{r_b-1}}{2} \cdot P_f^2 = s P_d^{-2} 2^n$ . Then  $y = \sqrt{s} 2^{n/2-r_b+1}/P$  and the data complexity is  $D = y \cdot 2^{r_b} = \sqrt{s} 2^{n/2+1}/P$ .

*Labels.* Due to the probabilistic extensions, there is a mix of concrete differences and truncated differences in the extended parts. For each cell of the internal state, we use two labels to describe its difference:  $(x, y) \in \{(0, 0), (1, 0), (1, 1)\}$ , where

- inactive, denoted by  $(x, y) = (0, 0)$  or  $\square$ ;
- active with a fixed non-zero difference, denoted by  $(x, y) = (1, 0)$ , or  $\blacksquare$ ;
- active with an arbitrary (truncated) difference, denoted by  $(x, y) = (1, 1)$  or  $\blacksquare$ .

Additionally, we use a label  $v$  to denote if the value of the cell is needed to verify the distinguisher. If it is, we denote by  $v = 1$  and  $v = 0$  otherwise. Then the labels will help to identify the boundaries as well as the probabilities  $P_b, P_f$  of the extension.

*Boundaries and  $P_b, P_f$ .* We take the forward extension as an example. The same techniques apply to the backward extension but in a reverse direction. There are several cases that happen with a probability.

- For the S-box layer, the probabilistic extension involves two cases:  $\blacksquare \rightarrow \blacksquare$  and  $\blacksquare \rightarrow \blacksquare$ . The number of such cases can be computed by

$$\sum_i (O_{i.x} - O_{i.y}), \tag{1}$$

where  $O_{i.x}, O_{i.y}$  are labels for the output cells of the S-box layer.

If the transition is  $\blacksquare \rightarrow \blacksquare$ , then the output of the S-box is needed for the verification, *i.e.*,  $O.v = 1$  for the output. Besides,  $O.v = I.v$ . Meanwhile, the fixed input difference acts as a one-cell filter.

- For the linear layer, where each output cell is a linear combination of some input cells, the probabilistic extension happens when truncated differences exist in the input but the output is a fixed difference. This fixed difference

happens with a probability. Let  $T = 1$  if some  $I_i.y = 1$  and  $T = 0$  if all  $I_i.y = 0$ . The number of such cases can be obtained by

$$\sum (T - O.y). \quad (2)$$

Conversely, each input cell is a linear combination of some output cells. If  $v = 1$  for the input cell, then the values for the involved output cells are also needed.

Now  $P_b, P_f$  can be computed (or estimated) by considering Eqs. (1) and (2) together. The outer parts are composed of those internal state cells with label  $v = 1$  as well as the operations and functions on them. This implies the boundaries. Then  $k_b, k_f$  are those key bits that are needed for determining the internal state cell with label  $v = 1$ .

*Guessing the Key and Filters.* The holistic key recovery strategy uses the guess-and-determine logic, which can be modeled as follows. For each key addition, if the key is guessed (a key cell in  $k_f$ ) and the output is known, then the input can be determined. Therefore, we introduce another label  $d$  for each state cell to denote if it is determined. For each key cell, we introduce a label  $g$  to denote if it is guessed. We then have  $I.d = 1$  when  $(K.g = 1) \wedge (O.d = 1)$  for  $I \oplus K = O$ . Then the determination can proceed backward naturally. For a linear operation, a filter is reached under the following conditions: (1) its output contains truncated cells, (2) the difference of an input cell can be determined, and (3) this input difference has a fixed difference. For the S-box, if the input can be determined and the input difference is fixed, then we have a filter. Finally, the number of filters  $r'_f$  can be computed by recording such cases and the number of guessed keys  $k'_f$  can be computed by counting distinct key cells with  $g = 1$ .

*Constraints for the Complexities.* The constraints for the data and memory complexities can be added, such as  $D < 2^n$ . The objective is to minimize the overall time complexity as described in Sect. 2.3. The source codes for the constraint programming model are available [here](#).

### 3.5 The Framework for Differential Attacks

Similar to the framework for rectangle attacks, a framework for differential attacks can be built, because the basic ideas and all the involved parameters are the same. The only difference lies in the computation of complexities.

## 4 Applications

In this section, we apply our new framework to **Deoxys-BC-384**, **SKINNY-128-256**, and **CRAFT** block ciphers. For **Deoxys-BC-384**, using the one-step framework for rectangle attacks obtains a 15-round rectangle attack for the first time, containing a 10-round inner part rather than the longest 11-round one. For **SKINNY-128-256**, we get a new 26-round rectangle attack with reduced data and time complexities. For **CRAFT**, we obtain a 21-round differential attack and a 23-round differential attack by the new framework.



#### 4.1 Application to Deoxys-BC-384

**Specification.** Deoxys-BC is an AES-based tweakable block cipher [JNPS16], based on the tweakable framework [JNP14]. The Deoxys authenticated encryption scheme makes use of two versions of the cipher as its internal primitive: Deoxys-BC-256 and Deoxys-BC-384. Both versions are ad-hoc 128-bit tweakable block ciphers which besides the two standard inputs, a plaintext  $P$  (or a ciphertext  $C$ ) and a key  $K$ , also take an additional input called a *tweak*  $T$ . The concatenation of the key and tweak states is called the *tweakey* state. For Deoxys-BC-384, the tweakey size is 384 bits.

Deoxys-BC is an AES-like design, *i.e.*, it is an iterative substitution-permutation network (SPN) that transforms the initial plaintext (viewed as a  $4 \times 4$  matrix of bytes) using the AES round function, with the main differences with AES being the number of rounds and the round subkeys that are used every round. Deoxys-BC-384 has 16 rounds.

Similarly to the AES, one round of Deoxys-BC has the following four transformations applied to the internal state in the order specified below:

- AddRoundTweakey – XOR the 128-bit round subtweakey to the internal state.
- SubBytes – Apply the 8-bit AES S-box to each of the 16 bytes of the internal state.
- ShiftRows – Rotate the 4-byte  $i$ -th row left by  $\rho[i]$  positions, where  $\rho = (0, 1, 2, 3)$ .
- MixColumns – Multiply the internal state by the  $4 \times 4$  constant MDS matrix of AES.

After the last round, a final AddRoundTweakey operation is performed to produce the ciphertext.

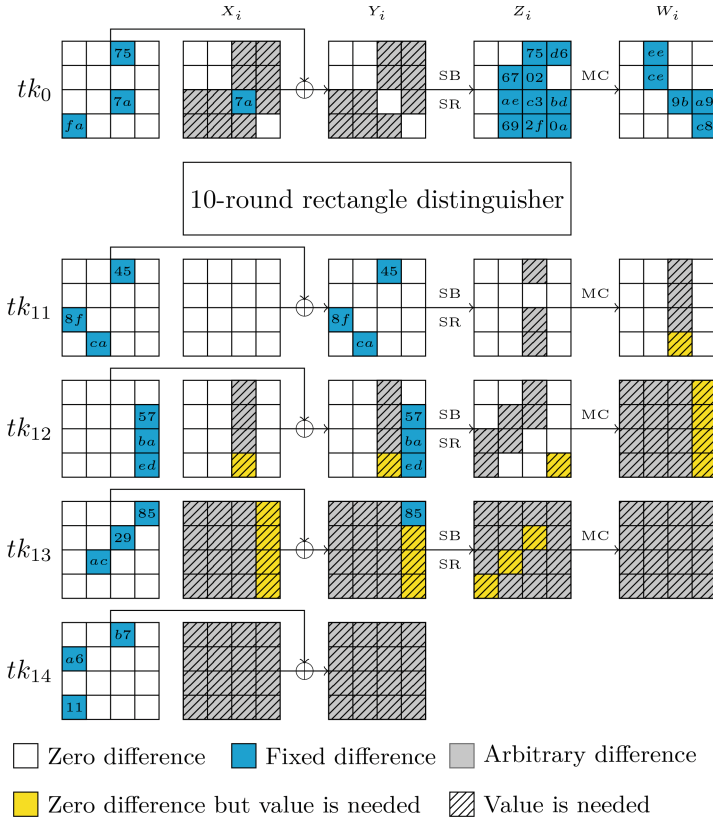
We denote the concatenation of the key  $K$  and the tweak  $T$  as  $KT$ , *i.e.*,  $KT = K||T$ . For Deoxys-BC-384, the size of  $KT$  is 384 bits, and we denote the first, second and third 128-bit words of  $KT$  by  $W_3$ ,  $W_2$  and  $W_1$ , respectively. Finally, we denote by  $STK_i$  the 128-bit *subtweakey* that is added to the state at round  $i$  during the AddRoundTweakey operation. For Deoxys-BC-384, a subtweakey is defined as  $STK_i = TK_i^1 \oplus TK_i^2 \oplus TK_i^3 \oplus RC_i$ . The tweakey schedule algorithm is defined as  $TK_{i+1}^1 = h(TK_i^1)$ ,  $TK_{i+1}^2 = h(LFSR_2(TK_i^2))$  and  $TK_{i+1}^3 = h(LFSR_3(TK_i^3))$ , where the byte permutation  $h$  is defined as

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 6 & 11 & 12 & 5 & 10 & 15 & 0 & 9 & 14 & 3 & 4 & 13 & 2 & 7 & 8 \end{pmatrix},$$

with the 16 bytes of a tweakey word numbered by the usual AES byte ordering.

**14-Round Rectangle Attack on Deoxys-BC-384.** By our new one-step framework, we get a 14-round rectangle attack on Deoxys-BC-384 with a 10-round inner part in  $E_d$ , 1-round in  $E_b$  and 3-round in  $E_f$ , as shown in Fig. 5. Detailed of the 10-round inner part refers to Fig. 11 in the full version of the paper. The probability of  $E_b$  is 1, the probability of  $E_d$  is  $P_d^2 = 2^{-14 \times 2 - 11.4 - 7 \times 2 - 14 \times 2} = 2^{-81.4}$

and the probability of  $E_f$  is  $P_f^2 = 2^{-8 \times 2}$ . Thus the probability of the whole rectangle attack is  $P^2 = P_b^2 P_d^2 P_f^2 = 2^{-97.4}$ , and other parameters of the attack are:  $n = 128, k = 384, m_b = r_b = 80, m_f = 8 \times (16 + 7 + 3) = 208$  and  $r_f = 8 \times 13 = 104$ .



**Fig. 5.** 14-round Rectangle Attack on Deoxys-BC-384

The best guessing parameters are  $m'_b = 8 \times 10 = 80, r'_b = 8 \times 10 = 80, m'_f = 8 \times (5 + 1) = 48, r'_f = 32, r_b^* = 0$  and  $r_f^* = 72$ , which means guessing 10 bytes of  $k_b$  and 6 bytes of  $k_f$ . The 6 bytes of  $k_f$  are  $eqtk_{14}[8, 9, 10, 11, 12]$  and  $eqtk_{13}[13]$  respectively. The complexities of our new attack are as follows<sup>1</sup>.

- The data complexity is  $D_R = 4 \cdot D = 4 \cdot y \cdot 2^{r_b} = 4 \cdot \sqrt{s} \cdot 2^{n/2} / P = 4 \cdot \sqrt{s} \cdot 2^{48.7+64} = \sqrt{s} \cdot 2^{114.7}$ .
- The memory complexity is  $M_R = D_R + D \cdot 2^{r_b^*} + 2^{t+m_b+m_f-m'_b-m'_f} = \sqrt{s} \cdot 2^{114.7} + \sqrt{s} \cdot 2^{112.7} + 2^{t+160}$ .

<sup>1</sup> We use a variant of formulas from Sect. 2.3 in the related-key setting for ciphers with a linear key schedule.

- The time complexity  $T_1 = 2^{m'_b+m'_f} \cdot D_R = \sqrt{s} \cdot 2^{128+114.7} = \sqrt{s} \cdot 2^{242.7}$ ;
- $T_2 = 2^{m'_b+m'_f} \cdot D = \sqrt{s} \cdot 2^{128+112.7} = \sqrt{s} \cdot 2^{240.7}$ ;
- $T_3 = 2^{m'_b+m'_f} \cdot D^2 \cdot 2^{2r_b^*} \cdot 2^{2r_f^*} \cdot 2^{-2n} \cdot \epsilon = s \cdot 2^{128+112.7 \times 2 + 72 \times 2 - 2 \times 128} \cdot \epsilon = s \cdot 2^{241.4} \cdot \epsilon$ ;
- $T_4 = 2^{m'_b+m'_f-t} \cdot 2^{k+t-m'_b-m'_f-h} = 2^{k-h}$ .

Processing a candidate quartet to retrieve the rest  $k_f$  can be realized by looking up tables. We pre-compute several tables as illustrated in Table 5, detailed refers to [SZY+22], so that  $\epsilon$  is equivalent to about  $2^{25}$  memory accesses which is around  $2^{25} \times \frac{1}{14} \times \frac{1}{16} = 2^{17.19}$  encryption. The time complexity and memory complexity of the tables are related to the involved state bits, the involved subkeys and the filters. The time complexity and memory complexity of the tables are all  $2^{128}$ . If we set  $s = 4, t = 0$ , then the data, memory, and time complexities of our attack are  $2^{115.7}$ ,  $2^{160}$  and  $2^{260.59}$  respectively.

**Table 5.** Precomputation tables for the 14-round attack on Deoxys-BC-384 where  $eqtk_i = \text{SR}^{-1} \circ \text{MC}^{-1}(\text{tk}_i)$  (let  $Q = s \cdot 2^{241.4}$ )

Tables	Involved key	Filters	Remaining quartets
1	$eqtk_{14}[0, 1, 2, 3]$	$\Delta Z_{12}[0, 1] = 0$	$Q$
2	$eqtk_{13}[15]$	$\Delta Y_{12}[15] = 0xed$	$Q \cdot 2^{-8}$
3	$eqtk_{14}[4, 5, 6, 7]$	$\Delta Z_{12}[4, 7] = 0$	$Q \cdot 2^{-8}$
4	$eqtk_{13}[14]$	$\Delta Y_{12}[14] = 0xba$	$Q \cdot 2^{-16}$
5	$eqtk_{13}[8, 9, 10]$	$\Delta Z_{11}[10] = 0$	$Q \cdot 2^{-8}$
6	$eqtk_{14}[13, 14, 15], eqtk_{13}[11]$	-	$Q \cdot 2^{24}$
7	$eqtk_{12}[2, 7, 8]$	$\Delta Y_{12}[2, 7, 8] = 0x8f, 0xca, 0x45$	$Q$

**Improved 14-Round Rectangle Attack by the Split-and-bunch Technique.** From the time complexity of the 14-round rectangle attack above, we can find that in subtable 6 there are 4 bytes involved subkeys and 0 filter. The number of quartets in this subtable reaches the maximum, making the time complexity of this step dominant.

With the same guessing strategy, by using the split-and-bunch technique, the number of the involved subkeys bits in  $E_f$  is  $m_f = 8 \times (13+6+3) = 176$ . And the number of the counts for a wrong key is  $e = D^2 \cdot 2^{m_e-2n-16} = s \cdot 2^{225.4+8-272} = s \cdot 2^{-38.6} \ll s$ . We get the new  $\epsilon$  as illustrated in Table 6. The time complexity and memory complexity of the tables are  $2^{128}$ . It is shown  $\epsilon$  is equivalent to about 2 memory accesses which is around  $2 \times \frac{1}{14} \times \frac{1}{16} = 2^{-6.81}$  encryption. The memory complexity is  $\sqrt{s} \cdot 2^{114.7} + \sqrt{s} \cdot 2^{112.7} + 2^{t+(176+80)-80-48}$ . If we set  $s = 4, t = 0$ , then the data, memory and time complexities of our attack are  $2^{115.7}$ ,  $2^{128}$  and  $2^{243.7}$  respectively.

**Table 6.** Precomputation tables for the improved 14-round attack on Deoxys-BC-384 where  $eqtk_i = SR^{-1} \circ MC^{-1}(tk_i)$  (let  $Q = s \cdot 2^{241.4}$ )

Tables	Involved key	Filters	Remaining quartets
1	$eqtk_{14}[0, 1, 2, 3]$	$\Delta Z_{12}[0, 1] = 0$	$Q$
2	$eqtk_{13}[15]$	$\Delta Y_{12}[15] = 0xed$	$Q \cdot 2^{-8}$
3	$eqtk_{14}[4, 5, 6, 7]$	$\Delta Z_{12}[4, 7] = 0$	$Q \cdot 2^{-8}$
4	$eqtk_{13}[14]$	$\Delta Y_{12}[14] = 0xba$	$Q \cdot 2^{-16}$
5	$eqtk_{13}[8, 9, 10]$	$\Delta Z_{11}[10] = 0$	$Q \cdot 2^{-8}$
6	$eqtk_{12}[2, 7, 8], W_{11}[11]$	$\Delta Y_{12}[2, 7, 8] = 0x8f, 0xca, 0x45$	$Q \cdot 2^{-24}$

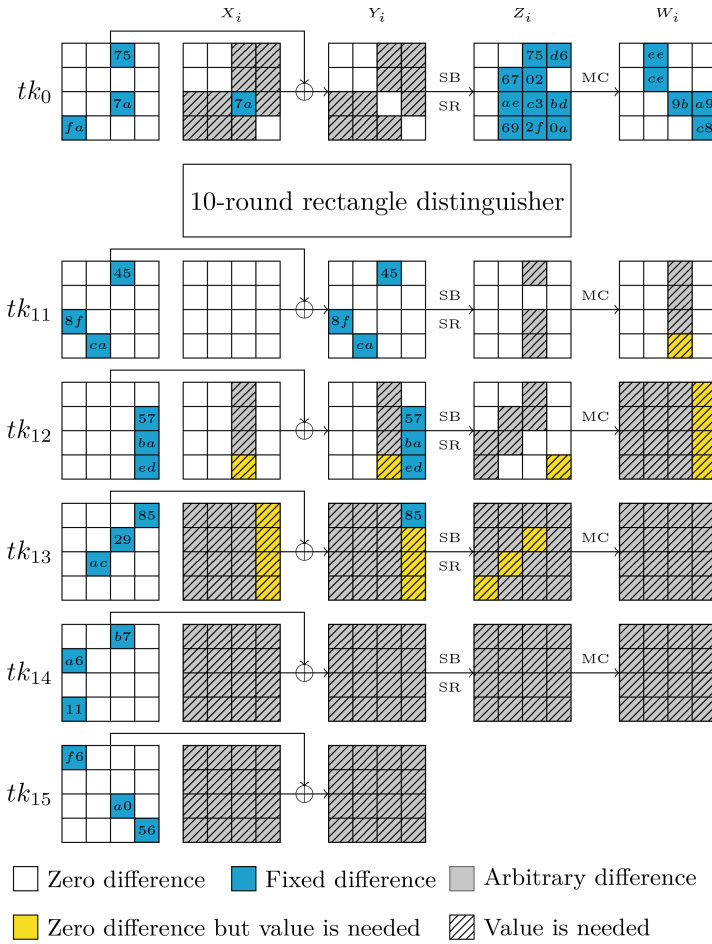
**15-Round Rectangle Attack on Deoxys-BC-384.** Using our new framework, we get a 15-round rectangle attack which is equal to add one round after the 14-round rectangle attack, as shown in Fig. 6.

In order to get an effective attack, we also use the split-and-bunch technique in the 15-round rectangle attack. The parameters of the attack are:  $m_b = r_b = 80, m_f = 8 \times (16 + 13 + 6 + 3) = 304$  and  $r_f = 128$ . Due to the key schedule,  $k_b \cup k_f$  contains 376 information bits. The best guessing parameters are  $m'_b = 80, r'_b = 80$  and  $m'_f = 8 \times (16 + 5 + 1) = 176, r'_f = 8 \times (4 + 3) = 56$ , which means guessing the whole subkey of  $k_b$ , 16 bytes of  $tk_{15}$ , 5 bytes of the equivalent subkey  $eqtk_{14}$  and 1 bytes of the equivalent subkey  $eqtk_{13}$ . The complexities of our 15-round attack are as follows:

- The data complexity is  $D_R = 4 \cdot D = 4 \cdot y \cdot 2^{r_b} = 4 \cdot \sqrt{s} \cdot 2^{n/2} / \tilde{P} = 4 \cdot \sqrt{s} \cdot 2^{48.7+64} = \sqrt{s} \cdot 2^{114.7}$ .
- The memory complexity is  $M_R = D_R + D \cdot 2^{r_b^*} + 2^{t+m_b+m_f-m'_b-m'_f} = \sqrt{s} \cdot 2^{114.7} + \sqrt{s} \cdot 2^{112.7} + 2^{t+120}$ .
- The time complexity  $T_1 = 2^{m'_b+m'_f} \cdot D_R = \sqrt{s} \cdot 2^{256+114.7} = \sqrt{s} \cdot 2^{370.7}$ ;
- $T_2 = 2^{m'_b+m'_f} \cdot D = \sqrt{s} \cdot 2^{256+112.7} = \sqrt{s} \cdot 2^{368.7}$ ;
- $T_3 = 2^{m'_b+m'_f} \cdot D^2 \cdot 2^{2r_b^*} \cdot 2^{2r_f^*} \cdot 2^{-2n} \cdot \epsilon = s \cdot 2^{256+112.7 \times 2 + 72 \times 2 - 2 \times 128} \cdot \epsilon = s \cdot 2^{369.4} \cdot \epsilon$ ;
- $T_4 = 2^{m'_b+m'_f-t} \cdot 2^{k+t-m'_b-m'_f-h} = 2^{k-h}$ .

Similar to the improved rectangle attack on 14-round Deoxys-BC-384, for  $s \cdot 2^{369.4}$  quartets,  $\epsilon$  is equivalent to about 2 memory accesses which is around  $2 \times \frac{1}{14} \times \frac{1}{16} = 2^{-6.81}$  encryption. The time complexity and memory complexity of the tables both are  $2^{128}$ . If we set  $s = 4, t = 0$  then the data, memory, and time complexities are  $2^{115.7}, 2^{128}$  and  $2^{371.7}$ , respectively.

*Comparison.* The comparison with the previous rectangle attacks is presented in Table 7. A note on how the new attacks are obtained is as follows. We search for the active patterns of the whole attack with the total time complexity of the attack as the objective function in one step and then instantiate these patterns. In order to balance the time complexity and data complexity, we impose some restrictions on the data complexity in the model.



**Fig. 6.** 15-round Rectangle Attack on Deoxys-BC-384

**Table 7.** Comparisons of rectangle attacks on Deoxys-BC-384

$P^2$	Rounds	$m_b, m_f$	$m'_b, m'_f$	Data	Memory	Time	Reference
$2^{-118.4}$	14	88, 168	88, 24	$2^{125.2}$	$2^{140}$	$2^{260}$	[DQSW22]
$2^{-97.4}$	14	80, 208	80, 48	$2^{115.7}$	$2^{160}$	$2^{260.59}$	This
$2^{-97.4}$	14	80, 176	80, 48	$2^{115.7}$	$2^{128}$	$2^{243.7}$	This
$2^{-97.4}$	15	80, 304	80, 176	$2^{115.7}$	$2^{128}$	$2^{371.7}$	This

### 4.2 Application to CRAFT

**Specification.** CRAFT is a lightweight tweakable block cipher that was introduced by Beierle *et al.* [BLMR19]. It supports 64-bit plaintexts, 128-bit keys, and 64-bit tweaks. Its round function is composed of involutory building blocks. The 64-bit input is arranged as a state of  $4 \times 4$  nibbles. The state is then going through 32 rounds  $\mathcal{R}_i, i \in 0, \dots, 31$ , to generate a 64-bit ciphertext. As depicted in Fig. 7, each round, excluding the last round, has five functions, *i.e.*, MixColumn (MC), AddRoundConstants (ARC), AddTweakey (ATK), PermuteNibbles (PN), and S-box (SB). The last round only includes MC, ARC and ATK, *i.e.*,  $\mathcal{R}_{31} = \text{ATK}_{31} \circ \text{ARC}_{31} \circ \text{MC}$ , while for any  $0 \leq i \leq 30, \mathcal{R}_i = \text{SB} \circ \text{PN} \circ \text{ATK}_i \circ \text{ARC}_i \circ \text{MC}$ .

The tweakkey schedule of CRAFT is rather simple. Given the secret key  $K = K_0 \| K_1$  and the tweak  $T \in \{0, 1\}^{64}$ , where  $K_i \in \{0, 1\}^{64}$ , four round tweakkeys  $TK_0 = K_0 \oplus T, TK_1 = K_1 \oplus T, TK_2 = K_0 \oplus Q(T)$  and  $TK_3 = K_1 \oplus Q(T)$  are generated, where  $Q$  is a nibble-wise permutation. Then at the round  $\mathcal{R}_i, TK_{i \% 4}$  is used as the subtweakey.

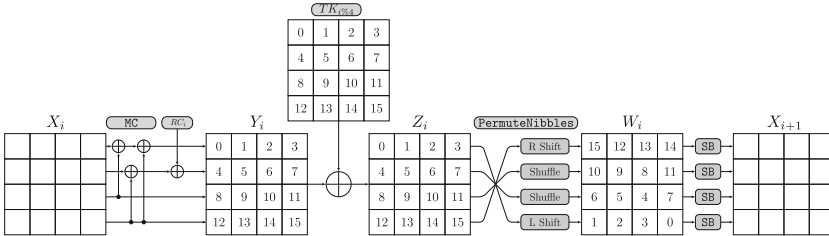
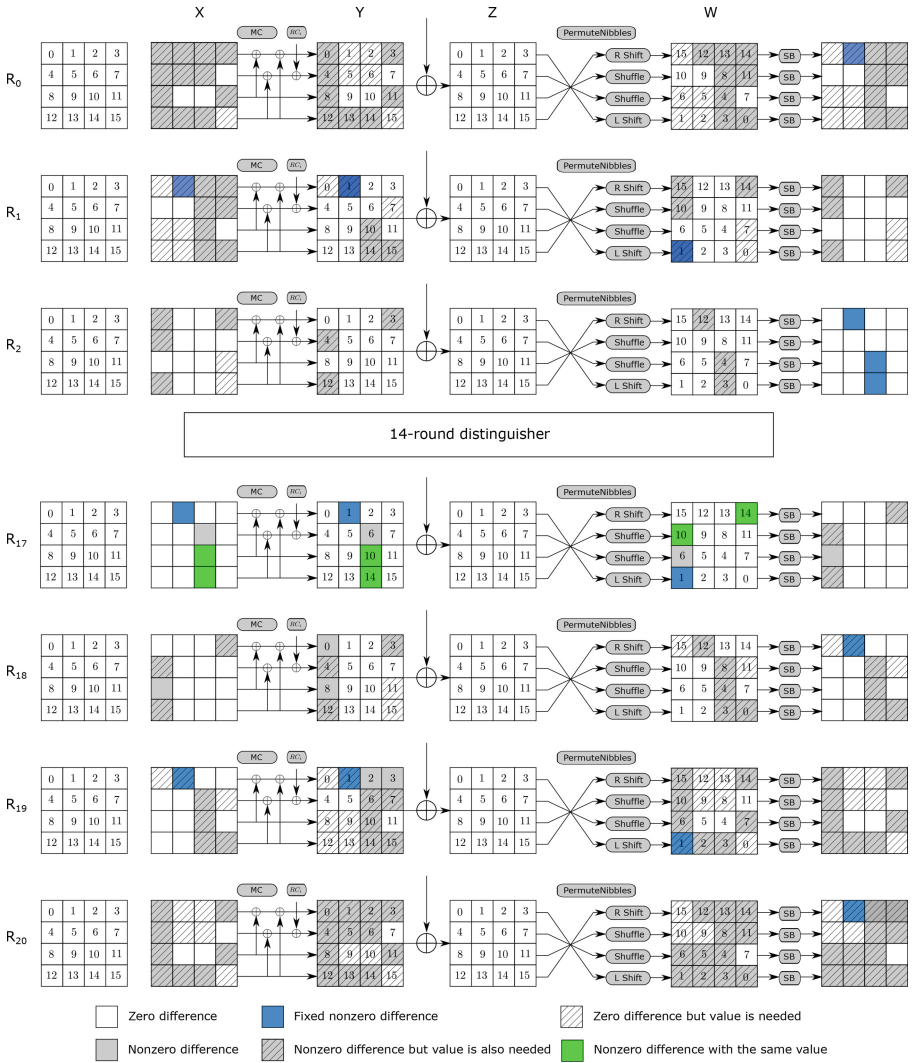


Fig. 7. A round of CRAFT

**21-Round Differential Attack on CRAFT.** By using our new framework, we obtain a 21-round differential attack on CRAFT, as depicted in Fig. 8, which contains a 14-round inner part, 3-round in  $E_b$  and 4-round in  $E_f$ . The input difference is  $0a00000000a000a0$  and the output difference is  $0a0000?000x000x0$ . The probability of the inner part is  $P = 2^{-54}$  and for a random permutation, the probability is  $2^{-56}$ . This inner part uses a 2-round invariant property of CRAFT proposed in [GSS+20] when the tweak  $T$  satisfies the conditions  $T[6] \oplus K[12] \in \{0x0, 0xa\}$  and  $T[12] \oplus K[12] \in \{0x0, 0xa\}$  the difference  $0xa$  propagates 2-round to  $0xa$  with probability 1, detailed can refer to Appendix C in the full version of the paper. This is a single-key model with some conditions in tweak  $T$ .

In this attack, we build  $2^x$  structures for each value of  $T[6]$  with the tweak  $T$  satisfying  $T[6] = T[12]$ . For each  $T[6]$ , we perform the following steps.



**Fig. 8.** 21-round Differential Attack on CRAFT

1. We build  $2^x$  structures that  $Y_0[0, 3, 4, 8, 11, 12, 13, 14]$  traverse all possible values while the other cells are fixed to some random constants. The data complexity is  $D_0 = 2^{x+32}$ . By using  $2^{x+32}$  plaintexts, there are  $N_0 = 2^x \cdot 2^{32} \times 2^{-1} = 2^{x+63}$  pairs of plaintexts. The expected number of the remaining pairs of ciphertexts are  $N_1 = N_0 \cdot 2^{-4 \times 8} = 2^{x+31}$ . The time complexity of this step is  $T_0 = D = 2^{x+32}$ .
2. For each of  $N_1 = 2^{x+31}$  pairs, determine the key candidates and increase the corresponding counters. We pre-compute several tables as shown in Table 8,

so that the time complexity of this step is  $T_1 = 2^{36} \cdot N_1 = 2^{x+67}$  which is about  $2^{x+67} \times \frac{1}{21} \times \frac{1}{16} \approx 2^{x+58.61}$  encryption.

3. Exhaustively search the remaining keys.

Set  $x = 24.99$ , then the data, memory, and time complexities are  $2^{4+24.99+32} = 2^{60.99}$ ,  $2^{92}$  and  $2^{87.60}$ , respectively.

**Table 8.** Precomputation tables for the 21-round attack on CRAFT.

Tables	Involved key	Filters	Remaining pairs
1	$K_0[12] = T[12]$	$\Delta X_1[1] = \Delta W_{19}[12] = a$	$2^{-8} \cdot N_1$
2	$K_0[0, 8, 11, 14]$	$\Delta Y_1[3, 7] = \Delta X_{19}[3, 7] = 0$	$2^{-8} \cdot N_1$
3	$K_0[13, 4, 3, 15]$	$\Delta Y_1[2, 6] = \Delta X_{19}[2] = 0$	$2^{-4} \cdot N_1$
4	$K_0[1, 2, 5, 6, 9, 10]$	-	$2^{20} \cdot N_1$
5	$K_1[1], K_1[9] \oplus K_1[13]$	-	$2^{28} \cdot N_1$
6	$K_1[6, 14, 10, 15]$	$\Delta X_2[0] = \Delta X_{18}[0] = 0, \Delta X_3[10] = a$	$2^{32} \cdot N_1$
7	$K_1[0, 7], K_1[8] \oplus K_1[12]$	$\Delta X_3[14] = a, \Delta W_{17}[3] = \Delta W_{17}[4]$	$2^{36} \cdot N_1$

**23-Round Differential Attack on CRAFT.** Expending one round before and one round after the 21-round differential attack, we obtain a 23-round differential attack on CRAFT. As for the 21-round attack, we perform the following steps for each  $T[6]$ . For 23-round attack,  $K_1[12] = T[6] = T[12]$ .

1. We build  $2^x$  structures that  $Y_0[2, 3, 6, 7, 8, 9, 10, 12, 13, 14, 15]$  traverse all possible values while the other cells are fixed to some random constants. The data complexity is  $D_0 = 2^{x+44}$ . By using  $2^{x+44}$  plaintexts, there are  $N_0 = 2^x \cdot 2^{44 \times 2 - 1} = 2^{x+87}$  pairs of plaintexts. The expected number of the remaining pairs of ciphertexts are  $N_1 = N_0 \cdot 2^{-4 \times 5} = 2^{x+67}$  pairs. The time complexity of this step is  $T_0 = D = 2^{x+44}$ .
2. For each of  $N_1 = 2^{x+87}$  pairs, determine the key candidates and increase the corresponding counters. We pre-compute several tables as shown in Table 9, so that the time complexity of this step is  $T_1 = 2^{40} \cdot N_1 = 2^{x+107}$  which is about  $2^{x+107} \times \frac{1}{23} \times \frac{1}{16} \approx 2^{x+98.47}$  encryption.
3. Exhaustively the remaining keys (Fig. 9).

Set  $x = 12.99$ , then the data, memory, and time complexities are  $2^{4+12.99+44} = 2^{60.99}$ ,  $2^{120}$  and  $2^{111.46}$ , respectively. The comparison with the previous attacks is presented in Table 10.



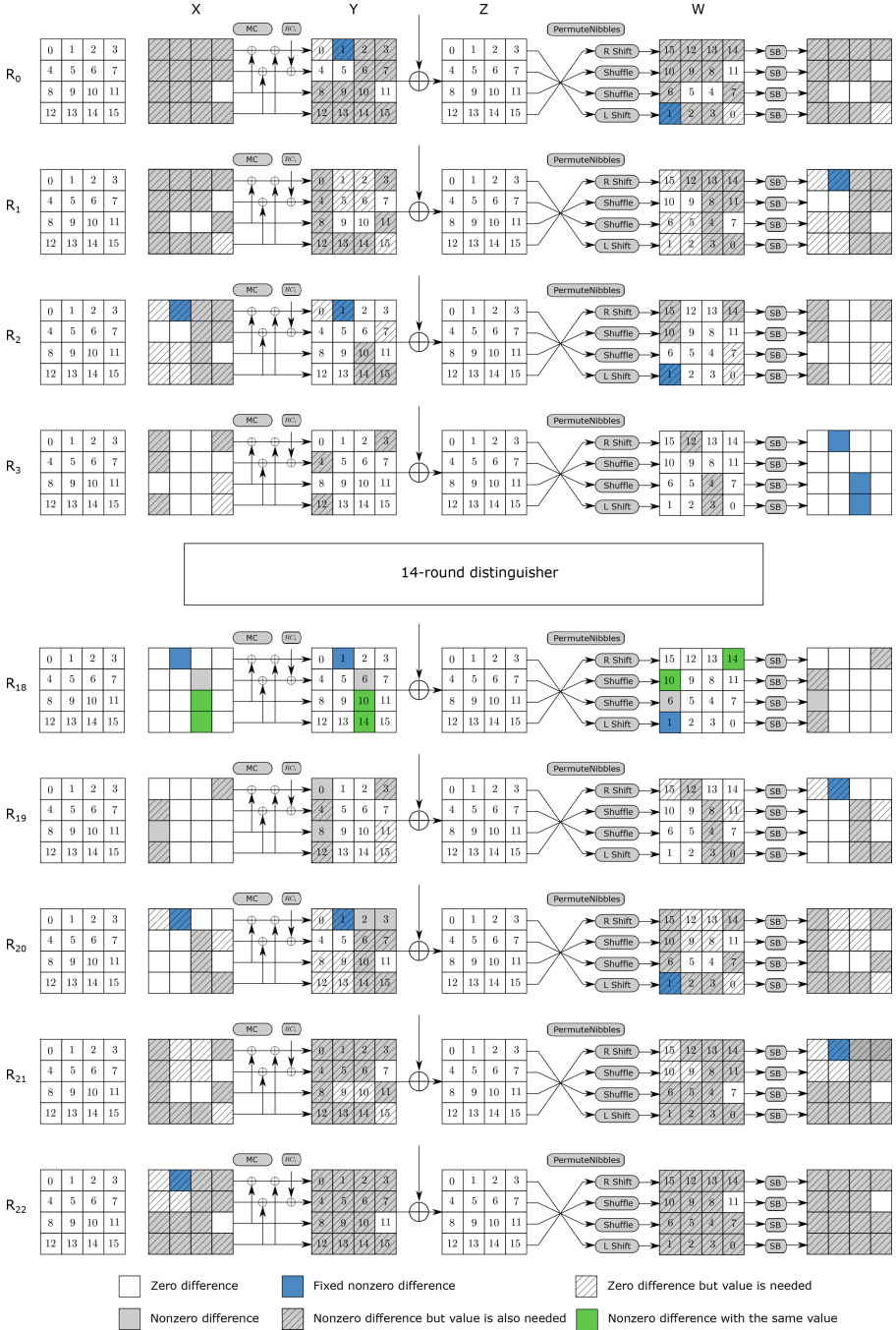


Fig. 9. 23-round Differential Attack on CRAFT

**Table 9.** Precomputation tables for the 23-round attack on CRAFT.

Tables	Involved keys	Filters	Remaining pairs
1	$K_0[9, 2, 10, 14, 12]$	$\Delta X_{21}[1, 5] = \Delta Y_1[1, 5] = 0$	$2^4 \cdot N_1$
2	$K_0[8, 13, 3, 11, 15]$	$\Delta X_{21}[2, 6] = \Delta Y_1[2, 6] = 0$	$2^8 \cdot N_1$
3	$K_0[0, 1, 4, 5, 6, 7]$	-	$2^{32} \cdot N_1$
4	$K_1[0, 8, 11, 14]$	$\Delta Y_2[3, 7] = \Delta X_{20}[3, 7] = 0$	$2^{32} \cdot N_1$
5	$K_1[13, 4, 3, 15]$	$\Delta Y_2[2, 6] = \Delta X_{20}[2] = 0$	$2^{36} \cdot N_1$
6	$K_1[2, 5, 9, 10]$	$\Delta Y_3[1] = \Delta X_{19}[0] = 0, \Delta X_4[10] = a$	$2^{40} \cdot N_1$
7	$K_1[1, 6]$	$\Delta X_4[14] = a, \Delta W_{18}[3] = \Delta W_{18}[4]$	$2^{40} \cdot N_1$

**Table 10.** Comparison of attacks on CRAFT

Rounds	Data	Memory	Time	Attack	Setting	Reference
23	$2^{74}$	$2^{51}$	$2^{94}$	D	WK(16-bit) & ST	[LR22]
26	$2^{73}$	$2^{60}$	$2^{105}$	D	WK(20-bit) & WT(12-bit)	[LR22]
20	$2^{62.89}$	$2^{49}$	$2^{120.43}$	ZC	SK & ST	[HSE23]
21	$2^{60.99}$	$2^{100}$	$2^{106.53}$	ID	SK & ST	[HSE23]
19	$2^{60.99}$	$2^{68}$	$2^{94.59}$	D	SK & WT(4-bit)	[GSS+20]
21	$2^{60.99}$	$2^{92}$	$2^{87.60}$	D	SK & WT(4-bit)	This
23	$2^{60.99}$	$2^{120}$	$2^{111.46}$	D	SK & WT(4-bit)	This

## 5 Conclusion

In this paper, we propose a one-step framework for finding the rectangle attacks with the purpose of reducing the overall complexities or attacking more rounds. Instead of extending the distinguisher forward and backward with probability 1, we propose to allow probabilistic propagations in the extended part. We treat the distinguisher and the extended part as a whole with a more flexible selection of propagation and involved keys to get better attacks. Moreover, we incorporate the holistic key recovery strategy into our one-step framework. Further, we introduce a technique, which is called the split-and-bunch technique, to reduce the time complexity. Applying our framework to Deoxys-BC-384, we obtain the first 15-round rectangle attack for Deoxys-BC-384, narrowing its security margin to only one round. Applying to SKINNY and ForkSkinny, we obtain a new rectangle attack with reduced data and time complexities. We also apply the main ideas of the framework to differential attacks on CRAFT block cipher, which achieves 2 more rounds than the previous best attacks.

**Further Works.** In this paper, we only apply the new framework to rectangle attacks and differential attacks. It would be a potential future work to explore the application of these ideas to other attacks.

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